Measurement of the Upsilon(nS) cross section at CDF

Michael D. Meier

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For the degree of  Doctor of Philosophy

Is approved by the final examining committee:

Matthew Jones
Chair

Andrew S. Hirsch

Ephraim Fischbach

Wei Xie

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Approved by Major Professor(s):  Matthew Jones

Approved by:  John P. Finley  7/25/2016
Head of the Departmental Graduate Program  Date
MEASUREMENT OF THE $\Upsilon(nS)$ CROSS SECTION AT CDF

A Dissertation

Submitted to the Faculty of Purdue University by Michael D. Meier

In Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

August 2016 Purdue University West Lafayette, Indiana
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ABBREVIATIONS

CDF  Collider Detector at Fermilab
CMP  Central Muon Upgrade
CMU  Central Muon Detector
CMX  Central Muon Extension
COT  Central Outer Tracker
CS   Collins-Soper
CSP  CMP scintillator counters
CSX  CMX scintillator counters
GJ   Gottfried-Jackson
IMU  Intermediate Muon Detection
ISL  Intermediate Silicon Layers
MC   Monte Carlo
NRQCD nonrelativistic quantum chromodynamics
pQCD perturbative quantum chromodynamics
QCD  quantum chromodynamics
SH   S-channel Helicity
SVX  Silicon Vertex Detector
XFT  eXtremely Fast Tracker
XTRP Extrapolation Unit
ABSTRACT

Meier, Michael D. Ph.D., Purdue University, August 2016. Measurement of the Υ(nS) Cross Section at CDF. Major Professor: Matthew Jones.

Since the bound $b\bar{b}$ system was first discovered, researchers have been trying to explain the production mechanism for quarkonium to learn more about this system. Several different theories try to describe quarkonium production, and while these theories agree with experimental measurements of production rates, theoretical predictions for quarkonium polarization vary. Careful measurement of the Υ(nS) angular distribution along with the Υ(nS) cross section can provide insight into the quarkonium production mechanism. This analysis measures the Υ(nS) cross section and Υ(1S) polarization parameters.
1. INTRODUCTION

The discovery of the $J/\psi$ meson as a $c\bar{c}$ bound state and the $\Upsilon$ meson as a $b\bar{b}$ bound state has motivated experimental and theoretical researchers to investigate the production mechanism for these quarkonium systems. Quantum chromodynamics (QCD) suggests that each quark has a “color” and that quarks can not exist separately as individual particles, but instead must be confined in particles, known as hadrons. Hadrons are combinations of quarks so that hadrons are colorless. One possible idea for quarkonium production that emerged is that gluons interact with each other and then fragment into a hadron that is in a colorless, color-singlet state.

Predictions of the quarkonium production rates were made using this color-singlet model. However, experimental measurements were made that exceeded the predicted production rates. Studying the production of the charm quark system found that prompt $J/\psi$ and $\psi(2S)$ production was higher than explained by color-singlet QCD predictions [1]. Furthermore, measurements of the bottom quark system found that $\Upsilon$ production was also higher than theoretical predictions [2].

This disagreement between experimental measurement and theoretical predictions lead to new ideas to explain the higher production rates. One such idea was that an intermediate quarkonium particle was produced with color, in a color-octet state, and that this particle eventually lost its color to become a colorless hadron in a final color-singlet state. The color-octet states introduced a color-octet matrix into the calculations, and the color-octet matrix had an infinite number of elements. These color-octet matrix elements could then be “tuned”, or set, so that calculations using color-octet terms would describe the observed quarkonium production rates. Each theory could use different values for the color-octet matrix elements, and even though the theories would agree on the production rates, they could each predict different
quarkonium polarization. As a result, a measurement of the polarization along with the cross section could help determine the correct theory for quarkonium production. Differences between early polarization measurements were thought to be caused by possible bias from the detector acceptance by using only one reference frame. It was suggested that the choice of coordinate system could influence the measured angular distribution more than previously thought, and this bias might explain differences in polarization for previous measurements [3–5]. To fix this issues, polarizations measurements should use multiple reference frames and also calculate frame-independent parameters that are rotationally invariant that then could be easily compared with other experiments [4, 5]. Once the invariant parameter is calculated, it can then be compared for each reference frame in the analysis to show that the analysis is self-consistent [5]. By using these suggestions, more accurate polarization measurements could be made that would not be influenced by experimental factors [5].

This analysis focuses on the measurement of the cross section and angular distribution of the $\Upsilon(nS)$ states at CDF Run II with $\sqrt{s} = 1.96$ TeV. In Chapter 2, the Standard Model of Particle Physics is presented along with a discussion of quantum chromodynamics including color-singlet and color-octet states. Also included in Chapter 2 is an explanation of the references frames and frame invariant parameter used in the analysis of the angular distribution. A discussion of the Physics of Heavy Quarkonia is found in Chapter 3 with leading theories on the quarkonium production mechanism and previous quarkonium measurements. Chapter 4 describes the CDF-II Detector used for this measurement and includes the muon system and trigger system. The Cross Section Analysis is presented in Chapter 5, and while Chapter 6 discusses the Angular Distribution Analysis. A summary of this analysis is included in Chapter 7.
2. THE STANDARD MODEL

Research into the structure of matter at the most fundamental level has lead to discoveries and theories about what matter fundamentally is. The Standard Model of Particle Physics explains what matter consists of at the most fundamental level and explains the forces that interact with matter. While there might be physics beyond the Standard Model, the Standard Model has so far held up to scrutiny and the discoveries of new particles.

2.1 Quarks and Leptons

At the most fundamental level, the Standard Model says that matter is made of six quarks, six leptons, and the anti-particles for each of these particles. Quarks are spin-1/2 particles that have varying mass and fractional charge of positive 2/3 or negative 1/3 times the fundamental charge ($e$). Quarks are never observed by themselves but instead are always confined within other particles, typically in quark and anti-quark pairs, known as mesons, or groups of three quarks or three anti-quarks, known as baryons. There are six “flavors” of quarks: up ($u$), down ($d$), charm ($c$), strange ($s$), top ($t$), and bottom ($b$). The up ($u$), charm ($c$), and top ($t$) quarks each have a charge of positive 2/3 times the fundamental charge ($e$) while the down ($d$), strange ($s$), and bottom ($b$) quarks each have a charge of negative 1/3 times the fundamental charge ($e$). Along with the six quark “flavors”, there is an anti-quark for each of the six “flavors” that has the opposite quantum numbers.

In addition to the “flavor”, quarks also have a “color” associated with them. There are three types of colors: red, blue, and green, along with three anti-colors. Hadrons are particles consisting of quarks and are separated into mesons and baryons. Mesons, quark and anti-quark pairs, are colorless and contain a quark of one color along an
Figure 2.1. The Standard Model. Fundamental Particles and Force Carriers in The Standard Model [6]

anti-quark with the anti-color. Baryons, groups of three quarks or three anti-quarks, are also colorless and have a quark with one of each of the three colors (red, blue, and green) or anti-quark with one of each of the three anti-colors (anti-red, anti-blue, and anti-green).

Leptons are also spin-1/2 particles with varying mass, but unlike quarks, leptons can exist by themselves as stand-alone particles. There are also six types of leptons: electron \((e)\), muon \((\mu)\), tau \((\tau)\), electron neutrino \((\nu_e)\), muon neutrino \((\nu_\mu)\), and tau neutrino \((\nu_\tau)\). Electrons \((e)\), muons \((\mu)\), taus \((\tau)\) each have varying mass and a negative charge equal to the fundamental charge \((e)\) while electron neutrinos \((\nu_e)\), muon neutrinos \((\nu_\mu)\), and tau neutrinos \((\nu_\tau)\) have very small mass and zero electric charge.
2.2 Interaction Forces and Force Carriers

Four interaction forces make up the Standard Model: the Electromagnetic force, the Strong Nuclear force, the Weak Nuclear force and the force of Gravity. Each of the interaction forces have a force carrier particle that is a boson and has either zero spin or an integer spin. The electromagnetic force is the interaction between charged particles and one of the interaction forces. Particles with similar charges repel, and particles with opposite charges attract. The Electromagnetic force is mediated by the electromagnetic carrier particle, the photon (\(\gamma\)). Another interaction force, the Strong Nuclear force, binds quarks together to form hadrons. The force carrier for the Strong Nuclear force are gluons (\(g\)). Gluons allow the exchange of colors between quarks so that a quark changes from one color to another. The Weak Nuclear force is the force responsible for decays from heavier quarks and leptons into lighter quarks and leptons and is the third interaction force. The Weak Nuclear force is mediated by the bosons: the \(W^+\) boson, the \(W^-\) boson, and the \(Z\) boson. Quarks and leptons can change types or “flavors” from the Weak Nuclear force and interaction with via bosons. The fourth interaction force is the force of Gravity, but the Standard Model does not completely incorporate gravity into the theory. The force carrier particle for Gravity is known as the Graviton, but the Graviton has not yet been found experimentally.

2.3 Mass and the Higgs Boson

The Standard Model also incorporates mass into the theory by explaining that the mass of a particle rises from the interaction of that particle with the Higgs Field. Quarks, leptons, and bosons all interact with the Higgs Field and have a measurable mass. Photons and gluons do not interact with the Higgs Field and therefore have zero mass. The Higgs Field was first theorized in 1964 and only recently confirmed with the discovery of the Higgs boson. The Higgs boson has a zero spin and a mass of 125.09 GeV [7].
2.4 Quantum Chromodynamics (QCD)

Quantum chromodynamics (QCD) describes interactions between quarks mediated by gluons and involving color changes of the quarks. Since there are six different “flavors” of quarks and three different colors, there are a total of eighteen quarks. Each quark with a specific color has an anti-quark so there are another eighteen anti-quarks for a total of thirty-six quarks and anti-quarks. Gluons carry both a color and anti-color, and there are eight possible different types of gluons. The eight types of gluons form the “color-octet”, and can be expressed by the following [8]:

\[
\begin{align*}
|1\rangle &= \frac{(r\bar{b} + b\bar{r})}{\sqrt{2}} \\
|2\rangle &= \frac{(r\bar{g} + g\bar{r})}{\sqrt{2}} \\
|3\rangle &= \frac{(b\bar{g} + g\bar{b})}{\sqrt{2}} \\
|4\rangle &= \frac{(r\bar{r} - b\bar{b})}{\sqrt{2}} \\
|5\rangle &= \frac{-i(r\bar{b} - b\bar{r})}{\sqrt{2}} \\
|6\rangle &= \frac{-i(r\bar{g} - g\bar{r})}{\sqrt{2}} \\
|7\rangle &= \frac{-i(b\bar{g} - g\bar{b})}{\sqrt{2}} \\
|8\rangle &= \frac{(r\bar{r} + b\bar{b} - 2g\bar{g})}{\sqrt{6}}
\end{align*}
\]

A ninth type of gluon would be a “color-singlet” and could be expressed by [8]:

\[
|9\rangle = \frac{r\bar{r} + b\bar{b} + g\bar{g}}{\sqrt{3}}
\]

However, since this ninth state would be colorless, it would not carry any color and thus would not be a gluon. Hadrons, combinations of quarks, are colorless and thus in a “color-singlet” state.

2.5 Angular Distribution

A two body particle decay is the decay of the original particle into two new particles. The decay must satisfy conservation of energy and conservation of momentum. One might assume that the angular distribution of the decay products would be uniform; however, the polarization of the original particle affects the angular distribution of the decay products.
The typical nomenclature for polarization of vector mesons is counterintuitive. For a photon, the electromagnetic wave oscillates transverse to the direction of the photon’s momentum and is transversely polarized [5]. However, the spin of the photon is actually along the direction of the momentum of the photon [5]. Thus, a transversely polarized vector meson actually means that the spin of the particle is along the particle’s momentum [5]. Likewise, longitudinal polarization refers to the electromagnetic wave of a photon oscillating in the same direction of the photon’s momentum, but the spin of the photon is actually perpendicular to the direction of the momentum of the photon.

The angular distribution is a function of the polar angle ($\theta$) and azimuthal angle ($\phi$) for one of the decay products, typically the positive muon for the $\Upsilon \rightarrow \mu^+\mu^-$ decay. Figure 2.2 shows the polar angle and azimuthal angle for a particle in a given reference frame.

The general observable two-dimensional angular distribution is given by

$$\frac{dN}{d \cos \theta d\phi} \approx \frac{1}{3 + \lambda_\theta} \left( 1 + \lambda_\theta \cos^2(\theta) + \lambda_\phi \sin^2(\theta) \cos(2\phi) + \lambda_{\theta\phi} \sin(2\theta) \cos(\phi) \right)$$

(2.3)
where \( \theta \) is the polar angle and \( \phi \) is the azimuthal angle but depend on the choice of the reference frame [5]. All three polarization parameters \((\lambda_\theta, \lambda_\phi, \text{ and } \lambda_{\theta \phi})\) are obtained from a single multi-parameter fit on a two-dimensional histogram.

The two-dimensional angular distribution can be integrated to obtain the one-dimensional angular distributions. Two polarization parameters \((\lambda_\theta \text{ and } \lambda_\phi)\) can then be obtained by fitting separate one-dimensional histograms. The first two one-dimensional angular distributions are given by

\[
\frac{dN}{d \cos \theta} \approx \frac{1}{3 + \lambda_\theta} \left( 1 + \lambda_\theta \cos^2(\theta) \right) \tag{2.4}
\]

\[
\frac{dN}{d \phi} \approx 1 + \left( \frac{2 \lambda_\phi}{3 + \lambda_\theta} \right) \cos(2\phi) \tag{2.5}
\]

where \( \theta \) is the polar angle and \( \phi \) is the azimuthal angle that depends on the choice of the reference frame [5].

The third polarization parameter \((\lambda_{\theta \phi})\) disappears in both of the previous equations after integration. However, this parameter can be obtained by defining a new variable \((\tilde{\phi})\) as

\[
\tilde{\phi} = \begin{cases} 
\phi - \frac{3\pi}{4}, & \text{for } \cos \theta < 0 \\
\phi - \frac{\pi}{4}, & \text{for } \cos \theta > 0
\end{cases} \tag{2.6}
\]

where \(\tilde{\phi}\) must lie in the range between 0 and \(2\pi\) [5]. Using this change of variable, the third polarization parameters \((\lambda_{\theta \phi})\) can be measured by fitting a separate one-dimensional histogram. The third one-dimensional angular distribution is given by

\[
\frac{dN}{d \tilde{\phi}} \approx 1 + \frac{\sqrt{2} \lambda_{\theta \phi}}{3 + \lambda_\theta} \cos \tilde{\phi} \tag{2.7}
\]

where \( \theta \) is the polar angle and \( \phi \) is the azimuthal angle and these angles depend on the choice of the reference frame [5].
2.5.1 Reference Frames

Several different reference frames are used to measure the polar and azimuthal angles for the angular distribution. All three reference frames share the same y-axis and have the z-axis rotated about the y-axis. In the S-channel helicity frame, the z-axis is selected along the direction of the particle’s momentum in the lab frame. The z-axis for the Gottfried-Jackson frame is chosen along one of the beam lines in the rest frame of the particle [9]. For the Collins-Soper frame, the z-axis is chosen in the rest frame of the particle along the bisector of the angle between a line along one beam and a line along the other beam through the collision point [10]. The z-axis for each of the S-channel helicity frame, the Gottfried-Jackson frame, and the Collins-Soper frame is shown in Figure 2.3.

2.5.2 Frame Invariant Parameter

The measurement of the polarization can be biased based on the chosen reference frame. As a result, it is beneficial to measure the polarization in multiple refer-
ence frames and then compare a frame invariant parameter. One form of the frame invariant parameter can be calculated by

\[ \tilde{\lambda} = \frac{\lambda_\theta + 3 \lambda_\phi}{1 - \lambda_\phi} \] (2.8)

where the polarization parameters have been measured in a specified frame [5]. A value of \( \tilde{\lambda} = +1 \) indicates transverse polarization while a value of \( \tilde{\lambda} = -1 \) suggests longitudinal polarization. A value of \( \tilde{\lambda} = 0 \) would indicate that on average the particle has no polarization, or is unpolarized. The measured frame invariant parameter (\( \tilde{\lambda} \)) from different reference frames can then be compared to cross check the measurement and discover any systematic errors.
3. PHYSICS OF HEAVY QUARKONIA

Heavy Quarkonia, or quarkonium, refers to the family of particles that involve heavy quarks, namely the charm (c) and bottom (b) quarks. Both the charm system, known as charmonium with a charm and anti-charm quark, and the bottom system, known as bottomonium with a bottom and anti-bottom quark, mimic the hydrogen system consisting of an electron and a proton. The top and anti-top quark do not form a bound system because of the large mass of the top quark. Just like the hydrogen system, these systems have different principal energy levels designated by “n”. Furthermore, there are separate states for the different orbitals. The lowest orbital is the “S” orbital with \( L = 0 \). This results in a single state per energy level, labeled as “nS”. The second orbital is the “P” orbital where \( L = 1 \), and these “P” states are actually triplet states, depending on the alignment of the spins of the quarks. The first state has \( J = 0 \), and the spin of the quarks is aligned opposite the direction of the orbital angular momentum so that the total angular momentum is zero. In the second state, the spin of the quarks are in opposite directions so that the spin angular momentum is zero. In this state, the total angular momentum comes from the orbital angular momentum which results in \( J = 1 \). The third state of the triplet has total angular momentum of \( J = 2 \), which means that the spin of each of the quarks is in the same direction as the spin of the orbital angular momentum. Figure 3.1 shows the lower energy states for bottomonium, the \( \bar{b}b \) bound system. Charmonium, the \( \bar{c}c \) bound system has similar states but with different values of mass for each of the states.

3.1 Quarkonium Theories and Production Mechanism

Although the \( J/\psi \) particle was first discovered in 1974 and the \( \Upsilon \) particle in 1977, the production mechanism for quarkonium is still not completely understood.
Theories for quarkonium production must deal with both “short distances”, dealing with the momentum scale $p$ (usually $p_T$) with distance $1/p$, and “long distances”, with momentum scale $m_Q v$, $m_Q v^2$, or $\Lambda_{QCD}$ [11]. Many of the quarkonium theories use “factorization” where the short distance with high momentum, perturbative effects are separated from the long distance with low momentum, non-perturbative effects [12]. In the following subsections, several of the quarkonium theories will be discussed.

### 3.1.1 Color-Singlet Model (CSM)

After the discovery of the $J/\psi$ particle, the color-singlet model (CSM) was proposed as one explanation for quarkonium production [13–19]. The color-singlet model says that quark and anti-quark pair will produce a quarkonium particle directly that will be colorless and in a color-singlet state [11]. Furthermore, the produced quarkonium and the original quark and anti-quark pair will have the same spin and quantum numbers [11]. The color-singlet model with relativistic corrections
and non-perturbative effects at low momentum agree with experimental measurements [20]. However, at higher momentum, studying the production of the charmonium (c\bar{c} bound) system found that prompt $J/\psi$ and $\psi(2S)$ production was higher than explained by color-singlet QCD predictions [1]. Furthermore, measurements of the bottomonium ($b\bar{b}$ bound) system found that $\Upsilon$ production was also higher than predictions made by the color-singlet QCD model [2]. However, next-to-leading order (NLO) corrections to the color-singlet model are large and have shown to be promising to describe experimental quarkonium production rates [21, 22].

3.1.2 Color-Evaporation Model (CEM)

The color-evaporation model (CEM) incorporates color-octet states in the quarkonium production process [23–26]. In the color-evaporation model, a quark and anti-quark pair first produce a quarkonium that is not colorless in the short distance range ($1/m_Q$), and furthermore, color is actually ignored at this range [27]. The color-evaporation model separates the production of the intermediate quarkonium, which is not in a colorless state (color-octet), and the materialization into the final colorless (color-singlet) quarkonium state over the long distance range ($1/\Lambda_{QCD}$) [27]. The intermediate particle in a color-octet state eventually evolves via the emission of a gluon into the final quarkonium particle in the color-singlet state [28].

The production of the quarkonium is perturbative and depends on the process, while the materialization into the final quarkonium state is non-perturbative and independent of the process [27]. The cross section of the final quarkonium can be calculated by

$$\sigma_{\text{final}} = \rho_{\text{final}} \sigma_{\text{production}}$$

(3.1)

where $\rho_{\text{final}}$ is assumed to be constant and is the fraction of produced quarkonium that materialize into the desired final state [27, 29]. While the color-evaporation model does generally describe experimental data [27], other models have shown to be a better statistical fit [11].
3.1.3 Nonrelativistic Quantum Chromodynamics (NRQCD)

Nonrelativistic Quantum Chromodynamics (NRQCD) is an effective field theory (EFT) that excludes relativistic states of order of the heavy quark mass \( m_Q \), but allows effects of short-lived fluctuations into the excluded relativistic states to be included in calculations [30,31]. Thus, NRQCD separates nonrelativistic physics into non-perturbative calculations discussed later and relativistic effects into coupling constants that can be calculated by a perturbation series of order \( \alpha_s(m_Q) \) [31]. The non-perturbative nonrelativistic physics can then be separated into different orders of \( v \), and so NRQCD allows calculations to be organized both in orders of \( v \) and in orders of \( \alpha_s(m_Q) \) [31]. The NRQCD Lagrangian is given by

\[
\mathcal{L}_{\text{NRQCD}} = \sum_n \frac{c_n(\alpha_s(m), \mu)}{m^n} \times O_n(\mu, mv, mv^2, ...)
\]  

(3.2)

where \( c_n \) are the Wilson coefficients of scale \( m_Q \) which includes the relativistic effects, \( O_n \) is an operator with dynamic matrix elements depending on the energy scale \( (mv, mv^2) \), and \( \mu \) is the NRQCD factorization scale [11]. Furthermore, the inclusive production cross section for a final quarkonium state \( H \) using the NRQCD factorization is given by

\[
\sigma(H) = \sum_n \frac{F_n(\Lambda)}{M_Q^{d_n-4}} \langle 0 | O_n^H(\Lambda) | 0 \rangle
\]  

(3.3)

where \( F_n \) are the short distance coefficients that depend on the kinematics of the production process but are independent of the final quarkonium state \( H \) and \( O_n^H(\Lambda) \) is the non-perturbative long distance operator [31]. \( O_1 \) refers to color-singlet structure of the operator while \( O_8 \) refers to the operator with color-octet structure [31].

With NRQCD factorization, the production process depends on the \( O_n \) operator, which has an infinite number of unknown matrix elements, but expansion in powers of \( v \) and \( \alpha_s \) can be rearranged and only calculated to a certain order [11]. The \( O_n \) operator contains both color-singlet and color-octet elements, but if only the color-singlet terms of leading order of \( v \) are kept, then the NRQCD factorization reduces back to the color-singlet model (CSM) [11]. The color-evaporation model (CEM)
requires certain relationships between the long distance matrix elements that NRQCD does not require [11]. For S-wave quarkonium, the NRQCD factorization agrees with the color-singlet model for production, but for P-wave quarkonium production, the NRQCD factorization shows that the color-singlet terms are incomplete and color-octet terms must be included [31,32]. While NRQCD has better agreement with experimental measurements than the color-singlet model or the color-evaporation model, in the perturbative expansion of the short distance coefficients, different orders of $\alpha_s$ might depend differently on $m_Q/p$, and so higher orders in the expansion might be more important to include than lower orders [11].

3.1.4 Fragmentation Function

Fragmentation is the method of quarkonium production where a parton with large transverse momentum ($p_T$) is first produced, and then the parton decays into the final quarkonium state [33]. At large enough $p_T$, quarkonium production is dominated by fragmentation instead of the short distance mechanism because the short distance mechanism is suppressed by powers of $m_Q/p_T$ even though fragmentation is of higher order in $\alpha_s$ [33]. As previously discussed, NRQCD factorization separates short distance and long distance calculations, but for the short distance expansions orders of $\alpha_s$ depend differently on powers of $m_Q/p$ [11]. Including fragmentation allows another step of separation by including the fragmentation function to organize powers of $m_Q/p$ before applying factorization formalism to separate the short and long distance mechanisms and to incorporate the color-singlet and color-octet mechanisms [34].

A fragmentation function describes the fragmentation of a parton into a quarkonium state and is represented by $D(z, \mu)$ where $z$ is the longitudinal momentum fraction and $\mu$ is a factorization scale [33]. Using a fragmentation function with NRQCD, the cross section formula can first be arranged in orders of $m_Q/p$ before the expansion in powers of $\alpha_s$ [11]. Thus arranging the cross section with the first term
being the leading order in $m_Q/p$, the second the next leading order, and so on, the cross section is given by

$$d\sigma_{A+B\rightarrow H+X}(p_T) = \sum_i d\hat{\sigma}_{A+B\rightarrow i+X}(p_T/z,\mu) \otimes D_{i\rightarrow H}(z,m_Q,\mu)$$

$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\rightarrow [Q\bar{Q}(\kappa)]+X}(P_{[Q\bar{Q}(\kappa)]} = p_T/z,\mu)$$

$$\otimes D_{[Q\bar{Q}(\kappa)]\rightarrow H}(z,m_Q,\mu) + \mathcal{O}(m_Q^4/p_T^4) \quad (3.4)$$

where $A$ and $B$ are the initial hadrons, $H$ is the final quarkonium state, $z$ is the momentum fraction, $\otimes$ is convolution of the momentum fraction, and $D(z,m_Q,\mu)$ is the fragmentation function [11]. While $D_{i\rightarrow H}$ is the fragmentation function for a parton of flavor $i$ to fragment into a final quarkonium state $H$, $D_{[Q\bar{Q}(\kappa)]\rightarrow H}$ is the fragmentation function for a quark anti-quark pair $(Q\bar{Q})$ with quantum numbers for spin and color $(\kappa)$ to fragment into a final quarkonium state $H$ [11]. Now applying NRQCD, the fragmentation function can be written as

$$D_{i\rightarrow H}(z,m_Q,\mu) = \sum_n d_{i\rightarrow q\bar{q}[n]}(z,m_Q,\mu) \langle \mathcal{O}_n^H \rangle$$

$$D_{[Q\bar{Q}(\kappa)]\rightarrow H}(z,m_Q,\mu) = \sum_n d_{[Q\bar{Q}(\kappa)]\rightarrow q\bar{q}[n]}(z,m_Q,\mu) \langle \mathcal{O}_n^H \rangle \quad (3.5)$$

where $d_{i\rightarrow q\bar{q}[n]}(z,m_Q,\mu)$ and $d_{[Q\bar{Q}(\kappa)]\rightarrow q\bar{q}[n]}$ are the short distance coefficients, $q\bar{q}[n]$ is a nonrelativistic state, and $\mathcal{O}_n^H$ are the NRQCD operators with long distance matrix elements (LDME) [11,35].

### 3.1.5 $k_T$ Factorization

Another approach to quarkonium production instead of the standard collinear factorization is the $k_T$ factorization method [11,36–38]. The standard collinear approach assumes that the momentum of all partons is in the same direction as the initial particle, which means there is zero transverse momentum $k_T$ [39]. At large energies, the longitudinal momentum fraction $x$ is small, and therefore, the transverse momentum $k_T$ is non-zero and must be considered [39]. In the $k_T$ factorization
approach, the quarkonium cross section is factorized into a cross section \( \bar{\sigma}(x, k_T, \mu) \) and a parton density function \( F(x, k_T, \mu) \), where both depend on the transverse momentum \( k_T \) [39, 40]. The quarkonium cross section is given by

\[
\sigma = \sum_{i,j} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} F_i(x_1, k_{T,1}^2, \mu^2) F_j(x_2, k_{T,2}^2, \mu^2) 
\times \hat{\sigma}_{i+j\to X}(k_{T,1}, k_{T,2}, x_1, x_2, s) \, dk_{T,1}^2 \, dk_{T,2}^2
\]

(3.6)

where \( i \) and \( j \) are initial partons, \( X \) is the final state, \( F(x, k_T, \mu) \) is the parton density function giving the probability of finding a parton with given \( x \), \( k_T \), and \( \mu \), and \( \hat{\sigma}_{i+j\to X} \) is the parton cross section giving the probability that initial partons \( i \) and \( j \) will form final state \( X \) [39, 41]. The ordinary collinear parton function \( f(x, \mu) \) can be obtained from the \( k_T \) factorization parton density function by

\[
f(x, \mu) = \int_0^{\mu^2} F(x, k_T^2, \mu^2) \, dk_T^2
\]

(3.7)

where \( x \) is the longitudinal momentum fraction, \( k_T \) is the transverse momentum, \( \mu \) is the factorization scale, and \( F(x, k_T, \mu) \) gives the probability of finding a gluon with given \( x \), \( k_T \), and \( \mu \) [39, 40].

For some energy scales, the transverse momentum \( k_T \) must be considered, and the \( k_T \) factorization approach has better predictions than the collinear factorization method [11, 42]. However, even though \( k_T \) factorization calculations are more accurate than collinear factorization methods, uncertainties on \( k_T \) factorization calculations are not yet well quantified and may be larger than the collinear approach [11].

### 3.1.6 Perturbative QCD (pQCD) Collinear Factorization

Perturbative QCD (pQCD) collinear factorization is another approach that first expands transverse momentum in powers of \( 1/p_T \) before applying the collinear factorization approach, also known as the fragmentation approach [43–47]. After the expansion in powers of transverse momentum, the terms can be factorized into perturbative
short distance parton functions and non-perturbative fragmentation functions \[48,49\]. Using this approach, the quarkonium cross section is given by

\[
d\sigma_H(p_T, m_Q) \approx \sum_f d\hat{\sigma}_f(p_T, z) \otimes D_{f \rightarrow H}(z, m_Q)
+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{[Q\bar{Q}(\kappa)]}(p_T, z, u, v) \otimes D_{[Q\bar{Q}(\kappa)] \rightarrow H}(z, u, v, m_Q) \tag{3.8}
\]

where \(z, u,\) and \(v\) are momentum fractions, \(D(z, m_Q)\) and \(\mathcal{D}(z, u, v, m_Q)\) are fragmentation functions, \(d\sigma_H\) is cross section to produce a parton \(f\), and \(\otimes\) is convolution of the momentum fractions \[44,48\]. The fragmentation function \(D_{f \rightarrow H}(z, m_Q)\) for parton \(f\) to fragment into a final quarkonium state \(H\), and \(\mathcal{D}_{[Q\bar{Q}(\kappa)] \rightarrow H}\) is the fragmentation function for a heavy quark anti-quark pair \((Q\bar{Q})\) to fragment into the same final quarkonium state \(H\) \[48,49\]. The first term shows the leading power (LP) in \(m_Q/p_T\) while the second term is the contribution of the next-to-leading power (NLP) \[48\]. Using pQCD factorization, allows the reorganization of NRQCD factorization terms when the transverse momentum is much larger than the heavy quarkonium mass \((p_T >> m_Q)\) \[44\].

### 3.2 Quarkonium Production and Polarization

While the color-singlet model (CSM) has been around since the 1960’s and the 1970’s, the first experimental measurements of the production rate of \(J/\psi\) and \(\Upsilon\) at CDF were drastically higher than QCD predictions \[1, 2\]. This difference between theories at the time and experimental measurement lead the to development of other theories, such as NRQCD, in the 1990’s \[11\]. The long distance matrix elements in NRQCD can be tuned to describe the production rates of quarkonium \[50\]. Calculations using \(k_T\) factorization with color-singlet contributions also shows agreement with quarkonium production \[11\]. By fitting the free parameters of each theory to the quarkonium cross sections, theories could make predictions for the polarization of quarkonium particles. However, current theories do not agree on quarkonium polarization, and slight adjustments to the long distance matrix elements or including more
terms in the expansions can change the calculated cross sections and polarizations. Furthermore, the excited $nP$ states decay into the $nS$ states via radiative decay that can also influence the measured polarization, and theories must account for the feed-down fraction. Better experimental measurements of the quarkonium polarization along with the cross section can provide the best way to determine the fundamental quarkonium production method [50].

In the charmonium system, several experimental measurements for the production rates and polarizations have been made. The $J/\psi$ cross section has been measured by CDF [1, 51], and the production ratio of $\chi_c$ excited states has been measured by CDF [52] and LHCb [53]. Measurements of the polarization for $J/\psi$ and $\psi(2S)$ have been made by CDF [54, 55].

Theoretical calculations for the charmonium system have also been made to compare with the experimental measurements. Calculations of $J/\psi$ production rates have been made using fragmentation [56], next-to-leading order (NLO) QCD via color-singlet [57], $k_T$ factorization with the color-singlet model [38, 58–60], and perturbative QCD (pQCD) [61]. Production rates for excited $\chi_c$ states with NRQCD have also been calculated [62]. Furthermore, calculations for $J/\psi$ polarizations have also been made using $k_T$ factorization with the color-singlet model [38], NRQCD factorization at next-to-leading order (NLO) [63–65], and $k_T$ factorization [60, 66]. Polarization calculates have also been made for the $\chi_{c1}$ and $\chi_{c2}$ excited states using NRQCD (with dominating color-octet terms) [67].

The theoretical calculations and experimental measurements for charmonium have allowed progress in discovering the quarkonium production mechanism. However, bottomonium is a better system for study than charmonium because the mass of the bottom quark is greater than the charm quark and both the relative velocity $v$ of the heavy quarks in quarkonium and $\alpha_s$ are smaller in bottomonium than charmonium which allows for the higher expansion terms to converge faster for bottomonium [68].

Several polarization calculations for the bottomonium system have been made using theories with different expansions. One approach to applying theory to calculate
Υ polarization is using $k_T$ factorization. When using $k_T$ factorization, it is found that the color-octet contribution is much smaller than in the collinear factorization approach, and the color-octet contributions can be ignored [69]. Υ can be produced directly via gluon-gluon fusion or via the radiative decay from a $\chi_b$ state. Using $k_T$ factorization calculations, the polarization parameter ($\alpha$), $\lambda_\theta$ in Equation 2.3, for both directly produced Υ and Υ from feeddown from $\chi_b$ decays is shown in Figure 3.2.

Other approaches use NRQCD for Υ polarization calculations with expansions to different orders. Using NRQCD collinear factorization leading order (LO) from 2007, the polarization parameter ($\alpha$), $\lambda_\theta$ in Equation 2.3, for Υ is shown in Figure 3.3 as a function of transverse momentum ($p_T$) [21]. Figure 3.3(a) shows Υ produced by $p\bar{p} \to \Upsilon + g$, while $p\bar{p} \to \Upsilon + Q\bar{Q}$ is shown in Figure 3.3(b). The second process ($p\bar{p} \to \Upsilon + Q\bar{Q}$), shown in Figure 3.3(b), dominates over other color-singlet terms at leading order (LO), so quarkonia produced at high transverse momentum $p_T$ should be unpolarized [21].
Calculations from 2008 extend the previous NRQCD results by including higher orders in the expansions. Figure 3.4 shows the representative diagrams for the $\Upsilon$ production process for NRQCD in leading order (LO), next-to-leading (NLO), and next-to-next-to-leading (NNLO). The calculations for direct $\Upsilon(1S)$ production as a function of $\Upsilon$ transverse momentum ($p_T$) along with a comparison to CDF Run I experimental measurements [71] are shown in Figure 3.5. At NRQCD leading order
(LO) (terms of $\alpha_s^3$), $\Upsilon(nS)$ states are predicted to be transversely polarized ($\alpha = 1$) but at next-to-leading order (NLO) ($\alpha_s^4$ corrections), the same states are calculated to be longitudinally polarized ($\alpha = -1$) [70]. Adding next-to-leading order (NLO) terms shows better agreement with measured $\Upsilon$ cross section than leading order (LO) calculations [70]. Estimating the contribution of next-to-next-to-leading order (NNLO) ($\alpha_s^5$ terms), further increases agreement with data and also shows longitudinal polarization ($\alpha = -1$) for $\Upsilon(nS)$ states [70]. Figure 3.6 shows a summary of the calculated polarization parameter ($\alpha$) for the $\Upsilon(nS)$ states using NRQCD.

The previous calculations were only for the $\Upsilon(1S)$ state, and a 2014 publication from 2014 extends the NRQCD next-to-leading order (NLO) polarization calculations to the $\Upsilon(2S)$ and $\Upsilon(3S)$ states [72]. Figure 3.7 show the NRQCD next-to-leading (NLO) calculations and comparison to experimental polarization measurements made by CDF and CMS [72]. The calculations in Figure 3.7 are shown with different factorization scales ($\mu = mv, \mu = \Lambda_{QCD}$, and $\mu = m_b$) [72]. For these calculations, the $\Upsilon(1S)$ state includes feeddown contributions from $\Upsilon(2S), \Upsilon(3S), \chi_{bJ}(1P)$, and $\chi_{bJ}(2P)$ states while the $\Upsilon(2S)$ state has feeddown from $\Upsilon(3S)$ and $\chi_{bJ}(2P)$ [72].
Figure 3.6. NRQCD Polarization for Direct $\Upsilon(1S')$ Production [70]. LO $- \alpha_s^3$ (blue dashed line), associated production $\Upsilon + b\bar{b} = \alpha_s^4$ (green dotted line), full NLO $- \alpha_s^3 + \alpha_s^4$ (black solid line), estimated NNLO $-\alpha_s^5$ (red band) [70].

Figure 3.7. NRQCD NLO Polarization Parameter ($\lambda$) for $\Upsilon(nS)$ Production at the Tevatron and the LHC [72]. Columns from left to right show $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$. Rows from top to bottom show data from CDF Run II ($|y| < 0.6$) [73], CMS ($|y| < 0.6$) [74], and CMS ($0.6 < |y| < 1.2$) [74]. NRQCD next-to-leading order (NLO) calculations shown for different factorization scales: $\mu = m_{\text{v}}$, $\mu = \Lambda_{\text{QCD}}$, and $\mu = m_{\text{b}}$ [72].
However, the calculations for the $\Upsilon(3S)$ state do not include any feeddown contributions [72]. The feeddown from excited states play an important role in both production and polarization measurements. Another publication from 2014 further extends the NRQCD next-to-leading order (NLO) calculations with predictions of feeddown fractions, including feeddown in the $\Upsilon(3S)$ state [68]. Furthermore, feeddown is included for $\Upsilon(mS)$ states with $m = 1, 2, 3$ contributions coming from $\chi_{bJ}(nP)$ states with $n = 1, 2, 3$ where $n \geq m$ [68]. Figure 3.8 shows the differential cross section calculations with feeddown and includes a comparison to cross section measurements from ATLAS, CMS, and CDF [68]. Figure 3.9 shows the $\Upsilon(nS)$ polarization calculations including feeding with comparison to data from CMS [68].

![Figure 3.8. NRQCD NLO $\Upsilon(nS)$ Differential Cross Section (with varying feeddown) [68]. Columns from left to right show $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$. Top row shows data from ATLAS ($\sqrt{s} = 7$ TeV and $|y| < 1.2$) [75] and bottom row shows data from CMS ($\sqrt{s} = 7$ TeV and $|y| < 2.4$) [76] and CDF ($\sqrt{s} = 1.8$ TeV and $|y| < 0.4$) [71]. Contributions from direct production (dashed black lines), total feeddown (dashed-dotted red lines), $\chi_{b1}(nP)$ (solid black lines), and $\chi_{b2}(nP)$ (dotted black lines) are shown [68].]
Figure 3.9. NRQCD NLO Polarization Parameter ($\lambda_\theta$) for $\Upsilon(nS)$ in the Helicity Frame (with varying feeddown) [68]. Columns from left to right show $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$. Top row shows data from CMS ($\sqrt{s} = 7$ TeV and $|y| < 0.6$) [77] and bottom row shows data from CMS ($\sqrt{s} = 7$ TeV and $0.6 < |y| < 1.2$) [77]. Contributions from direct production (dashed black lines), total feeddown (dashed-dotted blue lines), and total results (blue bands) are shown.

3.3 Previous $\Upsilon$ Measurements

Several experimental measurements for both production and polarization have been made in the bottomonium system. The first set of measurements was made at the Tevatron with $p\bar{p}$ collisions. CDF has two measurements of the $\Upsilon$ cross section during Run I at $\sqrt{s} = 1.8$ TeV with rapidity range, $|y| < 0.4$. CDF Run I measured the $\Upsilon(nS)$ production cross section in 1995 with a sample of 16.6 pb$^{-1}$ [2]. Table 3.1 summarizes the 1995 CDF Run I cross section measurement.

In 2002, CDF Run I measured both the $\Upsilon$ cross section and polarization with an integrated luminosity of 77 pb$^{-1}$ [71]. The cross section results from the CDF Run I measurement from 2002 are listed in Table 3.2. For Run II at the Tevatron at $\sqrt{s} = 1.96$ TeV, only D0 has published a result. D0 Run II made an $\Upsilon$ cross section measurement in 2005 with a varying rapidity range and a luminosity of 185 pb$^{-1}$ [78,79]. Table 3.3 summarizes the D0 Run II $\Upsilon$ cross section measurement.
Table 3.1
1995 CDF Run I Υ Cross Section Measurement [2]. Measurement with luminosity of 16.6 pb\(^{-1}\) at \(\sqrt{s} = 1.8\) TeV and with \(|y| < 0.4\) [2]

<table>
<thead>
<tr>
<th>(\Upsilon(nS)) State</th>
<th>(\frac{d\sigma(\Upsilon(nS))}{dy} \times B(\Upsilon(nS) \rightarrow \mu^+\mu^-)) (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Upsilon(1S))</td>
<td>753 ± 29 (stat.) ± 72 (syst.)</td>
</tr>
<tr>
<td>(\Upsilon(2S))</td>
<td>183 ± 18 (stat.) ± 24 (syst.)</td>
</tr>
<tr>
<td>(\Upsilon(3S))</td>
<td>101 ± 15 (stat.) ± 13 (syst.)</td>
</tr>
</tbody>
</table>

Table 3.2
2002 CDF Run I Υ Cross Section Measurement [71]. Measurement with luminosity of 77 pb\(^{-1}\) at \(\sqrt{s} = 1.8\) TeV and with \(|y| < 0.4\) [71]

<table>
<thead>
<tr>
<th>(\Upsilon(nS)) State</th>
<th>(\frac{d\sigma(\Upsilon(nS))}{dy} \times B(\Upsilon(nS) \rightarrow \mu^+\mu^-)) (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Upsilon(1S))</td>
<td>680 ± 15 (stat.) ± 18 (syst.) ± 26 (lumi.)</td>
</tr>
<tr>
<td>(\Upsilon(2S))</td>
<td>175 ± 9 (stat.) ± 8 (syst.)</td>
</tr>
<tr>
<td>(\Upsilon(3S))</td>
<td>97 ± 8 (stat.) ± 5 (syst.)</td>
</tr>
</tbody>
</table>

Table 3.3
2005 D0 Run II Υ Cross Section Measurement [78,79]. Measurement with luminosity of 185 pb\(^{-1}\) at \(\sqrt{s} = 1.96\) TeV [78,79]

<table>
<thead>
<tr>
<th>Rapidity Range</th>
<th>(\frac{d\sigma(\Upsilon(nS))}{dy} \times B(\Upsilon(nS) \rightarrow \mu^+\mu^-)) (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 - 0.6</td>
<td>628 ± 16 (stat.) ± 63 (syst.) ± 38 (lumi.)</td>
</tr>
<tr>
<td>0.6 - 1.2</td>
<td>654 ± 17 (stat.) ± 65 (syst.) ± 40 (lumi.)</td>
</tr>
<tr>
<td>1.2 - 1.8</td>
<td>515 ± 16 (stat.) ± 46 (syst.) ± 31 (lumi.)</td>
</tr>
<tr>
<td>0.0 - 1.8</td>
<td>597 ± 12 (stat.) ± 58 (syst.) ± 36 (lumi.)</td>
</tr>
</tbody>
</table>

The next set of measurements were done at the Large Hadron Collider (LHC) at \(\sqrt{s} = 7\) TeV in \(pp\) collisions. CMS measured the Υ cross section in 2011 with a luminosity of 3.1 pb\(^{-1}\), rapidity range \(|y| < 2\), and \(p_T < 30\) GeV [80]. Table 3.4 summarizes the 2011 CMS Υ cross section measurement. In 2013, CMS again measured the Υ cross section but with a luminosity of 35.8 pb\(^{-1}\), rapidity of \(|y| < 2.4\),
and $p_T < 50 \text{ GeV}$ [76]. Table 3.5 has the results for the 2013 CMS $\Upsilon$ cross section measurement. ATLAS measured the $\Upsilon$ cross section in 2013 with a luminosity of 1.8 fb$^{-1}$, rapidity of $|y| < 2.25$, and $p_T < 70 \text{ GeV}$ [75]. Table 3.6 lists the 2013 ATLAS $\Upsilon$ cross section measurement.

Several $\Upsilon$ polarization measurements have now also been made. With a luminosity of 77 pb$^{-1}$, CDF Run I measured the $\Upsilon(1S)$ polarization in 2002 at $\sqrt{s} = 1.8 \text{ TeV}$ with $|y| < 0.4$ and found the $\Upsilon(1S)$ to be unpolarized [71]. At $\sqrt{s} = 1.96 \text{ TeV}$, D0 Run II measured the $\Upsilon(1S)$ and $\Upsilon(2S)$ polarization in 2008 in a sample with luminosity of 1.3 fb$^{-1}$ [81]. The measurement done by D0 found longitudinal polarization for the $\Upsilon(1S)$ [81]. However, these first polarizations measurements were only measured in one reference frame, and the results could be biased due to the choice of the reference frame and the acceptance of the detector. Newer measurements were done in multiple reference frames and include the calculation of the frame invariant parameter to prevent bias from detector acceptance with the choice of a single reference frame.

The first full polarization for all $\Upsilon(nS)$ states was measured in 2012 by CDF Run II at $\sqrt{s} = 1.96 \text{ TeV}$ [73]. The CDF Run II measurement had a luminosity of 6.7 fb$^{-1}$ with $|y| < 0.6$ and $p_T < 40 \text{ GeV}$ and also found no evidence for polarization [73]. CMS measured the $\Upsilon(nS)$ polarization in 2013 using a luminosity of 4.9 fb$^{-1}$ at $\sqrt{s} = 7 \text{ TeV}$ [77]. The CMS polarization measurement found the $\Upsilon$ to be unpolarized, and suggested that this could be a result of including $\Upsilon$ produced in feeddown from an

<table>
<thead>
<tr>
<th>$\Upsilon(nS)$ State</th>
<th>$\sigma(pp \rightarrow \Upsilon(nS)X) \times B(\Upsilon(nS) \rightarrow \mu^+\mu^-)$ (nb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Upsilon(1S)$</td>
<td>$7.37 \pm 0.13 \text{ (stat.)}^{+0.61}_{-0.42} \text{ (syst.)} \pm 0.81 \text{ (lumi.)}$</td>
</tr>
<tr>
<td>$\Upsilon(2S)$</td>
<td>$1.90 \pm 0.09 \text{ (stat.)}^{+0.20}_{-0.14} \text{ (syst.)} \pm 0.24 \text{ (lumi.)}$</td>
</tr>
<tr>
<td>$\Upsilon(3S)$</td>
<td>$1.02 \pm 0.07 \text{ (stat.)}^{+0.11}_{-0.08} \text{ (syst.)} \pm 0.11 \text{ (lumi.)}$</td>
</tr>
</tbody>
</table>
Excited state [77]. Improved Υ polarization measurements with lower uncertainties will allow theories to develop to provide insight into the quarkonium production mechanism.
4. THE CDF-II DETECTOR

The Collider Detector at Fermilab (CDF) was upgraded in 2001 for Run IIa and upgraded again for Run IIb in 2005. The detector is made up of the tracking system, the solenoid, the calorimetry system, and the muon system. The tracking system allows the direction of particles to be determined and consists of the Silicon Inner Tracker and the Central Outer Tracker. The Silicon Inner Tracker has the Silicon Vertex Detector (SVX), which immediately surrounds the beam pipe, and the Intermediate Silicon Layers (ISL), which are outside of the SVX. Going radially outward from the beam pipe, the Central Outer Tracker (COT) is next, and then right outside of the COT is a solenoid that provides a 1.4 Tesla magnetic field. The magnetic field from the solenoid curves the path of particles so that the charge can be determined. The calorimeter system measures the energy deposited by particles and has a electromagnetic calorimeter surrounding the solenoid followed by the hadronic calorimeter. Finally, the muon system makes up the outside of the detector because muons are able to travel through all of the detector materials and are most likely the only particles to reach the muon systems at the outside of the detector. Figure 4.1 shows the side view of the CDF Detector with the main areas of the detector labeled.

4.1 Tracking System

4.1.1 Silicon Inner Tracker

The first part of the CDF Run II Detector tracking system is the Silicon Inner Tracker. The Silicon Inner Tracker is made up of eight total layers forming a barrel around the beam pipe. The first six layers are part of the Silicon Vertex Detector (SVX), while the last two layers form the Intermediate Silicon Layers (ISL).
The Silicon Vertex Detector (SVX) immediately surrounds the beam pipe and provides coverage from a radius of 1.9 cm to a radius 16.6 cm [82]. The SVX includes six axial layers, Layer 0 - Layer 5, and two small angle stereo layers [82]. The two small angle stereo layers measure the z-position of secondary vertices, which is important since the bottom quick has a long lifetime so that it decays at secondary vertex separate from the primary vertex. Each layer of the SVX contains several staves which run the length of the barrel and have multiple single-sided silicon sensors. The amount of material a particle encounters as it travels radially outward from the beam pipe depends on its path through the staves. The material budget for each stave can
be from as little as $1.08\% X_0$ to as much as $4.72\% X_0$ but the average over the stave area is $1.8\% X_0$ [82].

The Intermediate Silicon Layers (ISL) includes Layer 6 and Layer 7 for the silicon tracking system. The ISL covers a radius of 20 cm to 28 cm and uses double-sided silicon sensors. The material budget for the ISL is given as 2% $X_0$ [82].

Particles traveling through the silicon sensors in the SVX and ISL cause ionization, and electron-hole pairs develop in the silicon. The charges separate due to an applied electric field in the silicon and can be detected as a current. The current is measured by readout electronics, and a particle hit is registered. Readouts from the SVX and ISL are used to record hits from the silicon sensors, and then these silicon hits are used to reconstruct the track of a particle in the detector.
Figure 4.3. Inside View of the Central Outer Tracker (COT) for the CDF Run IIb Detector [83]

4.1.2 Central Outer Tracker

The second part of the detector tracking system is the Central Outer Tracker (COT). The COT is a gas-filled drift chamber, and the material structure of the COT is known as the CAN because of its cylindrical shape. A gas mixture of 50% Argon, 35% Ethane, and 15% $CF_4$ is used in the COT [84]. As particles move outward through the COT, charged particles ionize the gas in the drift chamber. Wires running through the COT record voltage from the ionization of the particles, and these hits are then used to determine the tracks of the particles.

The drift cells allow detection of particles from a radius of 44 cm to 132 cm and have a total material budget of $1.3% \times X_0$ across the entire COT [82]. While the drift cells provide coverage from 44 cm to 132 cm, the housing for the COT has an inner cylinder at a radius of 40 cm and an outer cylinder at a radius of 137 cm [84]. The
inner cylinder is made of a carbon fiber/epoxy composite that has an average thickness of 0.251 cm [85]. The side of the inner cylinder in contact with the gas mixture, the larger outside surface of the inner cylinder, is covered by a 25.4 μm thick aluminum sheet [85]. The total material budget for the inner cylinder is 0.99% $X_0$ for both the composite material and aluminum sheet [85]. The outer cylinder of the COT is made out of eight pieces of aluminum with a thickness of 0.953 cm in the center and 0.635 cm near the end plates [85].

4.2 Calorimeter Systems

The calorimeter system is located just outside of the solenoid and contains both an electromagnetic calorimeter and a hadronic calorimeter. The electromagnetic calorimeter is made of a plastic scintillator between pieces of lead and has a material budget of about 21 $X_0$ [84]. The hadronic calorimeter has both steel and scintillator in order to detect particles [84]. As particles travel through the calorimeter, they collide with the detector material and form a cascade of particles with lower energy. Fibers in the scintillators in the end cap and light guides in the central region transport the light to the photomultiplier tubes to measure the energy of the cascade particles. The total energy of the original particle can then be calculated from the energy measured from the cascade particles.

4.3 Muon Systems

The CDF detector uses four different muon systems to find and detect muons. The four muon systems are the Central Muon Detector (CMU), the Central Muon Upgrade (CMP), the Central Muon Extension (CMX), and the Intermediate Muon Detection (IMU). Each of the muon systems are similar with scintillators and steel absorbers. However, the Central Muon Detector (CMU) and the Central Muon Upgrade (CMP) cover the central region of the detector while the Central Muon Extension (CMX) and the Intermediate Muon Detection (IMU) cover forward regions of the detector.
All four muon systems work to record hits, and then the hits are used to reconstruct the path of muons through the detectors. The reconstruction of the muons hits are called muon stubs, and then the muon stubs are matched to particle tracks from the COT. The muon systems are discussed more in the following sections.

4.3.1 Central Muon Detector (CMU)

The first of the CDF muon systems is the Central Muon Detector (CMU), and the CMU covers the central region of the CDF Detector. The Central Muon Detector (CMU) is the original muon chamber from Run I and has 144 modules of drift tubes [84]. The CMU drift tubes have a length of 226 cm, and there are a total of 2304 drift tubes in CMU [84]. The CMU is located just beyond the Central Hadronic
Calorimeter and covers the region $|\eta| < 0.6$ [84]. Muons must have a minimum transverse momentum of 1.4 GeV/c to be detected in the CMU [84].

### 4.3.2 Central Muon Upgrade (CMP)

The Central Muon Upgrade (CMP) is the second muon system for the CDF Detector, and the CMP also detects muons in the central region of the CDF Detector. The Central Muon Upgrade (CMP) is located radially outward from the CMU after 60 cm of steel and forms a rectangular box shape around the cylindrical CMU [86]. The box shape leads to non-constant coverage of pseudorapidity as the azimuthal...
angle varies, as seen in Figure 4.5. The Central Muon Upgrade (CMP) covers a region of $|\eta| < 0.6$, which is approximately the same region as the CMU but at a larger radius from the beam pipe [84]. The CMP muon chambers are single-wire drift tubes made of aluminum and are arranged in four layer stacks [84]. The drift tubes in CMP have a length of 640 cm, and there are 1076 drift tubes used in CMP [84]. Scintillator counters, known as the CSP, are located on the outer surface of the CMP drift chambers [84]. Each of the CSP counters are twice the width of a CMP stack but are half the length [86]. The CMP detects muons with a minimum transverse momentum of 2.2 GeV/c [84].

### 4.3.3 Central Muon Extension (CMX)

The third muon system of the CDF Detector is the Central Muon Extension (CMX), and the CMX covers the forward regions of the CDF Detector. The Central Muon Extension (CMX) and scintillator counters (CSX) help to extend the coverage of muon detection to include the region $0.6 < |\eta| < 1.0$ [84]. The CMX is made of the same drift tubes as the CMP, but the CMX drift tubes are 180 cm in length [86], and there are a total of 2208 drift tubes [84]. The CMX drift tubes are arranged to form $15^\circ$ modules using 48 tubes, and four CSX scintillator counters are attached to the inside and outside of each CMX sector [86]. Both the CMP and CMX drift tubes are filled with a 50% Ar and 50% $C_2H_6$ gas mixture [86]. The minimum transverse momentum for muons to be detected by the CMX is 1.4 GeV/c [84].

### 4.3.4 Intermediate Muon Detection (IMU)

The Intermediate Muon Detection (IMU) is the fourth muon system in the CDF detector, and the IMU covers forward regions of the CDF Detector. The Intermediate Muon Detection (IMU) extends the muon coverage to $1.0 < |\eta| < 1.5$ [84]. The IMU has muon chambers with drift tubes like the CMP and scintillator counters like the CSP [84]. There are a total of 1728 drift tubes in the IMU, and the drift tubes
have a length of 363 cm [84]. The IMU will detect muons with a minimum transverse momentum between 1.4 GeV/c and 2.0 GeV/c [84].

4.4 Trigger System

The CDF Detector must handle the millions of proton and anti-proton collisions that happen every second. Since every single collision, or event, can not be saved, a trigger system is used to decide which events to save to tape. The speed of data being written to tape is less than 50 Hz, and the rate of collisions is approximately 7.6 MHz [84]. The trigger system for the Run IIb Detector is designed for a 396 ns bunch crossing. The trigger system is designed so that the dead time, where the detector is not able to record any data, is minimized. The trigger system used by the CDF detector has three levels, which helps to reduce the dead time. The decision for each level is made quickly, and the data is stored in memory so that new data from the next event is not lost as the decision to keep the event is being made. Figure 4.6 shows an overview of the trigger system and data acquisition process.

4.4.1 Level 1 Trigger

The Level 1 Trigger uses data from the tracking systems, calorimeters, and muon systems to determine if the event should be sent to the Level 2 Trigger. The decision time for the Level 1 Trigger is $5.5 \mu s$, and in order to achieve this quick of decision time, the Level 1 Trigger is a synchronous pipeline system so that all parts of the trigger process information at the same time [84]. The Level 1 Trigger reconstructs tracks using the eXtremely Fast Tracker (XFT), and tracks have a minimum transverse momentum of 1.5 GeV/c [84]. The XFT only uses hits from the four axial super layers of the COT to quickly reconstruct tracks [87]. The four axial super layers in the COT are positioned at approximately 30° angle relative to the radial [87].

After the XFT tracks are processed, the Extrapolation Unit (XTRP) uses the XFT track information to project the tracks to the radii of the calorimeter and muon
systems using look-up tables, and then the XTRP sends information to the Level 1 Trigger subsystems: the Level 1 Calorimeter trigger, Level 1 Muon trigger, and Level 1 Track trigger [84]. For the Level 1 Calorimeter trigger, the XTRP sends bits corresponding to different momentum cut-offs for each 15° calorimeter wedge. The Level 1 Calorimeter trigger is divided into object triggers, such as electrons, photons, and jets, using the energy in a single tower and global triggers, such as the total energy, using the sum of energy in all the towers [84]. For the Level 1 Muon trigger, the XTRP sends bits with the momentum threshold and azimuthal angle from the CMU and CMX look-up tables [84]. The Level 1 Muon trigger uses hits in the muon systems to form muon stubs, also known as primitives, and includes if the muon stub has a low, medium, or high transverse momentum [84]. The Level 1 Muon trigger

Figure 4.6. Diagram of Dataflow and Trigger System for Run IIb [82]
also uses the location of the muon stub within fixed 2.5° azimuthal (\(\phi\)) bins and four pseudorapidity (\(\eta\)) bins [84]. The Level 1 Track trigger uses the track information from the selected by the XTRP with a minimum transverse momentum along with the total number of tracks to check Level 1 triggers [84]. The decisions from each of the Level 1 trigger subsystems are fed into the Global Level 1 Trigger that determines if the event is sent to the Level 2 Trigger.

### 4.4.2 Level 2 Trigger

The Level 2 Trigger is an asynchronous system with several subsystems, and the average decision time for the Level 2 Trigger is 20 \(\mu\)s [82]. The Level 2 Triggers starts with the information from the Level 1 Trigger and subsystems. The Level 2 Trigger uses XFT track information from the XTRP from Level 1 and also uses the Level 1 Muon trigger information to trigger on matching XFT tracks and muon stubs. The Level 2 Calorimeter trigger uses the data gathered from the Level 1 Calorimeter trigger and runs a cluster finding algorithm that starts with seed towers above a certain threshold [84]. The Level 2 Central Shower Maximum (XCES) trigger sums four adjacent Central Shower Maximum Detector (CES) wires and then, along with the position, the results are then matched to a XFT track to see it meets trigger requirements [84]. The Silicon Vertex Tracker (SVT) processes hits from the SVX and uses XFT track information from the XTRP to determine track impact parameters for the Level 2 SVT trigger [84]. Each of the Level 2 trigger subsystems are relayed into the Global Level 2 Trigger to save or reject the event. Figure 4.7 shows the process for the Run II Trigger System, and includes how the detector information, Level 1 Trigger subsystems, and Level 2 Trigger subsystems work together.

### 4.4.3 Level 3 Trigger

Before applying the Level 1 Trigger and Level 2 Trigger, the rate of events occurring was 7.6 MHz, and after these triggers, it is reduced to about 300 Hz [88]. The
Level 3 Trigger must reduce the event rate to less than 50 Hz since this is the speed of writing data to tape. After an event is accepted by the Level 2 Trigger, the data is sent to the Event Builder system at the Level 3 PC farm. The Level 3 PC farm processes the data and reconstructs the event with full track reconstruction so that the Level 3 Trigger has the full detector information available, and the processing at the PC farm takes about one second per event [88]. Level 3 Triggers can then be applied to the full detector information to decide if the event should be saved to tape or rejected.
5. CROSS SECTION ANALYSIS

The measurement of the $\Upsilon(nS)$ cross section was done using data from the CDF-II Detector at Fermilab with center of mass energy at $\sqrt{s} = 1.96$ TeV. The $\Upsilon$ decays into two muons, $\mu^+$ and $\mu^-$, and $\Upsilon$ candidates are reconstructed by combining two muons. Section 5.1 discusses the data sample which has two different $\Upsilon$ triggers. The full selection cuts used to identify muons and $\Upsilon$ candidates are listed in Section 5.1.3. The $\Upsilon$ yield is determined from a fit to the reconstructed $\Upsilon$ candidates, and the fit is explained in Section 5.3. The cross section measurement is calculated in bins of transverse momentum ($p_T$) and also in separate run periods. Section 5.1.4 will describe the run periods used to show the cross section as a function of time.

The $\Upsilon(nS)$ cross section is given in Equation 5.1 below,

$$\frac{d\sigma}{dp_T \, dy} \cdot B(\Upsilon(nS) \rightarrow \mu^+\mu^-) = \frac{N_{\Upsilon(nS)}}{A \times \epsilon \cdot \int \mathcal{L} \, dt \cdot \Delta p_T \cdot \Delta y}$$  \hspace{1cm} (5.1)

where $B(\Upsilon(nS) \rightarrow \mu^+\mu^-)$ is the branching fraction, $N_{\Upsilon(nS)}$ is the $\Upsilon(nS)$ yield in the fitted peak, $A \times \epsilon$ is the acceptance times the efficiency, $\int \mathcal{L} \, dt$ is the integrated luminosity of the data sample, $\Delta p_T$ is the width of transverse momentum ($p_T$) bin, and $\Delta y$ is the width of the rapidity ($y$) bin.

The acceptance is defined as the probability that a candidate will be in the geometric acceptance of the detector, while the efficiency is the probability that a candidate will be found and reconstructed if it is in the geometric acceptance of the detector. Monte Carlo, explained in Section 5.6, is used to model the detector and calculate the acceptance. However, the acceptance must be measured along with the efficiency in Monte Carlo, which is discussed further in Section 5.6.1. As a result of this, the acceptance times the efficiency becomes more complicated. The full acceptance times the efficiency ($A \times \epsilon$) is given by

$$A \times \epsilon = (A \cdot \epsilon)_{MC} \cdot \frac{\epsilon_{data} \cdot \epsilon_{vertex}}{\epsilon_{MC}}$$  \hspace{1cm} (5.2)
where \((A \cdot \epsilon)_{MC}\) is the acceptance times the efficiency measured in Monte Carlo, \(\epsilon_{data}\) is the reconstruction efficiency in data, \(\epsilon_{MC}\) is the reconstruction efficiency in Monte Carlo, and \(\epsilon_{vertex}\) is the efficiency of the vertex cut. The Monte Carlo acceptance times efficiency is explained in Section 5.6.1, while the Monte Carlo efficiency is discussed in Section 5.6.2. The data efficiency and vertex efficiency are described in Section 5.7.

5.1 \(\Upsilon\) Data Sample

The data sample used for this analysis is the dimuon dataset (jbmm). The dimuon dataset requires two muons and a dimuon candidate mass greater than 5 Gev/c\(^2\). The exact requirements for the \(\Upsilon\) triggers selected for use in this analysis from the dimuon data sample are listed in the following section.

5.1.1 \(\Upsilon\) Triggers

Two trigger paths are used for this analysis, UPSILON_CMUP_C MU and UPSILON_CMUP_CMX. Basically, the first trigger path, UPSILON_CMUP_C MU, requires two central muons, while the second trigger path, UPSILON_CMUP_CMX, requires one central muon and one forward muon. The Level 1, Level 2, and Level 3 requirements for each of these triggers is listed below:

**UPSILON_CMUP_C MU (later referred to as CMUP-C MU)**

- L1.TWO.CMU1.5.LUMI280
  - 2 muon stubs in CMU with \(p_T > 1.5\) GeV/c
  - 2 XFT tracks with \(p_T > 1.52\) GeV/c
- L2.CMUP1.5.PT3.CMU1.5.PT1.5DPS
  - 1 CMUP muon with \(p_T > 3.04\) GeV/c
  - 1 CMU muon with \(p_T > 1.52\) GeV/c
- L3.UPSILON_CMUP3.CMU1.5
- 1 CMUP muon with $p_T > 3.0\,\text{GeV/c}$, $\Delta x(\text{CMU}) < 30\,\text{cm}$, $\Delta x(\text{CMP}) < 40\,\text{cm}$
- 1 CMU muon with $p_T > 1.5\,\text{GeV/c}$, $\Delta x(\text{CMU}) < 30\,\text{cm}$
- $8.0\,\text{GeV/c}^2 < m(\mu^+\mu^-) < 12.0\,\text{GeV/c}^2$

**UPSILON CMUP CMX (later referred to as CMUP-CMX)**

- **L1.TWO.CMX1.5.LUMI280**
  - 1 muon stubs in CMU with $p_T > 1.5\,\text{GeV/c}$, 1 muon stubs in CMX with $p_T > 1.5\,\text{GeV/c}$
  - 1 XFT tracks with $p_T > 1.52\,\text{GeV/c}$, 1 XFT tracks with $p_T > 2.04\,\text{GeV/c}$, signal in CSX
- **L2.CMUP1.5_PT3_CSX1.5_PT2_CSX_DPS**
  - 1 CMUP muon with $p_T > 3.04\,\text{GeV/c}$
  - 1 CMX muon with $p_T > 2.04\,\text{GeV/c}$
- **L3.UPSILON.CMUP3.CMX2**
  - 1 CMUP muon with $p_T > 3.0\,\text{GeV/c}$, $\Delta x(\text{CMU}) < 30\,\text{cm}$, $\Delta x(\text{CMP}) < 40\,\text{cm}$
  - 1 CMX muon with $p_T > 2.0\,\text{GeV/c}$, $\Delta x(\text{CMX}) < 50\,\text{cm}$
  - $8.0\,\text{GeV/c}^2 < m(\mu^+\mu^-) < 12.0\,\text{GeV/c}^2$

### 5.1.2 Pre-Selection Cuts

Loose pre-selection cuts are used to identify $\Upsilon(nS) \to \mu^+\mu^-$ candidates. The event must have fired one of the $\Upsilon$ triggers (CMUP-CMU or CMUP-CMX), and then the following pre-selection cuts are applied:

- $\Upsilon$ mass: $8 < m(\mu^+\mu^-) < 12\,\text{GeV/c}^2$
- Muons have opposite charge: $q(\mu_1) \cdot q(\mu_2) < 0$
- $|z_0(\mu_1)| < 60.0\,\text{cm}$ and $|z_0(\mu_2)| < 60.0\,\text{cm}$
- $|z_0(\mu_1) - z_0(\mu_2)| < 5.0\,\text{cm}$
- $|\Delta \phi_0| > 2.25^\circ$ between $\phi_0(\mu_1)$ and $\phi_0(\mu_2)$
\[ |p_T(\mu_1) - p_T(\mu_2)| < (p_T(\Upsilon) - 0.1 \text{ GeV/c}) \]

- \( N(\text{hits on COT axial super layers}) \geq 25 \)
- \( N(\text{hits on COT stereo super layers}) \geq 25 \)
- must pass vertex fit made with \( \mu_1 \) track and \( \mu_2 \) track

### 5.1.3 Selection Cuts

Selection cuts are used to reduce background and verify \( \Upsilon \) candidate. Candidates must have already passed the pre-selection cuts, and the event must have fired the trigger cuts. The selection cuts verify the trigger cuts by cutting tighter than the Level 3 trigger cuts listed previously. The selection cuts are listed separately for both CMUP-CMU and CMUP-CMX:

#### CMUP-CMU

- CMUP muon requirements
  - fiducial in CMU and CMP detectors
  - CMU Level 1 trigger: checks east or west CMU low \( p_T \) bit for wedge and tower
  - CMUP Level 1 trigger: checks CMU Level 1 trigger, then checks CMUP4 single muon trigger
  - CMU Level 2 trigger: checks CMU bits for range of CMU cells from XFT using lookup tables
  - CMUP Level 2 trigger: checks CMU Level 2 trigger, then checks CMP bits for range of CMP cells from XFT using lookup tables (CMUP muon matched to XFT track)
  - \( p_T > 4.05 \text{ GeV} \)
  - \( \Delta x(\text{CMU}) < 15 \text{ cm}, \Delta x(\text{CMP}) < 40 \text{ cm} \)
- CMU muon requirements
  - fiducial in CMU detector
- CMU Level 1 trigger: checks east or west CMU low $p_T$ bit for wedge and tower
- CMU Level 2 trigger: checks CMU bits for range of CMU cells from XFT using lookup tables (CMU muon matched to XFT track)
- has CMU hits and reconstructed as CDFMuon object
- $p_T > 3.05$ GeV
- $\Delta x({\text{CMU}}) < 15$ cm
  - must pass pre-selection cuts
  - must be in acceptance definition (See Section 5.2 and Section 5.5)

CMUP-CMX

- CMUP muon requirements
  - fiducial in CMU and CMP detectors
  - CMU Level 1 trigger: checks east or west CMU low $p_T$ bit for wedge and tower
  - CMUP Level 1 trigger: checks CMU Level 1 trigger, then checks CMUP4 single muon trigger
  - CMU Level 2 trigger: checks CMU bits for range of CMU cells from XFT using lookup tables
  - CMUP Level 2 trigger: checks CMU Level 2 trigger, then checks CMP bits for range of CMP cells from XFT using lookup tables (CMUP muon matched to XFT track)
  - $p_T > 4.05$ GeV
  - $\Delta x({\text{CMU}}) < 15$ cm, $\Delta x({\text{CMP}}) < 40$ cm

- CMX muon requirements
  - fiducial in CMX detector
  - CMX Level 1 trigger: checks east or west CMX low $p_T$ bit for wedge and tower
- CMX Level 2 trigger: checks CMX bits for range of CMX cells from XFT using lookup tables (CMX muon matched to XFT track)
- has CMX hits and reconstructed as CDFMuon object
- $p_T > 3.05$ GeV
- $\Delta x$(CMX) < 50 cm
  - must pass pre-selection cuts
  - must be in acceptance definition (See Section 5.2 and Section 5.5)

5.1.4 Luminosity

The data sample is divided up into 28 run periods, and each run period has an integrated luminosity of approximately 250 pb$^{-1}$. Table 5.1 lists the run periods, along with the corresponding CDF data taking period, range of runs, and integrated luminosity for both of the trigger paths used in this analysis, CMUP-CMU and CMUP-CMX.

5.2 Detector Acceptance

The acceptance depends on the geometric design of the detector. To successfully model the detector, any changes to the detector design must be included in the acceptance definition. Some regions of the detector are removed for all runs while other regions are only removed for a range of runs. The following regions of the detector are excluded from the acceptance:

CMU
- 1/3 of Wedge 8 West
- East end of Wedge 6 East with $z > 200$ cm

CMX
- Skip Wedge 15 and Wedge 20
Table 5.1
Run Periods and Luminosity. Run Periods with the integrated luminosity for both CMUP-CMU and CMUP-CMX listed for each run period

<table>
<thead>
<tr>
<th>Run Period</th>
<th>CDF Period</th>
<th>Run Range</th>
<th>CMUP-CMU (pb$^{-1}$)</th>
<th>CMUP-CMX (pb$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0,1,2</td>
<td>181013-196441</td>
<td>254.328</td>
<td>254.328</td>
</tr>
<tr>
<td>1</td>
<td>2,3,4</td>
<td>196471-201542</td>
<td>156.534</td>
<td>156.534</td>
</tr>
<tr>
<td>2</td>
<td>4,5,6,7,8</td>
<td>201543-219945</td>
<td>343.942</td>
<td>343.860</td>
</tr>
<tr>
<td>3</td>
<td>8,9,10</td>
<td>219946-228683</td>
<td>252.634</td>
<td>252.623</td>
</tr>
<tr>
<td>4</td>
<td>10,11</td>
<td>228691-234481</td>
<td>252.400</td>
<td>252.704</td>
</tr>
<tr>
<td>5</td>
<td>11,12</td>
<td>234572-240673</td>
<td>250.716</td>
<td>252.055</td>
</tr>
<tr>
<td>6</td>
<td>12,13</td>
<td>240788-245102</td>
<td>251.285</td>
<td>251.623</td>
</tr>
<tr>
<td>7</td>
<td>13,14,15,16</td>
<td>245210-257064</td>
<td>252.072</td>
<td>252.888</td>
</tr>
<tr>
<td>8</td>
<td>16,17,18</td>
<td>257200-261223</td>
<td>253.459</td>
<td>256.447</td>
</tr>
<tr>
<td>9</td>
<td>18</td>
<td>261225-263963</td>
<td>251.570</td>
<td>253.813</td>
</tr>
<tr>
<td>10</td>
<td>18,19,20</td>
<td>263979-266929</td>
<td>250.457</td>
<td>252.815</td>
</tr>
<tr>
<td>11</td>
<td>20,21</td>
<td>266964-270026</td>
<td>251.417</td>
<td>254.136</td>
</tr>
<tr>
<td>12</td>
<td>21,22</td>
<td>270028-271456</td>
<td>253.367</td>
<td>260.471</td>
</tr>
<tr>
<td>13</td>
<td>22,23</td>
<td>271482-273941</td>
<td>251.341</td>
<td>255.471</td>
</tr>
<tr>
<td>14</td>
<td>23,24,25</td>
<td>273943-276320</td>
<td>255.421</td>
<td>259.285</td>
</tr>
<tr>
<td>15</td>
<td>25,26</td>
<td>276395-284842</td>
<td>250.744</td>
<td>253.998</td>
</tr>
<tr>
<td>16</td>
<td>26,27</td>
<td>284843-286538</td>
<td>250.401</td>
<td>252.516</td>
</tr>
<tr>
<td>17</td>
<td>27,28</td>
<td>286625-288721</td>
<td>250.764</td>
<td>253.413</td>
</tr>
<tr>
<td>18</td>
<td>28,29</td>
<td>288745-290606</td>
<td>250.699</td>
<td>254.971</td>
</tr>
<tr>
<td>19</td>
<td>29,30</td>
<td>290607-292455</td>
<td>254.260</td>
<td>260.071</td>
</tr>
<tr>
<td>20</td>
<td>30,31</td>
<td>292491-294509</td>
<td>251.508</td>
<td>254.093</td>
</tr>
<tr>
<td>21</td>
<td>31,32</td>
<td>294510-298219</td>
<td>252.626</td>
<td>254.098</td>
</tr>
<tr>
<td>22</td>
<td>32,33</td>
<td>298235-300427</td>
<td>250.498</td>
<td>254.537</td>
</tr>
<tr>
<td>23</td>
<td>33,34</td>
<td>300428-302685</td>
<td>251.818</td>
<td>254.553</td>
</tr>
<tr>
<td>24</td>
<td>34,35</td>
<td>302686-305885</td>
<td>250.244</td>
<td>252.731</td>
</tr>
<tr>
<td>25</td>
<td>35,36</td>
<td>305886-307879</td>
<td>254.888</td>
<td>257.959</td>
</tr>
<tr>
<td>26</td>
<td>36,37</td>
<td>307894-310335</td>
<td>250.050</td>
<td>252.883</td>
</tr>
<tr>
<td>27</td>
<td>37,38</td>
<td>310359-312510</td>
<td>204.106</td>
<td>205.782</td>
</tr>
</tbody>
</table>

Total Luminosity 7,003.55 7,070.66
- Hole for the solenoid cryostat (Side 1 (East) Wedge 5 and Wedge 6)
- Mini-skirts (Wedge 15 to Wedge 20), missing before Run 227704
- Keystone wedges, Side 0 (West) Wedge 5 and Wedge 6, missing before Run 233112
- Small hole on Side 0 (West) Wedge 14 between Run 190695 and Run 210009

5.3 Fit of $\Upsilon$ Yield

The mass is calculated for each $\Upsilon$ candidate by using conservation of energy and conservation of momentum along with the vertex fit of the two muon tracks. The yield for the $\Upsilon(1S)$ signal, the $\Upsilon(2S)$ signal, and the $\Upsilon(3S)$ signal are calculated by fitting the background with a quadratic function and each of the signals with a Gaussian. The signal functions are normalized and are given by

$$f_{\text{signal}}(x, \sigma) = \frac{N_{\Upsilon(nS)}}{w_{\text{bin}}} \sqrt{\frac{2}{\pi\sigma}} e^{-\frac{(x-m)^2}{\sigma^2}}$$

where $N_{\Upsilon(nS)}$ is the $\Upsilon(nS)$ yield, $m$ is the measured value of the $\Upsilon(nS)$ mass, and $w_{\text{bin}}$ is the histogram bin width.

The $\Upsilon$ fits are done separately for CMUP-CMU and CMUP-CMX, and also each fit is done in bins of transverse momentum for each run period. Figure 5.1 shows an example of the fit done for CMUP-CMU for Run Period 10 in the eight separate bins of transverse momentum. A fit of CMUP-CMX for Run Period 10 in eight separate bins of transverse momentum is shown in Figure 5.2.

5.4 Time Dependence in Cross Section

While the full cross section measurement will need the detector acceptance and reconstruction efficiency, an estimated “cross section” can be calculated to see how the calculated cross section changes with time. The estimated “cross section” ($\sigma_{\text{estimated}}$) can be calculated for each separate run period by summing over the transverse momentum bins, as shown in the following formula:

$$\sigma_{\text{estimated}}(\text{run period}) = \frac{\sum_{p_T} N_{\Upsilon(nS)}((\text{run period}, p_T))}{L(\text{run period})}$$
Figure 5.1. Υ Yield Fit for CMUP-CMU Run Period 3
(a) $0 \text{ GeV/c} < p_T(\Upsilon) < 2 \text{ GeV/c}$

(b) $2 \text{ GeV/c} < p_T(\Upsilon) < 4 \text{ GeV/c}$

(c) $4 \text{ GeV/c} < p_T(\Upsilon) < 6 \text{ GeV/c}$

(d) $6 \text{ GeV/c} < p_T(\Upsilon) < 8 \text{ GeV/c}$

(e) $8 \text{ GeV/c} < p_T(\Upsilon) < 12 \text{ GeV/c}$

(f) $12 \text{ GeV/c} < p_T(\Upsilon) < 17 \text{ GeV/c}$

(g) $17 \text{ GeV/c} < p_T(\Upsilon) < 23 \text{ GeV/c}$

(h) $23 \text{ GeV/c} < p_T(\Upsilon) < 40 \text{ GeV/c}$

Figure 5.2. \( \Upsilon \) Yield Fit for CMUP-CMX Run Period 3
Figure 5.3. Estimated $\Upsilon(1S)$ “Cross Section” as a function of Run Period for CMUP-CMU and CMUP-CMX. CMUP-CMU data (blue) and CMUP-CMX data (red) with error bars shown for $0 < p_T < 40$ GeV/c. Plots have same data, but bottom plot has horizontal line fits (black lines) and linear fits. CMUP-CMU horizontal fit average is $51.76 \pm 0.12$ with $\chi^2 = 1066.71$, while CMUP-CMX is $28.12 \pm 0.08$ with $\chi^2 = 823.46$. Also shown are linear fits of CMUP-CMU (cyan line) with $\chi^2 = 124.08$ and of CMUP-CMX (pink line) with $\chi^2 = 492.53$. 

where $N_{\Upsilon(nS)}$ is the fitted yield for a specified run period and $p_T$ bin and $L$ is the integrated luminosity for the run period listed in Table 5.1. Figure 5.3 shows the estimated “cross section” for $\Upsilon(1S)$ as a function of run period. The estimated “cross section” does show time dependence, and the bottom plot in Figure 5.3 shows horizontal and linear fits for both CMUP-CMU and CMUP-CMX. One possible source of the time dependence is aging of the detector and broken or dead wires in the muon system. Identifying dead wires and removing them from the detector acceptance is discussed in Section 5.5.

### 5.5 Dead Wires and Detector Acceptance

In order to reduce the time dependence observed in the estimated “cross section”, shown in Figure 5.3, dead wires in the detectors of the muon system need to be identified and removed from the detector acceptance. The procedure for identifying and removing dead wires requires using Monte Carlo to determine the expected number of hits for a wire for a period of time and then to compute the Poisson probability of having the actual numbers of hits in the wire in data for the same time period given the expected number of hits.

The data sample used to identify dead wires in the detector is the Muon+SVT dataset (jbmnu). The Muon+SVT dataset requires one muon and one displaced SVT track in each event, and the full trigger requirements are listed in Section 5.7.1. This dataset is used to reconstruct the $J/\psi$ signal from the $J/\psi \rightarrow \mu^+ \mu^-$ decay. The muons from the $J/\psi$ signal are used to determine the total number of counts for the detector, $N_{\text{data,total}(run)}$, for each run.

The Monte Carlo used to map the occupancy of a wire in the detector is discussed further in Section 5.6. The Monte Carlo simulates the detector, and the occupancy of wires in a detector can vary because of the different amount of material in front of various regions of the detector. The Monte Carlo is assumed to be the same for
all runs, and so the first run period is used to find the total number of counts in the
detector, $N_{MC,\text{total}}$, as well as the number of counts in a single wire ($i$), $N_{MC,i}$.

The ratio of the number of counts in a given detector wire to the total number
of counts in the entire detector is assumed to be the same in both data and Monte
Carlo. The ratio of number of counts in a single wire to the total number of counts
in Monte Carlo is then used as a scale factor to determine the number of expected
counts in data. The number of expected counts in data for a specified wire and run,
$N_{\text{expected},i}$, is given by

$$N_{\text{expected},i}(\text{run}) = \frac{N_{MC,i}}{N_{MC,\text{total}}} N_{\text{data, total}}(\text{run})$$

(5.5)

where $i$ is the specified wire, $N_{MC,i}$ is the number of counts in Monte Carlo in
wire ($i$), $N_{MC,\text{total}}$ is the total number of counts in the detector in Monte Carlo,
and $N_{\text{data, total}}(\text{run})$ is the total number of counts in the detector for a specified run in
data.

If the number of expected counts is less than a minimum value ($N_{\text{expected},i} < \mu_{\text{min}}$),
then runs are combined together to form a “run index” until the minimum number
of expected counts is reached. Thus, the number of expected counts for a specified
wire and run index is given by

$$N_{\text{expected},i}(\text{run index}) = \sum_{\text{run}}^n \frac{N_{MC,i}}{N_{MC,\text{total}}} N_{\text{data, total}}(\text{run})$$

(5.6)

where $n$ is the number of runs until $N_{\text{expected},i} \geq \mu_{\text{min}}$. For this analysis, a minimum
value of 100 counts, $\mu_{\text{min}} = 100$, is required for the number of expected counts in a
wire.

Using muons from the $J/\psi$ signal in the Muon+SVT data sample, the actual
number of hits, $N_{\text{actual},i}(\text{run index})$, is also counted for a specified wire ($i$) in the
determined run index for the same set of runs. The cumulative Poisson probability is
then computed to determine the probability of finding the actual number of counts
for a wire, $N_{\text{actual},i}$, given the expected number of counts in that wire, $N_{\text{expected},i}$. The
Poisson probability distribution is given by

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

(5.7)
where \( \mu \) is the mean or expected count in this case and \( x \) is the actual count [89]. The cumulative Poisson probability is given by

\[
P(x) = e^{-\mu} \sum_{i=0}^{x} \frac{\mu^i}{i!}
\]  
where \( \mu \) is the expected count and \( x \) is the actual count [89].

A wire in the detector is considered dead if the calculated cumulative Poisson probability is less than a given probability. For this analysis, a probability of one-thousandth of a percent, \( P_{\text{dead}} = 0.001\% \), is used as the cut-off probability below which a wire is considered dead. Taking the negative natural log of the probability, the cut-off value \( (C_{\text{dead}}) \) is given as \( C_{\text{dead}} = -\ln(P) \approx 11.5129 \). Figure 5.4 shows the negative natural log of the Poisson probability for the CMP detector, while the same is shown in Figure 5.5 for the east side of the CMX detector and in Figure 5.6 for the west side of the CMX detector.

Using the cut-off probability to determine if a wire is dead, a list is made with the wire numbers and the range of runs in the run index when the wire is considered dead. A dead wire list is made for the CMP detector and CMX detector (east and west), and these lists are then included as part of the detector acceptance. The wire from a reconstructed muon found in a given run must not be listed as a dead wire in order for that muon to be included in the acceptance and used to reconstruct an \( \Upsilon \) candidate. A plot of the CMP dead wires is shown in Figure 5.7, the dead wires in CMX East are shown in Figure 5.8, and dead wires for CMX West are shown in Figure 5.9.

5.6 Monte Carlo

Monte Carlo is used to model the detector and simulate particles traveling through the detector, but the triggers are not included. Monte Carlo events are generated in several separate files for a certain range of transverse momentum \( (p_T) \), but only a subset of the generated range is selected for use. The events are generated with a transverse momentum \( (p_T) \) spectrum that is measured in data. Using a selected
Figure 5.4. CMP Dead Wire Probability shown for all wires. Dead wires with \(- \ln(P) > C_{\text{dead}}\) (shaded blue region), and working wires (white region).

Figure 5.5. CMX East Dead Wire Probability shown for all wires. Dead wires with \(- \ln(P) > C_{\text{dead}}\) (shaded blue region), and working wires (white region).
Figure 5.6. CMX West Dead Wire Probability shown for all wires. Dead wires with $-\ln(P) > C_{\text{dead}}$ (shaded blue region), and working wires (white region).

Figure 5.7. CMP Dead Wires shown in plot of run index vs wire numbers. Shaded regions represent dead wires (wires with $P \leq P_{\text{dead}}$).
Figure 5.8. CMX East Dead Wires shown in plot of run index vs wire numbers. Shaded regions represent dead wires (wires with $P \leq P_{\text{dead}}$).

Figure 5.9. CMX West Dead Wires shown in plot of run index vs wire numbers. Shaded regions represent dead wires (wires with $P \leq P_{\text{dead}}$).
Table 5.2
Generated and Selected $p_T$ Range for Monte Carlo Files

<table>
<thead>
<tr>
<th>Monte Carlo Files</th>
<th>Generated $p_T$ Range (GeV/c)</th>
<th>Selected $p_T$ Range (GeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>upt00i</td>
<td>0.0 - 2.5</td>
<td>0.0 - 2.0</td>
</tr>
<tr>
<td>upt15i</td>
<td>1.5 - 4.5</td>
<td>2.0 - 4.0</td>
</tr>
<tr>
<td>upt16i</td>
<td>3.5 - 6.5</td>
<td>4.0 - 6.0</td>
</tr>
<tr>
<td>upt19i</td>
<td>5.5 - 8.5</td>
<td>6.0 - 8.0</td>
</tr>
<tr>
<td>upt20i</td>
<td>7.5 - 12.5</td>
<td>8.0 - 12.0</td>
</tr>
<tr>
<td>upt21i</td>
<td>11.5 - 17.5</td>
<td>12.0 - 17.0</td>
</tr>
<tr>
<td>upt22i</td>
<td>16.5 - 23.5</td>
<td>17.0 - 23.0</td>
</tr>
<tr>
<td>upt23i</td>
<td>22.5 - 40.5</td>
<td>23.0 - 40.0</td>
</tr>
</tbody>
</table>

Subset of the generated $p_T$ range allows for any issues for generating the matching $p_T$ in data at the extremes of the generated range. Table 5.2 shows the generated $p_T$ range and selected $p_T$ range for Monte Carlo files.

The Monte Carlo must be modeled after the data and must distributions of parameters of Monte Carlo and data must match since the Monte Carlo is used to simulate the reconstruction of particles in the detector. As a result, the $\Upsilon$ $p_T$ spectrum in the Monte Carlo is generated to match the measured $\Upsilon$ $p_T$ spectrum in data. The $\Upsilon$ $p_T$ for Monte Carlo and data for each of the $p_T$ bins is shown in Figure 5.10 for CMUP-CMU and in Figure 5.20 for CMUP-CMX.

The Monte Carlo should also match the angular distribution of the $\Upsilon$ candidates in data, because the angular distribution affects the cross section measurement. The Monte Carlo is generated as unpolarized, but the Monte Carlo can be weighted with the measured angular distribution to match the data. The procedure for the measurement of the angular distribution is discussed in Chapter 6. This analysis uses the unpolarized Monte Carlo for the $\Upsilon(2S)$ and $\Upsilon(3S)$ states and the measured polarization parameters to re-weight the Monte Carlo for the $\Upsilon(1S)$ state. The systematic error due to the polarization in Monte Carlo is described in Section 5.8.
Figure 5.10. Υ \( p_T \) for CMUP-CMU in \( p_T(Υ) \) bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) 0 < \( p_T < 2 \), (b) 2 < \( p_T < 4 \), (c) 4 < \( p_T < 6 \), (d) 6 < \( p_T < 8 \). Bottom row (e) 8 < \( p_T < 12 \), (f) 12 < \( p_T < 17 \), (g) 17 < \( p_T < 23 \), (h) 23 < \( p_T < 40 \).)

Figure 5.11. CMUP muon \( p_T \) for CMUP-CMU in \( p_T(Υ) \) bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) 0 < \( p_T < 2 \), (b) 2 < \( p_T < 4 \), (c) 4 < \( p_T < 6 \), (d) 6 < \( p_T < 8 \). Bottom row (e) 8 < \( p_T < 12 \), (f) 12 < \( p_T < 17 \), (g) 17 < \( p_T < 23 \), (h) 23 < \( p_T < 40 \).)
Figure 5.12. CMUP muon $\Delta x$ for CMUP-CMU in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)

Figure 5.13. CMUP muon $\Delta x$ for CMUP-CMU in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)
Figure 5.14. CMUP muon $\phi_0$ for CMUP-CMU in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)

Figure 5.15. CMUP muon $\eta$ for CMUP-CMU in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)
Figure 5.16. CMU muon $p_T$ for CMUP-CMU in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)

Figure 5.17. CMU muon $\Delta x$ for CMUP-CMU in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)
Figure 5.18. CMU muon \( \phi_0 \) for CMUP-CMU in \( p_T(\Upsilon) \) bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) \( 0 < p_T < 2 \), (b) \( 2 < p_T < 4 \), (c) \( 4 < p_T < 6 \), (d) \( 6 < p_T < 8 \). Bottom row (e) \( 8 < p_T < 12 \), (f) \( 12 < p_T < 17 \), (g) \( 17 < p_T < 23 \), (h) \( 23 < p_T < 40 \).)

Figure 5.19. CMU muon \( \eta \) for CMUP-CMU in \( p_T(\Upsilon) \) bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) \( 0 < p_T < 2 \), (b) \( 2 < p_T < 4 \), (c) \( 4 < p_T < 6 \), (d) \( 6 < p_T < 8 \). Bottom row (e) \( 8 < p_T < 12 \), (f) \( 12 < p_T < 17 \), (g) \( 17 < p_T < 23 \), (h) \( 23 < p_T < 40 \).)
Figure 5.20. $\Upsilon$ $p_T$ for CMUP-CMX in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)

Figure 5.21. CMUP muon $p_T$ for CMUP-CMX in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)
Figure 5.22. CMUP muon $\Delta x$ for CMUP-CMX in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)

Figure 5.23. CMUP muon $\Delta x$ for CMUP-CMX in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)
Figure 5.24. CMUP muon $\phi_0$ for CMUP-CMX in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)

Figure 5.25. CMUP muon $\eta$ for CMUP-CMX in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)
Figure 5.26. CMX muon $p_T$ for CMUP-CMX in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)

Figure 5.27. CMX muon $\Delta x$ for CMUP-CMX in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)
Figure 5.28. CMX muon $\phi_0$ for CMUP-CMX in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)

Figure 5.29. CMX muon $\phi_0$ for CMUP-CMX in $p_T(\Upsilon)$ bins. Monte Carlo (red) and data (blue) after background subtraction. (Top row (a) $0 < p_T < 2$, (b) $2 < p_T < 4$, (c) $4 < p_T < 6$, (d) $6 < p_T < 8$. Bottom row (e) $8 < p_T < 12$, (f) $12 < p_T < 17$, (g) $17 < p_T < 23$, (h) $23 < p_T < 40$.)
Furthermore, distributions of parameters for muons in Monte Carlo should also be similar to the same distributions in data. For CMUP-CMU candidates, CMUP muon distributions shown are $p_T$ in Figure 5.11, $\Delta x$(CMU) in Figure 5.12, $\Delta x$(CMP) in Figure 5.13, $\phi$ in Figure 5.14, and $\eta$ in Figure 5.15, while CMU muon Figure 5.16, $\Delta x$(CMU) in Figure 5.17, $\phi$ in Figure 5.18, and $\eta$ in Figure 5.19. For CMUP-CMX candidates, CMUP muon distributions shown are $p_T$ in Figure 5.21, $\Delta x$(CMU) in Figure 5.22, $\Delta x$(CMP) in Figure 5.23, $\phi$ in Figure 5.24, and $\eta$ in Figure 5.25, while CMX muon Figure 5.26, $\Delta x$(CMX) in Figure 5.27, east $\phi$ in Figure 5.28, west $\phi$ in Figure 5.29, and $\eta$ in Figure 5.30.

The previously mentioned figures that contain plots for the $\Upsilon p_T$ and muon parameters in data are each made by using background subtraction. First, the signal regions are fit to determine the amount of signal and background. Next, the sideband distribution is normalized to the amount of background in the signal region. Then,
the normalized background distribution is subtracted off of the signal distribution to obtain the plots shown in the figures.

5.6.1 Acceptance and Efficiency Calculation in Monte Carlo

The detector acceptance \((A)\) is measured with a Monte Carlo sample in intervals of transverse momentum and rapidity. The acceptance is the fraction of Υ with two muons that are fiducial in the detector to the total number of generated Υ. The reconstruction efficiency \((\epsilon)\) is the ratio of reconstructed Υ candidates given that the Υ with two muons are fiducial in the detector. In order to remove dead wires from the detector acceptance, the acceptance times efficiency \(((A \cdot \epsilon)_{\text{MC}})\) must be measured in Monte Carlo instead of just measuring the acceptance in Monte Carlo. This is a result of not being able to accurately project the final wire number in the muon detectors where an inner track would hit. The inner tracks can be projected as fiducial in a muon detector, but predicting the correct wire number were the muon would hit is more challenging. As a result, the acceptance times efficiency is measured in Monte Carlo, and this requires a measurement of the efficiency in Monte Carlo. The acceptance times the efficiency has to be divided by the efficiency to obtain the acceptance.

Measured in bins of transverse momentum \((p_T)\) for each run period, the acceptance times efficiency in Monte Carlo \(((A \cdot \epsilon)_{\text{MC}})\) is given by

\[
(A \cdot \epsilon)_{\text{MC}} = \frac{N_{\text{MC}(\text{reco})}}{N_{\text{MC}(\text{gen})}}
\]

(5.9)

where \(N_{\text{MC}(\text{reco})}\) is the number of reconstructed Υ candidates and \(N_{\text{MC}(\text{gen})}\) is the number of generated Υ. To be considered a generated Υ, the following requirements must be met:

- rapidity: \(|y(\Upsilon)| < 0.6\)
- Υ vertex: \(|z(\Upsilon)| < 60.0\)

The acceptance times efficiency in Monte Carlo is calculated separately for the CMUP-CMU trigger path and CMUP-CMX trigger path.
The requirements for a reconstructed $\Upsilon$ candidate in Monte Carlo for the CMUP-CMU trigger path are the following:

- one CMUP muon with $p_T > 4$ GeV (track reconstructed as CdfMuon with hits in CMU and CMP)
  - fiducial in CMU and CMP
  - pass CMU and CMP acceptance
  - has CMU and CMP hits
  - $\Delta x$(CMU)$< 15$ cm
  - $\Delta x$(CMP)$< 40$ cm
- one CMX muon $p_T > 3$ GeV (track reconstructed as CdfMuon with hits in CMX)
  - fiducial in CMX
  - pass CMX acceptance
  - has CMX hits
  - $\Delta x$(CMX)$< 15$ cm

- muons both in valid regions of detector (in the detector acceptance definition)
- muons both from working wire numbers (not dead wires)
- also passes requirements for generated $\Upsilon$

The CMUP-CMX trigger path requirements for a reconstructed $\Upsilon$ candidate are given by:

- one CMUP muon with $p_T > 4$ GeV (track reconstructed as CdfMuon with hits in CMU and CMP)
  - fiducial in CMU and CMP
  - pass CMU and CMP acceptance
  - has CMU and CMP hits
  - $\Delta x$(CMU)$< 15$ cm
  - $\Delta x$(CMP)$< 40$ cm
- one CMX muon $p_T > 3$ GeV (track reconstructed as CdfMuon with hits in CMX)
– fiducial in CMX
– pass CMX acceptance
– has CMX hits
– $\Delta x$(CMX)< 50 cm

• muons both in valid regions of detector (in the detector acceptance definition)
• muons both from working wire numbers (not dead wires)
• also passes requirements for generated $\Upsilon$

Figure 5.31 shows the CMUP-CMU acceptance times efficiency in Monte Carlo as a function of run period for two $p_T$ bins, while the CMUP-CMX acceptance times efficiency is shown in Figure 5.32.

5.6.2 Efficiency Measurement in Monte Carlo

The reconstruction efficiency in Monte Carlo ($\epsilon_{MC}$), depends on the trigger path, CMUP-CMU or CMUP-CMX, and is the product of the CMUP efficiency times either the CMU efficiency or CMX efficiency. The Monte Carlo CMUP-CMU efficiency is given by

$$\epsilon_{MC}(\text{CMUP-CMU}) = \epsilon_{MC,CMUP} \cdot \epsilon_{MC,CMU}$$  \hspace{1cm} (5.10)

and the Monte Carlo CMUP-CMX efficiency is calculated by

$$\epsilon_{MC}(\text{CMUP-CMX}) = \epsilon_{MC,CMUP} \cdot \epsilon_{MC,CMX}$$  \hspace{1cm} (5.11)

The calculations for Monte Carlo CMUP efficiency ($\epsilon_{MC,CMUP}$), Monte Carlo CMU efficiency ($\epsilon_{MC,CMU}$), and Monte Carlo CMX efficiency ($\epsilon_{MC,CMX}$) all use the same basic formula and can be summarized by $\epsilon_{MC,\beta}$, where $\beta$ is either CMUP, CMU, or CMX depending on the detector. The Monte Carlo reconstruction efficiency for a specified muon detector ($\beta$) can be found by

$$\epsilon_{MC,\beta} = \frac{N_{MC}(\text{reco in } \beta)}{N_{MC}(\text{fid in } \beta)}$$  \hspace{1cm} (5.12)

where $N_{MC}(\text{reco in } \beta)$ is the number of muons reconstructed in muon detector $\beta$ and $N_{MC}(\text{fid in } \beta)$ is the number of muons that are fiducial in the same muon detector.
Figure 5.31. Monte Carlo Acceptance times Efficiency for CMUP-CMU as a function of Run Period

(a) 0 GeV/c < \( p_T(\Upsilon) < 2 \) GeV/c

(b) 2 GeV/c < \( p_T(\Upsilon) < 4 \) GeV/c

(c) 4 GeV/c < \( p_T(\Upsilon) < 6 \) GeV/c

(d) 6 GeV/c < \( p_T(\Upsilon) < 8 \) GeV/c

(e) 8 GeV/c < \( p_T(\Upsilon) < 12 \) GeV/c

(f) 12 GeV/c < \( p_T(\Upsilon) < 17 \) GeV/c

(g) 17 GeV/c < \( p_T(\Upsilon) < 23 \) GeV/c

(h) 23 GeV/c < \( p_T(\Upsilon) < 40 \) GeV/c
Figure 5.32. Monte Carlo Acceptance times Efficiency for CMUP-CMX as a function of Run Period
The number of muons that are fiducial in muon detector $\beta$, $N_{MC}(\text{fid in } \beta)$, counts the number of muons that have tracks that are:

- $p_T > 4 \text{ GeV/c}$ for CMUP or $p_T > 3 \text{ GeV/c}$ for CMU or CMX.
- fiducial in muon detector $\beta$

The extrapolation of the track to the muon detector is done using the Muon Fiducial Tool in the Muon software package, which extends the track to the radius of the muon detector and checks if the muon would hit inside the fiducial volume of the muon detector. For a muon to be counted as a reconstructed muon in muon detector $\beta$, $N_{MC}(\text{reco in } \beta)$, the following is required:

- has hits in muon detector $\beta$ (track reconstructed as CdfMuon)
- passes specific requirements for muon detector $\beta$
  - CMUP: $p_T(\mu) > 4 \text{ GeV/c}$, $\Delta x(\text{CMU}) < 15 \text{ cm}$ and $\Delta x(\text{CMP}) < 40 \text{ cm}$
  - CMU: $p_T(\mu) > 3 \text{ GeV/c}$, $\Delta x(\text{CMU}) < 15 \text{ cm}$
  - CMX: $p_T(\mu) > 3 \text{ GeV/c}$, $\Delta x(\text{CMX}) < 50 \text{ cm}$
- muons both in valid regions of detector (in the detector acceptance definition)
- muons both from working wire numbers (not dead wires)
- also passes requirements for fiducial muon listed previously

The requirements for a reconstructed muon include a check that the muons are in the detector acceptance definition and also a check if the wire is dead or not. The excluded regions of the detector are listed in Section 5.2, and an explanation of dead wires is in Section 5.5.

The muon reconstruction efficiency is calculated for each run period and does not depend on the transverse momentum of the $\Upsilon$. The muon reconstruction efficiency for muons with $p_T$ higher than 3 GeV/c is assumed to be independent of the muon’s transverse momentum. While the muon efficiency does depend on the muon’s transverse momentum at low $p_T$, muons used in this analysis are above the “turn on” point and so should not depend on the $p_T$ of the muon. The muon reconstruction efficiency
Figure 5.33. Monte Carlo CMUP Efficiency as a function of Run Period

Figure 5.34. Monte Carlo CMU Efficiency as a function of Run Period
Figure 5.35. Monte Carlo CMX Efficiency as a function of Run Period

as a function of run period is shown in Figure 5.33 for CMUP, in Figure 5.34 for CMU, and in Figure 5.35 for CMX.

5.7 Efficiency Measurement in Data

Equation 5.2 lists two efficiencies that are measured in data: the reconstruction efficiency ($\epsilon_{\text{data}}$) and the vertex efficiency ($\epsilon_{\text{vertex}}$). The reconstruction efficiency depends on the trigger path, CMUP-CMU or CMUP-CMX. The CMUP-CMU efficiency is can be found by

$$\epsilon_{\text{data}}(\text{CMUP-CMU}) = \epsilon_{\text{data,CMUP}} \cdot \epsilon_{\text{XFT}} \cdot \epsilon_{\text{data,CMU}} \cdot \epsilon_{\text{XFT}}$$  \hspace{1cm} (5.13)

while the CMUP-CMX efficiency is given by

$$\epsilon_{\text{data}}(\text{CMUP-CMX}) = \epsilon_{\text{data,CMUP}} \cdot \epsilon_{\text{XFT}} \cdot \epsilon_{\text{data,CMX}} \cdot \epsilon_{\text{XFT}}$$  \hspace{1cm} (5.14)

where $\epsilon_{\text{data,CMUP}}$ is the CMUP efficiency, $\epsilon_{\text{data,CMU}}$ is the CMU efficiency, $\epsilon_{\text{data,CMX}}$ is the CMX efficiency, and $\epsilon_{\text{XFT}}$ is the XFT efficiency. The reconstruction efficiency is
described in Section 5.7.2 and the XFT efficiency ($\epsilon_{\text{XFT}}$) in Section 5.7.3, while the vertex efficiency ($\epsilon_{\text{vertex}}$) will be discussed in Section 5.7.4.

5.7.1 Muon+SVT Data Sample

The data sample used to measure the efficiency is the Muon+SVT dataset (jbmu). The Muon+SVT dataset includes the B_SEMI_CMUP4_TRACK2_D120 trigger, which is used to measure the efficiency. The B_SEMI_CMUP4_TRACK2_D120 trigger requires one muon and one displaced SVT track, and the full trigger requirements are listed below:

B_SEMI_CMUP4_TRACK2_D120

- **L1_CMUP6_PT4_NCLC64**
  - one muon stub in CMU with $p_T > 6.0$ GeV/c
  - one XFT track with $p_T > 4.09$ GeV/c
- **L2_CMUP6_PT4_D0_&&_TRK2_D120_DPHI90_DPS**
  - one XFT track with $p_T > 4.09$ GeV/c and one CMUP muon
  - one SVT track with $p_T > 2.0$ GeV/c, $\chi^2 < 15$, and $\Delta\phi < 90^\circ$
- **L3_B_SEMI_CMUP4_TRACK2_D120**
  - one CMUP muon with $p_T > 4.0$ GeV/c, $\Delta x($CMU$) < 15$ cm, $\Delta x($CMP$) < 40$ cm
  - displaced track with $d_0 > 0.1$ m and with $p_T > 2.0$ GeV/c

5.7.2 Reconstruction and Trigger Efficiency

The reconstruction efficiency includes the trigger efficiency and is measured in the Muon+SVT sample by first reconstructing $J/\psi$ candidates by looping over all tracks. For each $J/\psi$ candidate, the trigger muon is identified as the biased muon and then the other muon in the event is unbiased. Basically, the measurement is done using tag and probe, where the tag is the trigger muon and the probe is the other muon.
that forms the $J/\psi$ candidate. The unbiased muon can then be checked to see if it was identified and reconstructed as a muon or not, and from this check, the efficiency can be calculated.

The requirements for $J/\psi$ candidates are listed below:

- muons have opposite charge: $q(\mu_1) \cdot q(\mu_2) < 0$
- $|z_0(\mu_1)| < 60.0$ cm and $|z_0(\mu_2)| < 60.0$ cm
- $|z_0(\mu_1) - z_0(\mu_2)| < 5.0$ cm
- $|\Delta \phi_0| > 2.25^\circ$ between $\phi_0(\mu_1)$ and $\phi_0(\mu_2)$
- both tracks are matched to XFT and XFT fiducial

One of the muons in the $J/\psi$ candidate must pass the requirements for the biased (tagged) trigger muon, given in the following:

- CMUP muon with hits in CMU and CMP
- $p_T(\mu) > 4.05$ GeV/$c$
- passed CMUP Level 1 trigger: checks CMU Level 1 trigger, then checks CMUP4 single muon trigger
- CMU Level 1 trigger: checks east or west CMU low $p_T$ bit for wedge and tower
- fiducial in CMU and CMP

If the trigger muon is identified in the $J/\psi$ candidate, then the other unbiased muon is analyzed. The unbiased (probe) muon also must pass a set of requirements. If either of these set of requirements is not met, then the candidate is skipped. The requirements for the unbiased muon depend on the muon detector $\beta$ and are given by:

- fiducial in muon detector $\beta$
- have $p_T(\mu) > 3.05$ GeV for CMU and CMX or $p_T(\mu) > 4.05$ GeV for CMUP

The unbiased (probe) muon is then checked to see whether or not it passes the following cuts, depending on the muon detector $\beta$. The efficiency could be calculated separately for each of the Level 1, Level 2, and Level 3 triggers, but to improve statistics, the reconstruction efficiency is calculated using requirements for the Level 1,
Level 2, and Level 3 triggers together. The required cuts for muon detector $\beta$ are listed in the following:

- has hits in muon detector $\beta$ (track reconstructed as CdfMuon)
- passes Level 1 trigger for muon detector $\beta$
- passes Level 2 trigger for muon detector $\beta$
- passes specific requirements for muon detector $\beta$
  - CMUP: $p_T(\mu) > 4.05$ GeV/c, $\Delta x$(CMU)$ < 15$ cm and $\Delta x$(CMP)$ < 40$ cm
  - CMU: $p_T(\mu) > 3.05$ GeV/c, $\Delta x$(CMU)$ < 15$ cm
  - CMX: $p_T(\mu) > 3.05$ GeV/c, $\Delta x$(CMX)$ < 50$ cm
- pass $\beta$ detector acceptance - must be in valid regions of the detector
- pass $\beta$ detector working wire number (not a dead wire)

The muon must be in the detector acceptance definition, listed in Section 5.2, and also the muon is not checked against the list of dead wires, explained in Section 5.5.

If the unbiased (probe) muon passes the previous requirements for muon detector $\beta$, then the $J/\psi$ candidate is added to the histogram of “Passed Cuts” for muon detector $\beta$. If the unbiased muon fails any of the requirements, the $J/\psi$ candidate is instead added to the histogram of “Failed Cuts” for muon detector $\beta$. The histograms for “Passed Cuts” and “Failed Cuts” are made for each run period, but are independent of the muon’s transverse momentum. As previously mentioned in Section 5.6.2, the muons used in this analysis have a higher transverse momentum than the efficiency “turn on” point, and so the efficiency is taken to be independent of the muon’s transverse momentum.

A simultaneous fit is done on the histograms for “Passed Cuts” and “Failed Cuts” using a double Gaussian function signal peak with a linear background. The simultaneous fit uses the same mean and same sigma for both the “Passed Cuts” and “Failed Cuts” cuts, but allows for signal yields and for different background parameters in each histogram. The $J/\psi$ signal yield for the “Passed Cuts” histogram, $N_{\text{passed}}$, and
Figure 5.36. CMUP $J/\psi$ Histograms for Run Period 10

(a) Passed Cuts  
(b) Failed Cuts

Figure 5.37. CMU $J/\psi$ Histograms for Run Period 10

(a) Passed Cuts  
(b) Failed Cuts
Figure 5.38. CMX $J/\psi$ Histograms for Run Period 10

(a) Passed Cuts

(b) Failed Cuts

Figure 5.39. CMUP Efficiency as a function of Run Period
Figure 5.40. CMU Efficiency as a function of Run Period

Figure 5.41. CMX Efficiency as a function of Run Period
the signal yield in the “Failed Cuts” histogram, $N_{\text{failed}}$, are used to calculate the reconstruction efficiency. The efficiency for muon detector $\beta$ is given by

$$\epsilon_{\text{data},\beta} = \frac{N_{\text{passed}}}{N_{\text{passed}} + N_{\text{failed}}}$$

(5.15)

This procedure is done for each run period for each of the muon detectors (CMUP, CMU, and CMX). For Run Period 10, the $J/\psi$ “Passed Cuts” and “Failed Cuts” histograms are shown for CMUP in Figure 5.36, for CMU in Figure 5.37, and for CMX in Figure 5.38. Figure 5.39 shows the CMUP efficiency as a function of run period, Figure 5.40 shows the CMU efficiency, and Figure 5.41 shows the CMX efficiency.

5.7.3 XFT Efficiency

The XFT efficiency must be included because the Muon+SVT dataset requires that the displaced track (unbiased muon) pass the XFT Level 1 trigger to be included in the data sample. As a result, the XFT efficiency ($\epsilon_{\text{XFT}}$) must be included in order to calculate the reconstruction efficiency. The XFT efficiency is measured by using the dimuon $J/\psi$ dataset (jpmm) and reconstructing the $B^{\pm} \to J/\psi K^{\pm}$ decay [90]. Kaon ($K^{\pm}$) candidates are only required to be fiducial in XFT, and so plots of the $B^{\pm}$ signal can be made separately for kaons passing and failing the requirements for the XFT Level 1 trigger, depending if the kaon candidates are matched to an XFT track [90]. The pass and fail histograms are then fit to measure $B^{\pm}$ signal yield, and the XFT efficiency is calculated [90].

The XFT efficiency is given by

$$\epsilon_{\text{XFT}} = 0.9648 \pm 0.0039$$

(5.16)

as explained above and calculated by CDF Note 10628 [90].

5.7.4 Vertex Efficiency

Cuts on the $z_0$ vertex position of the muons for the $\Upsilon$ candidates must be measured in data since the Monte Carlo does not accurately model the $z_0$ vertex. Two cuts
are made on the $z_0$ vertex for this analysis and so the efficiency for each cut must be included. The first cut requires the absolute value of the $z_0$ vertex for each muon to be less than 60 cm: $|z_0(\mu_1)| < 60.0$ cm and $|z_0(\mu_2)| < 60.0$ cm. The efficiency for this cut has been previously measured in CDF Note 7935 by using minimum bias events up to run 203799 and is given as: $\epsilon_{z_0} = 0.956 \pm 0.003$ [91]. The second cut requires the absolute value of the differences between the $z_0$ vertices of the two muons to be less than 5.0 cm: $|z_0(\mu_1) - z_0(\mu_2)| < 5.0$ cm. CDF Note 8289 has measured the efficiency for this cut to be: $\epsilon_{\Delta z_0} = 0.999 \pm 0.002$ [92].

The overall vertex efficiency is then given by

$$\epsilon_{\text{vertex}} = 0.9550 \pm 0.0035 \quad (5.17)$$

where $\epsilon_{\text{vertex}}$ is calculated by combining the $\epsilon_{z_0}$ and $\epsilon_{\Delta z_0}$ efficiency measurements [90].

### 5.8 Systematic Uncertainties

Several systematic uncertainties are measured in this cross section analysis. The first uncertainty is due to the dead wires in the detector, first discussed in Section 5.5. Selecting the cut off value for the dead wire probability as well as the procedure for determining the probability contribute to the systematic uncertainty. To measure this systematic uncertainty, several different values of the cut off probability were used, and the cross section was calculated. The largest difference for each calculated value was recorded as the uncertainty.

The next systematic uncertainty involves the measurement of the efficiency in data. This systematic uncertainty is quantified by varying the efficiency by $\pm \sigma$ and then repeating the cross section measurement. For each cross section value calculated, the largest deviation is then listed as the systematic error.

Another systematic uncertainty is due to the angular distribution in the Monte Carlo. The Monte Carlo is generated with an unpolarized angular distribution, while the data has an unknown angular distribution. In order to account for this systematic error, a longitudinal polarization ($\lambda_\theta = -1$) is used to re-weight the Monte Carlo, and
the cross section measurement is done. Next, a transverse polarization ($\lambda_\theta = +1$) is applied to the Monte Carlo, and the measurement is repeated. The largest difference between the polarized cross section, longitudinal or transverse, and the unpolarized cross section is the systematic error. Table 5.3 shows the value of the systematic error due to the unpolarized Monte Carlo.

The unpolarized Monte Carlo systematic error can be reduced by re-weighting the Monte Carlo with the measured angular distribution in data, and this is done for the $\Upsilon(1S)$ state. Chapter 6 discusses the procedure for fitting for the angular distribution, and the measured polarization parameters for the $\Upsilon(1S)$ state are given in Section 6.9. The $\Upsilon(1S)$ polarization parameters in the Collins-Soper frame, shown in Table 6.2, are used to re-weight the Monte Carlo in each bin of transverse momentum, using the generated Monte Carlo polar and azimuthal angles in the Collins-Soper frame. The systematic error for the measured polarization in the $\Upsilon(1S)$ state is quantified by varying the polarization parameters by $\pm\sigma$, and then calculating the cross section. The values calculated when the parameters are varied are compared to the cross section when using the measured polarization parameters, and the largest difference is the systematic error. Table 5.3 lists the polarization measurement systematic error for the $\Upsilon(1S)$ state.

The systematic uncertainty for the measurement of the luminosity is set at 6%. Table 5.3 shows a summary of each of the systematic errors for the total cross section.
for each $\Upsilon(nS)$ state. Except for the systematic error due to the luminosity measurement, the rest of the systematic uncertainties are added in quadrature, and the values are shown in the tables showing the results in Section 5.9. The systematic error for luminosity is listed separately in the tables.

5.9 $\Upsilon$ Cross Section Results

The $\Upsilon$ cross section is calculated using Equation 5.1 in bins of transverse momentum and run period. The acceptance, explained in Section 5.6.1, is calculated using unpolarized Monte Carlo for the $\Upsilon(2S)$ and $\Upsilon(3S)$ states, while the measured angular distribution is used for the $\Upsilon(1S)$ state. The Monte Carlo efficiency, described in Section 5.6.2, and the efficiency in data, explained in Section 5.7, are also used in the cross section measurement.

As explained in Section 5.5, dead wires in the detector have been removed from the measurement to help reduce the time dependence. The cross section is first summed over transverse momentum bins to see the time dependence. Figure 5.42 shows the calculated $\Upsilon(1S)$ cross section as a function of the run period with the bottom plot showing fits with horizontal and linear functions. The time dependence, discussed in Section 5.4, has been reduced by removing dead wires, and remaining time dependence might result from the measurement of the luminosity used in the calculations.

The $\Upsilon(nS)$ cross section results as a function of transverse momentum are calculated by summing over run periods. The measurements in bins of transverse momentum can then be summed for each $\Upsilon(nS)$ signal to calculate a total measurement. The full results from the cross section measurement are shown in Table 5.5, Table 5.6, and Table 5.7. Table 5.5 shows the $\Upsilon(nS)$ cross section measurement for CMUP-CMU, while Table 5.6 shows results for CMUP-CMX. Both of these tables include the statistical errors, systematic uncertainty, and luminosity uncertainty. The CMUP-CMU and CMUP-CMX cross section measurements are combined to calculate
Figure 5.42. $\Upsilon(1S)$ Cross Section as a function of Run Period for CMUP-CMU and CMUP-CMX. CMUP-CMU data (blue) and CMUP-CMX data (red) with error bars shown for $0 < p_T < 40$ GeV/c. Plots have same data, but bottom plot has horizontal line fits (black lines) and linear fits. CMUP-CMU horizontal fit average is $688.61 \pm 3.7$ with $\chi^2 = 148.62$, while CMUP-CMX is $785.84 \pm 6.8$ with $\chi^2 = 131.23$. Bottom plot also has linear fits of CMUP-CMU (cyan line) with $\chi^2 = 43.71$ and of CMUP-CMX (pink line) with $\chi^2 = 30.59$. 
Table 5.4
Summary of $\Upsilon(nS)$ Cross Section Measurement

<table>
<thead>
<tr>
<th>$\Upsilon(nS)$ State</th>
<th>$\frac{d\sigma(\Upsilon(nS))}{dy} \times B(\Upsilon(nS) \rightarrow \mu^+\mu^-)$ (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Upsilon(1S)$</td>
<td>$726.58 \pm 2.79$ (stat.) $\pm 62.33$ (syst.) $\pm 43.60$ (lumi.)</td>
</tr>
<tr>
<td>$\Upsilon(2S)$</td>
<td>$186.40 \pm 1.09$ (stat.) $\pm 33.13$ (syst.) $\pm 11.18$ (lumi.)</td>
</tr>
<tr>
<td>$\Upsilon(3S)$</td>
<td>$83.64 \pm 0.74$ (stat.) $\pm 15.66$ (syst.) $\pm 5.02$ (lumi.)</td>
</tr>
</tbody>
</table>

an average $\Upsilon(nS)$ cross section measurement that is shown in Table 5.7. A summary of the total cross section measurement calculated from an average of CMUP-CMU and CMUP-CMX is shown in Table 5.4.

The cross section analysis results are compared with previous measurements from CDF Run I with $|y| < 0.4$ for all $\Upsilon(nS)$ signals [71] and D0 Run II with $|y| < 0.6$ for the $\Upsilon(1S)$ signal [78, 79]. Figure 5.43 shows the $\Upsilon(1S)$ differential cross section measurement compared with previous results. Figure 5.44 shows the $\Upsilon(2S)$ measurement while the $\Upsilon(3S)$ results are in Figure 5.45.
Table 5.5
CMUP-CMU $\Upsilon(nS)$ Cross Section Measurement as a function of $p_T$.
Errors shown are statistical, systematic, and luminosity. (value ±
stat. ± syst. ± lumi.)

<table>
<thead>
<tr>
<th>$p_T$ Range ($GeV/c$)</th>
<th>$\Upsilon$ Yield</th>
<th>Cross Section $\frac{d\sigma}{dy}$ (pb)</th>
<th>Differential Cross Section $\frac{d^2\sigma}{dp_T dy}$ (pb/GeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Upsilon(1S)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 - 2.0</td>
<td>67118 ± 383</td>
<td>135.48 ± 1.30 ± 17.54 ± 8.13</td>
<td>67.74 ± 0.65 ± 8.77 ± 4.06</td>
</tr>
<tr>
<td>2.0 - 4.0</td>
<td>117850 ± 486</td>
<td>211.37 ± 1.77 ± 9.52 ± 12.68</td>
<td>105.69 ± 0.89 ± 4.76 ± 6.34</td>
</tr>
<tr>
<td>4.0 - 6.0</td>
<td>77048 ± 372</td>
<td>150.40 ± 1.37 ± 11.38 ± 9.02</td>
<td>75.20 ± 0.68 ± 5.69 ± 4.51</td>
</tr>
<tr>
<td>6.0 - 8.0</td>
<td>39427 ± 252</td>
<td>85.23 ± 0.89 ± 17.50 ± 5.11</td>
<td>42.61 ± 0.44 ± 8.75 ± 2.56</td>
</tr>
<tr>
<td>8.0 - 12.0</td>
<td>35685 ± 231</td>
<td>70.47 ± 0.69 ± 14.34 ± 4.23</td>
<td>17.06 ± 0.17 ± 3.59 ± 1.06</td>
</tr>
<tr>
<td>12.0 - 17.0</td>
<td>13758 ± 142</td>
<td>21.01 ± 0.26 ± 2.39 ± 1.26</td>
<td>4.20 ± 0.05 ± 0.48 ± 0.25</td>
</tr>
<tr>
<td>17.0 - 23.0</td>
<td>4057 ± 82</td>
<td>4.31 ± 0.09 ± 0.36 ± 0.26</td>
<td>0.72 ± 0.02 ± 0.06 ± 0.04</td>
</tr>
<tr>
<td>23.0 - 40.0</td>
<td>954 ± 60</td>
<td>0.83 ± 0.05 ± 0.11 ± 0.05</td>
<td>0.049 ± 0.003 ± 0.006 ± 0.003</td>
</tr>
<tr>
<td>Total</td>
<td>355899 ± 818</td>
<td>679.10 ± 2.84 ± 71.60 ± 40.75</td>
<td></td>
</tr>
<tr>
<td>$\Upsilon(2S)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 - 2.0</td>
<td>15167 ± 245</td>
<td>29.72 ± 0.55 ± 8.96 ± 1.78</td>
<td>14.86 ± 0.27 ± 4.48 ± 0.89</td>
</tr>
<tr>
<td>2.0 - 4.0</td>
<td>26725 ± 308</td>
<td>47.40 ± 0.68 ± 14.11 ± 2.84</td>
<td>23.70 ± 0.34 ± 7.05 ± 1.42</td>
</tr>
<tr>
<td>4.0 - 6.0</td>
<td>22023 ± 254</td>
<td>40.94 ± 0.59 ± 12.07 ± 2.46</td>
<td>20.47 ± 0.30 ± 6.03 ± 1.23</td>
</tr>
<tr>
<td>6.0 - 8.0</td>
<td>11330 ± 167</td>
<td>23.66 ± 0.41 ± 6.83 ± 1.42</td>
<td>11.83 ± 0.21 ± 3.42 ± 0.71</td>
</tr>
<tr>
<td>8.0 - 12.0</td>
<td>11173 ± 153</td>
<td>22.39 ± 0.36 ± 6.05 ± 1.34</td>
<td>5.58 ± 0.09 ± 1.51 ± 0.33</td>
</tr>
<tr>
<td>12.0 - 17.0</td>
<td>5044 ± 99</td>
<td>8.06 ± 0.17 ± 1.78 ± 0.48</td>
<td>1.61 ± 0.03 ± 0.36 ± 0.10</td>
</tr>
<tr>
<td>17.0 - 23.0</td>
<td>1233 ± 56</td>
<td>1.41 ± 0.06 ± 0.17 ± 0.08</td>
<td>0.24 ± 0.01 ± 0.03 ± 0.01</td>
</tr>
<tr>
<td>23.0 - 40.0</td>
<td>289 ± 47</td>
<td>0.25 ± 0.04 ± 0.03 ± 0.02</td>
<td>0.015 ± 0.002 ± 0.002 ± 0.001</td>
</tr>
<tr>
<td>Total</td>
<td>92984 ± 535</td>
<td>173.75 ± 1.20 ± 49.89 ± 10.43</td>
<td></td>
</tr>
<tr>
<td>$\Upsilon(3S)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 - 2.0</td>
<td>5924 ± 203</td>
<td>11.49 ± 0.41 ± 3.45 ± 0.69</td>
<td>5.74 ± 0.20 ± 1.72 ± 0.34</td>
</tr>
<tr>
<td>2.0 - 4.0</td>
<td>10842 ± 258</td>
<td>18.90 ± 0.48 ± 5.68 ± 1.13</td>
<td>9.45 ± 0.24 ± 2.84 ± 0.57</td>
</tr>
<tr>
<td>4.0 - 6.0</td>
<td>10904 ± 216</td>
<td>19.24 ± 0.41 ± 5.69 ± 1.15</td>
<td>9.62 ± 0.21 ± 2.84 ± 0.58</td>
</tr>
<tr>
<td>6.0 - 8.0</td>
<td>5772 ± 142</td>
<td>11.36 ± 0.30 ± 3.31 ± 0.68</td>
<td>5.68 ± 0.15 ± 1.66 ± 0.34</td>
</tr>
<tr>
<td>8.0 - 12.0</td>
<td>6225 ± 128</td>
<td>12.16 ± 0.27 ± 3.36 ± 0.73</td>
<td>3.04 ± 0.07 ± 0.84 ± 0.18</td>
</tr>
<tr>
<td>12.0 - 17.0</td>
<td>3024 ± 83</td>
<td>4.86 ± 0.14 ± 1.10 ± 0.29</td>
<td>0.97 ± 0.03 ± 0.22 ± 0.06</td>
</tr>
<tr>
<td>17.0 - 23.0</td>
<td>713 ± 48</td>
<td>0.83 ± 0.06 ± 0.11 ± 0.05</td>
<td>0.14 ± 0.01 ± 0.02 ± 0.01</td>
</tr>
<tr>
<td>23.0 - 40.0</td>
<td>207 ± 40</td>
<td>0.18 ± 0.04 ± 0.02 ± 0.01</td>
<td>0.011 ± 0.002 ± 0.001 ± 0.001</td>
</tr>
<tr>
<td>Total</td>
<td>43612 ± 449</td>
<td>79.02 ± 0.87 ± 22.67 ± 4.74</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.6

CMUP-CMX $\Upsilon(nS)$ Cross Section Measurement as a function of $p_T$.
Errors shown are statistical, systematic, and luminosity. (value $\pm$
stat. $\pm$ syst. $\pm$ lumi.)

<table>
<thead>
<tr>
<th>$p_T$ Range ($GeV/c$)</th>
<th>$\Upsilon$ Yield</th>
<th>Cross Section $\frac{d\sigma}{dp_T}$ (pb)</th>
<th>Differential Cross Section $\frac{d^2\sigma}{dp_T dq}$ (pb/GeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 - 2.0</td>
<td>35932 $\pm$ 259</td>
<td>156.72 $\pm$ 2.16 $\pm$ 19.08 $\pm$ 9.40</td>
<td>78.36 $\pm$ 1.08 $\pm$ 9.54 $\pm$ 4.70</td>
</tr>
<tr>
<td>2.0 - 4.0</td>
<td>57758 $\pm$ 324</td>
<td>243.80 $\pm$ 3.11 $\pm$ 26.88 $\pm$ 14.63</td>
<td>121.90 $\pm$ 1.56 $\pm$ 13.44 $\pm$ 7.31</td>
</tr>
<tr>
<td>4.0 - 6.0</td>
<td>36779 $\pm$ 241</td>
<td>168.11 $\pm$ 2.29 $\pm$ 20.35 $\pm$ 10.09</td>
<td>84.06 $\pm$ 1.15 $\pm$ 10.17 $\pm$ 5.04</td>
</tr>
<tr>
<td>6.0 - 8.0</td>
<td>21928 $\pm$ 175</td>
<td>99.86 $\pm$ 1.46 $\pm$ 18.23 $\pm$ 5.99</td>
<td>49.93 $\pm$ 0.73 $\pm$ 9.11 $\pm$ 3.00</td>
</tr>
<tr>
<td>8.0 - 12.0</td>
<td>21502 $\pm$ 169</td>
<td>79.28 $\pm$ 1.06 $\pm$ 15.07 $\pm$ 4.76</td>
<td>19.82 $\pm$ 0.27 $\pm$ 3.77 $\pm$ 1.19</td>
</tr>
<tr>
<td>12.0 - 17.0</td>
<td>9469 $\pm$ 113</td>
<td>22.28 $\pm$ 0.34 $\pm$ 6.37 $\pm$ 1.34</td>
<td>4.46 $\pm$ 0.07 $\pm$ 1.27 $\pm$ 0.27</td>
</tr>
<tr>
<td>17.0 - 23.0</td>
<td>1632 $\pm$ 59</td>
<td>3.45 $\pm$ 0.13 $\pm$ 1.45 $\pm$ 0.21</td>
<td>0.58 $\pm$ 0.02 $\pm$ 0.24 $\pm$ 0.03</td>
</tr>
<tr>
<td>23.0 - 40.0</td>
<td>248 $\pm$ 48</td>
<td>0.56 $\pm$ 0.11 $\pm$ 0.12 $\pm$ 0.03</td>
<td>0.033 $\pm$ 0.006 $\pm$ 0.007 $\pm$ 0.002</td>
</tr>
<tr>
<td>Total</td>
<td>185249 $\pm$ 555</td>
<td>774.07 $\pm$ 4.80 $\pm$ 102.05 $\pm$ 46.44</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\Upsilon$(2S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 - 2.0</td>
</tr>
<tr>
<td>2.0 - 4.0</td>
</tr>
<tr>
<td>4.0 - 6.0</td>
</tr>
<tr>
<td>6.0 - 8.0</td>
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<tr>
<td>8.0 - 12.0</td>
</tr>
<tr>
<td>12.0 - 17.0</td>
</tr>
<tr>
<td>17.0 - 23.0</td>
</tr>
<tr>
<td>23.0 - 40.0</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\Upsilon$(3S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 - 2.0</td>
</tr>
<tr>
<td>2.0 - 4.0</td>
</tr>
<tr>
<td>4.0 - 6.0</td>
</tr>
<tr>
<td>6.0 - 8.0</td>
</tr>
<tr>
<td>8.0 - 12.0</td>
</tr>
<tr>
<td>12.0 - 17.0</td>
</tr>
<tr>
<td>17.0 - 23.0</td>
</tr>
<tr>
<td>23.0 - 40.0</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>
Table 5.7
Average $\Upsilon(nS)$ Cross Section Measurement as a function of $p_T$. Errors shown are statistical, systematic, and luminosity. (value ± stat. ± syst. ± lumi.)

<table>
<thead>
<tr>
<th>$p_T$ Range (GeV/c)</th>
<th>Cross Section $\frac{d\sigma}{dy}$ (pb)</th>
<th>Differential Cross Section $\frac{d^2\sigma}{dp_Tdy}$ (pb/GeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Upsilon(1S)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 - 2.0</td>
<td>146.10 ± 1.26 ± 12.96 ± 8.77</td>
<td>73.05 ± 1.26 ± 6.48 ± 4.38</td>
</tr>
<tr>
<td>2.0 - 4.0</td>
<td>227.58 ± 1.79 ± 14.26 ± 13.66</td>
<td>113.79 ± 1.79 ± 7.13 ± 6.83</td>
</tr>
<tr>
<td>4.0 - 6.0</td>
<td>159.26 ± 1.34 ± 11.66 ± 9.56</td>
<td>79.63 ± 1.34 ± 5.83 ± 4.78</td>
</tr>
<tr>
<td>6.0 - 8.0</td>
<td>92.55 ± 0.86 ± 12.63 ± 5.55</td>
<td>46.27 ± 0.86 ± 6.32 ± 2.78</td>
</tr>
<tr>
<td>8.0 - 12.0</td>
<td>74.87 ± 0.63 ± 10.40 ± 4.49</td>
<td>18.72 ± 0.63 ± 2.60 ± 1.12</td>
</tr>
<tr>
<td>12.0 - 17.0</td>
<td>21.65 ± 0.21 ± 3.40 ± 1.30</td>
<td>4.33 ± 0.21 ± 0.68 ± 0.26</td>
</tr>
<tr>
<td>17.0 - 23.0</td>
<td>3.88 ± 0.08 ± 0.74 ± 0.23</td>
<td>0.65 ± 0.08 ± 0.12 ± 0.04</td>
</tr>
<tr>
<td>23.0 - 40.0</td>
<td>0.70 ± 0.06 ± 0.08 ± 0.04</td>
<td>0.041 ± 0.061 ± 0.005 ± 0.002</td>
</tr>
<tr>
<td>Total</td>
<td>726.58 ± 2.79 ± 62.33 ± 43.60</td>
<td></td>
</tr>
<tr>
<td>$\Upsilon(2S)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 - 2.0</td>
<td>31.47 ± 0.47 ± 5.93 ± 1.89</td>
<td>15.73 ± 0.47 ± 2.97 ± 0.94</td>
</tr>
<tr>
<td>2.0 - 4.0</td>
<td>53.14 ± 0.65 ± 9.93 ± 3.19</td>
<td>26.57 ± 0.65 ± 4.96 ± 1.59</td>
</tr>
<tr>
<td>4.0 - 6.0</td>
<td>42.96 ± 0.54 ± 7.88 ± 2.58</td>
<td>21.48 ± 0.54 ± 3.94 ± 1.29</td>
</tr>
<tr>
<td>6.0 - 8.0</td>
<td>26.00 ± 0.38 ± 4.71 ± 1.56</td>
<td>13.00 ± 0.38 ± 2.36 ± 0.78</td>
</tr>
<tr>
<td>8.0 - 12.0</td>
<td>23.76 ± 0.31 ± 4.81 ± 1.43</td>
<td>5.94 ± 0.31 ± 1.20 ± 0.36</td>
</tr>
<tr>
<td>12.0 - 17.0</td>
<td>7.57 ± 0.13 ± 1.97 ± 0.45</td>
<td>1.51 ± 0.13 ± 0.39 ± 0.09</td>
</tr>
<tr>
<td>17.0 - 23.0</td>
<td>1.23 ± 0.06 ± 0.27 ± 0.07</td>
<td>0.20 ± 0.06 ± 0.04 ± 0.01</td>
</tr>
<tr>
<td>23.0 - 40.0</td>
<td>0.29 ± 0.06 ± 0.06 ± 0.02</td>
<td>0.017 ± 0.056 ± 0.004 ± 0.001</td>
</tr>
<tr>
<td>Total</td>
<td>186.40 ± 1.09 ± 33.13 ± 11.18</td>
<td></td>
</tr>
<tr>
<td>$\Upsilon(3S)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0 - 2.0</td>
<td>12.20 ± 0.33 ± 2.44 ± 0.73</td>
<td>6.10 ± 0.33 ± 1.22 ± 0.37</td>
</tr>
<tr>
<td>2.0 - 4.0</td>
<td>20.71 ± 0.42 ± 3.74 ± 1.24</td>
<td>10.35 ± 0.42 ± 1.87 ± 0.62</td>
</tr>
<tr>
<td>4.0 - 6.0</td>
<td>19.92 ± 0.36 ± 3.88 ± 1.20</td>
<td>9.96 ± 0.36 ± 1.94 ± 0.60</td>
</tr>
<tr>
<td>6.0 - 8.0</td>
<td>12.66 ± 0.26 ± 2.82 ± 0.76</td>
<td>6.33 ± 0.26 ± 1.41 ± 0.38</td>
</tr>
<tr>
<td>8.0 - 12.0</td>
<td>12.88 ± 0.22 ± 2.95 ± 0.77</td>
<td>3.22 ± 0.22 ± 0.74 ± 0.19</td>
</tr>
<tr>
<td>12.0 - 17.0</td>
<td>4.43 ± 0.10 ± 1.27 ± 0.27</td>
<td>0.89 ± 0.10 ± 0.25 ± 0.05</td>
</tr>
<tr>
<td>17.0 - 23.0</td>
<td>0.68 ± 0.05 ± 0.16 ± 0.04</td>
<td>0.11 ± 0.05 ± 0.03 ± 0.01</td>
</tr>
<tr>
<td>23.0 - 40.0</td>
<td>0.16 ± 0.05 ± 0.04 ± 0.01</td>
<td>0.010 ± 0.047 ± 0.003 ± 0.001</td>
</tr>
<tr>
<td>Total</td>
<td>83.64 ± 0.74 ± 15.66 ± 5.02</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.43. $\Upsilon(1S)$ Differential Cross Section as a function of $p_T$. CMUP-CMU data (blue squares) and CMUP-CMX data (red circles) with $|y| < 0.6$. Previous measurements shown are CDF Run I (green triangles) with $|y| < 0.4$ [71] and D0 Run II (purple down arrows) with $|y| < 0.6$ [78,79]. (Total errors shown. Plots show same data but bottom plot has log scale.)
Figure 5.44. $\Upsilon(2S)$ Differential Cross Section as a function of $p_T$. CMUP-CMU data (blue squares) and CMUP-CMX data (red circles) with $|y| < 0.6$. Previous measurement shown is CDF Run I (green triangles) with $|y| < 0.4$ [71]. (Total errors shown. Plots show same data but bottom plot has log scale.)
Figure 5.45. $\Upsilon(3S)$ Differential Cross Section as a function of $p_T$. CMUP-CMU data (blue squares) and CMUP-CMX data (red circles) with $|y| < 0.6$. Previous measurement shown is CDF Run I (green triangles) with $|y| < 0.4$ [71]. (Total errors shown. Plots show same data but bottom plot has log scale.)
6. ANGULAR DISTRIBUTION ANALYSIS

The angular distribution of the $\Upsilon \to \mu^+ \mu^-$ decay depends on the polar angle ($\theta$) and azimuthal angle ($\phi$) of the positive muon ($\mu^+$). Figure 2.2 in Section 2.5 shows both the polar and azimuthal angle for a given reference frame. The choice of reference frame to determine the polar angle and azimuthal can also affect the results for the polarization parameters. This analysis measures the polarization parameters in the S-channel helicity frame (SH), the Gottfried-Jackson frame (GJ), and the Collins-Soper frame (CS), which are explained in Section 2.5.1 and shown in Figure 2.3.

Section 2.5 describes the angular distribution, and Equation 2.3, which gives the angular distribution as a function of the polar angle and azimuthal angle, is reproduced below

$$\frac{dN}{d\cos \theta \, d\phi} \approx \frac{1}{3 + \lambda_\theta} \left( 1 + \lambda_\theta \cos^2(\theta) + \lambda_\phi \sin^2(\theta) \cos(2\phi) + \lambda_{\theta\phi} \sin(2\theta) \cos(\phi) \right)$$  \hspace{1cm} (2.3 revisited)

The three polarization parameters ($\lambda_\theta$, $\lambda_\phi$, and $\lambda_{\theta\phi}$) are obtained from a fit using Equation 2.3. Furthermore, as discussed in Section 2.5.2, the frame invariant parameter ($\tilde{\lambda}$), given in Equation 2.8,

$$\tilde{\lambda} = \frac{\lambda_\theta + 3\lambda_\phi}{1 - \lambda_\phi}$$  \hspace{1cm} (2.8 revisited)

is also calculated to provide comparison of the measured polarization across multiple reference frames.

The angular distribution for the $\Upsilon(nS)$ signal regions undergo a fit based on Equation 2.3. However, the signal regions contain background in addition to the signal, and the angular distribution of the background can be entirely different than the angular distribution of the $\Upsilon(nS)$ signal. Section 6.1 describes the background and the mass regions used for the fit of the angular distribution. Section 6.2 discusses how the polarization of the background in the signal region can be quantified and separated.
from the polarization of the $\Upsilon$ signal. Both two-dimensional and one-dimensional fits are done to the angular distribution as a cross-check. The two-dimensional angular distribution fit is discussed further in Section 6.7. Section 6.6 further describes the one-dimensional fit.

6.1 Background and Mass Regions

The angular distribution of the background under the $\Upsilon(nS)$ signal must be quantified to accurately measure the $\Upsilon(nS)$ angular distribution. The dimuon background to the $\Upsilon(nS)$ signal is mostly made of semi-leptonic decays of bottom quark ($b$) hadrons from $b\bar{b}$ production [90]. Furthermore, the angular distribution of the background is much different at lower values of the dimuon candidate mass than at higher regions. Otherwise, background subtraction could be used to determine the angular distribution of the $\Upsilon(nS)$ signal.

The histogram of the $\Upsilon$ candidate invariant mass is divided into twelve mass bins with approximately equal width. This division gives three signal mass bins and nine background mass bins. Table 6.1 shows the twelve mass bins used for the fit of the $\Upsilon$ angular distribution.

6.2 Prompt Sample and Displaced Sample

The dimuon dataset (jbbmm), described in Section 5.1, that is used for the $\Upsilon$ cross section is used for the angular distribution analysis. Also, the same selection cuts listed in Section 5.1.3 are used for $\Upsilon$ candidates. In order to measure the angular distribution of the background, the $\Upsilon$ data sample is divided into two subsets: a prompt sample and a displaced track sample. The displaced track sample is dominated by background from decays of $b$ hadrons but still contains a small portion of the $\Upsilon(nS)$ signal. The prompt sample contains most of the $\Upsilon(nS)$ signal but still has some background.
Table 6.1
Mass Bins for the Υ Angular Distribution Fit

<table>
<thead>
<tr>
<th>Mass Bin</th>
<th>Mass Range (GeV/c²)</th>
<th>Region Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.30 - 8.50</td>
<td>background</td>
</tr>
<tr>
<td>2</td>
<td>8.50 - 8.70</td>
<td>background</td>
</tr>
<tr>
<td>3</td>
<td>8.70 - 9.00</td>
<td>background</td>
</tr>
<tr>
<td>4</td>
<td>9.00 - 9.25</td>
<td>background</td>
</tr>
<tr>
<td>5</td>
<td>9.25 - 9.65</td>
<td>Υ(1S) signal + background</td>
</tr>
<tr>
<td>6</td>
<td>9.65 - 9.85</td>
<td>background</td>
</tr>
<tr>
<td>7</td>
<td>9.85 - 10.15</td>
<td>Υ(2S) signal + background</td>
</tr>
<tr>
<td>8</td>
<td>10.15 - 10.50</td>
<td>Υ(3S) signal + background</td>
</tr>
<tr>
<td>9</td>
<td>10.50 - 10.80</td>
<td>background</td>
</tr>
<tr>
<td>10</td>
<td>10.80 - 11.10</td>
<td>background</td>
</tr>
<tr>
<td>11</td>
<td>11.10 - 11.40</td>
<td>background</td>
</tr>
<tr>
<td>12</td>
<td>11.40 - 11.70</td>
<td>background</td>
</tr>
</tbody>
</table>

The requirements for the displaced sample are for the Υ candidate to have one muon to pass the following cuts:

- track with $N$(silicon hits) ≥ 3
- track impact parameter $d_0 > 150$ μm

while all other Υ candidates are placed in the prompt sample.

The Υ mass peaks for both CMUP-CMU and CMUP-CMX are fit in both the prompt sample and displaced sample in bins of transverse momentum. The Υ($nS$) signals are fit with a Gaussian, and the background is fit with an exponential function or a gamma function. Figure 6.1 shows the fit of the background in the prompt and displaced samples for CMUP-CMU for $2 < p_T(Υ) < 4$ GeV/c, while CMUP-CMX is shown in Figure 6.2.

The signal and background fit is done simultaneously for the CMUP-CMU prompt and displaced samples. Another fit is then done for the CMUP-CMX prompt and displaced samples. The fraction of signal in the prompt sample out of the total
Figure 6.1. Background Fit for CMUP-CMU $\Upsilon$ Mass for $2 < p_T(\Upsilon) < 4$ GeV/c

Figure 6.2. Background Fit for CMUP-CMX $\Upsilon$ Mass for $2 < p_T(\Upsilon) < 4$ GeV/c
signal in both the prompt sample and displaced sample, called the \((f_{p,\text{sig}})\) signal fraction, is calculated from the fitted signal yields. The angular distribution fit uses the CMUP-CMU prompt signal fraction \((f_{p,\text{sig}}^{\text{CMU}})\) and the CMUP-CMX prompt signal fraction \((f_{p,\text{sig}}^{\text{CMX}})\). Furthermore, the ratio of the background in the prompt sample to the background in the displaced sample, called the prompt scale factor \((s_p)\), is also measured by using a linear function to fit the ratio of backgrounds in each sample, using the sidebands and ignoring the signal region. The linear fit is done separately for both CMUP-CMU and CMUP-CMX to measure the CMUP-CMU prompt scale factor \((s_p^{\text{CMU}})\) and the CMUP-CMX prompt scale factor \((s_p^{\text{CMX}})\).

### 6.3 Acceptance Templates

The angular distribution of the polar and azimuthal angles not only depend on the reference frame and kinematics of the decay, but also on the detector acceptance. Monte Carlo is used to model the detector acceptance to demonstrate the expected angular distribution for the unpolarized generated \(\Upsilon\) decays in Monte Carlo. These “acceptance templates” can then be used in the fit of the angular distribution as explained in Section 6.6 and Section 6.7.

The acceptance templates are made using the unpolarized Monte Carlo described in Section 5.6 as a function of polar and azimuthal angles. Templates are made in bins of transverse momentum for each \(\Upsilon(nS)\) signal and for both CMUP-CMU and CMUP-CMX. Furthermore, plots are made for the generated \(\Upsilon\) candidates and reconstructed \(\Upsilon\) candidates using the requirements listed in Section 5.6.1, and the ratio of reconstructed candidates to generated candidates in a specific bin of \(\cos\theta\) and \(\phi\) gives the Monte Carlo acceptance.

The reconstruction efficiency is included in the acceptance templates by using the efficiency for a candidate to weight each candidate entry. The reconstruction efficiency is ratio of the data efficiency to Monte Carlo efficiency and depends on the run period. The data efficiency is measured as explained in Section 5.7, and the
Monte Carlo efficiency is discussed in Section 5.6.2. The reconstruction efficiency is calculated for a candidate by looking up the corresponding efficiencies for the run period of the candidate and is given by

$$
\epsilon_{\text{reco}} = \frac{\epsilon_{\text{data,CMUP}} \cdot \epsilon_{\text{XFT}} \cdot \epsilon_{\text{data,CMX}} \cdot \epsilon_{\text{XFT}} \cdot \epsilon_{\text{vertex}}}{\epsilon_{\text{MC,CMUP}} \cdot \epsilon_{\text{MC,CMX}}} \quad (6.1)
$$

where $\beta$ is either CMU and CMX, $\epsilon_{\text{data,CMX}}$ is measured as described in Section 5.7.2, $\epsilon_{\text{XFT}}$ is the XFT efficiency discussed in Section 5.7.3, and $\epsilon_{\text{vertex}}$ is the vertex efficiency from Section 5.7.4.

### 6.4 Fit of Background Regions

The polarization fit for the background regions is a simultaneous fit of the CMUP-CMU prompt sample, CMUP-CMU displaced sample, CMUP-CMX prompt sample, and CMUP-CMX displaced sample. The angular distribution fit to the background regions can be written as

$$
\frac{dN}{d\Omega}(\vec{\alpha}) = P_{\text{bkg}}^{\text{CMU}}(\vec{\alpha}) + D_{\text{bkg}}^{\text{CMU}}(\vec{\alpha}) + P_{\text{bkg}}^{\text{CMX}}(\vec{\alpha}) + D_{\text{bkg}}^{\text{CMX}}(\vec{\alpha}) \quad (6.2)
$$

where $\vec{\alpha}$ represents either the variables in the one-dimensional fit ($\cos \theta, \phi$, or $\tilde{\phi}$), explained in Section 6.6, or the variables for the two-dimensional fit ($\cos \theta, \phi$), discussed in Section 6.7. The prompt CMUP-CMU term can be summarized as

$$
P_{\text{bkg}}^{\text{CMU}}(\vec{\alpha}) = N_d^{\text{CMU}} \cdot s_p^{\text{CMU}} \cdot w_{\text{bkg}}(\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)_{\text{bkg}}(\vec{\alpha}) \quad (6.3)
$$

and term for the CMUP-CMU displaced sample is given by

$$
D_{\text{bkg}}^{\text{CMU}}(\vec{\alpha}) = N_d^{\text{CMU}} \cdot w_{\text{bkg}}(\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)_{\text{bkg}}(\vec{\alpha}) \quad (6.4)
$$

where

- $N_d^{\text{CMU}}$ is the CMUP-CMU background yield in the displaced sample
- $s_p^{\text{CMU}}$ is the CMUP-CMU prompt scale factor (the factor of the background yield in the prompt sample compared to in the displaced sample)
- $w_{\text{bkg}}(\vec{\alpha}, \vec{\lambda})$ is the polarization fit for the background
• $(A \times \epsilon)_{\text{CMU}}^{\text{bkg}} (\vec{\alpha})$ is the CMUP-CMU acceptance template for the background

The term for CMUP-CMX prompt sample can be written as

$$P_{\text{bkg}}^{\text{CMX}} (\vec{\alpha}) = N_{d}^{\text{CMX}} \cdot s_{p}^{\text{CMX}} \cdot w_{\text{bkg}}^{\text{CMX}} (\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)_{\text{bkg}}^{\text{CMX}} (\vec{\alpha})$$

(6.5)

and the displaced sample for CMUP-CMX is

$$D_{\text{bkg}}^{\text{CMX}} (\vec{\alpha}) = N_{d}^{\text{CMX}} \cdot w_{\text{bkg}}^{\text{CMX}} (\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)_{\text{bkg}}^{\text{CMX}} (\vec{\alpha})$$

(6.6)

where

• $N_{d}^{\text{CMX}}$ is the CMUP-CMX background yield in the displaced sample  
• $s_{p}^{\text{CMX}}$ is the CMUP-CMX prompt scale factor (the factor of the background yield in the prompt sample compared to in the displaced sample) 
• $w_{\text{bkg}} (\vec{\alpha}, \vec{\lambda})$ is same function for the polarization fit for the background as in the CMUP-CMU terms  
• $(A \times \epsilon)_{\text{bkg}}^{\text{CMX}} (\vec{\alpha})$ is the CMUP-CMX acceptance template for the background

The complete function for the angular distribution fit of the background regions can be obtained by putting all of these terms together and is given by

$$\frac{dN}{d\Omega} (\vec{\alpha}) = N_{d}^{\text{CMU}} \cdot s_{p}^{\text{CMU}} \cdot w_{\text{bkg}}^{\text{CMU}} (\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)_{\text{bkg}}^{\text{CMU}} (\vec{\alpha})$$  
$$+ N_{d}^{\text{CMX}} \cdot w_{\text{bkg}}^{\text{CMX}} (\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)_{\text{bkg}}^{\text{CMX}} (\vec{\alpha})$$  
$$+ N_{d}^{\text{CMX}} \cdot s_{p}^{\text{CMX}} \cdot w_{\text{bkg}}^{\text{CMX}} (\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)_{\text{bkg}}^{\text{CMX}} (\vec{\alpha})$$  
$$+ N_{d}^{\text{CMX}} \cdot w_{\text{bkg}}^{\text{CMX}} (\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)_{\text{bkg}}^{\text{CMX}} (\vec{\alpha})$$

(6.7)

where $w_{\text{bkg}} (\vec{\alpha}, \vec{\lambda})$ is the same function in all terms and gives the polarization parameters $(\vec{\lambda})$. The polarization function, $w_{\text{bkg}} (\vec{\alpha}, \vec{\lambda})$, represents either the function for a one-dimensional fit, explained in Section 6.6, or for a two-dimensional fit, discussed in Section 6.7. The polarization parameters, $\vec{\lambda} = (\lambda_{\theta}, \lambda_{\phi}, \lambda_{\theta\phi})$, measured by doing the background fit are not bounded by the usual limits of $-1$ to $1$ because the background does not represent any physical polarization.
6.5 Fit of Signal Regions

The polarization fit of the signal regions is similar to the fit of the background regions but with additional terms for the $\Upsilon$ signal. The angular distribution fit for the signal regions is a simultaneous fit of the CMUP-CMU prompt sample, CMUP-CMU displaced sample, CMUP-CMX prompt sample, and CMUP-CMX displaced sample. The angular distribution fit to the signal regions can be summarized as

$$\frac{dN}{d\Omega}(\vec{\alpha}) = P_{\text{CMU}}^{\text{sig+bkg}}(\vec{\alpha}) + D_{\text{CMU}}^{\text{sig+bkg}}(\vec{\alpha}) + P_{\text{CMX}}^{\text{sig+bkg}}(\vec{\alpha}) + D_{\text{CMX}}^{\text{sig+bkg}}(\vec{\alpha}) \quad (6.8)$$

where $\vec{\alpha}$ represents either one-dimensional fit variable ($\cos \theta$, $\phi$, or $\tilde{\phi}$), further described in Section 6.6, or the two-dimensional fit variables ($\cos \theta$, $\phi$), explained in Section 6.7. Furthermore, the term for prompt CMUP-CMU sample is given by

$$P_{\text{CMU}}^{\text{sig+bkg}}(\vec{\alpha}) = N_{nS}^{\text{CMU}} \cdot f_{p,\text{sig}}^{\text{CMU}} \cdot w_{\text{sig}}(\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)^{\text{CMU}}_{\text{sig}}(\vec{\alpha})$$

$$+ N_d^{\text{CMU}} \cdot s_p^{\text{CMU}} \cdot w_{\text{bkg}}(\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)^{\text{CMU}}_{\text{bkg}}(\vec{\alpha}) \quad (6.9)$$

and the CMUP-CMU displaced sample is written as

$$D_{\text{CMU}}^{\text{sig+bkg}}(\vec{\alpha}) = N_{nS}^{\text{CMU}} \cdot (1 - f_{p,\text{sig}}^{\text{CMU}}) \cdot w_{\text{sig}}(\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)^{\text{CMU}}_{\text{sig}}(\vec{\alpha})$$

$$+ N_d^{\text{CMU}} \cdot w_{\text{bkg}}(\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)^{\text{CMU}}_{\text{bkg}}(\vec{\alpha}) \quad (6.10)$$

where

- $N_{nS}^{\text{CMU}}$ is the CMUP-CMU $\Upsilon(nS)$ signal yield
- $f_{p,\text{sig}}^{\text{CMU}}$ is the CMUP-CMU signal fraction in the prompt sample
- $w_{\text{sig}}(\vec{\alpha}, \vec{\lambda})$ is the polarization fit for the signal
- $(A \times \epsilon)^{\text{CMU}}_{\text{sig}}(\vec{\alpha})$ is the CMUP-CMU acceptance template for the signal
- $N_d^{\text{CMU}}$ is the CMUP-CMU background yield in the displaced sample
- $s_p^{\text{CMU}}$ is the CMUP-CMU prompt scale factor (the factor of the background yield in the prompt sample compared to in the displaced sample)
- $w_{\text{bkg}}(\vec{\alpha}, \vec{\lambda})$ is the polarization fit for the background
- $(A \times \epsilon)^{\text{CMU}}_{\text{bkg}}(\vec{\alpha})$ is the CMUP-CMU acceptance template for the background
The CMUP-CMX prompt sample term can be written as

\[ P_{\text{sig+bkg}}^{\text{CMX}}(\vec{\alpha}) = N_{nS}^{\text{CMX}} \cdot f_{p,\text{sig}}^{\text{CMX}} \cdot w_{\text{sig}}(\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)^{\text{CMX}}_{\text{sig}}(\vec{\alpha}) + N_d^{\text{CMX}} \cdot s_p^{\text{CMX}} \cdot w_{\text{bkg}}(\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)^{\text{CMX}}_{\text{bkg}}(\vec{\alpha}) \] (6.11)

while the displaced sample for CMUP-CMX displaced sample is

\[ D_{\text{sig+bkg}}^{\text{CMX}}(\vec{\alpha}) = N_{nS}^{\text{CMX}} \cdot (1 - f_{p,\text{sig}}^{\text{CMX}}) \cdot w_{\text{sig}}(\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)^{\text{CMX}}_{\text{sig}}(\vec{\alpha}) + N_d^{\text{CMX}} \cdot w_{\text{bkg}}(\vec{\alpha}, \vec{\lambda}) \cdot (A \times \epsilon)^{\text{CMX}}_{\text{bkg}}(\vec{\alpha}) \] (6.12)

where

- \( N_{nS}^{\text{CMX}} \) is the CMUP-CMX \( \Upsilon(nS) \) signal yield
- \( f_{p,\text{sig}}^{\text{CMX}} \) is the CMUP-CMX signal fraction in the prompt sample
- \( w_{\text{sig}}(\vec{\alpha}, \vec{\lambda}) \) is the polarization fit for the signal (same function and parameters as for CMUP-CMU)
- \( (A \times \epsilon)^{\text{CMX}}_{\text{sig}}(\vec{\alpha}) \) is the CMUP-CMU acceptance template for the signal
- \( N_d^{\text{CMX}} \) is the CMUP-CMX background yield in the displaced sample
- \( s_p^{\text{CMX}} \) is the CMUP-CMX prompt scale factor (the factor of the background yield in the prompt sample compared to in the displaced sample)
- \( w_{\text{bkg}}(\vec{\alpha}, \vec{\lambda}) \) is the polarization fit for the background (same function and parameters as for CMUP-CMU)
- \( (A \times \epsilon)^{\text{CMX}}_{\text{bkg}}(\vec{\alpha}) \) is the CMUP-CMU acceptance template for the background
Combining these terms together, the full angular distribution fit for the signal region becomes

\[
\frac{dN}{d\Omega}(\alpha) = N_{nS}^{CMU} \cdot f_{p,\text{sig}}^{CMU} \cdot w_{\text{sig}}(\bar{\alpha}, \bar{\lambda}) \cdot (A \times \epsilon)^{CMU}_{\text{sig}}(\bar{\alpha}) \\
+ N_{d}^{CMU} \cdot s_{p}^{CMU} \cdot w_{\text{bkg}}(\bar{\alpha}, \bar{\lambda}) \cdot (A \times \epsilon)^{CMU}_{\text{bkg}}(\bar{\alpha}) \\
+ N_{nS}^{CMU} \cdot (1 - f_{p,\text{sig}}^{CMU}) \cdot w_{\text{sig}}(\bar{\alpha}, \bar{\lambda}) \cdot (A \times \epsilon)^{CMU}_{\text{sig}}(\bar{\alpha}) \\
+ N_{d}^{CMU} \cdot w_{\text{bkg}}(\bar{\alpha}, \bar{\lambda}) \cdot (A \times \epsilon)^{CMU}_{\text{bkg}}(\bar{\alpha}) \\
+ N_{nS}^{CMX} \cdot f_{p,\text{sig}}^{CMX} \cdot w_{\text{sig}}(\bar{\alpha}, \bar{\lambda}) \cdot (A \times \epsilon)^{CMX}_{\text{sig}}(\bar{\alpha}) \\
+ N_{d}^{CMX} \cdot s_{p}^{CMX} \cdot w_{\text{bkg}}(\bar{\alpha}, \bar{\lambda}) \cdot (A \times \epsilon)^{CMX}_{\text{bkg}}(\bar{\alpha}) \\
+ N_{nS}^{CMX} \cdot (1 - f_{p,\text{sig}}^{CMX}) \cdot w_{\text{sig}}(\bar{\alpha}, \bar{\lambda}) \cdot (A \times \epsilon)^{CMX}_{\text{sig}}(\bar{\alpha}) \\
+ N_{d}^{CMX} \cdot w_{\text{bkg}}(\bar{\alpha}, \bar{\lambda}) \cdot (A \times \epsilon)^{CMX}_{\text{bkg}}(\bar{\alpha}) \tag{6.13}
\]

where \(w_{\text{sig}}(\bar{\alpha}, \bar{\lambda})\) gives the measured polarization parameters \((\bar{\lambda})\) for the signal and \(w_{\text{bkg}}(\bar{\alpha}, \bar{\lambda})\) measures the background polarization parameters \((\bar{\lambda})\). These polarization functions are either the one-dimensional fit function, described in Section 6.6, or the two-dimensional fit function, given in Section 6.7. The signal polarization parameters, \(\bar{\lambda} = (\lambda_{\theta}, \lambda_{\phi}, \lambda_{\theta\phi})\), are limited to the allowed values between \(-1\) and \(1\) because they represent the polarization of a physical decay. However, the polarization parameters for the background do not represent any physical polarization and are allowed beyond \(-1\) and \(1\).

6.6 One-Dimensional Fit of the Angular Distribution

The angular distribution one-dimensional fit requires fitting three histograms to obtain the three polarization parameters. The polarization function to fit \(\cos \theta\) is given by

\[
w(\cos \theta) = \frac{2}{3 + \lambda_{\theta}} \left(1 + \lambda_{\theta} \cos^{2}(\theta)\right) \tag{6.14}
\]

which allows the measurement of \(\lambda_{\theta}\). Next, the \(\phi\) polarization fit function is

\[
w(\phi) = \frac{2}{3} \left(1 + \frac{2 \lambda_{\phi}}{3 + \lambda_{\theta}} \cos(2\phi)\right) \tag{6.15}
\]
which obtains the value of $\lambda_\phi$. In order to fit for the third polarization parameter, a change of variable must be made. As discussed in Section 2.5, the new variable ($\tilde{\phi}$) is calculated using Equation 2.6, shown below:

\[
\tilde{\phi} = \begin{cases} 
\phi - \frac{3\pi}{4}, & \text{for } \cos \theta < 0 \\
\phi - \frac{\pi}{4}, & \text{for } \cos \theta > 0 
\end{cases} 
\] 

(2.6 revisited)

Finally, using this new variable ($\tilde{\phi}$), the third one-dimensional polarization fit function is

\[
w(\tilde{\phi}) = \frac{2}{3} \left( 1 + \frac{\sqrt{2} \lambda_{\theta\phi}}{3 + \lambda_\phi} \cos \tilde{\phi} \right) 
\]

(6.16)

which is used to measure the value of $\lambda_{\theta\phi}$.

By using the method discussed in Section 6.5, the three polarization parameters ($\lambda_\theta$, $\lambda_\phi$, and $\lambda_{\theta\phi}$) can be measured by using the three polarization fit functions described in this section and fitting the three separate distributions ($\cos \theta$, $\phi$, and $\tilde{\phi}$) using a simultaneous fit. Fits are made separately in each bin of transverse momentum and in each of the three reference frames used in the analysis. The fits to the prompt and displaced samples for both CMUP-CMU and CMUP-CMX for $0 < p_T(\Upsilon) < 2$ GeV/c are shown in the Collins-Soper frame in Figure 6.3, the S-channel helicity frame in Figure 6.5, and the Gottfried-Jackson frame in Figure 6.7. The same plots for $2 < p_T(\Upsilon) < 4$ GeV/c are shown in Figure 6.4 for the Collins-Soper frame, Figure 6.6 for the S-channel helicity frame, and Figure 6.8 for the Gottfried-Jackson frame.
Figure 6.3. $Y(1S)$ 1-D Fit in Collins-Soper Frame for $0 < p_T(Y) < 2$ GeV/c. Prompt sample (black line) with fit (red error bars) and displaced sample (blue line) with fit (green error bars).

Figure 6.4. $Y(1S)$ 1-D Fit in Collins-Soper Frame for $2 < p_T(Y) < 4$ GeV/c. Prompt sample (black line) with fit (red error bars) and displaced sample (blue line) with fit (green error bars).
Figure 6.5. $\Upsilon(1S)$ 1-D Fit in S-channel Helicity Frame for $0 < p_T(\Upsilon) < 2$ GeV/c. Prompt sample (black line) with fit (red error bars) and displaced sample (blue line) with fit (green error bars)

Figure 6.6. $\Upsilon(1S)$ 1-D Fit in S-channel Helicity Frame for $2 < p_T(\Upsilon) < 4$ GeV/c. Prompt sample (black line) with fit (red error bars) and displaced sample (blue line) with fit (green error bars)
Figure 6.7. $\Upsilon(1S)$ 1-D Fit in Gottfried-Jackson Frame for $0 < p_T(\Upsilon) < 2 \text{ GeV}/c$. Prompt sample (black line) with fit (red error bars) and displaced sample (blue line) with fit (green error bars).

Figure 6.8. $\Upsilon(1S)$ 1-D Fit in Gottfried-Jackson Frame for $2 < p_T(\Upsilon) < 4 \text{ GeV}/c$. Prompt sample (black line) with fit (red error bars) and displaced sample (blue line) with fit (green error bars).
6.7 Two-Dimensional Fit of the Angular Distribution

The full angular distribution in two-dimensions is given by

\[
\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2(\theta) + \lambda_\phi \sin^2(\theta) \cos(2\phi) + \lambda_{\theta\phi} \sin(2\theta) \cos(\phi) \\
+ \lambda_\phi^\perp \sin^2(\theta) \sin(2\phi) + \lambda_{\theta\phi}^\perp \sin(2\theta) \sin(\phi)
\]  

(6.17)

which can be simplified to

\[
\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2(\theta) + \lambda_\phi \sin^2(\theta) \cos(2\phi) + \lambda_{\theta\phi} \sin(2\theta) \cos(\phi)
\]  

(6.18)

by combining the interval for the polar and azimuthal angels \((\theta, \phi)\) with \((\theta, -\phi)\) and also combining with \((\theta, \phi)\) with \((\pi - \theta, \pi - \phi)\) \([73,90]\).

The polarization function for the two-dimensional fit is

\[
w(\cos \theta, \phi, \bar{\lambda}) = \frac{1}{3 + \lambda_\theta} \left( 1 + \lambda_\theta \cos^2(\theta) + \lambda_\phi \sin^2(\theta) \cos(2\phi) + \lambda_{\theta\phi} \sin(2\theta) \cos(\phi) \right)
\]  

(6.19)

Using this function and the procedure described in Section 6.5, the polarization parameters \((\lambda_\theta, \lambda_\phi, \text{and} \lambda_{\theta\phi})\) are measured. However, the two-dimensional fit must be projected to one-dimensional histograms for \(\cos \theta\) and \(\phi\) in order to see the fit. The one-dimensional projections for the two-dimensional fits to the prompt and displaced samples for both CMUP-CMU and CMUP-CMX for \(0 < p_T(Y) < 2\) GeV/c can be seen in Figure 6.9 for the Collins-Soper frame, Figure 6.11 for the S-channel helicity frame, and Figure 6.13 for the Gottfried-Jackson frame. For \(2 < p_T(Y) < 4\) GeV/c, the same plots are shown in the Collins-Soper frame in Figure 6.10, the S-channel helicity frame in Figure 6.12, and the Gottfried-Jackson frame in Figure 6.14.

6.8 Systematic Uncertainties

The angular distribution analysis has a few systematic uncertainties to consider. Fitting the background is essential to the procedure for measuring the angular distribution. The prompt scale factor, discussed in Section 6.2, is the ratio of the amount
Figure 6.9. $\Upsilon(1S)$ 2-D Fit in Collins-Soper Frame for $0 < p_T(\Upsilon) < 2$ GeV/c. Prompt sample (black line) with fit (red error bars) and displaced sample (blue line) with fit (green error bars)

Figure 6.10. $\Upsilon(1S)$ 2-D Fit in Collins-Soper Frame for $2 < p_T(\Upsilon) < 4$ GeV/c. Prompt sample (black line) with fit (red error bars) and displaced sample (blue line) with fit (green error bars)
Figure 6.11. $\Upsilon(1S)$ 2-D Fit in S-channel Helicity Frame for $0 < p_T(\Upsilon) < 2$ GeV/c. Prompt sample (black line) with fit (red error bars) and displaced sample (blue line) with fit (green error bars)

Figure 6.12. $\Upsilon(1S)$ 2-D Fit in S-channel Helicity Frame for $2 < p_T(\Upsilon) < 4$ GeV/c. Prompt sample (black line) with fit (red error bars) and displaced sample (blue line) with fit (green error bars)
Figure 6.13. $\Upsilon(1S)$ 2-D Fit in Gottfried-Jackson Frame for $0 < p_T(\Upsilon) < 2$ GeV/c. Prompt sample (black line) with fit (red error bars) and displaced sample (blue line) with fit (green error bars).

Figure 6.14. $\Upsilon(1S)$ 2-D Fit in Gottfried-Jackson Frame for $2 < p_T(\Upsilon) < 4$ GeV/c. Prompt sample (black line) with fit (red error bars) and displaced sample (blue line) with fit (green error bars).
of background in the prompt sample compared to the amount of background in the displaced sample. The prompt scale factor is calculated by fitting the ratio of the background amounts in the prompt and displaced samples by using only the sidebands, which excludes the signal mass regions. This allows a projection of the amount of expected background in the prompt sample in the signal regions. The procedure is done with linear fit to the ratio, and this fit is a source of systematic uncertainty. The fit is repeated using a quadratic fit to determine the prompt scale factor and the angular distribution measurement is repeated. The systematic error is given by any difference in the measured polarization parameters between using the quadratic fit and linear fit.

Another source of systematic uncertainty is the measured efficiency used in the analysis. The efficiency is varied by $\pm \sigma$ from the measured value, and the polarization fit is repeated. The largest difference in the polarization parameters quantifies the systematic uncertainty. The measured systematic error due to the efficiency used is much less than the statistical error on the polarization parameters.

The calculation of the frame invariant parameter ($\tilde{\lambda}$) can measure the systematic error between reference frames. The frame invariant parameter ($\tilde{\lambda}$) is compared between the S-channel helicity frame (SH), the Gottfried-Jackson frame (GJ), and the Collins-Soper frame (CS). The largest difference in the values between frames is recorded as a systematic error. Differences in the measured polarization parameters between the one-dimensional fit and two-dimensional fit also provide another measured systematic uncertainty. Systematic errors for all sources are added in quadrature, and the total systematic uncertainty appears in the results tables in Section 6.9.

6.9 $\Upsilon$ Angular Distribution Results

The fitting procedure described in this chapter is used to measure the $\Upsilon(1S)$ polarization parameters using the one-dimensional fit. The two-dimensional fit is also done and a cross check and is included in the systematic errors. Table 6.2 shows
the polarization parameters for the eight bins of transverse momentum ($p_T$) for the Collins-Soper frame, the S-channel helicity frame, and the Gottfried-Jackson frame. Fit results for the other $\Upsilon(nS)$ states are not shown as the fit procedure heavily relies on the accurately fitting the background polarization. Figure 6.15 shows the polarization parameters in each frame with statistical errors.

Furthermore, the frame invariant parameter ($\tilde{\lambda}$) is also calculated for the $\Upsilon(1S)$ state in the Collins-Soper frame, the S-channel helicity frame, and the Gottfried-Jackson frame. Measuring the frame invariant parameter always another cross check of the fit procedure as it is independent of the reference frame. Table 6.3 shows the frame invariant parameter in the Collins-Soper frame, the S-channel helicity frame, and the Gottfried-Jackson frame for each of the eight bins of transverse momentum ($p_T$). Figure 6.16 shows a comparison of the calculated frame invariant parameters from the three reference frames.
Table 6.2

$\Upsilon(1S)$ Polarization Parameters. Fitted polarization parameters ($\lambda_\theta$, $\lambda_\phi$, and $\lambda_{\theta\phi}$) for $p_T$ bins in Collins-Soper frame, S-channel helicity frame, and Gottfried-Jackson frame. Errors shown are statistical and systematic. (value ± stat. ± syst.)

<table>
<thead>
<tr>
<th>$p_T$ Range ($GeV/c$)</th>
<th>$\lambda_\theta$</th>
<th>Collins-Soper Frame</th>
<th>$\lambda_\phi$</th>
<th>$\lambda_{\theta\phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 - 2.0</td>
<td>$-0.0742 \pm 0.0342 \pm 0.3663$</td>
<td>$-0.0096 \pm 0.0194 \pm 0.0255$</td>
<td>$0.0261 \pm 0.0236 \pm 0.1321$</td>
<td></td>
</tr>
<tr>
<td>2.0 - 4.0</td>
<td>$-0.0626 \pm 0.0237 \pm 0.0594$</td>
<td>$-0.1349 \pm 0.0126 \pm 0.0484$</td>
<td>$0.0156 \pm 0.0167 \pm 0.1397$</td>
<td></td>
</tr>
<tr>
<td>4.0 - 6.0</td>
<td>$-0.084 \pm 0.0272 \pm 0.098$</td>
<td>$-0.1617 \pm 0.0145 \pm 0.0893$</td>
<td>$-0.0056 \pm 0.0188 \pm 0.177$</td>
<td></td>
</tr>
<tr>
<td>6.0 - 8.0</td>
<td>$-0.0345 \pm 0.0361 \pm 0.2588$</td>
<td>$-0.1087 \pm 0.0192 \pm 0.2043$</td>
<td>$0.0225 \pm 0.0237 \pm 0.1512$</td>
<td></td>
</tr>
<tr>
<td>8.0 - 12.0</td>
<td>$-0.0432 \pm 0.0336 \pm 0.2247$</td>
<td>$-0.0755 \pm 0.0187 \pm 0.2089$</td>
<td>$-0.0257 \pm 0.0237 \pm 0.1302$</td>
<td></td>
</tr>
<tr>
<td>12.0 - 17.0</td>
<td>$-0.0712 \pm 0.0378 \pm 0.1234$</td>
<td>$-0.0517 \pm 0.026 \pm 0.1362$</td>
<td>$0.0582 \pm 0.033 \pm 0.0574$</td>
<td></td>
</tr>
<tr>
<td>17.0 - 23.0</td>
<td>$-0.0142 \pm 0.0682 \pm 0.1237$</td>
<td>$-0.0832 \pm 0.0482 \pm 0.1087$</td>
<td>$-0.0787 \pm 0.0634 \pm 0.1504$</td>
<td></td>
</tr>
<tr>
<td>23.0 - 40.0</td>
<td>$-0.0246 \pm 0.1244 \pm 0.2262$</td>
<td>$0.0011 \pm 0.0823 \pm 0.2225$</td>
<td>$0.063 \pm 0.1177 \pm 0.2749$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p_T$ Range ($GeV/c$)</th>
<th>$\lambda_\theta$</th>
<th>S-channel Helicity Frame</th>
<th>$\lambda_\phi$</th>
<th>$\lambda_{\theta\phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 - 2.0</td>
<td>$-0.0769 \pm 0.0318 \pm 0.0942$</td>
<td>$-0.0001 \pm 0.0187 \pm 0.0307$</td>
<td>$-0.0186 \pm 0.0232 \pm 0.05$</td>
<td></td>
</tr>
<tr>
<td>2.0 - 4.0</td>
<td>$-0.2347 \pm 0.0193 \pm 0.0498$</td>
<td>$-0.0759 \pm 0.0122 \pm 0.0469$</td>
<td>$0.0018 \pm 0.0157 \pm 0.0542$</td>
<td></td>
</tr>
<tr>
<td>4.0 - 6.0</td>
<td>$-0.3283 \pm 0.0223 \pm 0.0886$</td>
<td>$-0.0979 \pm 0.0139 \pm 0.0796$</td>
<td>$0.0092 \pm 0.0174 \pm 0.0766$</td>
<td></td>
</tr>
<tr>
<td>6.0 - 8.0</td>
<td>$-0.2731 \pm 0.0322 \pm 0.2371$</td>
<td>$-0.0632 \pm 0.0182 \pm 0.1136$</td>
<td>$-0.0067 \pm 0.022 \pm 0.1292$</td>
<td></td>
</tr>
<tr>
<td>8.0 - 12.0</td>
<td>$-0.2155 \pm 0.0333 \pm 0.2364$</td>
<td>$-0.0476 \pm 0.0184 \pm 0.0928$</td>
<td>$-0.0117 \pm 0.0224 \pm 0.0915$</td>
<td></td>
</tr>
<tr>
<td>12.0 - 17.0</td>
<td>$-0.1275 \pm 0.048 \pm 0.1525$</td>
<td>$-0.0531 \pm 0.0259 \pm 0.0575$</td>
<td>$-0.0071 \pm 0.033 \pm 0.0594$</td>
<td></td>
</tr>
<tr>
<td>17.0 - 23.0</td>
<td>$-0.0841 \pm 0.0873 \pm 0.1298$</td>
<td>$-0.0183 \pm 0.0474 \pm 0.1069$</td>
<td>$-0.106 \pm 0.0621 \pm 0.1376$</td>
<td></td>
</tr>
<tr>
<td>23.0 - 40.0</td>
<td>$0.121 \pm 0.161 \pm 0.2253$</td>
<td>$-0.0153 \pm 0.0923 \pm 0.2253$</td>
<td>$-0.1555 \pm 0.1185 \pm 0.2484$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p_T$ Range ($GeV/c$)</th>
<th>$\lambda_\theta$</th>
<th>Gottfried-Jackson Frame</th>
<th>$\lambda_\phi$</th>
<th>$\lambda_{\theta\phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 - 2.0</td>
<td>$-0.0757 \pm 0.034 \pm 0.1217$</td>
<td>$-0.0065 \pm 0.0193 \pm 0.0252$</td>
<td>$-0.002 \pm 0.0236 \pm 0.0286$</td>
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</tr>
<tr>
<td>2.0 - 4.0</td>
<td>$-0.1181 \pm 0.0233 \pm 0.0628$</td>
<td>$-0.1338 \pm 0.0124 \pm 0.0535$</td>
<td>$0.0053 \pm 0.0166 \pm 0.0496$</td>
<td></td>
</tr>
<tr>
<td>4.0 - 6.0</td>
<td>$-0.1789 \pm 0.027 \pm 0.1272$</td>
<td>$-0.1581 \pm 0.0143 \pm 0.0766$</td>
<td>$0.0509 \pm 0.0187 \pm 0.0911$</td>
<td></td>
</tr>
<tr>
<td>6.0 - 8.0</td>
<td>$-0.1077 \pm 0.0356 \pm 0.2052$</td>
<td>$-0.1031 \pm 0.019 \pm 0.1476$</td>
<td>$0.0492 \pm 0.0233 \pm 0.1876$</td>
<td></td>
</tr>
<tr>
<td>8.0 - 12.0</td>
<td>$-0.1101 \pm 0.0351 \pm 0.0932$</td>
<td>$-0.0692 \pm 0.0187 \pm 0.127$</td>
<td>$0.053 \pm 0.0231 \pm 0.1633$</td>
<td></td>
</tr>
<tr>
<td>12.0 - 17.0</td>
<td>$-0.0944 \pm 0.0472 \pm 0.0789$</td>
<td>$-0.0584 \pm 0.0256 \pm 0.0717$</td>
<td>$0.0062 \pm 0.0334 \pm 0.1041$</td>
<td></td>
</tr>
<tr>
<td>17.0 - 23.0</td>
<td>$-0.0026 \pm 0.0867 \pm 0.118$</td>
<td>$-0.0485 \pm 0.0473 \pm 0.1071$</td>
<td>$0.1326 \pm 0.0636 \pm 0.1191$</td>
<td></td>
</tr>
<tr>
<td>23.0 - 40.0</td>
<td>$0.0179 \pm 0.1546 \pm 0.2667$</td>
<td>$-0.0582 \pm 0.088 \pm 0.2227$</td>
<td>$0.1509 \pm 0.1187 \pm 0.2885$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 6.15. $\Upsilon(1S)$ Polarization Parameters. Fitted polarization parameters ($\lambda_\theta$, $\lambda_\phi$, and $\lambda_{\theta,\phi}$) for $p_T$ bins in Collins-Soper frame, S-channel helicity frame, and Gottfried-Jackson frame. Only statistical errors are shown.
Table 6.3

Υ(1S) Frame Invariant Parameter. Frame invariant parameter ($\tilde{\lambda}$) for $p_T$ bins in Collins-Soper frame, S-channel helicity frame, and Gottfried-Jackson frame. Errors shown are statistical and systematic.

(\text{value} \pm \text{stat.} \pm \text{syst.})

<table>
<thead>
<tr>
<th>$p_T$ Range (GeV/c)</th>
<th>Collins-Soper Frame</th>
<th>S-channel Helicity Frame</th>
<th>Gottfried-Jackson Frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 - 2.0</td>
<td>$-0.1021 \pm 0.0715 \pm 0.374$</td>
<td>$-0.0772 \pm 0.0693 \pm 0.1427$</td>
<td>$-0.0946 \pm 0.0717 \pm 0.1132$</td>
</tr>
<tr>
<td>2.0 - 4.0</td>
<td>$-0.4117 \pm 0.0352 \pm 0.0551$</td>
<td>$-0.4298 \pm 0.036 \pm 0.0531$</td>
<td>$-0.4583 \pm 0.0347 \pm 0.1085$</td>
</tr>
<tr>
<td>4.0 - 6.0</td>
<td>$-0.49 \pm 0.0397 \pm 0.0893$</td>
<td>$-0.5665 \pm 0.0405 \pm 0.0774$</td>
<td>$-0.5641 \pm 0.0395 \pm 0.1163$</td>
</tr>
<tr>
<td>6.0 - 8.0</td>
<td>$-0.3253 \pm 0.0603 \pm 0.2099$</td>
<td>$-0.4351 \pm 0.0618 \pm 0.1731$</td>
<td>$-0.3781 \pm 0.0604 \pm 0.1331$</td>
</tr>
<tr>
<td>8.0 - 12.0</td>
<td>$-0.2507 \pm 0.0605 \pm 0.2632$</td>
<td>$-0.342 \pm 0.0647 \pm 0.1929$</td>
<td>$-0.2971 \pm 0.0633 \pm 0.2129$</td>
</tr>
<tr>
<td>12.0 - 17.0</td>
<td>$-0.2152 \pm 0.0767 \pm 0.2091$</td>
<td>$-0.2724 \pm 0.0852 \pm 0.1522$</td>
<td>$-0.2546 \pm 0.0832 \pm 0.1674$</td>
</tr>
<tr>
<td>17.0 - 23.0</td>
<td>$-0.2435 \pm 0.1276 \pm 0.109$</td>
<td>$-0.1366 \pm 0.1633 \pm 0.1286$</td>
<td>$-0.1413 \pm 0.1522 \pm 0.1134$</td>
</tr>
<tr>
<td>23.0 - 40.0</td>
<td>$-0.0211 \pm 0.2716 \pm 0.2296$</td>
<td>$0.0741 \pm 0.3128 \pm 0.2362$</td>
<td>$-0.1482 \pm 0.2715 \pm 0.2839$</td>
</tr>
</tbody>
</table>

Figure 6.16. Comparison of Υ(1S) Frame Invariant Parameter ($\tilde{\lambda}$) for $p_T$ bins in Collins-Soper frame (blue squares), S-channel helicity (red circles) frame, and Gottfried-Jackson frame (green triangles). Only statistical errors are shown.
7. CONCLUSION

Learning more about quarkonium production has been a goal in high energy physics for over twenty years. Several theories are able to explain quarkonium production but disagree on the expected polarization of the Υ meson. The bottomonium system with the \( \Upsilon \rightarrow \mu^+ \mu^- \) decay is useful in differentiating between production theories at lower transverse momentum than the charmonium system. Accurate measurements of the angular distribution and using the measured polarization in the measurement of the cross section can help to provide insight to the quarkonium production mechanism.

In this analysis, the angular distribution of the \( \Upsilon (1S) \) has been measured using a one-dimensional fit, and cross checked with a two-dimensional fit. The angular distribution has been measured in three separate reference frames, and the frame invariant parameter (\( \tilde{\lambda} \)) has also been calculated. The \( \Upsilon (1S) \) state is found be generally unpolarized in all three reference frames.

The measured \( \Upsilon (1S) \) polarization parameters were then used to re-weight the angular distribution of the Monte Carlo to match the data. Matching the angular distribution of the Monte Carlo to the angular distribution in data allowed the systematic uncertainty due to the polarization to be reduced. The angular distribution of the \( \Upsilon (2S) \) and \( \Upsilon (3S) \) states were not measured, and so unpolarized Monte Carlo was used for the acceptance calculations.

A procedure to calculate the probability that a wire in the muon chambers was dead was used to help remove dead wires from the detector acceptance. This helped to reduce time dependence observed in the cross section. The detector reconstruction efficiency was also measured as a function of run period by reconstructing \( J/\psi \) mesons in a separate data sample. Any remaining time dependent effects in the cross section may be due to the measurement of the luminosity.
The yields of the three ϒ(nS) states were fit in data for both the CMUP-CMU and CMUP-CMX trigger paths. Finally, using the acceptance and efficiency in Monte Carlo as well as the efficiency in data with the measured ϒ(nS) yields, the cross section was calculated in bins of transverse momentum. The ϒ(nS) cross section was measured for all three ϒ(nS) states using CDF Run II data at √s = 1.96 TeV.
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Michael Meier grew up in a small town in Oklahoma and attended Laverne High School in Laverne, Oklahoma. Michael attended the University of Oklahoma in Norman, Oklahoma and pursued a degree in Engineering Physics with a design sequence in Aerospace Engineering. Michael then attended Purdue University in West Lafayette, Indiana to pursue a graduate degree in Physics.