A DYNAMIC TIRE FORCE
MEASURING SYSTEM

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Joint
Highway
Research
Project

PURDUE UNIVERSITY
LAFAYETTE INDIANA

by

C.C. WILSON

Progress Report

A DYNAMIC TIRE FORCE MEASURING SYSTEM

TO: K. B. Woods, Director
Joint Highway Research Project

March 6, 1964

FROM: H. L. Michael, Associate Director
Joint Highway Research Project

Project: C-36-52F
File: 6-20-6

The attached Progress Report entitled "A Dynamic Tire Force Measuring System" by C. C. Wilson, Graduate Assistant on our staff, is submitted for the record. The report is on the Stresses and Deflections HPS project and will also be submitted to the State Highway Commission and the Bureau of Public Roads for their review and comments. The portion of the study covered by this report was performed under the direction of Prof. B. E. Quinn of the School of Mechanical Engineering.

Mr. Wilson also used this research for his Ph.D. degree which he will receive in June 1964. The report outlines the system which will be used for measuring the dynamic tire forces applied to the pavement in the field phase of the Stresses and Deflections project.

The report is presented for the record.

Respectfully submitted,

Harold L. Michael

Harold L. Michael, Secretary

HLM:bc

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Progress Report

A DYNAMIC TIRE FORCE MEASURING SYSTEM

by

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Project No: C-36-52F

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ABSTRACT

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The study of the dynamic interaction between vehicles and pavements is a relatively new field which has received increased attention in the last few years. Particular interest has been focused upon the development of instrumentation to measure the dynamic forces which occur between the tires of a vehicle and the pavement on which the vehicle is traveling.

One of the methods which has been used to give an indication of dynamic force is the measurement of the tire inflation pressure. An important basic problem associated with using tire pressure as a measure of dynamic force is that of designing the pressure measuring system in such a manner that the dynamic effects of the measuring system do not obscure the signal caused by the dynamic force input. Only if these dynamic effects are minimized can the tire pressure serve as an accurate indication of dynamic tire force.

In this investigation, a mathematical model was developed which identified the physical parameters controlling the dynamic behavior of the system. Conclusions as to the dimensions and the arrangement of various components of the

pressure measuring system were obtained from the theory. On the basis of these conclusions, a pressure measuring system was designed and constructed. Experiments conducted with the system verified the theoretical considerations.

Another basic problem associated with pressure measuring systems of this type is the elimination of gradual pressure changes which are not a function of the dynamic force. A comparison of two basically different methods of doing this was made. A continuous pressure equalization method was recommended and was therefore used in the pressure measuring system that was constructed.

In order to calibrate a pressure measuring system for the measurement of dynamic tire force, dynamic calibration methods are essential. Several methods of dynamic calibration are discussed.

As a measure of the adequacy of the calibration procedures, the result of applying a single calibration factor F/P (lb/psi) to pressure measurements is compared with the frequency dependent calibration relationship $F/P(f)$. The differences between the two methods were small enough to indicate that the use of a single calibration factor gives satisfactory results.

The pressure measuring system was used to measure dynamic forces in the pavement evaluation study of the National Cooperative Highway Research Program (NCHRP 1-2). Various methods of presenting dynamic tire force data from pavement tests are presented.

The results of this study indicate that useful measurements of dynamic tire forces by the use of a tire pressure measuring system can be made if the pressure measuring system is properly designed and calibrated.

CHAPTER I

SURVEY OF DEVELOPMENTS IN MEASURING DYNAMIC TIRE FORCE

The study of the dynamic interaction between vehicles and pavements is a relatively new field which has received increased attention in the last few years. Particular interest has been focused upon the development of instrumentation to measure the dynamic forces which occur between the tires of a vehicle and the pavement on which the vehicle is traveling. Before considering several methods of measuring dynamic force, the term "dynamic force" shall be defined.

When a vehicle is sitting motionless on a level pavement, the forces which interact between the vehicle and the pavement are due only to the weight of the vehicle and therefore are vertical forces. The component of the weight appearing at one tire is called the static tire force. When a vehicle begins to move, the interacting forces are no longer the static values because now additional forces are generated. Forces which are largely horizontal in direction may be generated as the vehicle moves by such causes as acceleration or braking of the

vehicle, steering action of the driver and wind loads on the vehicle. Forces which are largely vertical in direction are generated by motions of the vehicle which are induced by the unevenness of the pavement. These vertical forces are called dynamic tire forces (or just dynamic forces) and are defined with respect to the static tire force. An illustration of the relationship between static and dynamic forces is shown in Figure 1. It should be noted that the total vertical force exerted on the pavement is the sum of the static and dynamic tire forces. In other words, the dynamic force at any instant is defined as the change of force from the static value caused by the induced motion of the vehicle.

Since the study of dynamic forces is a relatively new field of interest, there is not a large body of literature about the subject. In particular, only a few investigators have reported on the measurement of dynamic forces because the technology of these measurements is just developing.

Various dynamic force measuring methods have been studied by investigators in this country. In 1957, Hopkins and Boswell¹ compared three different methods which were based upon the following techniques:

1. measurement of strain of the axle housing,
2. measurement of the sidewall deflection of the tire and
3. measurement of tire pressure.

---

1. Superscripts refer to references listed in the Bibliography.

Note: The total force of the tire upon the road at any time is the sum of the dynamic tire force and the static wheel load.

FIGURE 1. ILLUSTRATION OF THE RELATIONSHIP BETWEEN STATIC AND DYNAMIC FORCES

In spite of calibration difficulties, the measurement of tire pressure was recommended as the most practical method. To measure the tire pressure, they attached a pressure transducer (0 - 50 psig range) directly to the valve core of the tire and thus the pressure transducer rotated with the wheel. The electrical signal from this transducer was transferred to the recording instruments by means of five slip rings and brushes at the wheel. The resistance between the brushes and the slip rings was different depending on whether the slip rings were stationary or in motion. The investigators were then unable to calibrate their instrument statically with an electronic scale as they had desired, but could only compare the forces determined from tire pressure measurements with those determined from axle strain and sidewall tire deflection.

Three recommendations were made for improving the pressure system:

1. Use better quality slip rings.
2. Use a differential tire pressure transducer which should give a strong signal output compared to any slip ring noise.
3. Use a rotating pressure slip joint to eliminate the electrical slip rings between the pressure transducer and the recording equipment.

During the same period of time, investigators² at General Motors Proving Ground were working on a tire pressure

measuring system. This project was initiated after a meeting of the American Association of State Highway Officials (AASHO) Instrumentation Group in November 1955 when a suggestion was made that it might be feasible to measure dynamic load by observing tire air pressure variations. Their system made use of the differential pressure transducer and rotating joint suggested by Hopkins and Boswell. A static calibration was made by raising the truck off the scale by means of a jack and recording force and pressure as the static weight was lifted from the scale. Two methods of dynamic calibration were also used. In one method, the axle was raised by the jack and then the wheel was dropped onto the electronic scale by collapsing the jack mounting. In the second method, a one ton steel weight was dropped into the bed of the truck. In each case, both force and tire pressure were recorded. The information on these tests, which was made available to the Vehicle Dynamics Research Group at Purdue University, also included typical road records. Mention was made in the report of an "acoustical ring" which had been observed in the tire pressure records. The road records which were sent to Purdue University for study indicated that the "ring" was present.

Several German engineering schools were also investigating different methods for determining dynamic wheel loads during this period. In 1959, O. Bode and others³ reported on a comparison of methods developed at Aachen, Brunswick, Hanover, Darmstadt, and Munich. Of the six devices used,

only four of them were basically different. The Brunswick and Munich groups both measured tire deflection and then converted the tire deflections to force by using an experimentally determined load versus deflection curve for the tire. The methods of measuring the tire deflection, however, were quite different. In one method investigated at Brunswick, variations in distance between the axle and the road surface were measured by a capacitive device. Another method involved the measurement of tire bulging with an electric scanning device. The Munich investigators also measured the transverse bulging of the tire caused by the vertical tire deflection, but used two mechanical feelers in such a manner that the effect of side forces was largely eliminated.

The Aachen group measured accelerations of the axle and superstructure and used these accelerations to compute dynamic force. The device which was developed at Hanover measured the elastic deformations of the axle housing. The relationship between the dynamic wheel loads and these deformations was used to determine the dynamic forces.

All of the methods mentioned thus far give continuous traces of dynamic force. The Darmstadt group, however, developed a special test wheel in which the bending strains can be measured at two points in the rim base. This device only gives a correct indication of the loading twice for every revolution of the wheel. Therefore, the resulting sample of dynamic force can only be used for making statistical estimates of the dynamic forces which occurred.

It is interesting to note that, of the five different groups, none chose to measure the dynamic force by using tire pressure measurements.

Still other groups^{4,5} in Germany have reported methods of measuring dynamic force; but, in general, the methods were similar to those mentioned above and therefore will not be discussed in further detail.

In 1959, investigators^{6,7} in the United States who were conducting dynamic load tests on the AASHO Road Test, made use of tire pressure measurements as an indication of dynamic force. These tire pressure measurements were made on large two-and three-axle vehicles. The dynamic force measurements were to be used in conjunction with strain and deflection measurements of the bridges. Calibration difficulty was encountered as indicated by the following quotation:

"Additional uncertainties were associated with the scatter in the calibration curves, with drifting of the pressure records due to loss of air pressure in the tire pressure equipment, and with inaccuracies of the time scales on the records. Thus, quantitative evaluations of the tire and spring deflection records had to be considered as approximate only."⁶

The calibration methods which were developed for the AASHO Road Test⁷ are certainly of interest because of the

size of the vehicles which were used. These two- and three-axle vehicles are the largest to be used in any dynamic testing program reported in the literature. The pressure measuring equipment was calibrated by use of a mechanical oscillator mounted in the bed of the truck which induced vertical motion of the vehicle, and an electronic scale that measured the force at the tire surface. The oscillator was capable of exciting vertical motions of the vehicle in a frequency range which included the body motion frequencies of the vehicles (3 to 4 cps) but was limited to a maximum frequency of 8 to 10 cps. The calibration results from these steady state sinusoidal tests were reported as successful.

Impulsive tests were also conducted by driving the measuring wheel off a ramp onto the electronic scale and then comparing the signal from the pressure measuring system with that of the electronic scale. The results of these tests were reported as very erratic and not in agreement with the sinusoidal tests.

(Since dynamic forces of higher frequencies than 8 to 10 cps are induced under actual road testing conditions, it is desirable to be able to explain such discrepancies and, if possible, control them by proper design of the pressure measuring system. Possible causes of such behavior are discussed in Chapter IV.)

During the same period of time as the AASHO Tests, another group interested in measuring dynamic forces was working at the Michigan State Highway Department Laboratories at Lansing, Michigan. Although publication of their

work is unknown, these investigators were very generous with the results of their research. In 1961, members of the Vehicle Dynamics Research Group at Purdue University were invited to the Lansing Laboratories to inspect the pressure measuring system. In their system, the differential pressure gage and reference tank were conveniently mounted on a board which could be easily attached to their vehicle. This measuring system was adapted to the Purdue University test vehicles and was the basic type of system studied in this investigation. The components of this system are the same as those of the pressure measuring system used on the AASHO Road Test⁷. Since this basic configuration of pressure measuring system was the most satisfactory system available, it was the logical type to use for further investigation.

In addition to the recommendation of Hopkins and Boswell¹ there are certain advantages which encourage the use of tire inflation pressure as a means of measuring dynamic tire force. One of the advantages of this method is the proximity of the measuring device to the variable being measured. Since the force between the pavement and the tire tread is to be measured, the best location for a transducer would be in the tire tread. No known method has been developed which is capable of measuring the force in the tread. Although a measurement at the outer surface of the tread is not feasible, a measurement just at the back of the tread is possible by measuring the air pressure in the tire. Any device which is located

on the axle (such as a strain gage) has additional complications caused by the inertia of the components between the axle and the surface of the highway. In measuring the air pressure in the tire, the only inertia being neglected is that of the tire tread.

Another advantage of the tire pressure measuring system is that it is relatively easy to mount on a vehicle. The measuring tire and wheel, including rotating joint, can be previously balanced and assembled; therefore, the wheel need only to be mounted and the instrument board attached to the vehicle to be ready for testing. A more detailed description of this equipment will be given later.

A third advantage is the extreme sensitivity of the pressure measuring system. A dynamic force of ten pounds may be easily detected.

In order to use tire inflation pressure as a successful method of determining dynamic tire force, there are certain disadvantages which must be overcome. For example, it is important to recognize that the force which is being measured is the composite force that occurs over the area of contact between the tire tread and the pavement. The surface of the pavement must therefore be even enough that the tire tread can stay continuously in contact with the pavement in order that the indication of this force by the pressure change be valid. In other words the continuous area of contact which is maintained during the calibration tests, must be maintained during road tests in order for

the calibration relationship between force and pressure to be identical for both tests. This limitation of the method must be kept in mind if test data from faulted pavement are to be correctly evaluated.

Another problem which arises in using a pressure measuring system is that of the heating and cooling of the tire which causes relatively large pressure changes. Leakage, which has the same effect as a temperature change in the tire, is also a problem since a seemingly insignificant loss of air in the system can cause a measurable pressure change. Two methods of equalizing unwanted pressure changes are discussed in Chapter III.

A sometimes unrecognized problem in the design of a pressure measuring system is the possibility of a resonant condition resulting from a natural frequency of a component of the system being excited by the dynamic force. It is possible that mechanical as well as pneumatic oscillations could occur if any of the natural frequencies of these dynamic systems are within the range of the frequencies of the dynamic force. Pressure oscillations of the pneumatic system are considered in Chapter II and mechanical vibrations are discussed in Chapter III.

The most difficult problem in using a pressure measuring system is the determination of an accurate calibration relationship between the dynamic force and the corresponding pressure change. As other investigators have

indicated, it is possible to get quite different calibration relationships according to the method of calibration. Information as to the source of this erratic behavior could lead to the design of pressure measuring systems which would minimize this problem. The analysis and design of a pressure measuring system are considered in Chapters II and III respectively. Various calibration methods are discussed in Chapter IV.

The analysis of the pressure measuring system, which is discussed in the following chapter, was made in order to determine the design parameters which control the accuracy of the measurement of dynamic force. With these parameters known, pressure measuring systems may be designed for different types of vehicles with a minimum amount of time spent in changing the design for each new vehicle.

CHAPTER II

BASIC THEORY UNDERLYING THE USE OF TIRE INFLATION PRESSURE AS AN INDICATION OF DYNAMIC TIRE FORCE

At this point, it is in order to consider the construction of a typical pressure measuring system as shown schematically in Figure 2. It should be noted that the tire is an integral part of such a system. The force to be measured acts at the contact surface between the tire tread and the pavement, while the pressure signal is measured at the transducer.

The operation of this pressure system is accomplished in the following manner. Prior to making measurements, valve number 1 connected to the reference pressure tank is opened. Valve number 2, connected to the tire, is then opened thus establishing the static tire pressure in all parts of the pressure measuring system. In making the pressure measurements, valve number 1 is closed, thus subjecting side B to the original tire pressure established in the reference tank. As the vehicle moves down the highway, the pressure transducer responds to the pressure difference and transmits this information in the form of an electrical signal to an amplifier and a recording oscillograph.

FIGURE 2. SCHEMATIC DRAWING OF A PRESSURE MEASURING SYSTEM

The purpose of the normally-closed solenoid-operated valve is apparent when one considers the effect of heating or cooling of the tire. Any heating or cooling of the air in the tire will produce a difference in pressure relative to the original pressure introduced into the tank. The solenoid-operated valve provides a method for quickly equalizing this unwanted pressure difference.

Before a theoretical analysis is undertaken, it is well to consider several questions which should be answered in order that an accurate force transducer be designed. Of immediate concern is the following question. What kind of analysis is necessary to determine the important parameters of the pressure measuring system? In other words, what kind of mathematical model should be set up to explain the behavior of the pressure measuring system?

Once the behavior is explained, how much control can be exerted by proper design of the system? That is, can the important parameters be controlled within the physical constraints of the components of the system? Can the effects caused just by the tire (over which there is no control) be isolated from those effects from other components of the system over which there may be some control?

What type of calibration procedure is required to completely define the characteristics of the system? Is a static calibration satisfactory or is a dynamic calibration always required?

What are the basic limitations of using a pressure measuring system as a force transducer? What accuracy can be expected in predicting the dynamic force which occurs at the tire tread?

Can the transducer be applied to larger vehicles without major changes? Are there any specific difficulties which might be anticipated from the analysis? -

Finally, how may a summary of data recorded by the device be presented in order to have a measure of the dynamic force which could illustrate the effect that other variables such as speed have on the dynamic force? These questions will now be considered in detail.

It is instructive to trace a signal through the system. An increase in force at the tire surface causes a decrease in the volume of the tire. The pressure in the tire then increases which in turn initiates a pressure wave which travels with acoustic velocity through the tube and the rotating joint up to side A of the pressure transducer which is one of three parallel cavities connected to the main tube. The portion of the pressure wave which reaches the differential pressure transducer is then converted to an electrical signal by the transducer and is recorded by an oscillograph. As the wave reaches the terminal point of each cavity, it is reflected, and the magnitude of the reflection depends upon the manner in which the cavity is terminated. The reflected waves recombine in the common tube and travel back to the tire.

This process of wave motion continues until the waves are finally dissipated by friction.

It is important to realize the portion of the system over which control may be exerted. The dynamic force shown in Figure 2 causes a pressure change "p" in the tire cavity. It is this pressure change "p" which should be measured as accurately as possible. Unfortunately, as soon as any connections are made to the tire in order to measure the magnitude of this pressure change, a dynamic problem arises because the pressure transducer which measures at point "A" is at one end of a pneumatic system and the pressure "p" which is to be measured is at the other end. The dynamic properties of this pneumatic system definitely affect the magnitude of the pressure signal which is recorded by the pressure transducer at point "A" in Figure 2. The principal problem, then, is to find the controlling parameters of this pneumatic system which is connected to the tire and then to design a system which will minimize the undesirable dynamic effects. Accordingly, the first step in the analysis will be to determine the parameters which govern the dynamic behavior of the system.

In Figure 3, a schematic drawing is presented in which is shown a cross-sectional view of the active side of the pressure measuring system (valve number 1 and the solenoid-operated valve are closed during operation). The schematic drawing in Figure 3 illustrates the relative volume of the components of the system, but it does not show the changes

FIGURE 3. SCHEMATIC DRAWING OF THE ACTIVE SIDE OF THE SYSTEM

in the internal dimensions in the fittings, rotating joint, and valves. The cross-sectional areas of the passageways of the system change at the connections of each of the components just mentioned, and are by no means as constant as might be indicated by the schematic drawing. A full-scale drawing of valve number 2 with its fittings is shown beside the schematic to illustrate this point.

In order to predict the dynamic behavior of the air in the system, a suitable mathematical model must be determined. The complexity of the model which would completely describe this dynamic behavior cannot be fully appreciated until one considers the solution of a much simpler system.

For example, the system shown in Figure 4 has been studied by a number of investigators. Note that there is only one volume (the receiver volume) which is considered and that the cross-sectional area of the tubing is constant throughout. Schuder⁸ derived equations for the pressure-time relationship at the receiver end of this system, given that a sudden (step) increase of pressure had occurred at the sending end. The two basic differential equations which govern transient flow in pipes were used in this derivation. They are

$$\frac{\partial u}{\partial x} = -\frac{1}{\rho c^2} \frac{\partial p}{\partial t} \quad (1)$$

$$\frac{\partial p}{\partial x} = -\rho \frac{\partial u}{\partial t} - Ru \quad (2)$$

FIGURE 4. DIAGRAM OF A SIMPLE SYSTEM

where: u = fluid velocity
 x = distance from sending end
 ρ = fluid density
 c = acoustic velocity
 t = time
 p = pressure
 R = tubing resistance.

These equations are based on the conservation of mass, energy, and momentum for the one-dimensional, adiabatic flow of a compressible fluid with friction. Their derivation is included in an Appendix to Schuder's thesis. The tubing resistance R was taken as a constant and was evaluated on the assumption of fully developed laminar flow. Pressure and temperature changes were assumed to be small so that the density and the acoustic velocity could be treated as constants and evaluated at the initial temperature and mean pressure of the system.

By assuming laminar flow, the tubing resistance R was obtained from the Hagen-Poiseuille law as follows:

$$\delta p = \frac{32\mu u L}{d^2} = RuL, \quad R = 32\mu/d^2 \quad (3)$$

where: d = the inside diameter of the tubing (in.²)
 μ = the dynamic viscosity of the fluid (lb.sec./in.²)
 u = the velocity of the fluid (in./sec.)
 L = the length of tube (in.)
 R = the tubing resistance (lb.sec./in.⁴).
 δp = pressure change in length L , (psi)

The resulting solution for the pressure at the receiving end of the tube as a function of time was

$$p(L,t) = p_m - 2p_m e^{-Rt/2\rho} \sum_{+a_1}^{+a_\infty} \left[\cos \frac{\Theta t}{2} + \frac{R}{\rho \Theta} \sin \frac{\Theta t}{2} \right] \frac{a \left[\left(\frac{Q}{aL} + 1 \right) \sin a + \frac{Qa}{aL} \cos a \right]}{a} \quad (4)$$

where

$$\Theta = \sqrt{\left(\frac{2ac}{L} \right)^2 - \left(\frac{R}{\rho} \right)^2} \quad (5)$$

and a is obtained by solving the equation

$$a \tan a = aL/Q. \quad (6)$$

The variables which were not defined previously are illustrated in Figure 4. Schuder obtained values for a_1 through a_6 by solving equation (6) graphically.

As Schuder observed, the series converges very slowly for systems which overshoot (see Figure 4) and therefore the solution (4) can become very unwieldy when applied to an oscillatory system. Also, the effects of varying the system parameters Q , L , and R are not easily observed because of the complexity of the solution.

In order to make this solution less cumbersome and therefore more useful, another investigator⁹ derived an approximate solution from Schuder's work which is valid if the Laplace operator " s " is small enough. Using the approximation, the equations describing the system were

reduced to the form of a second-order linear differential equation. As a result, expressions for the damping ratio,

$$r = 1/2 \frac{RL}{\rho c} \sqrt{1/2 + Q/aL} \quad (7)$$

and the undamped natural angular frequency,

$$\omega = \frac{c/L}{\sqrt{1/2 + Q/aL}} \quad (8)$$

were obtained.

The particular advantage of this approximate solution is that the parameters involved in r and ω are completely identified with the physical system. Conclusions concerning the effect of changing these parameters may be easily reached.

Another method of analyzing the system shown in Figure 4 was used by an investigator¹⁰ at the National Bureau of Standards. In this theoretical investigation, an oscillatory pressure was applied to the sending end of the tube and the attenuation and lag of that signal as it appeared at the receiving end was calculated. These attenuation factors and lag values are conveniently presented by graphs of dimensionless parameters for the use of the designer of pneumatic instrument lines. If a designer had

an application which matched the system shown in Figure 4 well enough, he could pick the diameter of tubing which would result in minimum attenuation and delay by using the information in this paper.

All of the preceding discussion was concerned with a system with a long straight tube ending in a single receiving volume. Referring to Figure 3, it is not apparent as to how the theory just discussed might be applied to the pressure measuring system shown there. In fact, it is evident that the theory does not apply because of the number of cavities and passageways which make up the pressure measuring system.

In 1878, Lord Rayleigh¹¹ considered the dynamics of a more complicated system which consisted of two chambers and three passageways as shown in Figure 5. In his analysis, the air in each passageway was assumed to act as an incompressible fluid and therefore to behave as if it were a rigid mass. The inertia of the air in the chambers was considered to be negligible and the density was assumed to be uniform. As a result, the air in these chambers was assumed to act purely as a spring. The equivalent spring-mass system is shown in Figure 5 below the appropriate section of the system.

It is interesting to note Lord Rayleigh's comment concerning this simplification:

"In flowing through the aperture under the operation of a difference of pressure on the

FIGURE 5. A MORE COMPLEX SYSTEM CONSIDERED BY RAYLEIGH AND THE EQUIVALENT SPRING-MASS SYSTEM

two sides, or in virtue of its own inertia after such pressure has ceased, the air moves approximately as an incompressible fluid would do under like circumstances provided that the space through which the kinetic energy is sensible be very small in comparison with the length of the wave. The suppositions on which we are about to proceed are not of course strictly correct as applied to actual resonators such as are used in experiment, but they are near enough to the mark to afford an instructive view of the subject and in many cases a foundation for a sufficiently accurate calculation of the pitch. They become rigorous only in the limit when the wave-length is indefinitely great in comparison with the dimensions of the vessel."

In the case of the pressure measuring system, the lumped parameter system model is certainly instructive as it indicates the role of the different components of the system.

Consideration of the system shown in Figure 5 will illustrate the value of this approach. The kinetic energy of the air in the passageways may be expressed as

$$K.E. = 1/2 \rho \left[\frac{\dot{x}_1^2}{a_1/L_1} + \frac{\dot{x}_2^2}{a_2/L_2} + \frac{\dot{x}_3^2}{a_3/L_3} \right] \quad (9)$$

where: ρ = the mass density of the air (lb.sec.²/in.⁴)

K.E. = the kinetic energy (in-lb)

$\dot{X}$ = the volumetric rate of flow of the air (in.³/sec.)

a = the cross-sectional area of a passageway (in.²)

L = the length of a passageway (in.).

The kinetic energy of the spring-mass system may be expressed as

$$\text{K.E.} = 1/2 M_1 \dot{X}_1^2 + 1/2 M_2 \dot{X}_2^2 + M_3 \dot{X}_3^2 \quad (10)$$

where: M = mass(lb-sec²/in)

$\dot{X}$ = the velocity of a mass(in/sec).

The potential energy stored in a single chamber during a compression or expansion of the air in the chamber may be expressed by

$$\text{P.E.} = 1/2 \rho c^2 \left[\frac{(\text{change in volume})^2}{\text{volume}} \right] \quad (11)$$

Therefore the potential energy stored by the air in the chambers shown in Figure 5 may be expressed by

$$\text{P.E.} = 1/2 \rho c^2 \left[\frac{(X_2 - X_1)^2}{Q_2} + \frac{(X_3 - X_2)^2}{Q_3} \right] \quad (12)$$

where: ρ = the mass density of the air

X = the volume of the channels

Q = the volume of the air in the chambers

c = the acoustic velocity $([\gamma p / \rho]^{1/2})$.

The potential energy stored in the springs of the spring-mass system may be expressed by

$$P.E. = 1/2 K_1(X_2 - X_1)^2 + 1/2 K_2(X_3 - X_2)^2 \quad (13)$$

where: K = a spring constant

X = the displacement of a mass.

By comparing equations (9) and (10) term by term, the mass term in (10) may be expressed as

$$M_1 = \rho L_1 / a_1 \quad (14)$$

By comparing equations (12) and (13) term by term, an expression for the spring constant of the spring-mass system may be derived.

$$K_1 = \rho c^2 / Q_2 \quad \text{and} \quad K_2 = \rho c^2 / Q_3 \quad (15)$$

An important conclusion may be reached by examination of the form of equations (15). That is, the equivalent spring

constants of the system are inversely proportional to the volume of the chambers. Therefore the relative stiffness of the two chambers may be readily compared by the ratio of K_1/K_2 i.e.

$$K_1/K_2 = (\rho c^2/Q_2)/(\rho c^2/Q_3) = Q_3/Q_2 \quad (16)$$

Comparisons of this type may be made of the components of the pressure measuring system shown in Figure 3. Immediately it may be seen that the equivalent stiffness of the tire volume is negligible compared to the stiffness of the other cavities of the system since the tire volume is approximately one cubic foot while the other volumes are less than one cubic inch. Thus, the oscillations between the tire and the pressure transducer are not governed by the tire volume, and the chamber which represents the tire as shown in Figure 3 may be treated as if it had infinite volume.

The analysis of the pressure measuring system has been simplified by elimination of the tire volume as a variable. An equivalent spring-mass system for this new system as shown in Figure 6 may be used to advantage. The spring-mass system shown closest to the schematic drawing of the pressure measuring system was determined by replacing each length of tubing by a mass, and by replacing each enlargement of the interior by a spring. An approximation was made for the tubes connecting the pressure transducer, the solenoid-operated valve, and the valve number 1 by lumping

FIGURE 6. AN EQUIVALENT SPRING-MASS SYSTEM FOR THE PRESSURE MEASURING SYSTEM

the air in these tubes into one mass and converting the volumes of the three cavities into three parallel springs. The justification for this approximation is more apparent when the second equivalent system shown in Figure 6 is considered. If only the fundamental mode of oscillation is considered, (the same assumption that the investigator¹⁰ made when the Laplace operator "s" was restricted to small values), all masses of this undamped three degree of freedom system move in phase and the springs which represent the rotating joint and valve number 2 are not deflected. In other words, considering the physical system, the column of air in the tubing oscillates back and forth between the cavities of the pressure transducer and valves and the tire cavity which for all practical purposes has an infinite volume.

For the fundamental mode of oscillation, the masses of air in the rotating joint and valve number 2 also possess some kinetic energy. A convenient way of accounting for this kinetic energy is to calculate an equivalent mass of air which is moving with the velocity of the air in the tube. For example, the actual mass of air contained in the rotating joint may be reduced to an equivalent mass of air in the tube by multiplying by the squared ratio of the average velocity in the rotating joint to the tube velocity. This equivalent mass of air may be, in turn, converted to an equivalent length of tube. To be specific, the equivalent length of tube which represents the mass of air in the

rotating joint is .55 inches. The enlargement shown in Figure 3 which represents the rotating joint may then be replaced by .55 inches of tube for purposes of calculating the fundamental frequency of oscillation. A similar replacement may be made for valve number 2.

The calculation of the fundamental frequency of the system may be made by substituting the previously determined quantities for the spring constant K and the mass M into

$$f = (K/M)^{1/2} / 2\pi \quad (17)$$

or

$$f = (c/2\pi)(a/QL)^{1/2} \quad (18)$$

where: f = the fundamental frequency (cps)

c = the velocity of sound (in./sec.)

a = the cross-sectional area of the tubing (in.²)

Q = the total volume of the pressure transducer (in.³)

solenoid-operated valve, and valve number 1, and

L = the total length of tubing (in.).

The cross-sectional area of the tubing was used even though the area is reduced in the fittings and the valves, because the tubing area is present over most of the length of internal passageway and therefore controls the kinetic energy of the air in its slug-like flow.

When the numerical values for the pressure measuring system are substituted into equation (18), it is apparent that the simplified model is not able to predict accurately the fundamental frequency of oscillation since the predicted frequency is 40 percent higher than what has been observed experimentally. Evidently, the volume Q is too small and the length L is too short for the system parameters to be lumped, therefore a method must be devised in which the elasticity of the air in the tubing and the mass of air in the cavities may be taken into account.

Although the lumped-parameter model does not give an accurate prediction of system behavior, the simplification of converting the system into a single tube connected to a single chamber is invaluable. Again Lord Rayleigh has a suggested method of solution in treating such a system so that the distribution of mass and elasticity within the system may be considered. If the volume Q is replaced with an equivalent length L' of tubing which has the same cross-sectional area " a " as the other tubing in the system, the model of the system is now just a tube of length $L + L'$ where L' is calculated from

$$L' = Q/a \quad (19)$$

The fundamental mode of vibration for a single tube closed on one end and open on the other is well known. A node must

exist at the closed end and an anti-node must be present at the open end. Therefore, the wave length must be equal to four times the length of the tube. The equation for the fundamental frequency is simply

$$f' = (c/4)/(L + L') \quad (20)$$

or

$$f = \left(\frac{c}{4L}\right) / (1 + Q/aL) \quad (21)$$

If the volume is too large for the above treatment to apply, Lord Rayleigh derived another expression by which the fundamental frequency may be calculated. The fundamental frequency f is found by solving the following equation:

$$\text{TAN}(2\pi fL/c) = c/(2\pi fQ/a) \quad (22)$$

where the variables are the same as were used previously.

An equation of this form may be solved readily by plotting each side of the equation versus the dimensionless parameter $(2\pi fL/ca)$ and determining the intersection point of the two functions. If accuracy greater than that of the graphical method is desired, a few iterations about the point obtained graphically will determine the intersection point to any desired accuracy.

In the case of the pressure measuring system, the simpler form i.e. equation (21), predicts the experimental

results adequately. Figure 7 illustrates the accuracy of the prediction of the fundamental frequency by equation (21). The experimental values were obtained by subjecting the system to impulsive excitation and observing the period of the resulting oscillation. The equivalent length was varied by connecting various lengths of tube between valve number 2 and the rotating joint. The prediction of the fundamental frequency by equation (8) is also shown in Figure 7 as a dashed line. Note that within the range of experimental values both equations (8) and (21) predict the fundamental frequency within the accuracy needed for design purposes. The largest deviation from the experimental values occurred at the equivalent length of 49.5 inches where equation (8) predicted a frequency value 18 percent higher than the experimental value which agreed with the prediction of equation (21).

Having discussed three relatively simple methods of predicting the fundamental frequency of oscillation of the air in a tube which is open at one end and is connected to a chamber at the other, a comparison of the predictions of the methods should be of interest. Equations (8), (18), and (21) may all be expressed in the same dimensionless form if the frequency is expressed in rad./sec. and the equations are written as follows:

$$\frac{\omega}{(c/L)} = (Q/aL)^{-1/2} \quad (\text{lumped parameters}) \quad (18')$$

$$\frac{\omega}{(c/L)} = \frac{(\pi/2)}{(1+Q/aL)} \quad (\text{equivalent length}) \quad (21')$$

$$\frac{\omega}{(c/L)} = (1/2 + Q/aL)^{-1/2} \quad (\text{equivalent 2nd order system}) \quad (8')$$

FIGURE 7. PREDICTED FUNDAMENTAL FREQUENCY AND EXPERIMENTAL RESULTS

The words in parentheses identify the assumptions made in deriving the equations. These three equations are plotted in Figure 8 for purposes of comparison. The abscissa is the dimensionless parameter Q/aL or L'/L where L' is the equivalent length of tube calculated by dividing the volume of the chamber by the area of the tube. This parameter L'/L is a measure of the size of the chamber relative to the length of the tube. It is interesting to note that only in the range of relatively large values of volume Q , or $(Q/aL) > 1$, does the lumped parameter equation (18') approach equation (8'). This result is not surprising since the velocity is small enough in the chamber to neglect the kinetic energy and treat the air in the chamber as a pure spring only if the chamber volume (and therefore area of cross section) is large relative to the tube area. Equation (21') and (3') run almost parallel to each other with a maximum difference in magnitude of approximately 11 percent for small values of Q/aL until they approach each other and finally cross near the value Q/aL of .75. For values of Q/aL greater than unity, the solutions diverge. The solution (21'), which is based on replacing the volume Q by an equivalent tube length, would be expected to lose its validity as this volume Q becomes large with respect to the tube volume (aL).

To conclude the discussion of the prediction of the fundamental frequency of oscillation, it should be pointed

FIGURE 8. COMPARISON OF THREE METHODS OF PREDICTING FUNDAMENTAL FREQUENCY

out that equation (21) appears to be the most suitable of the methods for analyzing the pressure measuring system if accuracy and simplicity are considered.

It is interesting to note that the method of determining equation (20) may be used as an approximate method of determining the magnitude of the second natural frequency of the pressure measuring system. In deriving equation (20), it was noted that the length of the tube was four times the wave length of the fundamental frequency. Because of the end conditions of the tube, the tube length is only three-fourths of the wave length of the second harmonic. The result of dividing the acoustic velocity by the wave length of the second harmonic is that the second natural frequency is three times the fundamental frequency of the pressure measuring system. The importance of this calculation is that it provides an order of magnitude estimate of the second natural frequency. It illustrates that if the system can be designed so that the fundamental frequency is considerably higher than the exciting frequencies, the second natural frequency will certainly cause no resonance problems.

In order to have adequate design information about the pressure measuring system, not only the fundamental frequency must be known, but also some procedure for estimating the damping of the fundamental mode of oscillation must be available. Equations (7), (3) and (8) may be

combined to give the damping ratio

$$r = 8(\mu/\rho) / (\pi d^2 f). \quad (23)$$

By use of the perfect gas law, equation (23) may be expressed in terms of pressure and temperature instead of density i.e.

$$r = 8\mu RT / (\pi p d^2 f). \quad (24)$$

where: r = the damping ratio of an equivalent second order system

R = the universal gas constant ($\text{in}^2/\text{°R}$)

μ = the dynamic viscosity of the air (lb-sec/in^2)

p = the absolute pressure (lb/in^2)

T = the absolute temperature (°R)

f = the fundamental frequency (cyc/sec) and

d = the diameter of constant diameter tubing (in)
which has the same resistance as the actual
passageways.

Equation (23) is plotted in Figure 9 using the actual tube diameter, and a density which corresponds to a pressure of 30 psig and a temperature of 75 degrees Fahrenheit. Several measured values of damping factor are shown for a range of fundamental frequencies from 26 cps to 46 cps. Only the

FIGURE 9. DAMPING RATIO PREDICTED BY EQUATION (23) AND EXPERIMENTAL RESULTS

two values measured at the highest frequency appeared to agree with the theory. Fortunately these frequencies correspond to the operating range of the pressure measuring system; therefore, the equation may be of value at least to illustrate qualitatively the effects of the various parameters on the damping of the system. It is certainly important to realize the inverse relationship between damping and fundamental frequency.

Equation (24) points out the inverse relationship between damping and tire pressure. This relationship indicates that the damping device which would provide proper damping for an automobile pressure measuring system which operates at 30 psig would not provide satisfactory damping if it were applied to a larger vehicle which operates at a pressure of 75 psig. Another conclusion which may be drawn from equation (24) is that the system is not extremely sensitive to temperature changes since the absolute temperature appears in the equation.

Another form of equation (23) may be useful when it is solved for the diameter

$$d = 4 [(\mu/\rho) / r(2\pi f)]^{1/2} \quad (25)$$

This equation can serve as a starting point for determination of the diameter of a restriction to be put in the pressure measuring system in order to provide a certain damping factor.

Of course, the correct diameter could not be calculated exactly to give the desired damping because the mathematical model is not able to account for the complexity of the physical system. However, as will be shown in the next chapter, such a calculation can facilitate the selection of the most satisfactory diameter.

In summarizing the dynamic effects of the pressure measuring system, it has been shown that the system is capable of oscillating at its fundamental natural frequency and a suitable mathematical model has been presented which accurately predicts this fundamental frequency in terms of the physical parameters of the system. The parameters associated with the damping of this fundamental mode of oscillation have been identified by a mathematical model which for the most part predicts only qualitative results.

Another question which may be considered separately from the dynamic problem just considered is that of the relationship between a force increase at the tread of the tire and a pressure increase within the tire. It should be realized that no control may be exerted over this relationship because it is determined by the characteristics of the tire. It is important, however, that the part of the behavior of the complete system which is governed by this relationship be known.

An approximate relationship is derived in the Appendix for the "calibration factor" of the pressure measuring

system usually denoted as F/P , the ratio of dynamic force to dynamic pressure. As indicated in the Appendix, this ratio F/P is actually the slope of the total force versus total pressure curve and may be written as the derivative dF/dp . This relationship is repeated below.

$$F/P = \frac{dF}{dp} = 4 \left(\frac{w^2 R V_o}{\gamma F_o^2} \right) \left(\frac{p_g^3}{p} \right) \left[1 - \left(\frac{F_o}{2 p_g w R} \right)^2 \right]^{1/2} \quad (26)$$

where: w = the width of the tire (in)
 R = the outer radius of the tire tread (in)
 V_o = the volume of the tire (in³)
 F_o = the static wheel force (lb)
 p = the absolute pressure in the tire (lb/in²)
 p_g = the gage pressure in the tire (lb/in²)
 γ = a polytropic constant with value between 1.0 and 1.4 (γ is closer to 1.0 for frequencies less than 100 cps.)

Equation (26) is plotted in Figure 10 versus gage pressure of the tire. This approximation, although only based on the geometry of the tire and the thermodynamic properties of the air in the tire, illustrates a similar trend to the experimentally determined calibration factors which are also shown in Figure 10. The large increase in

FIGURE 10. APPROXIMATE CALIBRATION FACTOR

the ratio F/P as pressure increases indicates that the pressure measuring system is increasingly less sensitive at higher pressures as less change in pressure results from a given increase in force. This effect was also observed experimentally by the AASHO investigators⁷.

Another important fact is also evident from an examination of Figure 10. The slope of the curve is quite large, therefore making it imperative that test runs be made at the identical inflation pressure at which the system was calibrated, and that the pressure not be allowed to change during a test because of thermal effects. It is also interesting to note that if large pressure changes resulted from dynamic forces, a pressure measuring system would not be a feasible way to measure dynamic forces because the calibration factor would not remain constant. However, since the pressure changes due to dynamic forces usually are less than .3 psi, it is possible to use an average F/P ratio over such a small range of pressure values. The fact that the F/P curve shown in Figure 10 is not constant with pressure produces the result which shows the calibration factor will be dependent upon the force amplitude. That is, the calibration factor which is determined experimentally in a low force range will be lower than the calibration factor obtained in a higher force range. This conclusion suggests that calibration tests should measure the F/P factor in the range of forces expected on the road in order to obtain reliable results.

Equation (26) is plotted in Figure 11 for two different tires, a 7.00-14 passenger car tire and a 6.50-16 truck tire. The two curves indicate the increase in F/P ratio which may be expected as volume and overall tire dimensions are increased. This trend was also observed experimentally by the AASHO investigators⁷ in their investigation of various sizes of truck tires. Note that although the F/P ratio is directly proportional to the volume of the tire, it is also directly proportional to the tire radius and to the tire width squared.

As a final conclusion drawn from equation (26), consider the manner in which the F/P ratio depends upon the static tire force. Not only does static tire force appear squared in the denominator, but it also appears squared under the radical where a fraction is subtracted from unity. The result is that the F/P ratio is quite sensitive to change of the static tire force as shown in Figure 12. The important conclusion which may be drawn is that the static tire force must remain the same in road tests as it was in the calibration tests in order for the calibration constant F/P to be valid.

In this chapter, the theory which describes the operation of a basic type of pressure measuring system has been presented. In the next chapter, this theory will be utilized in order to design a pressure measuring system which should serve satisfactorily as a direct dynamic force transducer.

FIGURE 11. EFFECT OF THE TIRE DIMENSIONS ON THE APPROXIMATE CALIBRATION FACTOR

FIGURE 12. EFFECT OF THE STATIC TIRE FORCE ON THE APPROXIMATE CALIBRATION FACTOR

CHAPTER III

DESIGN OF A PRESSURE MEASURING SYSTEM BASED
ON THEORETICAL CONSIDERATIONS

As a first step in the design process, the desired performance of the system must be defined. Referring to Figure 2, a perfect system would transmit instantaneously a signal to the differential pressure transducer that would be directly proportional to the applied dynamic force for all frequencies of the dynamic force. Another way of expressing this condition is to say that the system should have a flat frequency response characteristic between the dynamic force and the recorded pressure signal over the range of frequencies for which the dynamic force has appreciable amplitude. This ideal characteristic of a constant ratio of dynamic force to pressure is illustrated as F/P in Figure 13 along with an undesirable characteristic that is frequency dependent. If this ideal characteristic is to be approached, there can be no resonant frequencies of either pneumatic or mechanical components within the range of the exciting frequencies.

It is important to recognize the parts of the system over which there is control and those parts over which

FIGURE 13. CALIBRATION CURVES FOR THE PRESSURE MEASURING SYSTEM

there is no control. In terms of the pressure measuring system shown in Figure 2, control may be exerted over the parts of the system which carry the pressure signal to the pressure transducer. By changing the arrangement of the components that are mounted on the instrument board, the lengths of tubing and the internal volume of the pressure measuring system are changed, which in turn changes the dynamic characteristics of the pressure measuring system. The sizes of the various components are subject to change including internal dimensions such as tube diameter and diameters of restrictions that might be introduced within the tubes. The tire itself is not a parameter over which control may be exerted; therefore the relationship between the dynamic force and the dynamic pressure change " p " within the tire as shown in Figure 2 is fixed for a particular tire operating at a particular inflation pressure. The pressure measuring system should be designed to minimize any distortion of the dynamic pressure change " p " and therefore enable " p " to be recorded accurately. If the dynamic pressure change " p " were not proportional to the dynamic force, there is little that could be done in the interconnecting pneumatic system to make the output signal proportional to the dynamic force.

The theory available to aid the design of the pressure measuring system was presented in the preceding chapter and gives information about the dynamic properties of the pressure measuring system in terms of its first two natural

frequencies and the damping ratio of the fundamental mode of oscillation. This information enables the designer to control the pressure measuring system in such a way as to minimize any resonant effects and to select damping for the system.

In order to visualize the effect of the dynamics of the pressure measuring system on the calibration factor F/P , consider the inverse of this factor P/F which will be called the pressure measuring system characteristic. All natural frequencies of the pressure measuring system will be associated with peaks on a P/F versus frequency plot as opposed to valleys on a F/P plot. A pressure measuring system characteristic P/F is shown in Figure 14 and illustrates a system in which the fundamental frequency of oscillation of the pressure measuring system is within the range of the exciting frequencies. It is evident that, in this case, large pressure signals can result from force excitation in the frequency range of the fundamental frequency of oscillation of the pressure measuring system while relatively small pressure signals can be produced by the same magnitude of force excitation in other frequency ranges. The pressure measuring system must be designed to remove the peaks from the P/F curve or at least move them out of the range of the exciting frequencies by making the fundamental frequency of oscillation as high as possible.

It is desirable then to have the fundamental frequency of this dynamic system as high as possible for two reasons. Firstly, this frequency should be separated from

FIGURE 14. EFFECT OF FUNDAMENTAL FREQUENCY ON PRESSURE MEASURING SYSTEM CHARACTERISTIC

the range of frequencies to be measured. If this desired separation of frequencies is possible, any spurious signals which might arise because of excitation of the fundamental frequency of the pressure measuring system may be recognized as such and even filtered out if that is desired. (Note that if the fundamental frequency is high enough to be removed from the range of exciting frequencies, the second natural frequency can cause no trouble). Secondly, in order for the response of the system to be rapid, the frequency should be high since the time of response (time to reach a steady state value after a step disturbance) is inversely proportional to the frequency.

Also, it is evident that some damping must be introduced because the pressure measuring system has very small damping; and as the fundamental frequency is raised, the damping decreases as illustrated by Figure 9.

Equation (21), which is plotted in Figure 7, indicates that a decrease in the length of tube L will increase the fundamental frequency of oscillation. Referring to the physical system shown in Figure 3, this suggested reduction indicates that the tube between valve number 2 and the rotating joint must be shortened as much as possible. The minimum length of this tube is governed by the mounting position of the instrument board shown in Figure 2 and the allowance which must be made for the vertical motion of the wheel.

The volume Q also appears in the denominator of equation (21) which suggests that any reduction in this volume will increase the fundamental frequency. Visualization of the system as a lumped parameter system is particularly valuable in making design decisions about the parallel chambers and passageways in the vicinity of the differential pressure transducer. Referring to Figure 6, it may be seen that these parallel components may be represented as springs which have a spring rate that is inversely proportional to the volume of the particular component as shown by equation (15). Any decrease in these volumes increases their spring constants which in turn increases the frequency of the complete system. The volumes of these components are summed in the term Q in equation (21) which explains the reason for increase in frequency when Q is reduced. An immediate consequence of considering the spring rate of these parallel passageways is the conclusion that valve number 1 should be moved as close as possible to the "T" fitting leading to the solenoid-operated valve in order to minimize the effect of this parallel volume. Another conclusion which may be drawn is that the two "T" fittings which lead to the solenoid-operated valve and the pressure transducer must be as close together as possible to reduce the volume of this passageway. A more important conclusion is the importance of the relatively large internal volume of the solenoid-operated valve. This internal volume is .45 in.³ which corresponds to an equivalent length of

tubing of 19.3 inches. By removing this large internal volume, the same effect is realized as would result if the tube between the rotating joint and valve number 2 were shortened by 19.3 inches.

There are two basic methods of reducing the effect of the internal volume of the solenoid-operated valve. One method would be to fill most of the interior with a material in such a manner that there would not be any interference with moving parts of the valve. Another method would be to replace the valve with a tube containing a restriction that would allow a controlled rate of flow. The method which utilizes a small continuous flow of air in parallel with the differential pressure transducer in order to equalize unwanted pressure changes is called the continuous equalization method. The method of equalization used in the pressure measuring system shown in Figure 2 is called the discrete equalization method. It was given this name because the solenoid-operated valve may be opened for discrete time intervals to equalize the pressure difference across the differential pressure transducer. A comparison of these two methods will be made in order to determine which method would be the more advantageous way to equalize the unwanted thermal effects and at the same time answer the equation about the large internal volume of the solenoid-operated valve.

One advantage of the continuous equalization method has already been mentioned, that is, the higher frequency

which is possible to obtain because of the reduction of the internal volume through removal of the solenoid-operated valve. Another advantage which is apparent, particularly during a road test, is that changes in tire pressure due to temperature changes within the tire are continuously equalized and therefore complete temperature equilibrium of the system does not have to be established before each test. To appreciate this point, consider the situation which would occur during a test in which the solenoid-operated valve is the method of equalization. Assume that the gain of the amplifiers has been set so that the largest variation in pressure does not cause the recorder pen to leave the paper as the vehicle travels over the test section of pavement. The solenoid-operated valve is actuated just before the vehicle enters the test section; and if the spring loaded valve closes at the instant that the pressure is close to the static pressure in the tire, the variations about this static pressure value are centered on the recorder paper (if the pressure happens to be at a peak value at the instant the valve is closed, the valve must be actuated again to achieve the proper equalization). If there is now any heating effect still taking place within the tire, the average value of the pressure signal will gradually increase until the pressure variations cause the recording pen to move off the paper. If temperature equilibrium in the tire has not been carefully established before the test section was reached, the usual result is that the test

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must be repeated or the solenoid-operated valve must be actuated during the test. Equalization during the test is undesirable because, in that case, different sections of the record have different base values. Because of the time saved during road tests, the continuous equalization method offers distinct advantages over the discrete equalization method.

A disadvantage of the continuous equalization method is that the system will not register a constant force as such, but the recording pen will gradually return to the equilibrium position as the pressure difference is equalized even though the constant force is still present. This condition is not detrimental in making dynamic measurements, but it does not allow a static calibration factor of F/P to be measured unless a valve is placed beyond the restriction expressly for the purpose of stopping the equalization process.

Another requirement of the continuous equalization method which will not be listed as a disadvantage but is a definite limitation of the method, is that the size of the restriction must be carefully chosen. If the equalization rate is too fast, the low frequencies which are to be recorded will be distorted by the equalization. If the size of the restriction is too small and accordingly the equalization rate is too slow, compensation for changes in temperature during testing will not be satisfactory.

Of course, the distortion problem is the more important condition and must therefore govern the design decision. The maximum size of restriction must be determined for the individual vehicle based on its lowest frequency of body motion. For example, the lowest frequency of body motion for vehicle number 210 (a 1962 Chevrolet sedan) was approximately one cps. To be conservative, a restriction was chosen so that within one half the period of this lowest frequency oscillation constant pressure difference would be reduced less than three percent. This equalization rate is illustrated in Figure 15. All other frequencies would be distorted less than this value. This rate of equalization was found to be adequate to balance the temperature effects even when the pavement was at a temperature of over 100°F. An expression for the rate of equalization may be obtained by applying the theory presented in reference 12.

Another comparison which may be made between the two methods is the control which may be exerted over the system in case of an emergency, such as a tire puncture. The solenoid-operated valve offers the advantage of immediate equalization if the operator realizes the emergency and activates the solenoid to hold the valve open. If the operator does not realize the situation in time, then there is a possibility of damage to the differential pressure gage which is designed for a maximum differential pressure of only 2.5 psi. In the case of the continuous

FIGURE 15. EQUALIZATION RATE OF RESTRICTION

equalization device, there is no way to completely open the passageway across the pressure transducer from within the vehicle; but the large pressure difference will be equalized by the flow of air from the reference tank through the restriction. In case of rapid loss of air from the tire, this equalization would be too slow to prevent damage to the transducer. Therefore, the system utilizing the solenoid operated valve has the advantage when emergency equalization is considered.

To summarize this comparison, the advantage of ease of operation under test conditions which the continuous equalization system offered outweighed its disadvantages with the result that this method of equalization was chosen as the one to be used in this investigation. Actual test experience with the continuous equalization system justified this decision as the testing time was reduced by approximately one half as compared with similar tests using the discrete equalization system.

Another major design decision was the selection and placement of a restriction in the system in order to damp the fundamental frequency of the pressure measuring system. The first question which had to be answered was that of the proper size of restriction to be used in order to provide the proper amount of damping. As a first approximation to the correct size of restriction the curve showing the relationship between damping ratio and diameter from equation (23) is shown in Figure 16. For a damping ratio in the

FIGURE 16. DAMPING RATIO AS A FUNCTION OF DIAMETER

range of .7 to 1.0, the restriction diameter should be in the range of .020 inches to .017 inches according to Figure 16. The final diameter was chosen on the basis of impulsive tests in which the force and pressure records were compared until the most satisfactory match was obtained and the fundamental frequency did not appreciably distort the pressure record.

The second question involves the placement of the restriction in the system. Just where in the system should the flow be restricted in order that the damping be most effective? Equation (23) yields no information about a placement of a restriction (of course, it should not be expected to do that since it was derived for a constant diameter). Again the approximate spring mass system which is shown in Figure 6 is valuable. The largest element of mass is the length of the tube between valve number 2 and the rotating joint. It is logical to put a damper on the largest mass in the system for the most effective damping, therefore the restriction should be put in that particular passageway. Experiments showed that this placement of the restriction was more effective than at other positions, in particular, much more effective than a restriction placed just before the pressure transducer.

The final selection of a restriction to provide adequate damping of the pressure measuring system may be made quickly by observing the effect of a particular restriction when the tire is excited by a pulse. Figure 17 illustrates the pressure measuring system response for two different

RESTRICTION A

RESTRICTION B

FIGURE 17. RESPONSE OF PRESSURE SIGNAL TO IMPULSIVE EXCITATION OF THE TIRE FOR TWO DAMPING RESTRICTIONS

restrictions, restriction B providing more damping than restriction A. These records, in particular the lower one, may be regarded as typical records of a pressure measuring system which is operating in the desired manner.

The final design of the instrument board components of the pressure measuring system incorporating continuous equalization and required damping is shown in Figure 18.

In concluding the design considerations of the pressure measuring system, the possibility of mechanical vibrations that might excite the pressure measuring system was investigated. Of particular interest were those components that have natural frequencies close to the fundamental frequency of oscillation of the pressure measuring system. The determination of the precise magnitude of the fundamental frequency of the pressure measuring system, then, was of importance. After removing the restriction from the pressure measuring system, impulsive excitation of the tire inflation pressure revealed this fundamental frequency as shown in Figure 19. The oscillation shown in Figure 19 was caused only by a rapid change in inflation pressure and was not due to any force or motion of the tire or any of the components of the pressure measuring system. The frequency of this oscillation may be measured from the oscillograph trace as 69 cps.

Thus, for this particular pressure measuring system, mechanical vibrations with frequencies near 69 cps may cause oscillations of the pressure measuring system.

FIGURE 18. FINAL DESIGN OF THE PRESSURE MEASURING SYSTEM

FIGURE 19. OSCILLATION OF PRESSURE SIGNAL AFTER IMPULSIVE EXCITATION OF TIRE PRESSURE

The components which could be suspected of having natural frequencies near this value are the tubes, the instrument board, and the rotating joint. Mechanical vibration of the tubes is a problem that was solved by the investigators at Michigan State Highway Department. They found that copper tubing could not be used because of the induced oscillations, but that polyethylene tubing was essentially vibration free because of its high damping qualities.

The instrument board was made out of fiber board which has high damping qualities and was mounted rigidly to the automobile body in order to minimize vibrational amplitudes by forcing the frequency of any oscillations to a very high value. There were no indications from experimental work that any oscillations were induced into the pressure measuring system from the instrument board.

The component most likely to be suspected of relatively low frequency mechanical vibrations is the rotating joint and its associated mounting. As shown in Figure 2, the rotating joint was mounted on three turnbuckles which were attached to the rim of the test wheel. The center of gravity of the rotating joint was approximately two inches away from the plane of the turnbuckles, a construction that enabled the axis of the rotating joint to turn through an angle for any loading through the center of gravity. Since the rotating joint is in an acceleration field caused by the motion of the wheel, there is an inertia loading through the center of gravity of the joint under pavement testing

conditions. The turnbuckles afford a path for vibrations to be transmitted into the rim of the wheel and thereby into the sidewall of the tire which in turn could excite the pressure measuring system.

The natural frequency of the rotating joint could have been predicted by vibration theory, but the analysis would hardly have been worth the effort if the mounting of the rotating joint had to be changed in order to eliminate the possibility of vibration transmission to the pressure measuring system. As in many cases which involve vibrations of complex physical systems, an experimental approach was the most expedient method of accurately determining the natural frequency of the rotating joint as well as the natural frequencies of other components of the system.

An accelerometer was mounted on the rotating joint, the joint was struck a sharp blow, and the resulting oscillations recorded as shown in Figure 20. The frequency of the oscillations of the rotating joint and accelerometer was 68.5 cps and the forced oscillation of the pressure measuring system was virtually identical at 68.0 cps as measured from Figure 20. It is important to note the very light damping associated with the mechanical vibration, a condition that is certainly detrimental. There was no damping introduced into the pressure measuring system for this test. The frequency of oscillation of the rotating joint under these test conditions is almost identical to the fundamental frequency of the pressure measuring system which is the worst possible condition that might arise.

FIGURE 20. OSCILLATION OF THE ROTATING JOINT AFTER IMPULSIVE
EXCITATION OF THE ROTATING JOINT

In order to determine the effectiveness of the damping of the pressure measuring system and to test the system under more realistic conditions, a force impulse was applied to the tire and the resulting force and pressure variations recorded as shown in Figure 21. A beating phenomenon is apparent in the pressure trace indicating that the pressure measuring system is responding to a frequency that is close to its own fundamental frequency. Since the pressure signal is still oscillating after the force has decreased to zero, it is apparent that the force on the tire is not sustaining the pressure oscillation.

The beat frequency calculated from Figure 21 is four cycles per second. Using the theory of coupled vibrations, this value of four cycles per second is the difference between the two natural frequencies of the rotating joint and the pressure measuring system. Figures 19 and 20 indicate, however, that the difference between the two frequencies is not as much as four cycles per second. The additional mass of the accelerometer mounted on the rotating joint during the test, for which the results are shown in Figure 20, evidently lowered the natural frequency of the mechanical vibration of the rotating joint to the magnitude of 68 cps which was computed from Figure 20. Figure 22 substantiates this assumption by indicating that the natural frequency of the rotating joint is approximately 72.5 cps. The upper trace of Figure 22 shows the oscillation of the rim of the wheel after impulsive excitation of the tire. The lower trace shows the response of the pressure measuring system.

FIGURE 21. BEATING OF THE PRESSURE SIGNAL AFTER IMPULSIVE
EXCITATION OF THE TIRE

FIGURE 22. OSCILLATION OF THE RIM OF THE WHEEL AFTER IMPULSIVE EXCITATION OF THE TIRE

Both of these oscillations are at a frequency of 72.5 cps and are the result of the oscillation of the rotating joint which must be oscillating at its own natural frequency. The conclusion is that the rotating joint natural frequency is 72.5 cps which is approximately four cycles per second above the fundamental frequency of the pressure measuring system.

In order to determine if the rim of the wheel can magnify the oscillations of the rotating joint, the response of the rim to an impulsive blow was measured and shown in Figure 23. An accelerometer was mounted on the rim to measure this oscillation which has a small amplitude of displacement. The rim responded with a frequency of oscillation of 145 cps which indicated that rim vibration was not a source of magnification. The response of the pressure measuring system is also shown in Figure 23. It is interesting to note that the frequency of this response is 230 cps, which is approximately three times the fundamental frequency of 69 cps. The factor of three is worthy of note because it is the ratio of the second natural frequency to the first (or fundamental) for fluid in a tube which has one open end and one closed end. Therefore, the mathematical model which was developed in Chapter II to predict the fundamental frequency is able to predict approximately the second natural frequency of the pressure measuring system.

To summarize the results of this investigation, the vibration will be traced through the system. The source of the mechanical vibration was the rotating joint oscillating on its mounting. This vibration was transmitted

FIGURE 23. OSCILLATION OF THE RIM OF THE WHEEL AFTER IMPULSIVE EXCITATION OF THE RIM

through the turnbuckles to the rim of the tire which in turn excited the sidewall of the tire. The sidewall vibration caused the air within the tire to pick up the oscillation and transmit it to the pressure measuring system. The pressure measuring system, which has its fundamental frequency very close to this frequency, was very receptive to this frequency even though the system is heavily damped. As Figure 20 shows, the mechanical vibration of the rotating joint has very little damping which accounts for the apparent lack of damping in the pressure measuring system that was illustrated by Figure 21. Referring to Figure 21 again, the beating of the pressure signal may be explained by the excitation of the sustained mechanical vibration of the rotating joint at a frequency which was very close to the fundamental frequency of the pressure measuring system. The combination of these two oscillations within the pressure measuring system resulted in the beating which is characteristic of the sum of two harmonic motions.

It is apparent that a mechanical vibration of the rotating joint in resonance with the fundamental frequency of the pressure measuring system cannot be tolerated. The pressure signal will always be corrupted with this superimposed oscillation not only during road tests but also during calibration tests of the pressure measuring system. There are two feasible ways to separate these frequencies in order to get them out of resonance with each other. One method involves lowering the fundamental frequency of the

pressure measuring system and the other involves raising the frequency of the mechanical vibration of the rotating joint. Since much effort has been exerted to raise the fundamental frequency of the pressure measuring system to its present value, it is obvious that the second suggestion is the one which should be followed.

One of the principal disadvantages of the mounting of the rotating joint was that the turnbuckles which were attached to the rim provided a path which enabled the joint vibrations to feed into the rim and sidewall of the tire. Instead of making the mounting stiffer by using larger turnbuckles, which would still allow vibrations to be transmitted to the tire, a new design was developed which eliminated the connection at the rim. This design enabled the rotating joint to be mounted directly to the wheel studs, thereby removing the path by which vibration might be transmitted to the rim. This mounting was constructed of aluminum alloy tubing and plexiglas in order that it would be light and stiff so as to have a high natural frequency. The subsequent tests with this mounting showed no distortion from mechanical vibration. For comparison, records of identical tests are shown in Figure 24, one with the turnbuckle mounting and one with the wheel stud mounting. The effect of the mechanical vibration is apparent in the record for the turnbuckle mounting.

In summary, it is appropriate to point out that all frequencies of mechanical vibration of the components must

SYSTEM RESPONSE WITH TURNBUCKLE MOUNTING

SYSTEM RESPONSE WITH WHEEL STUD MOUNTING

FIGURE 24. ILLUSTRATION OF THE REMOVAL OF MECHANICAL VIBRATION EFFECTS FROM THE PRESSURE SIGNAL

be separated from the fundamental frequency of oscillation of the pressure measuring system in order to keep the pressure recording free from spurious signals arising from these vibrations. Because of the acceleration environment in which this equipment has to operate, it is imperative that all sources of vibration be eliminated which might cause erroneous signals to be present in the record.

Up to this point, the discussion has been concerned with the removal of the dynamic effects of the pneumatic and mechanical components of the pressure measuring system in order that an accurate measurement of the dynamic pressure variations in the tire would be possible. The next step is the calibration of the pressure measuring system which is the determination of the relationship between dynamic force and pressure. These calibration methods and results are considered in the next chapter.

CHAPTER IV

CALIBRATION OF A PRESSURE MEASURING SYSTEM

If a pressure measuring system is to be used successfully as a method for measuring dynamic force, the calibration method must be chosen carefully and the calibration tests executed precisely. The methods of calibration that may be selected in order to determine the relationship between the tire pressure measurement and the dynamic force applied to the tire will be considered now.

In general, a pressure measuring system may be calibrated either statically or dynamically. In one static calibration method that may be utilized, an increasing force is applied to the tire by loading the vehicle and then the load increase and the corresponding pressure increase are recorded. The ratio of the load change to the pressure change is then taken as an F/P calibration factor to be applied to pressure measurements that are made during road tests. The principal objection which must be raised to static testing is that there is no way for the dynamic properties of the pressure measuring system to be identified. In fact, it is possible for a pressure measuring system that has an "ideal" perfectly linear static calibration curve to exhibit wildly erratic results under dynamic conditions

because a resonant condition has been excited within the system. The amount of damping associated with the natural frequencies of the pressure measuring system may not be determined easily without dynamic tests. In addition, mechanical vibrations which might excite the pressure measuring system during pavement testing cannot be easily recognized without dynamic testing.

In general, there are two methods of dynamic calibration that are available for obtaining the relationship between dynamic force and dynamic pressure: one method involves using steady state sinusoidal tests and the other method involves using transient tests. Of the sinusoidal tests, there are two principal test procedures, each requiring somewhat different equipment. In one method which utilizes excitation at the wheel, the test tire is displaced with sinusoidal motion while force at the tire tread and pressure are simultaneously recorded. The most troublesome problem associated with this calibration method is the selection of displacement amplitudes. If a constant input displacement is selected and the amplitude of displacement chosen in order to obtain a relatively large force amplitude for the low frequencies, the tire is likely to lose contact with the force measuring platform when the wheel-hop frequency is attained. The net result is that the amplitudes of the input displacement must be decreased with increasing frequencies because the forces that are generated are proportional to the square of the frequency.

What criteria then should be used for selecting the amplitude or displacement? Should the displacement amplitude be selected so that the forces are always in a range corresponding to the dynamic force range on an "average" highway? Should the displacement amplitude be selected to produce force amplitudes which approximate the force amplitudes that are generated within each corresponding frequency band for "average" highway testing conditions? Should the displacement amplitudes be selected on the basis of the displacement amplitudes obtained from harmonic analysis of a "typical" highway? For each question above, a different displacement amplitude versus frequency relationship could be selected. As equation (26) indicates (see Figure 10), the F/P calibration factor is amplitude dependent to a certain extent. To minimize the effect of this nonlinearity, calibration conditions must be as near the operating conditions as possible. But how can a single amplitude at a single frequency be made to correspond to a multitude of superimposed frequencies which exist under actual highway test conditions? It is apparent that the answers to these questions are not obvious. No one criterion for selecting amplitudes has been found that will satisfy all the requirements for a "perfect" calibration test.

Another method of steady state sinusoidal testing utilizes a mechanical oscillator which is mounted in the back of the vehicle. This oscillator applies a vertical force to the vehicle which sets the vehicle in motion at

the frequency of excitation. The force that is generated at the tire is measured by an electronic scale, and the corresponding pressure is recorded by the pressure measuring system. The problem again arises as to what amplitude of unbalanced force should be applied in order to best simulate the amplitude-frequency relationship which exists under actual highway test conditions. Again a criterion which is all encompassing is not available.

In tests of this type where the oscillator occupies a large portion of the bed of the vehicle, it is important that the correct amount of additional load be placed on the vehicle during calibration tests in order that the static tire force at the test tire be identical during calibration and subsequent highway tests. Figure 12 illustrates the change of calibration factor that occurs when the static tire force is changed and therefore indicates the need for constant static tire force. Investigators who have used this method also have reported difficulty in applying frequencies any greater than 12 cps because of the suspension characteristics of the vehicle. Since the wheelhop frequency for many vehicles is above this value, the calibration factor cannot be measured at one of the resonant frequencies of oscillation of the vehicle. It is possible that the fundamental frequency of the pressure measuring system could be close to the wheelhop frequency which could cause extremely large pressure variations whenever the wheelhop frequency was excited by the dynamic force. The result would be that much larger forces would be indicated by the pressure measuring system than are actually

being applied. One requirement, then, for an adequate calibration method is that it be able to excite the range of frequencies that includes all of the natural frequencies of the vehicle.

Just as in the case of the steady state tests, there are two basic transient testing methods. In one method, an impulse is applied to the vehicle and in the other method, an impulse is applied to the tire. A method of applying an impulse to the vehicle is that of dropping a weight into the bed of the vehicle and then recording the subsequent force and pressure variations. The force is measured by an electronic scale on which the test tire is resting.

There are two ways of applying impulsive excitation to the tire. The first method that will be mentioned makes use of the standard electronic scale for measuring weights of vehicles. A hydraulic jack with a collapsing base is used to enable the load which the jack is supporting to be released rapidly. The procedure is as follows: the jack and collapsing mount are placed under the axle of the vehicle and a portion of the static tire force is removed by raising the axle a small distance. The jack mounting is then collapsed, causing the vehicle to undergo a transient oscillation which in turn excites the pressure measuring system and the electronic scale that is under the tire.

The second method of exciting the tire makes use of a specially designed electronic scale on which the test wheel rests. This scale was designed so that it can give various

impulsive displacements to the test tire. As the test tire receives the impulsive displacements, the dynamic force and pressure measurements are recorded. A harmonic analysis of these transient records of force and pressure converts the measurements from the time domain into the frequency domain where the ratio of dynamic force to dynamic pressure yields the calibration factor F/P as a function of frequency. This latter method of calibration was used in this investigation.

The method of calibration by using transient excitation is not without problems, however. Just as the steady state calibration method raised unanswered questions about the amplitudes of the harmonic motion, the transient method raises the question of the form of the impulsive excitation. In theory a step input of displacement would excite all frequencies of the system and therefore should be ideal to define the dynamic characteristics of the pressure measuring system throughout the complete frequency range. However, in practice, the result of applying an approximate step function of displacement to the tire is that there is insufficient amplitude within certain frequency ranges of the pressure and force response. If the frequency content of the pressure and force records is low in a particular frequency range, the determination of the transfer function F/P within that range is subject to error because the ratio of the small amplitudes is sensitive to very small variations in force or pressure. Although the approximate step excitation is deficient, particularly in the low frequency range, it has

the advantage that the high frequency range may be readily excited. This high frequency range includes the wheelhop frequency and may include the fundamental frequency of the pressure measuring system. The performance of the pressure measuring system then may be accurately evaluated in this critical frequency range. In order to excite the low frequency ranges, an approximate sine pulse displacement is more suitable than the approximate step because of the higher amplitudes present in the low frequency ranges. The period of the pulse required to excite the low frequency ranges depends upon the lowest natural frequency of the vehicle. Therefore, a pulse that would properly excite the low frequency range of one vehicle might not be satisfactory to use with another vehicle.

To excite the complete frequency range of importance to the pressure measuring system, a shaped pulse may be required. Of course, the shape of the pulse would depend upon the characteristics of the vehicle being tested. It may be possible to shape the pulse in such a manner that the amplitude distribution of the pulse would be similar to that encountered on the highway. Preliminary work with a passenger vehicle indicates that the use of a shaped pulse is promising and therefore continued study is being undertaken to apply this method to a range of larger vehicles.

The amount of damping present in a pressure measuring system radically affects the calibration factor of the system.

-

To illustrate this effect, consider Figure 25 where both the calibration relationship F/P and its inverse P/F , the pressure measuring system characteristic, are shown. The solid line represents a pressure measuring system that contains no restriction to damp its fundamental frequency of oscillation. Note the peak of the pressure measuring system characteristic that occurs at the fundamental frequency and the corresponding dip of the calibration relationship. The dashed line indicates the effect of introducing damping into the system by the addition of a damping restriction. The addition of damping into the system cannot produce the "ideal" straight-line F/P relationship because of the nonlinearities of the system, but a definite improvement of the calibration relationship is realized. Figure 26 illustrates a time domain description of the effect of insufficient damping of the pressure measuring system. The oscillations that appear in the pressure record but do not appear in the force record are due to the excitation of the fundamental frequency of oscillation of the pressure measuring system. The frequency analysis of records such as these yields the curves for insufficient damping shown in Figure 25.

If a pressure measuring system that has insufficient damping is used for highway tests, the pressure signal will be distorted in the same manner that the calibration pressure record shown in Figure 26 was distorted. A spectral analysis of pressure measurements taken with such a system gives an indication of the amount of power associated with oscillations

SYSTEM
CHARACTERISTIC
P/F
(PSI/LB)

CALIBRATION
FACTOR
F/P
(LB/PSI)

FIGURE 25. EFFECT OF DAMPING ON CALIBRATION RELATIONSHIP

FIGURE 26. IMPULSIVE CALIBRATION TEST FOR A PRESSURE MEASURING SYSTEM WITH INSUFFICIENT DAMPING

of the pressure measuring system as well as the power associated with the natural frequencies of the vehicle.

Power spectral density functions of pressure measurements are shown in Figures 27 and 28 where Figure 27 illustrates insufficient damping of the pressure measuring system and Figure 28 illustrates sufficient damping. Note the peak at 35 cps that appears in Figure 27 but does not appear in Figure 28. This peak represents the effect of the insufficient damping of the pressure measuring system in introducing pressure signals that are not representative of the dynamic force on the road. An even more subtle effect of insufficient damping of the pressure measuring system occurs if the fundamental frequency of the pressure measuring system is higher than the frequency range used in the spectral analysis (in this case, for frequencies greater than 45 cps). Pressure amplitudes above this frequency range are not ignored but are aliased back into the lower frequency ranges. It is apparent from the examples just considered that proper damping of the pressure measuring system is extremely important if the pressure measuring system is to be used effectively as a device to indicate dynamic force.

Considering Figure 25, it is apparent that damping is an important factor in making the calibration relationship less frequency dependent; however, the ideal flat characteristic has yet to be attained. With a relatively flat calibration relationship as shown in Figure 25, the question arises as to the effect of using a single average

PSD
(PSI²/CPS)

VEHICLE V05

S.R. 26

38 MPH

29 PSI

10⁻⁴

10⁻⁵

10⁻⁶

BODY MOTION FREQUENCIES

WHEELHOP FREQUENCY

FUNDAMENTAL
FREQUENCY OF PRESSURE
MEASURING SYSTEM

FREQUENCY (CPS)

FIGURE 27. SPECTRAL ANALYSIS OF PRESSURE MEASURING SYSTEM RESPONSE TO HIGHWAY PROFILE (INSUFFICIENT DAMPING)

calibration factor estimated from the calibration relationship. Is this calibration relationship flat enough so that an average factor applied to the pressure signal will give an accurate estimate of the dynamic force?

Not only does the problem of determining the proper calibration relationship exist, but there is also the problem of determining the overall accuracy of the system. In other words, how accurately can the dynamic force be measured? The following examples may serve to indicate the accuracy of the instrument as a direct force transducer.

Consider the time domain comparison of the force and pressure records taken from a calibration test as shown in Figure 29. The upper trace is the actual force measured at the tire tread, and the lower trace is the signal from the pressure measuring system. Superimposed on the force trace is the product of a single average calibration factor F/P and the pressure trace shown in the lower trace. Therefore, this dashed curve, which is the predicted force, may be compared directly with the measured force. The percent difference of the predicted force and the measured force calculated at the first positive peak is six percent. The time lag of the pressure signal behind the force signal at the first negative peak is .011 seconds. To visualize the magnitude of this time lag, consider that a vehicle moving at a speed of 60 miles per hour travels approximately one foot during the time required for the signal to travel from the tire surface to the differential pressure gage.

LEGEND:

- SIMULTANEOUS FORCE AND PRESSURE RECORDS
- (PRESSURE RECORD) $\times (F/P)_{avg}$.

FIGURE 29. REPRODUCTION OF A FORCE RECORD BY USING A SINGLE CALIBRATION FACTOR.

Another indication of the effect of using an average calibration factor may be made by comparing the use of an average F/P ratio with the use of the frequency dependent calibration relationship. The calibration relationship may be applied to pressure measurements taken from highway tests by transforming the pressure versus time records into the frequency domain, operating on the resulting pressure versus frequency record by the calibration relationship $[F/P(f)]$, and then transforming the result back into the time domain to get the dynamic force versus time record. This comparison is made in Figure 30 where the solid curve is the dynamic force relationship obtained by a transformation into the frequency domain, and the dashed curve is the force record calculated by a linear transformation in the time domain using a single calibration factor. In this case, the percent difference between the two curves at the maximum force magnitude is two percent. The lag of the dashed curve relative to the solid curve is approximately constant at .008 seconds which corresponds to a distance of 0.7 feet at 60 mph.

Another method by which the calibration relationship and the single calibration factor may be compared is by using each to convert a tire pressure power spectrum into a dynamic force power spectrum. In Figure 31, a force power spectrum (solid curve) has been derived from the pressure power spectrum shown in Figure 28 by multiplying the ordinates of the pressure power spectrum by the square of the calibration relationship for each frequency band. The

FIGURE 30. COMPARISON OF DYNAMIC TIRE FORCE RECORDS

dashed curve is an estimate of the force power spectrum obtained by converting an approximate force versus time record into the frequency domain by making a spectral analysis. The effect of using the single calibration factor should be noted particularly in the two principal frequency ranges which correspond to the body motion and to the wheelhop natural frequencies of the vehicle. The use of the single calibration factor magnifies the force amplitudes in the body motion frequency range and attenuates the force amplitudes in the wheelhop frequency range. The percent difference in the root mean square dynamic force (which is the square root of the area under the power spectral density function) is 6.5 percent. The use of the single calibration factor resulted in over estimating the RMS force by 6.5 percent as compared with the use of the complete calibration relationship.

The foregoing discussion has been concerned with the calibration of a pressure measuring system for an automobile tire that is operating within the rated pressure range. The question arises as to the use of this same system at inflation pressures other than the design pressure of the tire. The F/P ratio will increase with higher inflation pressures as indicated in Figure 10. The question as to the shape of the curve of the F/P relationship versus frequency for different pressures can best be determined experimentally.

In order to answer the question concerning the effect of inflation pressure on the calibration relationship, a series

of tests was run for inflation pressures of 20, 30, 40, and 50 psi for an automobile tire that had a recommended cold inflation pressure of 24 psi. The results of these tests are shown in Figure 32. It is apparent that as inflation pressure increases above the recommended pressure, the calibration relationship becomes more frequency dependent. The use of a single calibration factor for these higher pressures is increasingly inaccurate as the pressure increases. It is thus recommended that the frequency domain transformation of pressure data by using the calibration relationship be used in calculating dynamic force from pressure measurements for inflation pressures appreciably above the manufacturer's recommended tire inflation pressure.

It is thus evident that the design of a pressure measuring system can have an appreciable effect upon the calibration relationship of the system. With careful selection of the parameters of the system, the calibration relationship may be made flat enough so that a single calibration factor may be applied to the pressure measurements without the introduction of excessive error. Therefore, the pressure measuring system may serve as a direct force transducer providing the inflation pressure of the tire is close to the rated value.

However, the ideal flat calibration relationship has not been attained. This ideal relationship is extremely

FIGURE 32. EFFECT OF TIRE PRESSURE ON THE CALIBRATION RELATIONSHIP

difficult to obtain, particularly because of the non-linear behavior of the tire. Even the approximate relationship for F/P shown in Figure 10 predicts that the pressure signal will be amplitude dependent, a phenomenon that has been observed experimentally. In order to partially account for these non-linear effects, an average calibration relationship may be obtained from a series of calibration tests utilizing different impulsive excitation. This average calibration relationship can be applied in the frequency domain to the pressure measurements in order to convert them to dynamic force measurements.

The calibration techniques which were employed to determine the pressure measuring system characteristics for the passenger vehicle used in this investigation were based principally on judgement and experience with passenger vehicles. In particular, the impulsive excitations were selected to excite the significant frequency ranges of the vehicle. The experience gained with this vehicle may not be applicable to larger vehicles which have different suspension systems.

The particular advantage of impulsive testing is that the actual tests may be conducted in a very short time although the processing of the data is somewhat lengthy. If a test does not completely define the frequency range of interest, the result is usually not known until the data are processed. If the results of the calibration test are then not satisfactory, the test must be conducted again with a more suitable impulsive input. The advantage of knowing beforehand adequate impulsive inputs for any vehicle to be tested is apparent. It is evident that there is a need for continued research in this area.

CHAPTER V

THE ANALYSIS OF HIGHWAY TEST DATA

It is appropriate to consider methods by which the output records of a pressure measuring system may be analyzed to yield information about the dynamic forces applied to a highway by the vehicle.

Two records of the output of the pressure measuring system are shown in Figure 33. The upper trace, which was recorded at a slow oscillograph speed, serves as a convenient visual observation of the dynamic force from which the principal frequencies occurring in this record can easily be identified (It is assumed in this discussion that the pressure measuring system exhibits no resonant condition which might be excited, that the system is damped, and that it has a relatively flat calibration relationship). The principal frequencies in this record correspond to the natural frequencies of the test vehicle; that is, the body motion frequency is approximately one cycle per second and the wheel-hop frequency is approximately fourteen cycles per second. For a more detailed analysis, the record shown by the lower trace of Figure 33 is used. This record was taken at five times the recording speed of the upper trace. The abscissa of the record may be easily changed from time to distance

FIGURE 33. OSCILLOGRAPH RECORDS OF DYNAMIC FORCE

along the highway by introducing the velocity of the vehicle. By applying the single calibration factor F/P (lb/psi), the pressure scale may be converted into a force scale. If markers are put on the highway to produce a pressure indication on the record at a known location, the dynamic force may be determined at any point on the test section. If the static tire force is known, the total force on the pavement may be calculated at any position.

These records may also be processed to indicate how often certain magnitudes of the dynamic force were applied to the pavement section. This information may be found by first reading values of force from the record at equal intervals of distance. These values are then assorted so that the number of values in various ranges of tire force is determined. This number is divided by the total number of force readings included in the analysis of the record to obtain the fraction of the values in the force interval under consideration.

The force interval which was selected for each test section corresponds to $1/20$ of the magnitude of the absolute maximum force that was recorded. The twenty intervals between zero and the absolute maximum force correspond to the twenty intervals between zero and the edge of the recorder paper as shown in Figure 33.

The results of applying this procedure to two different pavement sections are shown in Figure 34. One of these pavement sections was very rough and the other very

FIGURE 34. DISTRIBUTION OF DYNAMIC TIRE FORCES FOR PAVEMENT SECTIONS N35 AND N85

smooth, as is evident from the range of forces that occurred. The force interval for the smooth section is correspondingly smaller than the interval for the rough section because of this difference in force range. The standard deviation or root mean square (RMS) force was also calculated for this distribution. It is interesting to note that there is almost a factor of six between the RMS force values for the rough and smooth pavements which indicates that the dynamic force might serve well as a measure of pavement condition.¹³

The ordinate of the histogram shown in Figure 34 may also be expressed as the fraction of time in the force interval. This representation is exact if the spacing of the data points is small enough so that the oscillograph trace remains within the selected force interval for the length of time represented by the distance between two data points. As the data spacing is increased beyond a minimum interval, the representation becomes less exact.

A second method of analysis of highway test data is the power spectral density analysis that was illustrated in Chapter IV. This method of analysis indicates the amplitude distribution with respect to the frequencies present in the record. The RMS dynamic force may be obtained by extracting the square root of the area under the power spectral density function.

Referring to Figure 31, it is important to realize that the force power spectral density function may be obtained directly from the pressure record by the use of a

single calibration factor F/P or it may be obtained from the pressure power spectral density function by the introduction of the complete calibration relationship in the frequency domain. The introduction of the calibration relationship yields a more accurate result.

As an example of the use of these methods of analysis, consider the results of two of a series of tests conducted over a 300 foot test section with a passenger vehicle. The vehicle was operated at a different velocity for each test. The distribution of the dynamic forces for the extreme speeds of approximately 30 and 60 mph are shown in Figure 35. It is interesting to note that as the vehicle velocity changed, the distribution of the dynamic forces changed. As the velocity increased, the maximum force increased. In addition, the distribution developed an appreciable skewness toward the positive forces. This skewed phenomena was also observed during the tests and may be seen in the force records shown in Figure 30. The largest positive forces, which had a maximum positive value of approximately 350 pounds, occurred between 48 and 62 feet from the starting point of the test section. The dynamic forces were all negative between 70 and 115 feet from the origin of the test section. The maximum absolute value of these negative forces was only approximately 195 pounds as compared with the 350 pound magnitude of the positive forces.

Further examination of Figures 30 and 33 reveals that the dynamic force consists of two principal frequencies, the low body motion frequency of approximately one cycle per second and the higher wheelhop frequency of approximately

FIGURE 35. DISTRIBUTION OF DYNAMIC TIRE FORCES AT DIFFERENT SPEEDS

fourteen cycles per second. It is of interest to observe in Figure 30 that the dynamic force variation associated with the wheelhop is much larger when the force associated with the body motion is positive, i.e. when the total force due to the static weight and the body motion is increased.

A power spectral density analysis of the two velocity tests reveals information about the frequencies present in the dynamic force records. The dynamic force power spectrum for the test vehicle traveling at 30 mph is shown by the dotted line in Figure 36. Two peaks of appreciable magnitude are indicated by this curve. The first peak, occurring over a range of low frequencies, indicates that the motion of the sprung mass (body motion) of the vehicle makes an appreciable contribution to the total mean square value of the dynamic tire force. A second peak, occurring over a range of higher frequencies, indicates the contribution to this value that results from motion of the unsprung mass of the vehicle (wheelhop).

The power spectral density function of the dynamic tire force for the same vehicle traveling over the same section of pavement at 60 mph is shown by the solid line in Figure 36. It is evident that the area under this curve is greater, indicating that the RMS force is greater at 60 mph than at 30 mph. The RMS force values are shown in Figure 36.

Of considerable interest is the change in the shape of the curve. The peak associated with the low frequencies

FORCE
PSD
(LB²/CPS)

INDIANA S.R. 26
SECTIONS 17-20
VEHICLE NO. 210
TIRE PRESSURE =
30 PSI

FIGURE 36. POWER SPECTRAL DENSITY ANALYSIS OF DYNAMIC TIRE FORCE RECORDS

is not the same at both speeds, indicating that the motion of the sprung mass (body motion) changed appreciably. The motion of the unsprung mass (wheelhop) also was increased at the higher vehicle speed as evidenced by the large increase in the ordinates of the curve in the region of the wheelhop frequency.

Another change of considerable interest is also evident. In the power spectrum curve for 30 mph, a small intermediate peak can be seen approximately midway between the two large peaks. This peak is due to a large amount of excitation coming from the pavement since no natural frequencies exist in the suspension system of the test vehicle at this frequency. When the vehicle speed is doubled, the frequency of excitation from the highway is doubled. At 60 mph, the excitation that caused the small intermediate peak has doubled in frequency causing an additional excitation at the wheelhop frequency. Since the vehicle is very responsive at this frequency, a great increase in tire force is produced as shown by the curve.

In summarizing the effect of the velocity of the vehicle on the resulting dynamic forces, it can be seen that the RMS force serves as a valuable statistic. The RMS forces are plotted against vehicle velocity for four test velocities of 30, 40, 50, and 60 mph in Figure 37. The high and low values on this curve correspond to the test results shown in Figures 35 and 36.

The RMS force statistic may also be used to compare the dynamic forces generated when a vehicle is operated

FIGURE 37. ROOT MEAN SQUARE VALUE OF DYNAMIC TIRE FORCE VERSUS VEHICLE VELOCITY

over different pavements. Figure 38 illustrates the dynamic forces generated when the test vehicle was driven over several pavements that varied in condition as well as type of construction. Each pavement was assigned one of three subjective ratings (smooth, average, rough) as determined by the individuals who operated the test vehicle. It is interesting to note that RMS dynamic forces ranging from 32 to 341 pounds were obtained. A large range of values is thus encountered when this statistic is employed as a criterion.

The methods just presented are certainly not the only methods that might be applied for analyzing dynamic force measurements. However, these methods have proved to be the most useful for the data analysis for this project.

CHAPTER VI

SUMMARY AND CONCLUSIONS

This investigation has been concerned primarily with the design and construction of a pressure measuring system for the purpose of measuring the dynamic forces between a vehicle tire and the pavement. The pressure measuring system as defined in this paper includes the vehicle tire and the associated equipment required for measuring the tire inflation pressure. The following observations may be made about the design and use of a pressure measuring system as a direct force transducer.

1. An effective pressure measuring system can be designed so that the dynamic forces do not excite any resonant condition within the pressure measuring system. The system can then be adequately damped by the introduction of an added restriction into the system as previously shown. The mechanical components can also be designed so that they exhibit no natural frequencies within the frequency range that is of interest. The theory to guide such a design is developed in Chapter II. The design of both pneumatic and mechanical components to accomplish the desired end is discussed in Chapter III.

2. In order to measure the adequacy of the design of a pressure measuring system, dynamic calibration tests are required. By using inputs having a larger frequency range than is encountered under actual road test conditions, the pressure measuring system can be thoroughly tested and adequately calibrated.

3. With careful selection of the parameters of the system, the calibration relationship can be made flat enough so that a single calibration factor can be applied to the pressure measurements without the introduction of excessive error.

4. If the inflation pressure of the tire is considerably larger than the recommended value, the calibration relationship of the pressure measuring system is decidedly frequency dependent. Therefore, the use of the frequency domain calibration relationship rather than the single calibration factor is necessary if the system is to be used at inflation pressures other than the recommended value.

5. The ideal flat calibration relationship is extremely difficult to obtain, particularly because of the non-linear behavior of the tire. The pressure measuring system is also amplitude dependent. In order to partially account for these non-linear effects, it is recommended that an average calibration relationship be obtained from a series of calibration tests utilizing different impulsive excitations.

6. The continuous method of equalizing the long-term pressure variations is particularly advantageous as compared with the discrete equalization method because of the substantially reduced testing time which results when continuous equalization is used.

7. The methods of designing the pressure measuring system as presented in this paper are applicable to pressure measuring systems to be used with larger vehicles. The basic theory considered in Chapter II indicates that the volume of the tire is not a limiting factor as far as the dynamic behavior of the system is concerned. It is expected, however, that the frequency characteristics of the system attributed to the tire may be different in the case of truck tires with heavy treads.

On the basis of this study, the following recommendations for areas of further study are made:

1. A calibration procedure is needed that is capable of producing significant excitation at all frequencies and which would apply to all vehicles. Therefore, no matter what the frequency characteristics of the particular system and vehicle to be calibrated, a test or series of tests could be conducted which would completely define the system characteristics.

2. The calibration equipment required to produce an adequate range of impulsive inputs for all vehicles to be tested should be completed. The basic equipment for calibration of large vehicles was developed by McLemore¹⁴ who

designed the calibrator so that only slight modifications would be necessary to accommodate a range of impulsive inputs once this range is fully defined.

3. The development of equipment for dual wheels which would allow the measurement of the dynamic force at either one wheel, both wheels, or each wheel separately would be advantageous.

4. The continued study of methods of compensating for the frequency dependence of the calibration relationship is still an area of considerable interest.

5. The development of a compact, easily portable set of instruments which would include a power source, visual recorder, and a magnetic tape recorder would be a great contribution to the measurement of dynamic forces. The tape recording of test results would enable rapid data reduction in digitalizing the data for further processing as well as direct calculation of power spectral density functions and RMS forces by electronic instruments.

The results of this investigation have been quite gratifying. The use of a pressure measuring system for measuring dynamic tire forces appears to be a promising and useful engineering tool.

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BIBLIOGRAPHY

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APPENDIX

APPENDIX

An Approximate F/P Ratio for a Pneumatic Tire

Consider the force developed at the contact area "A" between a tire and the pavement as shown in Figure 39. If the stiffness of the tire body is neglected, the force at the contact surface is due only to the inflation pressure and may be expressed as

$$F = p_g A = pA - p_a A \quad (A.1)$$

where: F = total force over the contact area (lb)

A = contact area (in^2)

p = absolute inflation pressure (psia)

p_a = atmospheric pressure (psia)

p_g = gage inflation pressure (psia).

The calibration factor F/P is actually the derivative dF/dp evaluated at the static values of force and inflation pressure. This derivative is

$$\frac{dF}{dp} = \frac{d}{dp}(pA - p_a A) = A + p_g \frac{dA}{dp} \quad (A.2)$$

The derivative dA/dp may be expressed in terms of the geometrical properties of the tire as shown in Figure 39.

FIGURE 39. SCHEMATIC DRAWING OF A PNEUMATIC TIRE

The following relationships may be derived from Figure 39:

$$A = wL \quad (A.3)$$

$$L = 2R \sin(\phi/2) \quad (A.4)$$

$$a = \frac{R^2}{2} \phi - \frac{R}{2} L \cos(\phi/2). \quad (A.5)$$

Differentiating equations (A.3) and (A.4),

$$\frac{dA}{dp} = w \frac{dL}{dp} = wR \cos(\phi/2) \frac{d\phi}{dp}. \quad (A.6)$$

Equation (A.5) can be simplified by substituting equation (A.4) into (A.5),

$$a = \frac{R^2}{2} \phi - \left(\frac{R}{2}\right) 2RL \sin(\phi/2) \cos(\phi/2)$$

or

$$a = \frac{R^2}{2} (\phi - \sin \phi), \quad (A.7)$$

since

$$\sin \phi = 2 \sin(\phi/2) \cos(\phi/2).$$

Considering equation (A.6), it is evident that the relationship between " ϕ " and " p " must be found in order to evaluate the derivative dA/dp . In order to make this evaluation, the relationship between tire pressure and tire volume must be considered. Neglecting any bulging of the sidewall when the tire is loaded, the volume of air in the tire is assumed to be

$$V = V_1 - aw \quad (A.8)$$

where

V = internal volume of the tire

V_1 = initial internal volume of the tire when no force is applied to the tire,

and a and w are defined in Figure 39.

Assuming that the air in the tire follows a polytropic compression process, the following relationship is valid:

$$pV^\gamma = \text{Constant} = p_1 V_1^\gamma \quad (\text{A.9})$$

where

γ = a constant such that $1 < \gamma < 1.4$, and the subscript 1 indicates that there is no force applied to the tire.

From equation (A.9)

$$V = (p_1^{1/\gamma} V_1) p^{-1/\gamma} = Q p^{-1/\gamma} \quad (\text{A.10})$$

where

$$Q = p_1^{1/\gamma} V_1. \quad (\text{A.11})$$

Substituting equations (A.10) and (A.7) into equation (A.8) yields

$$Q p^{-1/\gamma} = V_1 - \frac{wR^2}{2} [\phi - \sin \phi] \quad (\text{A.12})$$

which is the desired function relating " ϕ " and " p ".

By differentiating equation (A.12) with respect to inflation pressure,

$$\frac{d\phi}{dp} = \frac{Q p^{-\left(\frac{\gamma+1}{\gamma}\right)}}{\frac{wR^2}{2} (1 - \cos \phi)} \quad (\text{A.13})$$

An expression for the derivative dF/dp results if equations (A.6), (A.11), and (A.13) are substituted into equation (A.2).

$$\frac{dF}{dp} = A + \left(\frac{2V_1}{\gamma R} \right) \left(\frac{\cos \phi/2}{1 - \cos \phi} \right) \left(\frac{p_1}{p} \right)^{1/\gamma} \left(\frac{p_g}{p} \right) \quad (A.14)$$

The angle ϕ may be expressed in terms of the static wheel load F_o and the geometrical parameters shown in Figure 39.

$$A = F_o/p_g = wL = w[2R \sin(\phi/2)] \quad (A.15)$$

Therefore

$$\sin(\phi/2) = F_o/(p_g 2wR) = B \quad (A.16)$$

By the Pythagorean Theorem,

$$\cos(\phi/2) = \left(1 - B^2 \right)^{1/2} \quad (A.17)$$

Also

$$\cos \phi = 1 - 2[\sin(\phi/2)]^2 = 1 - 2B^2 \quad (A.18)$$

Combining equations (A.15), (A.17), and (A.18) with (A.14),

$$\frac{dF}{dp} = \left[\frac{F_o}{p_g} \right] + \frac{V_1}{\gamma R} \left[\frac{(1-B^2)^{1/2}}{B^2} \right] \left[\frac{p_1}{p} \right]^{1/\gamma} \left[\frac{p_g}{p} \right] \quad (A.19)$$

where

$$B = F_o/(2p_g wR) = A/(2wR) = L/2R. \quad (A.20)$$

The polytropic constant γ may be taken as approximately equal to unity since the frequency range of importance is relatively low (less than 100 cps). This low frequency range allows the assumption of an isothermal compression to be made. The ratio (p_1/p) in equation (A.19) is approximately equal to unity because the difference between the initial inflation pressure p_1 and the inflation pressure under load is approximately .5 psi for a passenger tire with an inflation pressure of 30 psig.

Equation (A.20) illustrates that the dimensionless parameter B may be reduced to the ratio of the contact length to the tire diameter. This important dimensionless parameter may be measured directly from a tire in the loaded condition.

Equation (26) (which is identical to equation (A.19)) is plotted in Figures 10, 11, and 12 and is compared with experimental measurements in Figure 10.

It can be observed in Figure 10 that the curve representing the approximate F/P ratio is lower than the line joining the experimental values of F/P. It is interesting to note that if the assumption that led to equation (A.8) is altered, the approximate F/P ratio is increased and thereby compares more favorably with the experimental data. That is, if a volume change due to the bulging of the sidewall is considered, the term " a_w " in equation (A.8) will be reduced. Since this reduced term appears in the denominator of equation (A.19), the F/P ratio will therefore increase.

Other refinements of this approximate relationship which can be studied are the consideration of the mass effect of the tire tread under dynamic conditions and the consideration of the stiffness of the tire body. These extensions of the theory are suggested areas for further study.

VITA

VITA

The author, Clement Card Wilson, was born on August 11, 1933, in High Point, North Carolina. He was graduated from University High School, Johnson City, Tennessee, in June 1951. He attended The University of Tennessee on the Cooperative Program after his freshman year at East Tennessee State University. His cooperative work was for the Tennessee Eastman Company, Kingsport, Tennessee. He was graduated from The University of Tennessee in June 1956 with a Bachelor of Science degree in Mechanical Engineering.

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