**FLEXIBLE PAVEMENT DEFLECTIONS—
METHODS OF ANALYSIS AND INTERPRETATION**

FEB. 1962

NO. 4

**Joint
Highway
Research
Project**

by
E. J. YODER

**PURDUE UNIVERSITY
LAFAYETTE INDIANA**

Technical Paper

FLEXIBLE PAVEMENT DEFLECTIONS - METHODS
OF ANALYSIS AND INTERPRETATION

TO: K. B. Woods, Director
Joint Highway Research Project

February 14, 1962

FROM: H. L. Michael, Associate Director
Joint Highway Research Project

File: 6-20-6
Project: C-36-52F

Attached is a technical paper entitled "Flexible Pavement Deflections - Methods of Analysis and Interpretation" by Professor E. J. Yoder of our staff. The paper was presented at the recent meeting of the Association of Asphalt Paving Technologists in New Orleans.

The paper discusses the several methods of analysis and interpretation which can be used in the evaluation of flexible pavements.

The paper is presented to the Board for the record and for approval of submission to the AAPT for possible publication.

Respectfully submitted,

Harold L. Michael

Harold L. Michael, Secretary

HLM:kmc

Attachment

Copies:

F. L. Ashbaucher
J. R. Cooper
W. L. Dolch
W. H. Goetz
F. F. Havey
F. S. Hill
G. A. Leonards

J. F. McLaughlin
R. D. Miles
R. E. Mills
M. B. Scott
J. V. Smythe
J. L. Waling
E. J. Yoder

Digitized by the Internet Archive
in 2011 with funding from

LYRASIS members and Sloan Foundation; Indiana Department of Transportation

Technical Paper

FLEXIBLE PAVEMENT DEFLECTIONS - METHODS
OF ANALYSIS AND INTERPRETATION

by

Eldon J. Yoder
Research Engineer

Joint Highway Research Project

File No: 6-20-6

Project No: C-36-52F

Purdue University
Lafayette, Indiana

February 14, 1962

UNITED STATES DEPARTMENT OF AGRICULTURE
NATIONAL AGRICULTURAL STATISTICS SERVICE

1

1997-1998
1997-1998

1997-1998
1997-1998
1997-1998

1997-1998
1997-1998

1997-1998

FLEXIBLE PAVEMENT DEFLECTIONS - METHODS OF ANALYSIS OF INTERPRETATION

by

Eldon J. Yoder

Joint Highway Research Project
Purdue University

Synopsis

The amount a flexible pavement deflects under load indicates, in part, its adequacy insofar as structural capacity is concerned. Repeated deflection may cause the pavement to crack and distort as a result of fatigue, excessive bending stresses, accumulated plastic deformation and other factors.

The deflection of a flexible pavement is partly elastic in character, but it is also made up of plastic strains. Elastic strains are regained upon removal of an applied load whereas plastic strains are not. Thus, the accumulation of non-recoverable plastic strains with repeated applications of load can result in distortion of the paving surface.

Many engineers use deflection measurements to evaluate the adequacy of existing pavements. The literature contains numerous references of deflection measurements, including the work done at the WASHO and AASHO Road Tests. Deflection measurements are but one tool that can be used by the researcher to formulate concepts regarding the behavior of flexible and rigid pavements. Deflections measurements are subject to many limitations and therefore must be considered to be a means towards an end rather than an end within themselves.

The primary purpose of determining the deflection of an existing pavement, insofar as structural adequacy is concerned, is to obtain basic

data, either by inference or direct measurement, relative to the stress-strain properties of the pavement materials. Mere measurement of gross deflection at the pavement surface may not yield the desired results. Such factors as radius of bending and the viscoelastic properties of the pavement components also have their effect.

Several methods of analysis can be adopted.

1. Measurement of gross deflection and correlation of same with performance.
2. Measurement and analysis of gross deflection along with the deflection of each component layer of the pavement.
3. Analysis of deflection profiles and contours.
4. Determination of unit deformation of each layer (total deflection divided by height.)
5. Determination of a constant or constants which define the stress-strain properties of the materials.
6. Analysis of the fundamental frequency responses of the system.

The significance of numerical deflection values depends upon the properties of the pavement. Pavements which have high values of moduli of elasticity can tolerate less deflection, for a given load, than those with low moduli. Deflection patterns must be considered in some situations to get the most out of deflection measurements.

FLEXIBLE PAVEMENT DEFLECTIONS -- METHODS OF ANALYSIS AND INTERPRETATION

by

Eldon J. Yoder

Research Engineer and Associate Professor
of Highway Engineering
Purdue University

INTRODUCTION

The amount a pavement deflects under load indicates, in part, its adequacy insofar as its structural capacity is concerned. Large deflections indicate an over-stressed condition which may cause the pavement surface to crack. Repeated deflections may cause the pavement to crack and distort as a result of fatigue, or accumulative plastic deformation.

The performance of the flexible pavement is influenced by many factors. These include gross load, tire pressure, repetition of load, thickness and durability of the various pavement components, and the elastic-plastic properties of the pavement components, (particularly the subgrade soil). Pavement distress may be influenced by a combination of the above factors. Pavement failure may result from excessive shear stresses, consolidation of one or more of the pavement components, lateral shoving, bending or a combination of these.

Historically, pavement design has been approached from two broad differing points of view. First, the practicing engineer most often uses concepts of design based primarily on pavement performance. Second, the approach used by researchers and educators is based principally upon theoretical concepts. Experience has indicated that neither of these concepts by itself gives the optimum answer. Complete reliance upon

pavement performance represents a static condition where one must wait a relatively long period of time before new concepts can be proven out. On the other hand, theoretical equations are generally based upon simplified assumptions of elasticity and many times are still too cumbersome to be used in practice.

The inevitable result of the above has been that most design techniques are approached on an empirical basis, although several are "quasi-theoretical". Under this latter approach, the design engineer utilizes theoretical concepts as much as he can, but these concepts are modified in various ways to make them usable in practice.

In the ideal case the design engineer should relate pavement stress to the ability of the paving materials to resist these stresses. However, not only are the computed stresses under load open to question due to the non-homogeneity of the problem, but in addition stresses are very difficult to measure in the prototype pavement. In contrast to this, measurement of pavement deflection under load is relatively easy to accomplish. Since stresses are related to strains, the amount of pavement deflection under load indicates to some extent its ability to resist the applied forces.

Engineers have for many years used the principle (at least in an indirect sense) that deflection of a pavement under load indicates its load carrying capacity. Several methods of flexible pavement design are based upon limiting deflection criteria. These include, for example, procedures adopted by the Kansas State Highway Department and the Navy Department. Both of these methods are predicated in part upon theoretical consideration which relate pavement stresses and deflections to the applied load. It is necessary in these analyses to make certain simplifying assumptions regarding the shape of the tire imprint on the pavement surface, the

relationship between tire pressure and contact pressure and homogeneity of the structural system. In addition, a simple elastic model is used in deriving the mathematical equations for stresses and strains.

Many engineers have used deflection measurements to evaluate the adequacy of existing pavements. Deflection measurements are but one tool which can be utilized by the researcher to formulate concepts regarding the behavior of flexible and rigid pavements. Deflection measurements are subject to certain limitations and therefore must be considered to be a means toward an end rather than an end within themselves.

Elasticity of the paving structure is perhaps the most important factor the engineer must consider when dealing with deflection measurements. Pavement deformation is dependent upon time. Rapidly applied loads generally cause less deflection than slowly applied loads. Also, flexible pavements are layered systems and they are not homogeneous. Nevertheless, principles derived from the theories of elasticity and viscoelasticity have great use in pointing out to the engineer the inter-relationship of the factors affecting pavement performance. For this reason it is desirable for the engineer to have an understanding of the theories so that he can more truly appreciate the significance of pavement deflections.

PURPOSE OF MEASURING DEFLECTION

The primary purpose of determining the deflection of an existing pavement, insofar as structural adequacy is concerned, is to obtain basic data either by inference or by direct measurement relative to the stress-strain properties of a pavement structure. Sometimes measurement of gross deflection at a pavement surface may not yield the desired results. Such factors as radius of bending and component deflection have their effect.

Pavement Distress

One of the biggest questions that must ultimately be considered by the practicing engineer is "What constitutes a failure?". The complexity of this question probably affects the variation in design thicknesses obtained by the various design methods as much as any other single point.

There is no exact definition in existence at the present time which states the ultimate desired performance of pavements. Engineers vary widely in their concepts of acceptable performance. If one is willing to accept the assumption that the purpose of the pavement is to carry vehicles over it through all weather conditions with maximum comfort and minimum inconvenience to user, design criteria immediately imply relatively smooth surfaces, accident free roads, and economic operation of vehicles over the pavement. This leaves the definition of ultimate failure open to the opinion of the pavement user. The concept that the pavement user should dictate performance criteria was adopted by personnel at the AASHO Road Test for evaluating the performance of their test pavements.

If criteria for ultimate performance of pavements are open to the opinion of the user, allowable structural behavior (in this particular case pavement deflection) also becomes largely a matter of opinion. Because of this opinion aspect, engineers differ widely in their concepts of allowable deflections. Likewise, methods of analyzing deflection data are not standardized. This in turn has resulted in some confusion among engineers relative to the significance of pavement deflections and how deflection criteria can be used in pavement evaluation and design.

Use of Deflection in Design Criteria

As was previously stated several agencies currently use design methods based on limiting deflection criteria. These, by and large, are quasi-theoretical

methods that rely heavily upon empirical data. It will be pointed out in subsequent paragraphs that assignment of allowable deflections is open to some question. This is because of the fact that deflection patterns, component deflections and, in particular, radii of bending can be extremely important and in many cases override inferences that may be drawn from total deflection.

The movement of a pavement under load is determined in the final analysis by the stress-strain properties of the paving materials. This is true whether one assumes an elastic pavement or whether viscoelastic methods of analysis are used. For design purposes, theory is generally used for determining stresses in the pavement structure and from this the thickness of pavement to keep the deflection within a certain allowable amount is determined.

Use in Pavement Performance and Evaluation Studies

Deflection measurements are extremely useful in evaluating the potential performance of existing pavements. These data, by and large, are used in an empirical manner in which relative values of pavement deflection are correlated with performance. Data of this type are useful in studying the effects of time and climate on the pavement structure.

Use as a Research Tool

Researchers have found measurement of deflection to be a very useful tool. Data of this type can be used to evaluate the fundamental stress-strain properties of paving materials and these in turn can be utilized for setting up new design procedures.

METHODS OF ANALYSIS

Pavement deflection values can be used in a variety of ways. The method of analysis should depend upon the intended usage of the data. In some cases detailed data are not only unwieldy but are too complicated to handle at the present time. As a general rule, measurements which yield the maximum amount of information with the least amount of effort are used.

Several methods of analysis can be adopted.

1. Measurement of gross deflection and correlation of same with performance,
2. Measurement and analysis of gross deflection along with the deflection of each component layer of the pavement,
3. Analysis of deflection profiles and contours,
4. Determination of unit deformation of each layer (total deflection divided by height),
5. Determination of a constant or constants which define the stress-strain properties of the materials,
6. Analysis of the fundamental frequency responses of the system.

Correlation of surface deflection with performance is perhaps the most widely used method at the present time. It offers the possibility of accumulating a large amount of data in a relatively short period of time. Determination of the deflection of the pavement components yields relative data which can be used in a qualitative sense. However, since component deflection is dependent upon depth as well as type of material, it is necessary to analyze the data in light of the depth below the pavement surface.

Utilization of deflection profiles offers some potential in the analysis of the data. From structural mechanics it is known that bending stresses are related to radii of bending. Pavements which have small radii of bending under load will crack more frequently than those with large radii of bending.

Use of unit deformation is subject to the restriction that deflection is dependent upon depth below the surface as well as type of material. Thus it becomes necessary to exercise a degree of caution in computing unit deformation since it does not take into account stresses that exist on any given layer of the pavement.

Another method of analysis deals with determination of certain elastic constants which define the stress-strain properties of the pavement materials. The constant which first comes to mind is the modulus of elasticity (sometimes called modulus of deformation); Poisson's ratio is also a significant property of the material that must be considered.

Prototype flexible pavements are not truly elastic and response of a pavement to load is time dependent. There is a need to know something of the fundamental response characteristics of the pavement.

RELATIONSHIP BETWEEN DEFLECTION AND STRESS

In the final analysis, stresses, and the pavement's ability to resist these stresses, determine the pavement's load carrying capacity. Pavement deflection has a definite relation to stresses and, for a given load, gives an indication of the stress resisting properties of the structure. Deflection is determined in large part by the stiffness of the pavement and subgrade. Deflection is time dependent, although it has been shown that for any single application of a moving load, pavement deformations are

primarily elastic (30). This principle has been used by several investigators* in the analysis of flexible pavement behavior.

Pavement Stiffness

As used herein the term "stiffness" has specific meaning. Stiffness is the quality of a pavement which enables it to resist bending. It encompasses the property of elasticity and takes into account thickness.

Considering a hypothetical stiffness scale, one can visualize at one end of the scale a completely rigid material which is infinitely thick and has a relatively high modulus of elasticity. On the other end of the scale is a plastic material which is relatively thin. According to common usage just two broad categories of pavements are considered. These are termed "rigid" and "flexible". It must be recognized first that the rigid pavement is not truly rigid in the sense that it doesn't deform under load, but rather it is implied that its relative stiffness is much greater than that of the conventional flexible pavement.

Design concepts adopted by theoreticians differ widely for the two types of pavements. In the analysis of rigid pavements "slab action" is considered whereas in the case of flexible pavements primary consideration is given to vertical stress distribution through the subgrade. The classical theory of elasticity generally ignores vertical strains in the rigid pavement slab. In contrast, vertical strains are given detailed treatment in the analysis of flexible pavements. Restrained movement (warping and/or shrinkage and contraction) is extremely important in rigid pavements but has practically no effect on the stresses in flexible pavements.

* See reference 1 for example.

Stresses in subgrades under flexible pavements are of paramount importance whereas they are neglected in the classical rigid pavement analysis.

It should be emphasized that the definitions of "rigid" and "flexible" pavements are purely arbitrary and relative. There is a condition between these two extremes where the pavement can be termed semi-rigid or semi-flexible. Most pavements constructed at the present time (both rigid and flexible) probably fall in the semi-class between the two extremes. This further complicates the problem since it is necessary for the designer first to make an assumption whether his pavement structure is rigid or flexible and then to approach the problem of design on this basis.

Elastic Behavior

By definition, stress in an elastic material is proportional to strain below the material's proportional limit. It has been known for many years that the modulus of elasticity of practically all paving materials (particularly soils) is dependent on rate of loading. Pister and Monismith (25) and others have reported on the viscoelastic behavior of paving materials. On the other hand, most investigators have assumed elasticity in the conventional sense (1, 6, 35). It is perhaps academic at this point to present arguments pro and con relative to the assumptions pertaining to elastic behavior but it must be recognized that flexible pavements are layered systems that behave, in the strictest sense, in only a partially elastic manner. Nevertheless, there is considerable justification in the literature for assuming quasi-elastic behavior.

Figure 1 illustrates the relationship between modulus of elasticity and strain of a loaded element. Poisson's ratio, which is the ratio of strain in the direction of the applied stress to the strain in a direction

90° to the applied stress, is a tenuous value that is extremely difficult to evaluate.

Considering the special triaxial condition where $\sigma_1 = \sigma_2 = \sigma_3$, the strain in the vertical direction is

$$\delta = \frac{1}{E} (\sigma_1 - \mu\sigma_2 - \mu\sigma_3) \quad (1)$$

$$\Delta = \frac{\mu\sigma}{E} (1 - 2\mu) \quad (2)$$

The change in volume per unit volume is

$$\frac{\Delta V}{V} = \frac{\frac{3\sigma}{E} (1 - 2\mu)}{1} \quad (3)$$

For a Poisson's ratio of 0.5, the quantity $\frac{\Delta V}{V}$ becomes zero and there is no volume change whereas for Poisson's ratio of zero, $\frac{\Delta V}{V}$ is $\frac{3\sigma}{E}$. This is recognized as the unidirectional strain problem.

Poisson's ratio for a pavement just opened to traffic is probably some value between zero and 0.5. After an extended period of time, the value probably approaches that indicating no volume change (0.5). This latter value is the one generally used in the theoretical equations.

The modulus of elasticity of a material is the ratio of unit stress to the corresponding unit strain for any value of stress below the proportional elastic limit. Most soils and granular mixtures do not have definite elastic limits, since they do not have stress-strain curves which are straight lines. Likewise the moduli are time dependant, increasing as rate of strain increases. Baker (1) suggests the potential use of an instantaneous modulus which can be determined experimentally by performing a series of tests under varying rates of strain. There appears to be

considerable justification for this concept, especially when dealing with moving loads on a highway. It should be recognized, however, that rate of strain can be standardized when dealing with measured values of pavement deflection. This principle will be discussed further in subsequent paragraphs.

Classical Analysis of Flexible Pavements

Consideration will be directed next to the classical methods of stress analysis in flexible pavements. Much has been written about this by various individuals; these articles will not be discussed but rather a bibliography, listing several pertinent references is given at the end of this report.

In standard soil mechanics problems dealing with foundations two design requirements must be met, (1) design against shear failure and (2) design against consolidation of the soil. These analyses are carried out for a single application of a static load. In flexible pavements consideration is given to shear and consolidation but emphasis is also placed on safety against lateral movement, analysis of contact stresses and analogies dealing with slab action. The problem is further compounded by the fact that in design of pavements it is necessary to deal with repeated loads which are moving, quality of paving materials, climate and many other factors. It is important to recognize that in actual practice the factors given above are interrelated and cannot in fact be separated.

Perhaps the oldest method of analysis deals with calculation of stresses in ideal elastic masses (Figure 2) based upon the Boussinesq stress problem. In this analysis it is assumed that the pavement is a single layer which is ideally elastic, homogeneous and isotropic. This

analysis was later modified to account for layered systems by Dr. Burmister (6, 7). The Burmister solution was extended by Fox (15). Tabulated stresses for the three layer problem have recently been presented by Jones (21).

Deflection equations for flexible pavements take the following general form.

$$\Delta = \frac{p a F}{E} \quad (4)$$

where Δ is deflection

p is contact pressure

a is radius of contact (circular)

E is modulus of elasticity of the subgrade

F is a dimensionless factor accounting for depth and the ratio of moduli of elasticity, pavement layers and subgrade

Equation 4 is the general form of the elastic deflection equation under the center of a circular load. Equation 5* gives a quantity defined as the plate rigidity of the pavement layer. Combination of the flexible equation (equation 4) and the stiffness equation (equation 5) was first accomplished by Palmer and Barber (24); these concepts were later expanded by Baker (1).

Researchers are continually attempting to check theoretical stresses and strains with the actual values which exist in flexible pavement systems. As a rule the measured stress and deflection patterns follow the ideal (16, 20) but the exact numerical values cannot be checked against the theoretical values. (See Figure 11 for a typical example). Hence, it becomes immediately apparent that some modification of theoretical concepts must be made before they can be applied to the design of pavements.

* See section of this paper dealing with slab action.

Deflection profiles in a pavement take the characteristic shape indicated in Figure 3. Surface deflection is made up of cumulative deflections and for the general case a large portion of the deflection takes place in the subgrade. The pavement may "heave" between wheels depending upon the stiffness of the pavement materials.

For a given loading condition and thickness of pavement, deflection bears a direct relationship to stress. However, the modulus of elasticity also influences the amount a pavement will deflect. Hence, the general statement can be made that deflection can be used as a guide for determining the ability of a pavement to resist stresses since deflection reflects, for a given load and thickness, the pavement's modulus of elasticity. Also, layer deflections can be extremely useful in the analysis of the problem. These factors will be discussed in greater detail in subsequent paragraphs.

Slab Action

The present trend in design of flexible pavements is towards building stiffer pavements. This is accomplished by one of several means. In some cases the base course materials are stabilized with cementing agents and in addition bituminous pavements are thicker than they were several years ago. As a result, these pavements probably initially approach a rigid condition. However, if structural integrity of the pavement is lost by cracking, the pavement reverts to the flexible condition.

Figure 4 illustrates in diagrammatic form the stress inducing factors for a rigid slab resting on an elastic foundation. As the slab deforms under load, reactive pressures are built up under the slab. These reactive pressures in turn limit deflection. The primary implication in the analysis is that the pavement structure is capable of resisting bending. This, it

is seen, is in direct contrast to the equations dealing with deformation of an elastic-flexible system.

Certain assumptions are made in the classical analysis of rigid pavements which should be recalled. First it is assumed that the reactive pressure under the slab is proportional to deflection and that the reactive pressures are vertical and vertical only. Next it is assumed that slab strains in the vertical direction are negligible and it is implied that the paving material can take tensile stresses. The fundamental equation which defines the stiffness of the pavement itself is as given in equation 5.

$$D = \frac{Eh^3}{12(1-\mu^2)} \quad (5)$$

Stiffness of a slab varies as the cube of the thickness and as the first power of the modulus of elasticity. The relative stiffness of the slab to the subgrade is as given in equation 6.

$$l = 4 \sqrt{\frac{Eh^3}{12(1-\mu^2)k}} \quad (6)$$

where

E = Modulus of elasticity of the slab

h = Thickness of the slab

μ = Poisson's ratio of the paving layer

k = Modulus of subgrade reaction

It is doubtful whether flexible pavements are ever rigid in the strictest sense except in certain instances. Nevertheless, practically all pavements exhibit some degree of stiffness and the factors considered in these analyses are of interest to the engineer.

Stiffness of a slab is extremely important since it implies the ability to resist tension. Cracking of a bituminous pavement indicates that either shear or tension failures have occurred. Tension in the pavement depends upon the degree of sharpness of the bent surface. Long gently deflected surfaces have less stress (for a given deflection) than surfaces bent over a sharp radius (Figure 5). The above principle is utilized in structural design of foundations and design of dowel bars in concrete pavement. In the former case, considering a long and narrow rectangular footing, the major steel reinforcement is placed parallel to the short dimension. Likewise, heavier dowel bars are required over stiff subgrades than over yielding subgrades due to the sharply bent surface over the stiff subgrades.

For a given deflection and paving material, stiff subgrades result in higher slab stresses than do soft subgrades. By the same token, for a given deflection, and for a given subgrade, stress varies directly with the rigidity of the slab.

Equation 6 is the well known radius of relative stiffness first defined by Westergaard. The assumptions behind the derivation (notably those of continuous contact between the slab and base and complete structural integrity of the slab) probably are not completely fulfilled in prototype pavements for a variety of reasons. First, in the case of rigid slabs, warping and/or consolidation of the base or subgrade can negate the assumption relative to pavement contact and secondly, cracking destroys structural continuity of the slab itself.

For a constant value of Poisson's ratio and constant rigidity equivalent thicknesses of pavement can be obtained from equation 5 as follows:

$$\frac{E_1 h_1^3}{12(1-\mu^2)} = D = \frac{E_2 h_2^3}{12(1-\mu^2)} \quad (7)$$

$$h_1 = h_2 \sqrt[3]{\frac{E_2}{E_1}} \quad (8)$$

Equations 5 and 8 account for the moduli of elasticity and thickness of the pavement, but in addition, equation 6 accounts for the physical property of the subgrade. Equation 8 is the stiffness equation of Palmer and Barber (24). Hveem (20), personnel at the WASHO Road Test (30) and others have presented a simplification of the stiffness concept by using the simple ratio of load over deflection

$$S = \frac{P}{\Delta} \quad (9)$$

where P is the wheel load and Δ is the deflection (see Figure 8 for an example of use).

Insofar as pavement deflection is concerned, the important principle here is that the physical properties of the pavement components as well as thickness and load, determine the relationship between pavement stress and deflection, and, therefore, the allowable deflection to preclude tension failures in the pavement.

These principles are demonstrated in Figure 6. In the figure the three lower curves were calculated for a pavement having a modulus of elasticity equal to 500,000 psi and for various values of modulus of subgrade reaction. The numbers on the curves indicate the pavement thickness used for the calculations.

Comparing the two upper curves with the three lower curves it will be noted that for a given deflection stress varies directly as the pavement's modulus of elasticity. Stated in another way, as stiffness of the pavement increases the pavement "takes up more and more of the load". On

the other hand, as stiffness of the subgrade increases the subgrade itself is stressed more.

The important consideration shown in Figure 6 is that permissible deflection is a function of the properties of the pavement and subgrade. If one assumes an allowable stress criteria, permissible deflections must invariably be smaller for stiff subgrades than for lesser stiff ones. Likewise, allowable deflections must also be a function of the thickness and type of pavement.

EXTRINSIC FACTORS INFLUENCING DEFLECTION

In addition to the physical properties of the pavement, base, subbase and subgrade, many other factors influence pavement deflection. No doubt some of the factors which will be discussed in subsequent paragraphs are secondary and of little importance to the practicing engineer. Nevertheless, as in many other tests it is necessary to isolate insofar as is practical the external conditions which might affect the test values. Thus, even though for the general case these factors can be handled by keeping them constant, the engineer should know something of their relationship to deflections so that they can be put in their proper perspective.

Effect of Speed

Previous paragraphs of this paper have implied that the pavement structure is at least partially elastic. However, it has been well established that flexible pavements are viscoelastic and that their deflection under load is dependent upon time. This is further complicated by the fact that the load imposed on the pavement is dependent upon speed of loading. Quinn (26) and others have demonstrated that the load imposed

on the pavement under a moving vehicle bears a direct relationship with speed of vehicle. In the lower left hand portion of Figure 7 is shown, in its simplest form, the variation of contact load with speed for a smooth hard wheel moving over a bump on a plane surface. Increase in load is due to acceleration in the vertical direction as the wheel moves over the bump. This is the simplest representation of the problem that can be made since the true load acting on the pavement is a function of the frequency response of the vehicle. Frequency response in turn is a function of wave length and amplitude of the pavement undulations as well as speed. All other things being equal the load between the wheel of a vehicle and the pavement increases as speed of the vehicle increases.

In contrast to this, theories of viscoelasticity indicate that the deflection of a pavement will vary inversely with the speed of loading. In its simplest form this can be demonstrated with the Maxwell model shown in the lower right hand portion of Figure 7. Various investigators have assumed mathematical models which place these simple models in series and parallel; however, the principles of time dependent deflection can be represented simply by a spring connected in series with a dashpot.

The upper portion of Figure 7 demonstrates the principles outlined above. It is to be noted that as speed of vehicle increases pavement deflections decrease but that as speed increases load imposed upon the pavement also increases. This necessitates making several simplifying assumptions if one is to attempt relating stress and strain in the pavement system. This can generally be circumvented by holding speed constant; creep speed is generally accepted as the ideal speed of test.

Temperature

The effect of temperature on deflection is significant in some cases. The stiffness of asphaltic pavement, of course, varies inversely with temperature. Deflection then bears a direct relationship with temperature (Figure 8).

There is some evidence from field performance that a flexible pavement approaches the rigid condition at low temperatures. Failure of some flexible surfaces during the spring months, for example, results from cracking much as is the case for rigid pavements. Fortunately, this works to the advantage of the pavement structure during the spring melt period in that if the temperature of the pavement is relatively low at the same time that the subgrade is in its weakest condition, increased rigidity of the asphaltic concrete strengthens the structure, partially compensating for low subgrade stability.

When making deflection measurements it is necessary to make every attempt to hold temperature as constant as is possible. The writer was involved in a study of flexible pavement behavior in a tropical climate wherein pavement deflections were measured by means of a Benkelman Beam. The surface in question was a sand-asphalt mixture constructed over a cement stabilized base course. The surface of this particular pavement was relatively thin and whenever measurements were made during the hot afternoon hours the pavement surface had a distinct tendency to heave between the dual wheels of the test truck. The net result was that the measurements invariably showed an apparent residual effect due to the reversed movement of the soft material between the dual wheels. This problem was circumvented by restricting all field measurements to the early morning hours.

Effect of Position of Load

Position of load has a great effect on measured deflection particularly when comparing deflections in the outer wheel path with those in the inner wheel path. (See Figure 9). This can be presumed to be due to added supporting power of the pavement when considering the interior position. This also accounts for low supporting capacity at the edge of the pavement; the effect of edge condition on bearing capacity has been established for many years.

Previous Traffic History

Much research has been accomplished in recent years regarding the effect of repeated loads on strength of pavement materials. In particular, it has been shown that the strength of some soils under repeated loads is directly influenced by the stress history of the soil. Some subgrades that have been subjected to repeated application of load have increased strength over identical soils that have no stress history due to several factors. These include densification, and rearrangement of the soil particles. On the other hand, other soils show reduced strength after repeated loading. Recent research has indicated that previous stress history has a pronounced effect on the strength of base course materials.

Time, incidently, also has its effect on the properties of the asphaltic surface. Asphalt may become progressively harder with time and this in turn is reflected in the stiffness of the pavement. It is believed that hardening of the asphaltic surface is responsible for cracking that occurs on some pavements. Figure 10 illustrates longitudinal cracking that has occurred on a test pavement in Indiana. The exact cause of these cracks is not known at the present time. It is believed, however, that fatigue and/or shrinkage are the primary causes. Deflection measurements have indicated that excessive movement has occurred in the base and subbase of

this pavement but in addition, extraction tests definitely indicate a general hardening of the asphaltic surface. It is to be noted that the crack pattern on this pavement is distinctly longitudinal. This factor will be discussed in subsequent paragraphs under Deflection Patterns.

Cracking of a pavement surface has the effect of changing the elastic properties of the paving materials. Obviously stiffness of the pavement is decreased in the vicinity of a crack which in turn implies different permissible deflections at these locations. Cracks in the pavement surface are potentially dangerous from the standpoint of loss of supporting power because of discontinuity in the slab as well as potential softening of the underlying materials from surface infiltration of water.

Effect of Climate

Climate, as used herein, refers to seasonal variations in rainfall and temperature which influence the properties of subgrades, bases, and subbases. It has long been established that most subgrades show a loss in supporting power during the frost melt period. Table 1 shows data relative to fall and spring measurements on a flexible pavement. A general increase in the measured deflections under load is noted for all components of the pavement, particularly the plastic subgrades.

It is not the intent here to discuss the matter of pavement supporting power during the frost melt period. It has been known for a long time that a major portion of the distress to a pavement can occur under very few repetitions of load during the frost melt period. This was forcibly demonstrated on the WASHO Road Test.

Deflection measurements are a very useful tool in determining the effect of climate on pavement supporting capacity. Pavement deflection

under spring time conditions are generally greater than those under other conditions when the subgrade is dry and possesses greater rigidity.

Statistical Variation with Subgrade Type

Unfortunately from the standpoint of the researcher truly homogeneous pavements never exist. Construction practices are such that considerable variation can be found along the length of pavement. In addition, in situ soils are seldom uniform along the length of the pavement. Thus, when evaluating a pavement with deflection measurements, it becomes necessary to obtain some sort of a statistical average of the deflection measurements. Work done in Canada by Davis, McLeod, and Bliss (12) demonstrates that considerable variation can exist in measured deflections for many reasons. Therefore for deflection measurements to be of utmost significance to the engineer he must make many measurements and use a statistical average or some percentile of the measured values for evaluation purposes. The percentile value which should be used depends upon the variability of each particular problem. This necessitates making several measurements and determining from these the variability that exists at the location and an estimation of the resulting degree of accuracy. It is beyond the scope of this paper to discuss statistical variation but it is necessary to recognize this factor in testing of this type.

SIGNIFICANCE OF MEASURED NUMERICAL VALUES

Previous paragraphs of this paper have presented a discussion of many of the factors which influence pavement deflection. It has been brought out that the relationship between stress and deflection is variable depending upon the properties of the pavement and underlying layers. It has also been demonstrated that numerical values of deflection are dependent

upon speed of loading, temperature, position of load, previous traffic history, climate and normal variation in test values.

Due to the many factors which affect deflection measurements, it is apparent that the true significance of making tests can be realized only if the factors affecting the measurements are thoroughly understood. Fortunately, for a given situation many of the factors are constant, permitting generalization of the analysis. Generalization, however, can at times complicate the picture rather than simplify it.

Allowable Gross Deflection

It has been demonstrated that gross deflection is dependent upon the properties of the paving materials. In general, allowable deflection decreases as stiffness of the pavement increases (see Figure 6). Hveem (20) has recognized this principle in establishing tentative values of permissible deflection. These are tabulated in Table 2.

It has been recognized for many years that permissible deflection of a rigid pavement is less than that of the conventional flexible pavement. This necessitates the need for estimating the relative rigidity of the paving structure. Certainly cement stabilized bases approach the rigid condition; however, if they crack they may revert to the flexible condition. Likewise, thick asphaltic surfaces, particularly at low temperature, probably approach the rigid condition. In turn rigidity is a function of thickness of the surface which indicates that permissible deflection can be greater for thin surfaces than for relatively thick surfaces.

It is believed that in general for conventional flexible pavements, deflections greater than 0.02 inches can cause distress although there is evidence on hand to indicate that some times deflections in excess of 0.005 inches can be detrimental. It is apparent that there is a need for research

into the matter of determining numerical values of permissible deflection.

For the conventional pavement, a large portion of the gross deflection occurs in the subgrade. This can be used to great advantage in evaluating the supporting power of subgrade materials during climate variations (particularly spring melt period). This principle is demonstrated in Table 1 of this report.

In summary, gross deflections can be significant if they are evaluated in light of the relative stiffness of the paving layers. As stiffness of the materials increase permissible deflections decrease and it becomes necessary to make an estimate of the properties of the components before an analysis can be attempted.

Deflection Patterns

For the general case the deflection patterns throughout a flexible system assume unique characteristics. Measured deflection patterns vary with depth in the same manner as do theoretical values. However, numerical values of deflection generally cannot be correlated with theoretical values. This is due primarily to the non-homogeneity of the system. Figure 11 demonstrates the above fact. Deflection of a pavement particle decreases with depth and as a rule the major cumulative deflection occurs in the subgrade.

The transverse and longitudinal profiles of bending can be used to great advantage in determining the significance of total vertical deflection. For the general case, transverse bending is sharper than longitudinal bending (see Figure 12). This implies that the critical stress is in the transverse direction and explains why map cracking sometimes initially assumes a general longitudinal direction.

Figure 13 substantiates the hypothesis that permissible gross deflection should vary with the stiffness of the subgrade. In this figure values of deflection for a flexible pavement built over three subgrades is shown. This pavement consists of a five inch asphaltic concrete surface constructed over a waterbound macadam base which in turn rests upon a relatively cohesionless sand subbase. This particular pavement has shown considerable distress from cracking and it is significant to note that some of the worst cracking has occurred over the gravel subgrade. The relative steepness of the deflected surface over the granular subgrade compared to that over the clay subgrade indicated that higher stress existed in the former case than in the latter.

Still another method of presenting data of this type is demonstrated in Figure 14. Contours of bending yield information relative to gross deflection as well as to the transverse and longitudinal profiles. Typically, deflection is distributed over a greater longitudinal distance than transverse. In addition, it will be noted that the contours are not symmetrical about the center. The contour of bending ahead of a moving vehicle will vary from that behind due principally to time effects. It has been demonstrated by several investigators that a wave sometimes actually forms ahead of a moving wheel resulting in negative measured deflections at some distance ahead of the wheel.

Undoubtedly determination of the complete contour of deflection under a wheel load is too time consuming for most routine work. However, the Benkelman Beam has been used satisfactorily for this purpose. It merely requires that as the load is moved toward the probe, readings be taken at various time intervals. The transverse profiles must be obtained by using more than one beam.

Component Deflections

Relative deflection of a pavement layer decreases as depth increases. Therefore, to make use of component deflections it is necessary to take into account decreased stress with increased depth. Measurement of component deflections has been accomplished by many investigators. The results of these measurements have in many cases pointed up weak layers that exist in the pavement system. Considering the curves in Figure 11, it is noted that measured and calculated values assume a characteristic shape with depth. This suggests a general relationship between surface and component deflection.

Component deflection can be used to estimate relative values of elasticity for the paving components (Figure 15). It should be pointed out that it is necessary to make several simplifying assumptions regarding stress distribution before computations such as these can be made and that data of this type are subject to many limitations. Nevertheless they point out the significance of determining the component deflections for specific cases. For the particular case cited in Figure 15 no correlation could be found between gross deflection and pavement distress but a relatively good degree of correlation was established between component deflections and pavement distress. It was shown that subbase and base course deflections were primarily responsible for the cracking of the pavement surface.

Correlation with Other Test Values

Several investigators have attempted correlations between pavement deflection and related test values. These correlations are generally restrictive in that they do not take into account properties of the pavement system. They do have use, however, in evaluating pavements of a

given type if deflection is analyzed in light of thickness and quality of paving materials.

Relative Values for Performance and Evaluation

Perhaps the biggest use that can be made of pavement deflection is in making condition and evaluation surveys. Data can be obtained during the spring of the year, for example, to determine the suitability of one design as opposed to another. Likewise potential failures can be predicted, within limits, from gross deflection and/or deflection pattern measurements.

Deflection as a Research Tool

The utility of deflection measurements as a research tool is immediately apparent. New design techniques and evaluation of present design procedures are greatly facilitated if they are substantiated with deflection information. Researchers are not confronted with the need for gaining a vast amount of information on many pavements and can generally concentrate their efforts on detailed information from several carefully selected pavement. Instrumentation involved in research is generally quite complicated and involves great skill in electronics. At this point it is well to bring out that there is a need for conducting more research in which stresses and strains are measured simultaneously. Only in this manner can the true significance of pavement deflections be forthcoming.

SUMMARY

The amount a pavement deflects under load determines in part its structural accuracy. Deflection in turn, however, is affected by many factors. Theory as well as practice has demonstrated that permissible

deflection is dependent upon the properties of materials. Likewise, in some cases to be of most benefit it is necessary to analyze the data in light of profiles or contours of deflection.

Permissible deflection varies with the type of pavement materials. Relatively stiff pavements can tolerate less deflection under load than can pavements of less stiffness. Therefore it is necessary to make an estimate of the pavement properties before allowable deflections are used. Permissible deflection also varies with the thickness of the pavement.

Deflection is a useful tool that can be used by the engineer for establishing new design criteria as well as evaluating criteria in existence at the present time. Its usefulness as a research tool is significant, particularly in connection with obtaining detailed information relative to component deflections and deflection patterns.

It is suggested that there is a need for conducting research in which pavement stresses and strains are measured simultaneously under controlled loading conditions. By doing this it will be possible for the engineer to check theoretical hypothesis regarding the elastic and viscoelastic behavior of pavements.

LIST OF REFERENCES

1. Baker, Robert F. and Kratch S. Papazian, "The Effect of Stiffness Ratio on Pavement Stress Analysis," Proceedings, Highway Research Board, 1960.
2. Baker, Robert F., "Pavement Deflections Using Photogrammetric Techniques," Proceedings, Highway Research Board, 1957.
3. Benkelman, A. C., "Analysis of Flexible Pavement Deflection and Behavior Data," Highway Research Board Bulletin 210, 1959.
4. Buchanan, S. J. and Khuri, F. I., "Elastic and Plastic Properties of Soils and Their Influence on the Continuous Support of Rigid Pavements," Ohio River Division Laboratories, Mariemont, Ohio.
5. Burmister, D. M., "Evaluation of Pavement Systems of the WASHO Road Test by Layered Systems Methods," Highway Research Board Bulletin 177, 1958.
6. Burmister, D. M., "Stress and Displacement Characteristics of a Two Layer Rigid Base Soil System: Influence Diagrams and Practical Applications," Proceedings, Highway Research Board, 1956.
7. Burmister, D. M., "The Theory of Stresses and Displacements in Layered Systems and Application to the Design of Airport Runways," Proceedings, Highway Research Board, 1943.
8. Carey, W. M., Jr., "Benkelman Beam Pavement-Deflection Indicator," Abstracts, Highway Research Board, September 1954.
9. Carlton, P. F. and Behrmann, R. M., "A Model Study of Rigid Pavement Behavior Under Corner and Edge Loadings," Proceedings, Highway Research Board, 1956.
10. Childs, L. D., "A Study of Slab Action in Concrete Pavements Under Static Loads," Proceedings, Highway Research Board, 1947.
11. Corps of Engineers, "Investigations of Pressures and Deflections for Flexible Pavements," Waterways Experiment Station, Corps of Engineers Technical Memorandum 3-323, March 1950, October 1951, September 1953, December 1954.
12. Davis, M. M., N. W. McLeod and E. J. Bliss, "Symposium on Pavement Design and Evaluation - Report and Discussion of Preliminary Results," Proceedings, Canadian Good Roads Association, 1960.
13. Ewell, W. W. and Shigeo Okubo, "Deflections in Slabs and Elastic Foundations," Proceedings, Highway Research Board, 1950.

14. Foster, C. R. and Ahlvin, R. G., "Stresses and Deflections Induced by a Uniform Circular Load," Proceedings, Highway Research Board, 1954.
15. Fox, L., "Computation of Traffic Stresses in a Simple Road Structure," Road Research Technical Paper No. 9, Road Research Laboratory, London, England, 1948.
16. Geldmacher et al, "Subgrade Support Characteristics as Indicated by Measurement of Deflection and Strain," Proceedings, Highway Research Board, 1957.
17. Hicks, L. D., "Flexible Pavement Deflection Study in North Carolina," Proceedings, Highway Research Board, 1960.
18. Haskin, B. C. and E. H. Lee, "Flexible Surfaces on Viscoelastic Subgrades," Journal of the Engineering Mechanics Division, American Society of Civil Engineers, 1959.
19. Huculak, N. A., "Evaluation of Pavement Failures with the Benkelman Beam," Proceedings, Canadian Good Roads Association, 1960.
20. Hveem, F. N., "Pavement Deflections and Fatigue Failures," Design and Testing of Flexible Pavements, Highway Research Board Bulletin 114, 1955.
21. Jones, A., "Tables of Stresses in Three-Layer Elastic Systems," Paper presented to Highway Research Board, January 1962.
22. Larew, H. G., "Flexible Pavement Design - Literature Survey," A Report of the Virginia Council of Highway Investigation and Research, Charlottesville, Virginia, 1957.
23. Mack, Charles, "Deformation Mechanism and Bearing Strength of Bituminous Pavements," Proceedings, Highway Research Board, 1954.
24. Palmer, L. A. and Barber, E. S., "Soil Displacement Under a Circular Loaded Area," Proceedings, Highway Research Board, 1940.
25. Pister, Karl S. and Carl L. Monismith, "Analysis of Viscoelastic Flexible Pavements," Highway Research Board Bulletin 269, 1960.
26. Quinn, Bayard E. and David R. Thompson, "Effect of Pavement Condition Upon Dynamic Vehicle Reactions," Paper presented to the Highway Research Board, January 1962.
27. Sebastyan, G. Y., "The Benkelman Beam Deflection as a Measure of Pavement Strength," Proceedings, Canadian Good Roads Association, 1960.
28. Seed, H. B. and McNeill, Robert L., "Soil Deformations in Normal Compression and Repeated Loading Tests," Highway Research Board Bulletin 141, 1956.

29. Spangler, N. G., "The Structural Design of Flexible Pavements," Proceedings, Highway Research Board, 1942.
30. The WASHO Road Test, Highway Research Board, Special Report 22.
31. Walker, R. D., "Significance of Layer Deflections in Evaluating Flexible Pavements," Joint Highway Research Project, Purdue University, Lafayette, Indiana, January 25, 1961.
32. Williams, Stuart and Lee, Allan, "Load-Deflection Study of Selected High-Type Flexible Pavement in Maryland," Flexible Pavement Studies and Georgia Design Practice, Highway Research Board Bulletin 177, 1958.
33. Winnitoy, W. E., "The CGRA Benkelman Beam Procedure and Its Application in Testing Flexible Pavements," Proceedings, Canadian Good Roads Association, 1960.
34. Yoder, Eldon J. and Richard D. Walker, "Some Concepts on the Use of Deflection Measurements for Evaluating Flexible Pavements," Proceedings, Purdue Road School, 1960.
35. Westergaard, H. M., "New Formulas for Stresses in Concrete Pavements of Airfields," Transactions, ASCE, 1948.

TABLE 1. EFFECT OF SEASON ON COMPONENT DEFLECTIONS*

Subgrade Type (AASHO Class.)	Component Deflections -0.001 inches					
	Subgrade		Subbase		Base	
	Fall	Spring	Fall	Spring	Fall	Spring
A-1	5.1	9.3	3.3	4.3	1.4	4.0
A-2	4.6	8.4	1.4	5.0	3.2	3.7
A-4	3.7	6.9	4.1	10.8	0.9	5.3
A-6	4.1	14.4	3.2	5.0	1.9	8.4

* Measurements made under inner dual wheel, 24,000 axle load.
Pavement consists of 5" A.C., 8" Macadam base, 5" Subbase.

TABLE 2. SUGGESTED PERMISSIBLE DEFLECTIONS*

Thickness of Pavement	Type of Pavement	Max. Permissible Deflection for Design Purposes (Tentative)
8-in.	Portland Cement Concrete	0.012-in.
6-in.	Cement Treated Base (Surfaced with Bituminous Pavement)	0.012-in.
4-in.	Asphalt Concrete	0.017-in.
3-in.	Plant Mix on Gravel Base	0.020-in.
2-in.	Plant Mix on Gravel Base	0.025-in.
1-in.	Road Mix on Gravel Base	0.036-in.
1/2-in.	Surface Treatment	0.050-in.

* From Hveem (reference 20)

Note that Mr. Hveem terms these values "tentative" and he recognized that other factors must be considered.

$$\delta_n = \frac{\sigma_n}{E} - \mu \frac{\sigma_n'}{E} - \mu \frac{\sigma_n''}{E}$$

FIG.1 ELASTIC ELEMENT

FIG. 2. SKETCH ILLUSTRATING SIMPLE STRESS DISTRIBUTION THROUGH A FLEXIBLE PAVEMENT.

FIG.3 DEFLECTION PROFILES

$$\sigma_B = k \Delta > \sigma_{SG}$$

FIG. 4 DEFLECTION, REACTIVE PRESSURE AND STRESSES - RIGID PAVEMENT

FIG. 5 EFFECT OF RADIUS OF BENDING ON STRESSES

FIG. 6 RELATIONSHIP BETWEEN DEFLECTION & STRESS

(a)

AFTER QUINN, WASHO ROAD
TEST AND OTHERS

FIG. 7 EFFECT OF SPEED ON LOAD AND DEFLECTION

FIG. 8 EFFECT OF TEMPERATURE ON DEFLECTION

FIG. 9 EXAMPLE OF TRANSVERSE DEFLECTION UNDER REAR AXLE OF A TRUCK, 5" AC, TOTAL PAVEMENT THICKNESS 18"

FIG. 10

MEASURED VALUES FROM GELDMACHER ET AL.

FIG. II DEFLECTION PATTERNS

- NOTES: 1. 9,000 LB. DUAL WHEEL (4,500 LB. PER TIRE)
2. PAVEMENT CONSISTS OF:
 5" ASPHALT CONCRETE
 8" WATER BOUND MACADAM
 5" SANDY SUBBASE
3. SUBGRADE — A-6 AASHO CLASS

FIG. 12 TYPICAL PROFILES OF DEFLECTIONS, INTERIOR LOAD

FIG. 13 DEFLECTION AND SUBGRADE TYPE

- NOTES: 1. 9,000 LB DUAL WHEEL (4,500 LB PER TIRE)
2. CONTOUR INTERVAL VARIES AND IS IN 0.001 INCHES
3. NUMBERS ON CONTOURS ARE DEFLECTIONS IN 0.001 IN.
4. POINTS PLOTTED FROM PRINCIPLE OF RECIPROCAL DISPLACEMENTS

FIG. 14 CONTOURS OF DEFLECTION

RELATIVE MODULUS OF PAVEMENT LAYERS
11250 POUND DUAL WHEEL LOAD

FIGURE 15