1993

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Report Number:
93-031

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CSD-TR-93-031
May 1993

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H. BYUN, E.N. HOUSTIS AND E.A. VAVALIS †

Abstract

In this paper we describe the algorithm used, discuss the implementation and present the performance of the Basic Linear Algebra Subroutine (BLAS) sgemv for distributed-memory multiprocessors. The basic assumption is that the matrix and the vectors are row distributed among processors. Performance data from nCUBE II, iPSC/860 and iPSC DELTA machines are presented.

1. Introduction. In this study we present data that describe various aspects of the performance of the Basic Linear Algebra Subroutine (BLAS) sdgemv on three distributed memory multiprocessor systems namely the nCUBE II, the iPSC/860 and the iPSC DELTA. sdgemv is a member of a set of parallel BLAS routines we have implemented on such machines [1]. The software methodology utilized to parallelize the BLAS routines assumes that each processor performs the appropriate local operations by calling the corresponding uniprocessor BLAS routines ([9], [5], [4]). The local results are "combined" by PICL [6] routines to generate the final answer. It is worth noticing that the global combine operations can use any multiprocessor connection topology that PICL supports.

The rest of the paper is organized as follows. Section 3 consists of a list of tables that present raw timing data measuring the total elapse and total (communication and idle) overhead time required by the sdgemv routine to perform the matrix-vector operations on the three machines considered. Using the data given in Section 3 we present, in Section 4 the Gflops achieved on the three machines for different matrix sizes and connection topologies, and in Section 5 the data that show the differences observed when we used the optimized uniprocessor BLAS routines instead of FORTRAN BLAS on the iPSC/860. Finally in Section 6 we give the utilization and concurrency profiles and in Section 7 spacetime execution diagrams. The data in the latter two sections were obtained using PARAGRAPH [7].

2. The algorithm and its implementation. In this section we discuss the parallel implementation of the matrix $\times$ vector operations Ax or A^Tx. Throughout, we assume that the matrix A and the vector x is distributed among processors row-wise. This distribution is defined by a vector $idist(i), i = 1, nprocs + 1$ where $nprocs$ is the number of processors and $idist(i)$ denotes the global index of the first row of A

* This work was partially supported by NSF grants 9123502-CDQ and 9202536-CCR, AFOSR F49620-92-J-0069 and PRF 6902003. This research was performed in part using the Intel Touchstone Delta System operated by Caltech on behalf of the Concurrent Supercomputer Consortium. Access to this facility was provided by Purdue University.

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(first element of x) that belongs to processor i. Thus, we store rows from $idist(i)$ to $idist(i+1)-1$ of the matrix A on the local memory of processor i together with the associated elements of x. *idist* is the only global information needed and all other variables are local to each processor.

For the implementation of the Ax operation, we follow the methodology described in the previous section. As an example we give in Figure 1 the *actual* code for the sdgemv routine for full matrices. We assume that the data are distributed on a wrap around linear array of processors. The level two BLAS routine sgemv gets as input the matrix $A \in R^{m \times n}$, the vectors $x, y \in R^m$ and the scalars α and β. It computes the vector $y = \alpha Ax + \beta y$ or $\alpha A^T x + \beta y$. In the case of matrix A, each node calculates the part of the product that involves local data by utilizing the uniprocessor level two BLAS routine sgemv. Then it broadcasts its local vector x to all other processors by calling the PICL routine bcast1, in which the reception and forwarding of the message are decoupled. Thus, processors that participate in this broadcast call sgemv which computes the part of the product associated with the incoming vector x before forwarding it. Finally, we restore the original value of the vector x by reading it from a nearest neighbor. It is important to notice that the only memory overhead for the routine *sdgemv* is the integer array *idist* of length $nprocs + 1$.

In the case of the A^Tx operation, each processor i calls *sgemv* to compute $\alpha A_i^T x$ and stores this result to a buffer. The entries of all these buffers are added componentwise using the PICL routine *gsum0* and the result is stored in the buffer at processor 1. Finally a call to the level one BLAS routine saxpy is used to accumulate the term βy to the buffer. The above described procedure is repeated $nprocs - 1$ more times as shown in Figure 1.

We have experiment with two different interconnection topologies that PICL supports, namely the ring connectivity and the full connectivity. It is worth noticing that in the case of banded or sparse matrices we were able to successfully follow the above approach, coupled with appropriate data structures, with only a few basic differences [2].

3. Raw time data. The uniprocessor BLAS routines available to us on the iPSCs [8] were used unless it is stated otherwise in the caption of the tables. All times are in milli-seconds.

4. Gflops achieved. In Figures 2 and 3 we present the Gflops achieved on the three machines using ring topology for both the non-transposed and transposed cases respectively. It is worth noticing that the "theoretical peak" (determined by counting the number of floating point operations in full precision that can be completed during a cycle) for the nCUBE II, the iPSC 860 and for the iPSC DELTA is .15, 2.6 and 20 Gflops respectively [3].

5. The affect of using optimized BLAS. In Figure 4 we present the speedup achieved on the iPSC/860 by using optimized uniprocessor BLAS routines [8] for the non-transposed case. As we see the performance increases four to six times. It is also apparent that the performance drops as the number of processors increases. To further

FIG. 1. *The fortran code of subroutine* sdgemv.

```
      subroutine sdgemv(trans,m,n,alpha,a,lda,x,beta,y,tmp,idst)

      integer idst(1)
      real    a(lda,1),x(1),y(1),tmp(1)
      character trans

      integer nprocs, me, host, top, ord, dir
      common  /open/ nprocs,me,host
      common  /setarc/ top, ord, dir

      integer bytes, mtype, root, lnx, jidx, node_no

      lnx = idst(me+2) - idst(me+1)
      bytes = 4 * lnx

      if( ( lsame(trans,'N') ) .or. ( lsame(trans,'n') ) ) then
         jidx = idst(me+1)
         call sgemv(trans,m,n,alpha,a(1,jidx),lda,x,1,beta,y,1)
         mtype = 1000 + me
         call  bcast0(x,bytes,mtype,me)
         do 100 i=1,nprocs-1
            root  = mod(me-i+nprocs, nprocs)
            mtype = 1000 + root
            jidx = idst(root+1)
            call bcast0(x,bytes,mtype,root)
            call sgemv(trans,m,n,alpha,a(1,jidx),lda,x,1,1.,y,1)
 100     continue
         root = mod(me+1+nprocs, nprocs)
         call send0(x,bytes,root+1000,root)
         call recv0(x,bytes,me+1000)
      endif
      if( ( lsame(trans,'T') ) .or. ( lsame(trans,'t') ) ) then
         mtype = 4000
         do 200 node_no = 0, nprocs-1
            istart = idst(node_no+1)
            call sgemv(trans,m,n,alpha,a(1,istart),lda,x,1,0.,tmp,1)
            call gsum0(tmp,lnx,4,mtype,node_no)
            if (node_no .eq. me)  then
              if (beta .ne. 0.) call saxpy(lnx,beta,y,1,tmp,1)
      call scopy(lnx,tmp,1,y,1)
    endif
 200     continue
      endif

      return
      end
```

TABLE 1

The total elapse and overhead timings on the nCUBE II for matrices A of size $n \times n$.

	1	2		4		8		16		32		64	
n	T_e	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o
800	2834	1424	3	717	2	366	3						
1200		3194	3	1605	3	812	3	418	6				
1600				2846	3	1435	4	732	6	386	12		
2400						3210	5	1624	7	836	12	454	22
3200								2869	8	1463	12	771	24
4800										3248	14	1672	23
6400												2926	25

TABLE 2

The total elapse and overhead time on the nCUBE II for matrices A^T of size $n \times n$.

	1	2		4		8		16		32		64	
n	T_e	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o
800	2615	1316	3	666	3	343	5						
1200		2950	4	1487	4	756	6	395	6				
1600				2632	6	1333	7	686	10	370	15		
2400						2974	9	1514	12	790	17	442	27
3200								2666	14	1373	19	749	30
4800										3028	24	1581	36
6400												2747	40

TABLE 3

The total elapse and overhead timings on the iPSC/860 for matrices A of size $n \times n$. This implementation is based on full connectivity.

	1	2		4		8		16		32		64	
n	T_e	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o
1600	201	106	4	60	4	38	6						
2400		234	6	125	5	74	11	53	11				
3200				214	8	122	12	80	16	63	24		
4800						254	13	151	22	110	27	92	43
6400								250	27	162	36	129	52
9600										302	48	220	63
12800												310	85

TABLE 4

The total elapse and overhead timings on the iPSC/860 for matrices A of size n × n. This implementation is based on ring topology.

	1	2		4		8		16		32		64	
n	T_e	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o
1600	201	106	3	59	6	38	6						
2400		233	6	124	5	72	6	52	11				
3200				213	8	118	8	76	9	62	22		
4800						249	10	145	13	105	16	92	34
6400								238	16	152	24	125	42
9600										281	19	207	41
12800												286	43

TABLE 5

The total elapse and overhead timings on the iPSC/860 for matrices A of size n × n. This implementation is based on full connectivity and the uni-processors level two BLAS used were written in FORTRAN.

	1	2		4		8		16		32		64	
n	T_e	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o
3200				1873	6	924	10	468	10	251	22		
4800						2087	12	1042	16	537	17	303	34
6400								1849	10	937	17	502	33
9600										2072	21	1069	34
12800												1852	42

TABLE 6

The total elapse and overhead timings on the iPSC/860 for matrices A of size n × n. This implementation is based on ring connectivity and the uni-processors level two BLAS used were written in FORTRAN.

	1	2		4		8		16		32		64	
n	T_e	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o
3200				1741	8	870	16	455	23	260	35		
4800						1957	34	994	34	535	41	330	74
6400								1744	35	912	51	523	81
9600										1987	60	1071	85
12800												1825	90

TABLE 7

The total elapse and overhead timings on the iPSC/860 for matrices A^T of size $n \times n$. This implementation is based on full connectivity.

	1	2		4		8		16		32		64	
n	T_e	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o
1600	198	104	3	59	4	36	7						
2400		230	6	124	8	69	8	48	9				
3200				197	7	114	8	76	18	63	20		
4800						250	13	145	17	101	22	101	39
6400								239	20	157	16	131	38
9600										293	35	204	47
12800												310	49

TABLE 8

The total elapse and overhead timings on the iPSC/860 for matrices A^T of size $n \times n$. This implementation is based on ring topology.

	1	2		4		8		16		32		64	
n	T_e	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o
1600	198	104	3	62	7	44	6						
2400		230	7	127	12	81	14	61	19				
3200				218	13	129	20	93	30	84	39		
4800						262	26	166	37	127	47	137	77
6400								264	46	188	62	170	87
9600										337	73	254	99
12800												378	110

TABLE 9

Total elapse and overhead measured times the DELTA for matrices A of size $n \times n$.

	1	4		8		16		32		64		128		256		512	
n	T_e	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o
1600	100	31	2	22	3												
2400		62	3	39	5	32	6										
3200		104	6	62	3	44	7	36	8								
4800				125	5	79	7	62	13	51	20						
6400						125	10	86	11	71	21	83	38				
9600								163	18	129	30	107	41	141	120		
12800										182	31	152	50	176	113	317	224
19200												277	81	226	114	329	233
25600														325	116	412	253
38400																548	343

TABLE 10
Total elapse and overhead measured times on the DELTA for matrices A^T of size $n \times n$.

	1	4		8		16		32		64		128		256		512	
n	T_e	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o	T_e	T_o
1600	78	27	3	21	4												
2400		54	4	36	5	27	6										
3200		90	6	56	5	40	8	36	9								
4800				109	9	73	6	54	14	58	18						
6400						112	13	83	14	73	16	77	29				
9600								146	21	109	26	115	32	153	55		
12800										166	25	148	35	167	58	255	110
19200												227	54	241	60	288	112
25600														312	68	331	113
38400																489	126

examine this we present in Figure 5 the total overhead time involved. This overhead measures communication and synchronization time and obviously should not depend on how we perform the floating point arithmetic. As the data in Figure 5 surprisingly show that this is not correct since the use of optimized BLAS might double or half the overhead time depending on the interconnection topology. This is due to cash memory management that both the BLAS and the communication router use. We will present a complete analysis of that phenomenon elsewhere.

5.1. Concurrency and utilization diagrams. In Figures 6 and 7 we present the concurrency profile and the utilization summary diagrams we obtained using PARAGRAPH on the 64 node nCUBE II and iPSC/860 respectively. The matrix sizes $n = 6400$ and $n = 12800$ correspond to the largest problem we could fit on these machines. Notice that the nCUBE II machine we use is configured with 4 Mbytes of memory per node while the iPSC/860 with 16 Mbytes per node.

It is worth noticing the erratic utilization behavior on the iPSC/860. This is due to the non-deterministic routing message mechanism. The full analysis of this behavior is under way and it will be given elsewhere.

5.2. Spacetime diagrams. In Figures 8 and 9 we give the spacetime execution diagrams associated with the problems and machine configurations considered in Figures 6 and 7 respectively. We only give the final time intervals since initially (intervals [0,121], [0-70] [0-90] and [0-70]) all processors are doing only floating point operations calculating the local part of the matrix vector operation. Notice that the erratic utilization behavior on the iPSC/860 mentioned in the previous section can be noticed here too.

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FIG. 2. *The achieved Gflops for* $A \in R^{n \times n}$.

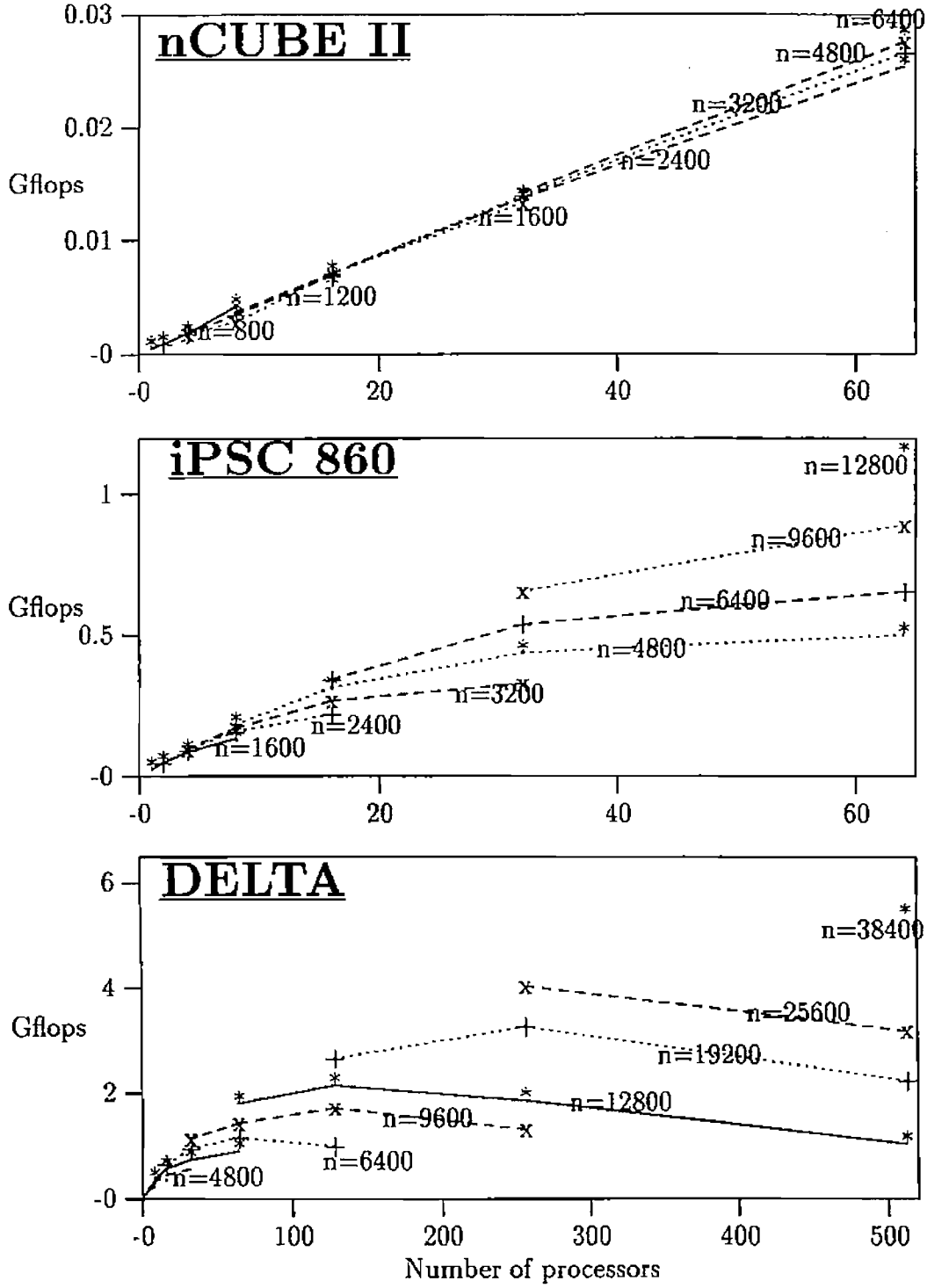


FIG. 3. *The achieved Gflops for* $A^T \in R^{n \times n}$.

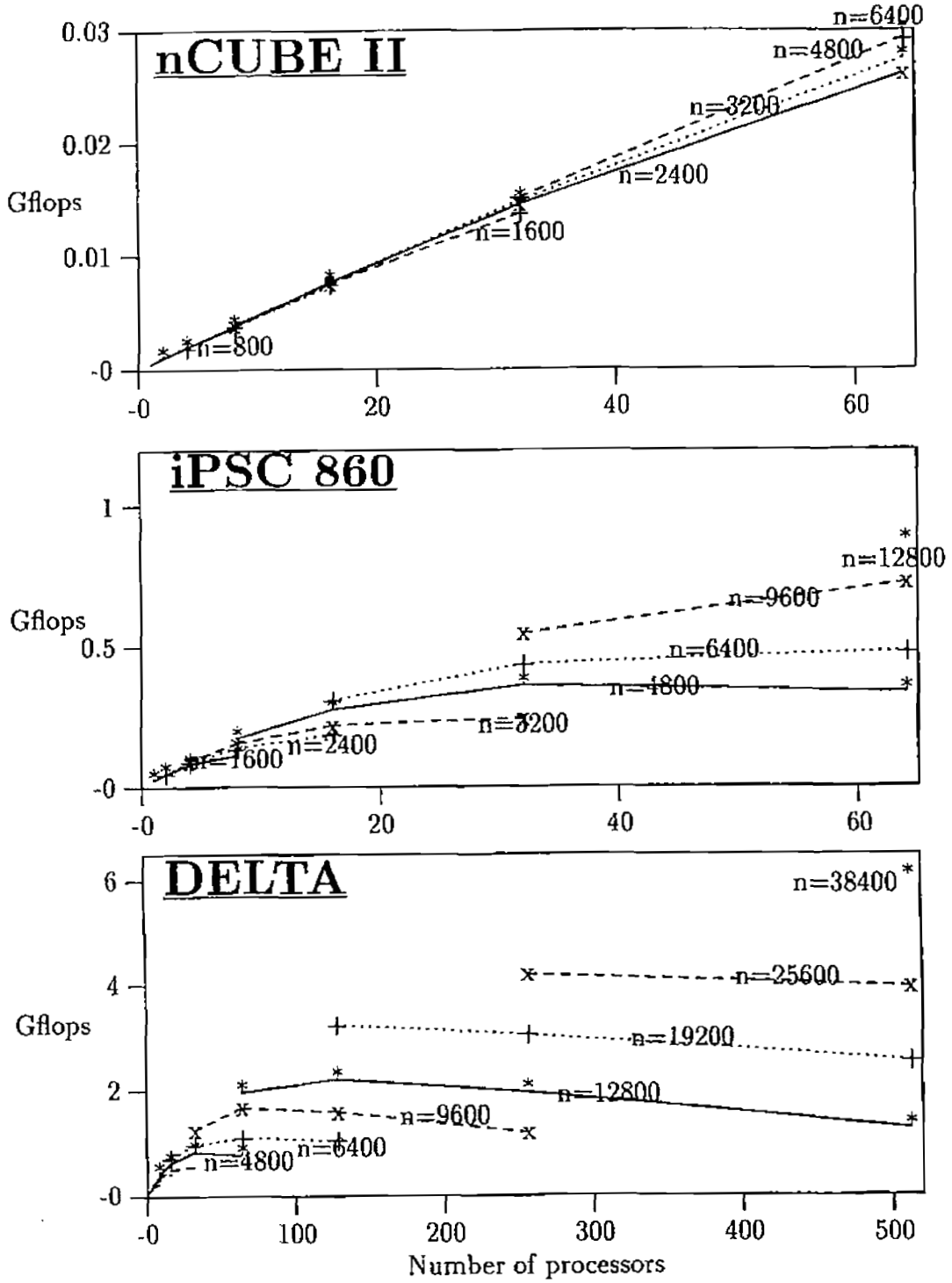


FIG. 4. *The achieved speedup on the iPSC/860 with full (F) and ring (R) connectivity and* $A \in R^{n \times n}$.

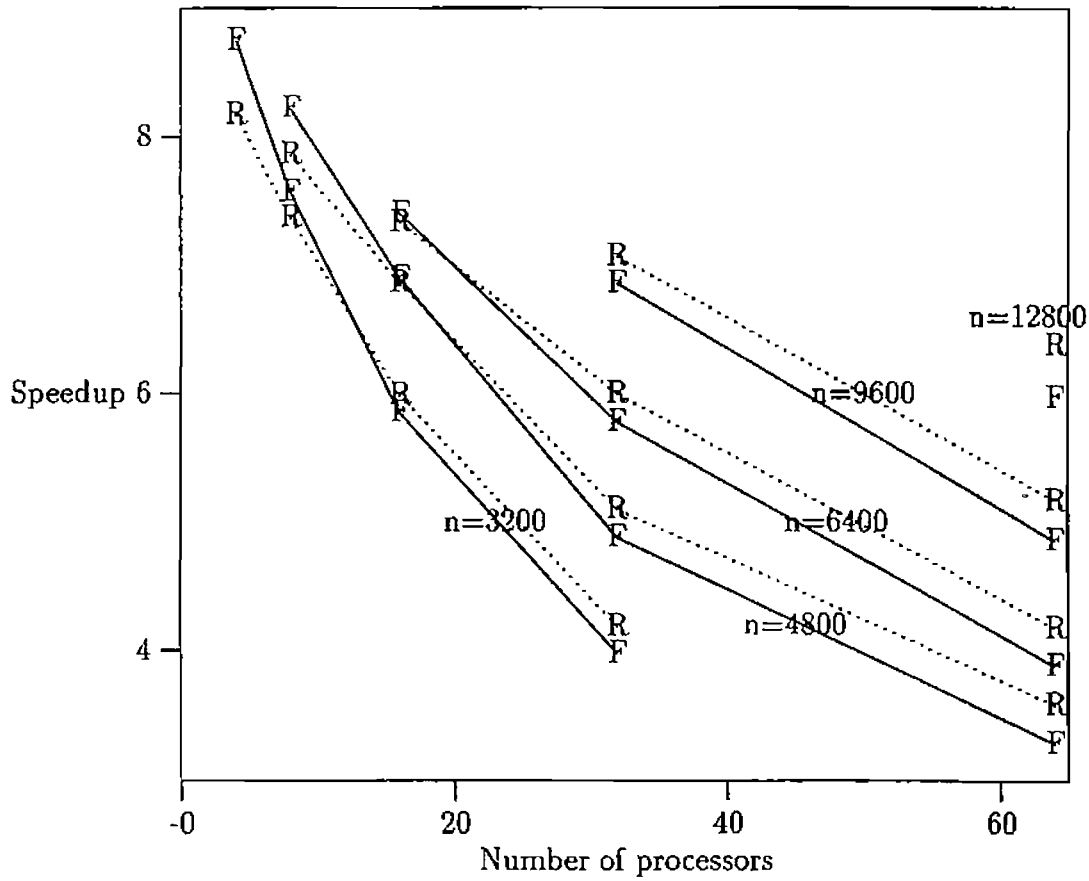


FIG. 5. *The total overhead time (in milli-seconds) on the iPSC/860 with full (F) and ring (R) connectivity, FORTRAN and optimized uniprocessor BLAS routines and* $A \in R^{n \times n}$.

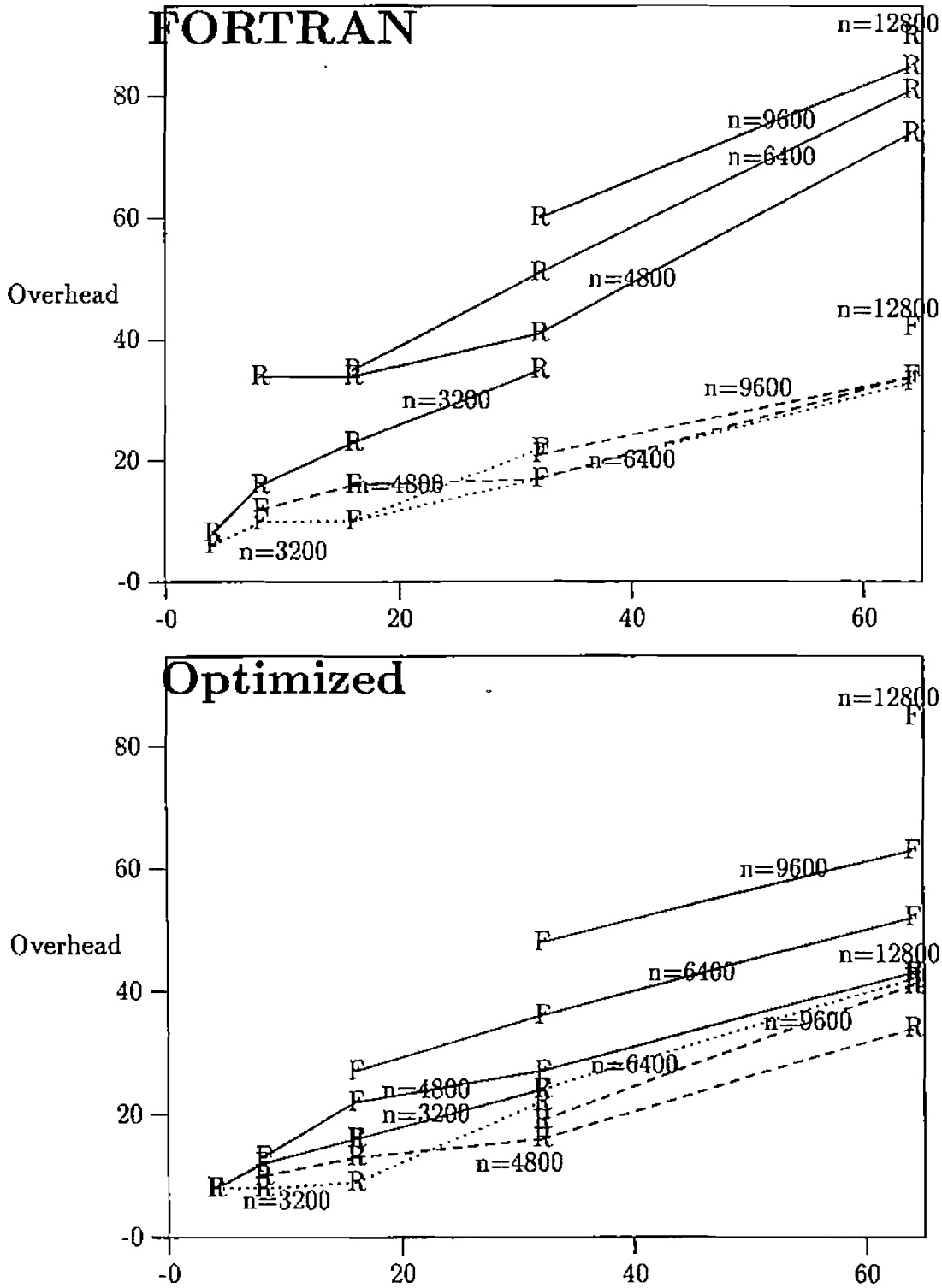
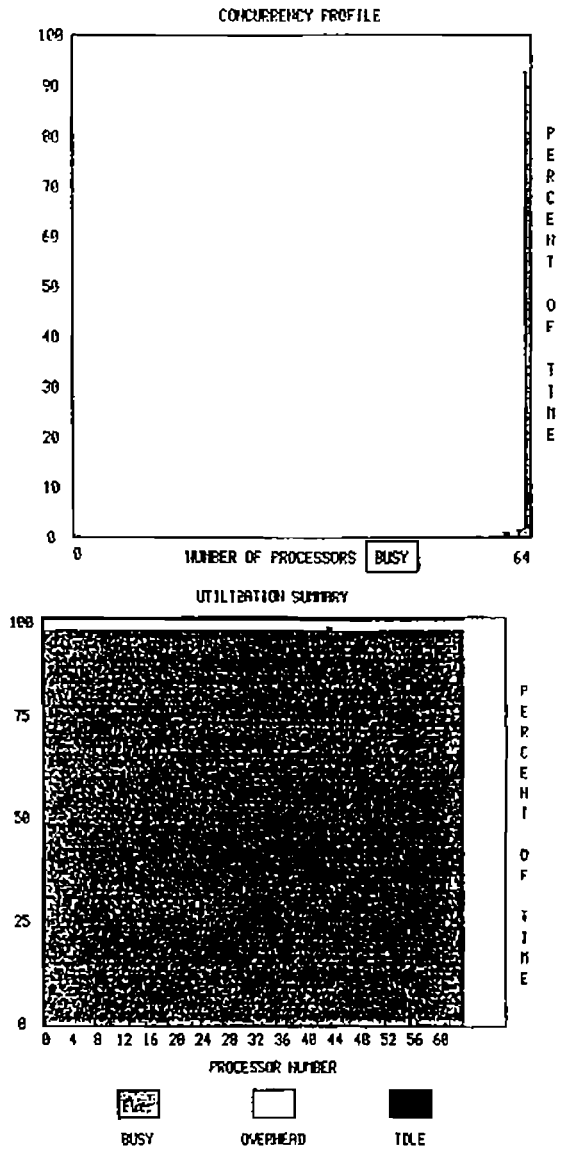


FIG. 6. *Profile and utilization diagrams on the 64 node nCUBE II for matrices of size $n = 6400$. Ring connectivity were used.*

$y = \alpha Ax + \beta y$



$y = \alpha A^T x + \beta y$

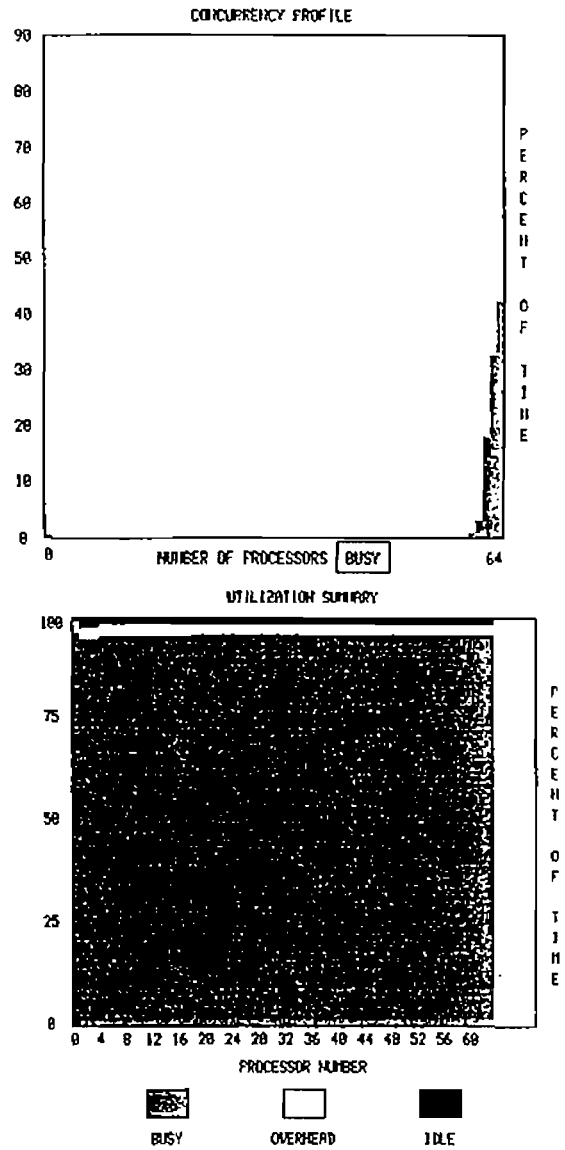


FIG. 7. *Profile and utilization diagrams on the 64 node iPSC/860 for matrices of size* $n = 128000$. *Ring connectivity were used.*

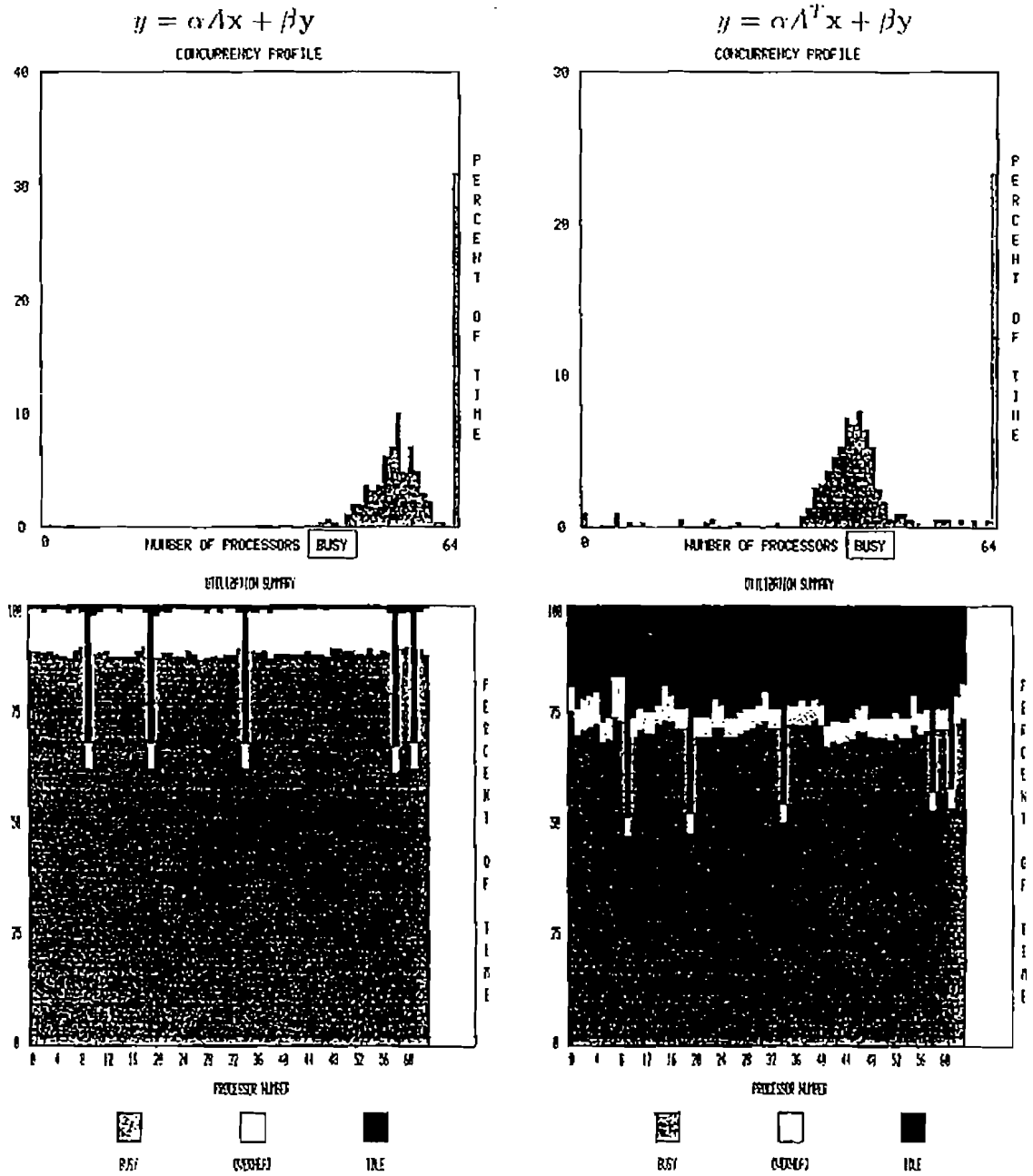
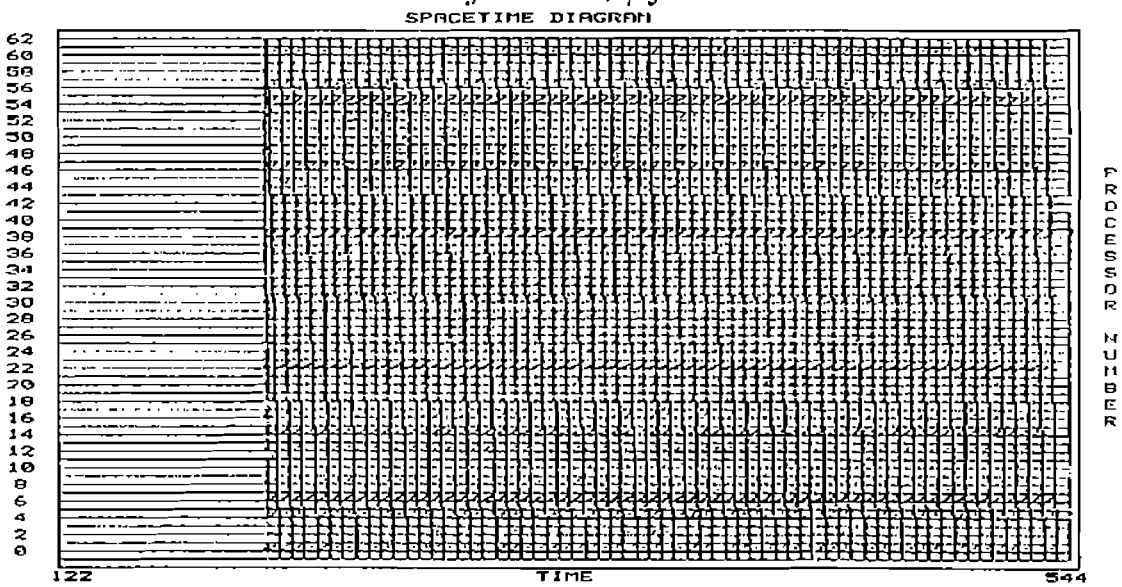


FIG. 8. *Timespace diagrams on the nCUBE II for the data considered in Figure 6.*

$y = \alpha \Lambda x + \beta y$



$y = \alpha \Lambda^T x + \beta y$

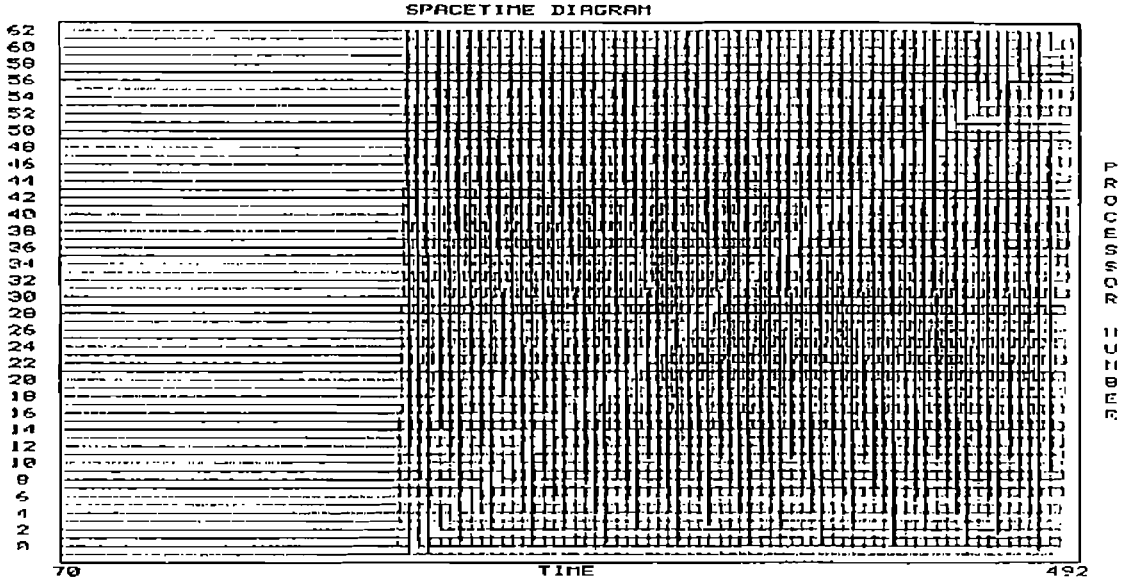
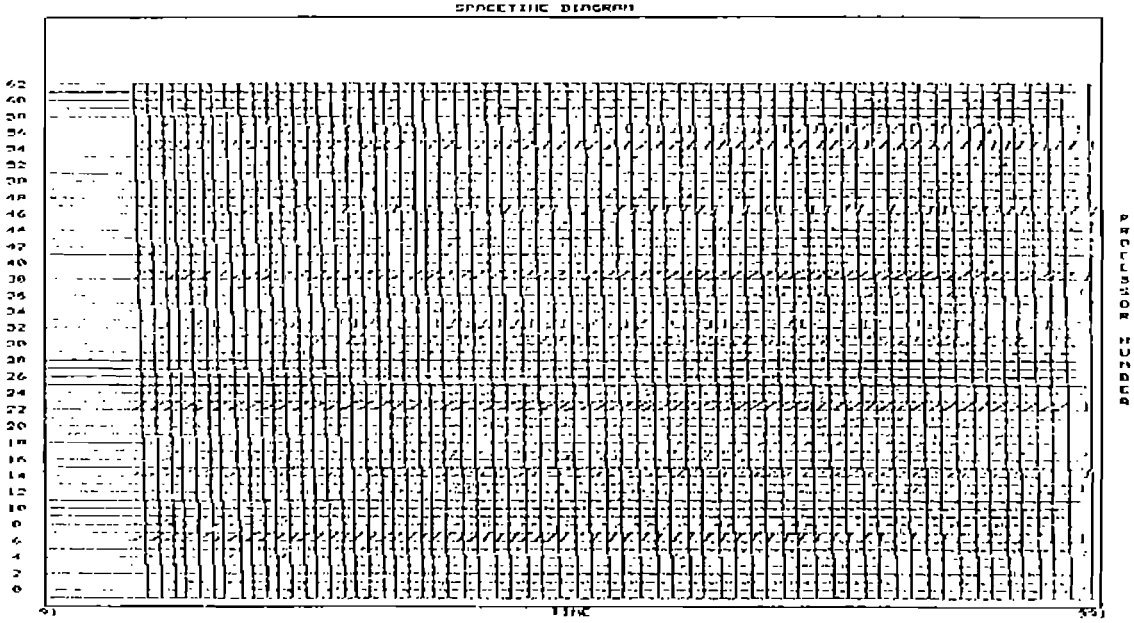


FIG. 9. *Timespace diagrams on the iPSC/860 for the data considered in Figure 7.*

$$y = \alpha A x + \beta y$$



$$y = \alpha A^T x + \beta y$$