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C. Wang
University of Cincinnati

J. Kim
University of Cincinnati

ANALYSIS OF IMPACT FORCE IN COMPRESSOR VALVES

Chen-hua Wang and Jay Kim
Dept. of Mechanical, Industrial, and Nuclear Engineering
University of Cincinnati
Cincinnati, OH 45221-0072

ABSTRACT

A new procedure has been developed to calculate the impact force which occurs when the valve reed contacts the valve seat. The procedure includes the dynamic models of both the reed and the seat. It has been typical to idealize the latter as a lumped parameter spring assuming that its stiffness is given. It is shown that the wave response of the valve seat makes a large influence on the impact force, therefore such models are not proper. A very simple configuration of the valve and valve seat is used to make the related discussions and derivation clear to understand. The method developed in this work can be easily extended to realistic configurations as necessary.

I. INTRODUCTION

Impact stress occurring when the valve tip contacts the valve seat has been recognized as one of the most common causes of the valve failure[1]. However, it is also one of the least understood problems because of the unique aspects of the problem. They are, compared with other common stress evaluation problems,

1. The contact force should be obtained through a complicated iteration method.
2. After the contact force is obtained, the wave equation should be solved by using the result of the contact force to obtain the stresses.
3. Related failure criterion is not well established because the endurance stress is always obtained from steady-state type experiments.
4. Direct experimental evaluation is not possible.

Because of these difficulties, usually the "set velocity" or the "impact velocity" was used for the valve design instead of the impact stress. Obviously, this is only because there has been no better alternatives.

This work is to resolve the first phase of the problems listed above. Many references[3]-[5] can be found for this type of work, where the problem is modeled as a beam contacting an equivalent linear spring. If one tries to utilize some of the reported results or analysis techniques, one would immediately finds that there is no published report on how to determine the spring constant to model the valve seat. Most work in this area simply assumes that the value is given. Nonlinear spring constants by the Hertz's contact theory have been used in some contact problems[2], which seems to be valid only when both contacting objects have relatively comparable minimum dimensions. In our case, the Hertzian deformation process doesn't seem to be applicable, because the reed is much thinner compared with the seat.

II. PROBLEM DESCRIPTION

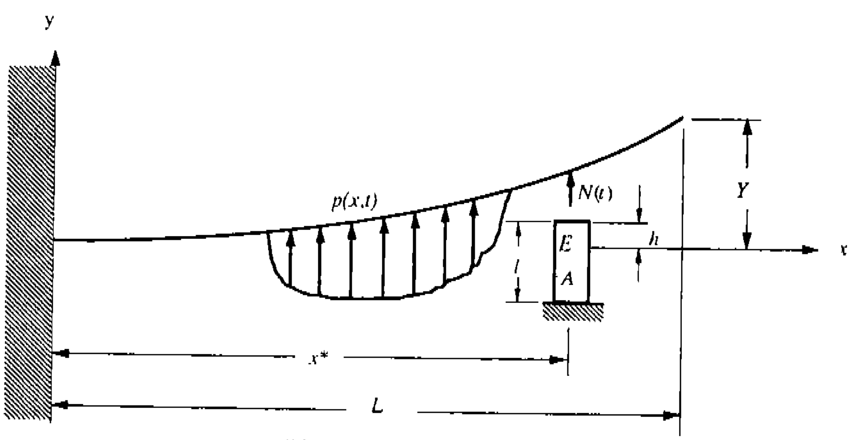

Figure 1. idealized impact model

The problem considered in this work is shown in Figure 1. A cantilever beam which represents the valve is released from a position of an initial deflection to strike the one dimensional rod. An external force $p(x,t)$ which represents the valve force may also be considered. The reasons to employ this simple model is to develop the analysis procedure mostly based on analytical, exact solutions except the numerical integration part. This helps us to understand the related mechanics more clearly. The procedure can be generalized to more realistic geometry as necessary only at the expense of more computing resources as it will be discussed.

III. CONVENTIONAL SOLUTION PROCEDURE

In most cases of the impact problems, the stopper is modeled by a spring with a constant stiffness K. In such a case, the equation of motion of the beam shown in Figure 1 becomes

$$EI\frac{\partial^4 y}{\partial x^4}+\rho A\frac{\partial^2 y}{\partial t^2}=p(x,t)+N(t)\delta(x-x^*) \tag{1}$$

where $N(t)=K(h-y)$ when $y<h$

$N(t)=0$ when $y>h$

Using a modal expansion procedure in equation (1) would lead to

$$y(x,t)=\sum_{n=1}^{\infty}\phi_n(x)\eta_n(t)+\sum_{n=1}^{\infty}\frac{\phi_n(x)}{m\omega_n}\int_0^t\int_0^L\phi_n(x)[p(x,\tau)+N(\tau)]dx\sin\omega_n(t-\tau)d\tau \tag{2}$$

where, m is the total mass of the beam, ω_n and ϕ_n are the n^{th} natural frequency and mode shape of the beam. $\eta_n(t)$ is the modal participation factor during the initial period before the impact.

During the contact, the following equation is obtained when $p(x,t)$=0.[7]

$$\sum_{n=1}^{\infty}\phi_n(x^*)\eta_n(t)+\sum_{n=1}^{\infty}\frac{\phi_n^2(x^*)}{m\omega_n}\int_0^t N(\tau)\sin\omega_n(t-\tau)d\tau=h-\frac{N(t)}{K} \tag{3}$$

Numerical integration can be used iteratively to obtain the contact force $N(t)$ from equation (3). Detail procedure can be found in reference[4].

Although it is complicated, the solution procedure is well established and routine. However, the estimated contact force becomes completely different by selecting a different value for the spring constant K. To the extent of our survey, no previous work in this area has discussed on how to obtain the spring constant.

IV. NEW SOLUTION METHOD

Dynamic characteristics of the stopper can be represented by the wave equation. For the one-dimensional rod shown in figure 2, the equation becomes

$$\frac{\partial^2 u}{\partial x^2}=\frac{1}{c_o^2}\frac{\partial^2 u}{\partial t^2} \tag{4}$$

where, $u(x,t)$ is the displacement of the rod, c_o is the speed of sound given as $\sqrt{E/\rho}$. E and ρ are the Young's modulus and density of the rod material.

The initial conditions and boundary conditions are given as

$$\begin{cases}u(x,0)=0\\ \dot{u}(x,0)=0\end{cases} \tag{5}$$

and

$$\begin{cases}u(0,t)=0\\ EA\dfrac{\partial}{\partial x}u(l,t)=F(t)\end{cases} \tag{6}$$

where E is the Young's modulus of the rod, and A the cross sectional area of the rod.

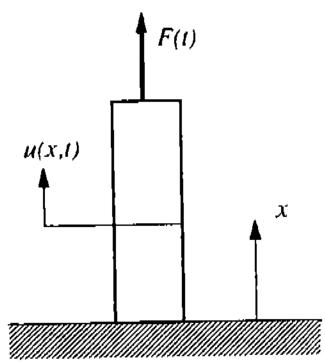

Figure 2. Rod geometry

The Green's function $g(x,t)$ of the wave motion of the rod is obtained by solving equation (4) by taking $F(t)$=1$\delta(t)$, where $\delta(t)$ is the Dirac delta function. This function becomes[6]

$$g(x,t)=\frac{1}{\rho c_o A}\left(\left\{H\left(t-\frac{(l-x)}{c_o}\right)-H\left(t-\frac{(l+x)}{c_o}\right)\right\}-\left\{H\left(t-\frac{(3l-x)}{c_o}\right)-H\left(t-\frac{(3l+x)}{c_o}\right)\right\}\right.$$
$$\left.+\left\{H\left(t-\frac{(5l-x)}{c_o}\right)-H\left(t-\frac{(5l+x)}{c_o}\right)\right\}-\left\{H\left(t-\frac{(7l-x)}{c_o}\right)-H\left(t-\frac{(7l+x)}{c_o}\right)\right\}+\cdots\right) \tag{7}$$

where ρ and l are the density and length of the rod, c_o the sound speed, and $H(t)$ the unit step function.

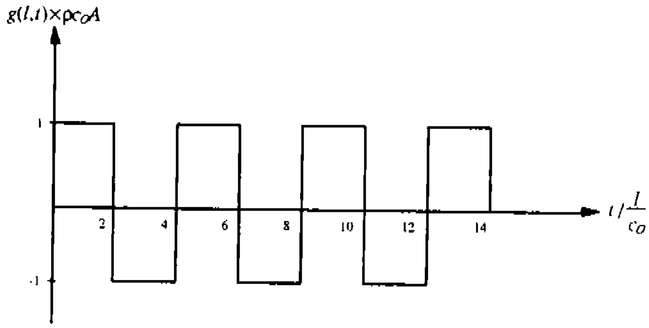

Figure 3. Driving point Green's function of the rod at $x=l$

Actually, $g(x,t)$ is the response of the rod at location x subjected to a unit Dirac delta function type input at the free end. Figure 3 is the time history of the Green's function at $x=l$. Instead of equation (7), a modal expansion form is used for the Green's function in this work as,

$$g(x,t)=\sum_{n=1}^{\infty}\frac{\phi_{rn}(l)\phi_{rn}(x)}{m_r\omega_{rn}}\sin\omega_{rn}t \qquad (8)$$

where, m_r is the mass of the rod, ω_{rn} and ϕ_{rn} are the n^{th} natural frequency and mode shape of the rod as shown in figure 2.

The response of the rod due to the external force $F(t)$ shown in figure 2 is now represented as a convolution integral,

$$u(x,t)=\int_0^t F(\tau)g(x,t-\tau)d\tau \qquad (9)$$

Hence, the displacement of the rod at the free end becomes

$$u(l,t)=\sum_{n=1}^{\infty}\frac{\phi_{rn}^2(l)}{m_r\omega_{rn}}\int_0^t F(\tau)\sin\omega_{rn}(t-\tau)d\tau \qquad (10)$$

Equation (10) can be written in a discretized form

$$u(l,t_i)=\sum_{n=1}^{\infty}\frac{\phi_{rn}^2(l)}{m_r\omega_{rn}}\Delta t\left\{\left[\sin\omega_{rn}t_i\sum_{k=1}^{i}F(t_k)\cos\omega_{rn}t_k-\cos\omega_{rn}t_i\sum_{k=1}^{i}F(t_k)\sin\omega_{rn}t_k\right]+F(t_i)\sin\omega_{rn}\Delta t\right\} \qquad (11)$$

where t_i and t_k represent the i^{th} and k^{th} time step respectively, and Δt is the time step used.

The deflection of the beam at the contact point is obtained from equation (2) with $x=x^*$. Writing the equation in a discretrized form,

$$y(x^*,t_i)=\sum_{n=1}^{\infty}\phi_n(x^*)\eta_n(t_i)+\sum_{n=1}^{\infty}\frac{\phi_n^2(x^*)}{m\omega_n}\Delta t\left\{\left[\sin\omega_n t_i\sum_{k=1}^{i}N(t_k)\cos\omega_n t_k-\cos\omega_n t_i\sum_{k=1}^{i}N(t_k)\sin\omega_n t_k\right]+N(t_i)\sin\omega_n\Delta t\right\} \qquad (12)$$

where, m is the mass of the beam, ω_n and ϕ_n are the n^{th} natural frequency and mode shape of the beam.

The contact force $N(t_i)$ can be computed from equations (11) and (12) letting $-F(t_i)=N(t_i)$ and $u(l,t_i)=y(x^*,t_i)$ providing h=0. Figure 4 shows the procedures to obtain the contact force and the displacements of the beam and the rod.

The wave effect in the beam(valve reed) has been neglected in this procedure, because it has a much thinner thickness than the rod length. The wave traverses across the beam many times to develop the same deflection across the thickness before any significant response occurs in the stopper.

V. NUMERICAL RESULTS

Table 1 shows the numerical data of the problem considered here. The initial conditions of the beam is taken as $y(L,0)=1mm$, and $\dot{y}(x,0)=0$. Figure 5 and 6 show the responses of the beam deflection and the contact force calculated by the procedure suggested here compared with the result obtained by the constant spring assumption. The spring constant was taken from the static stiffness, $K=EA/l$.

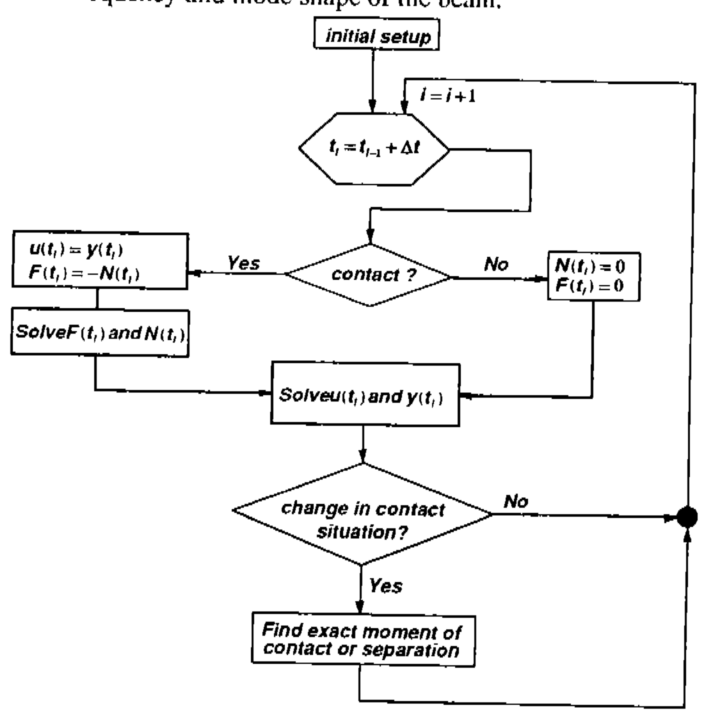

Figure 4. numerical procedure

Figure 5 is the case where l=3*mm* and figure 6 is the case where l=30*mm*. From the figures, it is noted that the contact force is in general over-estimated when the constant K model based on the static deflection is used. The error neglecting the wave effect in the stopper becomes larger as the rod becomes longer as seen in figure 6 compared with figure 5. The reason is believed that the development of the full static deflection takes longer in the longer rod which makes the wave effect more significant.

	Beam	Rod
Young's modulus (N/m^2)	200×10^{9}	200×10^{9}
Mass density (Kg/m^3)	7.85×10^{3}	7.85×10^{3}
Length (m)	2.54×10^{-2}	3.00×10^{-3} and 30.00×10^{-3}
Width (m)	1.00×10^{-2}	1.00×10^{-2}
Thickness (m)	3.05×10^{-4}	4.17×10^{-7}
Damping ratio (%)	3	10

Table. 1

Actual valve seat is of much more complicated shape in the compressors. However, the procedure suggested in this work can be used exactly the same way only except that a different Green's function of the wave motion of the valve seat should be evaluated. Finite element analysis method has to be used for the purpose to obtain the Green's function in a form of a time series, which is considered as a matter of computing resources because some commercial packages have such capabilities.

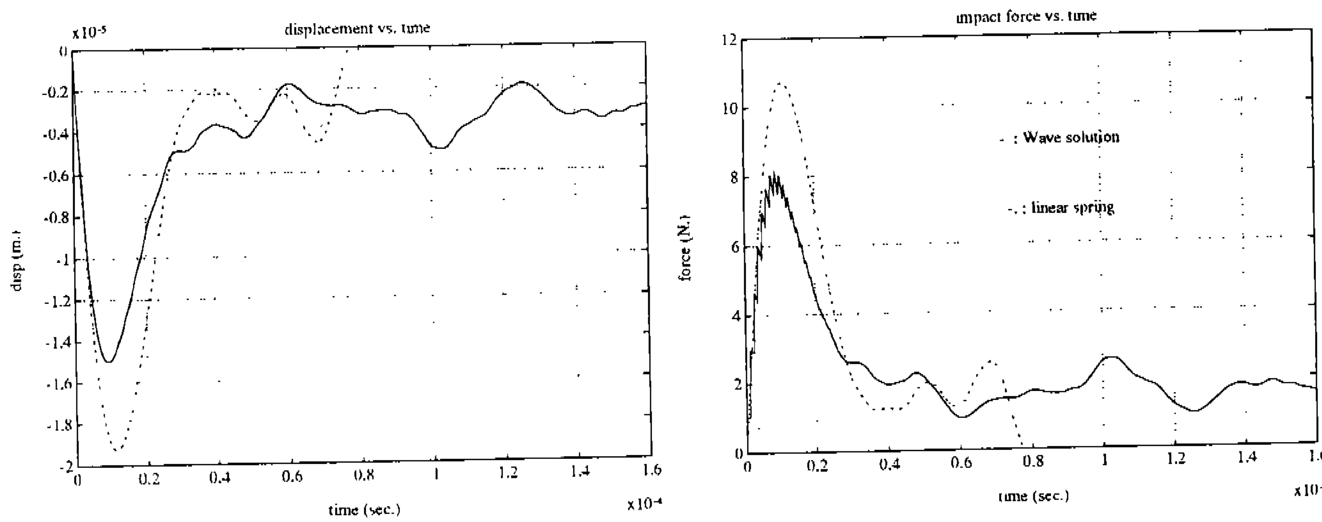

Figure 5. Rod length is 3 mm

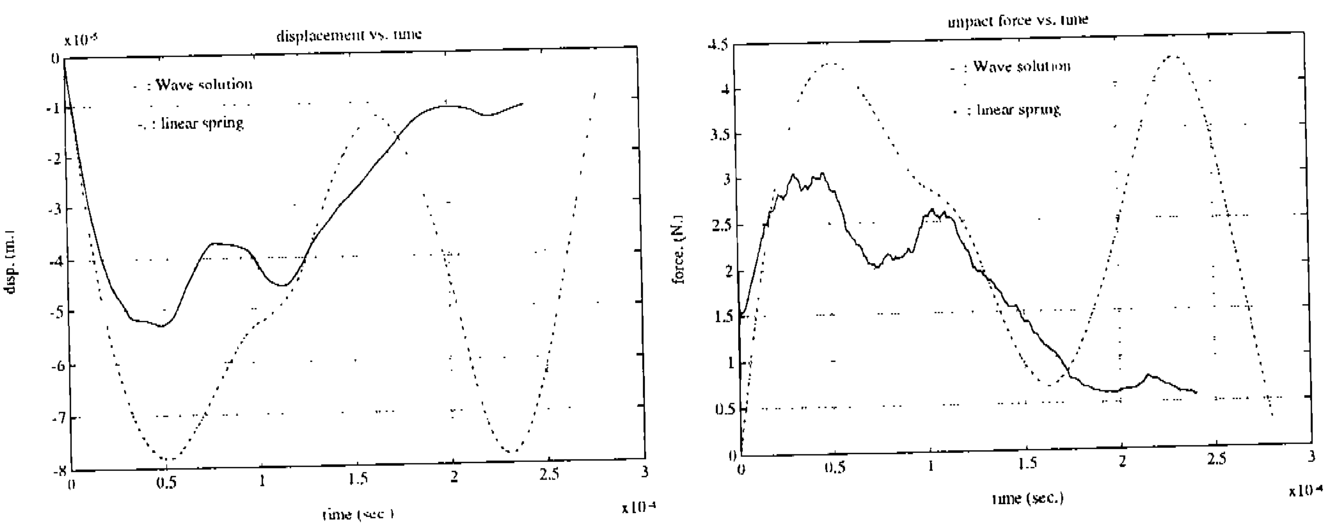

Figure 6 Rod length is 30 mm

VI. CONCLUDING REMARKS

In this work, a new procedure to calculate the contact force between the valve reed and the valve seat is suggested. It is shown that both the valve reed and the valve seat should be included in the model for accurate estimation of the contact force. The procedure developed in this work can be generalized for the analysis of practical cases with complicated geometry. In such cases, the Green's function of the wave response of the seat will have to be obtained by the Finite Element Method.

For the complete design analysis of the valve impact failure, following steps are considered necessary.

1. Application of the procedure suggested in this work to a realistic geometry.
2. Analysis of the stress wave propagation utilizing the contact force estimated in step 1.
3. Study to establish the failure criterion for high speed impact problems.

The first two subjects are currently being studied by the authors.

VII. REFERENCE

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