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Abstract

Assessing stability of multidimensional systems is notoriously difficult. We consider the standard discrete-time slotted ALOHA system with a finite number of buffered users. Stability study of such a system was initiated in 1979 by Tsybakov and Mikhailov. Several bounds on the stability region have been established up-to-date, however, the exact stability region is known only for the symmetric system and two users ALOHA. This paper proves necessary and sufficient conditions for stability of the ALOHA system, hence solves the problem posed by Tsybakov and Mikhailov. We accomplish this by means of a novel technique based on three simple observations. Namely, isolating single queue from the system, applying Loynes' stability criteria for such an isolated queue, and using stochastic dominance and mathematical induction to verify the required stationarity assumptions in the Loynes' criterion. We also point out that our technique can be used to assess stability regions for other multidimensional systems. We illustrate it by providing also the stability region for a buffered system with conflict resolution algorithms. In another paper (Georgiadis and Szpankowski (1992)) we have used a similar technique to establish stability criteria for the token passing ring system.

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1. INTRODUCTION

A fundamental issue in the design of any distributed system is its *stability*, loosely defined as its ability to possess required properties in the presence of some disturbances. Hereafter, by stability we understand an ability of a system to keep a quantity of interest (e.g., queue length, waiting time, etc.) in a bounded region, or more precisely the existence of the limiting distribution for a quantity of interest. Important examples of distributed systems are local area networks (e.g., ALOHA system, Ethernet, FDDI ring, token ring), multiprocessor systems (e.g., concurrent execution of tasks on multiprocessors), distributed computations (cooperative problem solving by sets of distributed processors), etc. More general and thus more important examples are multidimensional queuing systems with applications which include the ALOHA system (Fayolle *et al.* (1977), Tsybakov and Mikhailov (1979), Szpankowski (1986), Rao and Ephremides (1989), Borovkov (1989), etc.), backoff protocol for multiaccess channels (Aldous (1988)), Kelly (1985), Goodman *et al.* (1988)), data base systems with concurrent processing (Tsitsiklis *et al.* (1986), Baccelli and Liu (1992)), and so forth.

In this paper we concentrate on the buffered ALOHA system, propose a new method of evaluating its stability, and show that this new approach can be extended to a larger class of distributed systems. Stability analysis of the buffered ALOHA system was initiated by Tsybakov and Mikhailov (1979) who obtained a simple bound for the stability region, and exact sufficient and necessary conditions for the ergodicity of the symmetric system (e.g., all input rates and probability of transmissions are the same). These authors used the *stochastic dominance technique* to derive their bound. This was simplified and generalized in Szpankowski (1988) who derived some improved bounds for the stability region, and some new bounds for the instability region. The *Lyapunov test function* approach (cf. Tweedie (1976)) was first adopted to the stability of the ALOHA by Falin (1981) who derived another bound for the stability region in the case of very asymmetric traffic (e.g., very different input rates and probability of transmissions for various users). This was further improved in Szpankowski (1988). Recently Rao and Ephremides (1989), using the stochastic dominance method constructed the best up-to-date bound for not-too-asymmetric buffered ALOHA system.

Finally, Anantharam (1991) – for very simple model of the arrival process – computed the ergodicity region for another formulation of the stability problem. Namely, the stability region considered therein contains every input rate vector for which *there exists* such a vector of transmission probabilities resulting in the stable ALOHA system. This is a different stability problem, and it was first investigated by Tsybakov and Mikhailov (1979) (see also Rao and Ephremides (1989)). It is easy to notice that stability region of this kind is an envelope of stability regions that we plan to investigate, and while the latter do not have a closed-form solution for stability condition, the former one enjoys a simple solution.

The exact stability region for the ALOHA model is known *only* for $M = 2$ users system, and the symmetric model. The case $M = 2$ was solved by Tsybakov and Mikhailov (1979) by applying general stability criteria for two-dimensional homogeneous Markov chains derived by Malyshev in his seminal paper Malyshev (1972). These general stability criteria have been extended to higher dimensions by Mensikov (1974), and Malyshev and Mensikov (1981). For two-dimensional homogeneous Markov chains Vaninskii and Lazareva (1988), Fayolle (1989) and Rosenkrantz (1989) relaxed some of Malyshev's restrictions (e.g., boundness of the arrival process). It should be said, however, that the above criteria for higher than two-dimensional Markov chains are *very difficult* to apply in practice. Despite the fact that these criteria are known for almost twenty years, very few real systems have been analyzed through this approach (see Karatzoglu and Ephremides (1989) for an application).

In this paper, we solve the stability problem of the ALOHA system originally posed by Tsybakov and Mikhailov (1979), that is, we provide exact stability region by establishing necessary and sufficient condition for stability of the ALOHA system. Our approach to the stability problem of the ALOHA (and some other distributed) systems is novel, and it was already outlined in Szpankowski (1990). This technique was recently rigorized in Georgiadis and Szpankowski (1992) where sufficient and necessary stability condition was established for the token passing ring. Our technique is based on three simple observations. Namely: (i) we show that stability of an M-dimensional multiqueue system can be reduced to stability of an isolated single queue; (ii) we apply an old result of Loynes (1962) that allows to assess stability of a general $G|G|1$ queue with stationary arrival and service processes; (iii) finally, to verify a technical stationarity requirement in Loynes' criteria we apply the stochastic dominance technique and mathematical induction. It should be stressed, however, that within this general framework every multidimensional model requires subtle but significant modifications that are often far from obvious.

From the work of Malyshev (1972), and Malyshev and Mensikov (1981), it is known that stability of an M-dimensional Markov chain depends on the stability of lower dimensional imbeded Markov chains. This is consistent with our findings. In the ALOHA case, stability criteria depends on the probability of whether users in a smaller copy of the ALOHA model are empty or not. Therefore, no closed-form solution exists. This is an *inherent* characteristic of stability conditions for the ALOHA system, and many other multidimensional

Markov chains (cf. Malyshev and Mensikov (1981)).

This paper is organized as follows. In the next section we present our main result that provides sufficient and necessary conditions for stability of the buffered ALOHA system. We also present explicit solutions to the stability problem for $M = 2$ and $M = 3$. The proof of our main result is delayed till Section 3 that also presents a detailed description of our new approach. The proof of the sufficient condition resembles the one given in Georgiadis and Szpankowski (1992), but the necessary part requires entirely new approach that might be applicable to the stability analysis of several other distributed systems. The last section shows how the proposed technique can be extended to other multiqueue systems. In particular, we discuss without a detailed proof stability conditions for the buffered system with conflict resolution algorithm (cf. Capetanakis (1979), Szpankowski (1987), Paterakis *et al.* (1987)).

2. MAIN RESULTS

This section presents our main results concerning stability of the buffered ALOHA system. The proof of Theorem 1 below is presented in the next section. In fact, the ALOHA system serves as a motivating example for a more general stability analysis of some multiqueue distributed systems, as indicated in Section 4.

2.1 Model Formulation

We start with a short description of the buffered ALOHA system. The system consists of M distributed users, each having an infinite buffer for storing fixed-length packets. The packets are transmitted through a broadcast channel. The channel is slotted, and a slot duration is equal to a packet transmission time. Each nonempty user transmits a packet with the probability r_i in a slot, where $i \in \mathcal{M}$ and $\mathcal{M} = \{1, 2, \ldots, M\}$ is the set of users. If two or more users transmit simultaneously, then a collision occurs and the packets must be retransmitted in future. When exactly one packet is transmitted in a slot, then a successful transmission takes place, the packet is removed from its queue, and another packet, if the queue is nonempty, gets its chance to be served. The arrival process is i.i.d. with respect to slots, and arrival processes are independent from a user to a user.

Let N_j^t represent the queue length in the jth user at the beginning of the tth slot, where $t = 0, 1, \ldots$ is a nonnegative integer that indexes slots. Under the above assumptions, the M-dimensional process $\mathbf{N}^t = (N_1^t, N_2^t, \ldots, N_M^t)$ is a *Markov chain* (cf. Tsybakov and Mikhailov (1979), Szpankowski (1986)). To see this, we note that the jth queue evolves according to the following stochastic equation

$$N_j^{t+1} = (N_j^t - Y_j^t)^+ + X_j^t \tag{2.1}$$

where X_j^t represents the number of new customers arriving during the tth slot to the jth user. We assume that X_j^t has its first moment λ_j finite, that is, $\lambda_j = EX_j^t < \infty$. The random variable Y_j^t takes only two values, namely $Y_j^t = 1$ when a transmission from the jth user is successful, and $Y_j^t = 0$ otherwise. In the above, $x^+ = \max\{0, x\}$.

The random variable Y_j^t depends on the M-dimensional vector $\mathbf{N}^t = (N_1^t, \ldots, N_M^t)$, and as easy to see (cf. Szpankowski (1986)) for every $j \in \mathcal{M}$ we have

$$Y_j^t = R_j^t \left(1 - \sum_{k \in \mathcal{M} - \{j\}} R_k^t \chi(N_k^t) \right)^+ . \tag{2.2}$$

In the above, the transmission decision variable R_k^t is equal to one when the kth user attempts to transmit in the tth slot and zero otherwise, that is, $1 - \Pr\{R_j^t = 0\} = \Pr\{R_j^t = 1\} = r_j$. Also, by definition $\chi(x) = 1$ for $x > 0$ and $\chi(0) = 0$. In words, (2.1) and (2.2) imply directly that $\mathbf{N}^t$ is a Markov chain. Our task is to find conditions under which this Markov chain is ergodic (stable).

2.2 Stability Criteria

Before we establish the announced stability conditions, we first formalize the notion of stability. For stability of a multidimensional processes $\mathbf{N}^t = (N_1^t, \ldots, N_M^t)$ (not necessarily Markovian process) one usually requires the existence of a *honest* limiting distribution of $\mathbf{N}^t$ as $t \to \infty$. In other words, $\mathbf{N}^t$ is stable if for $\mathbf{x} \in \mathbb{N}_0^M$, where $\mathbb{N}_0$ is a set of nonnegative integers, the following holds

$$\lim_{t \to \infty} \Pr\{\mathbf{N}^t < \mathbf{x}\} = F(\mathbf{x}) \quad \text{and} \quad \lim_{\mathbf{x} \to \infty} F(\mathbf{x}) = 1 \tag{2.3}$$

where $F(\mathbf{x})$ is the limiting distribution function, and by $\mathbf{x} \to \infty$ we understand that $x_j \to \infty$ for all $j \in \mathcal{M} = \{1, \ldots, M\}$. If a weaker condition holds, namely,

$$\lim_{\mathbf{x} \to \infty} \lim_{t \to \infty} \inf \ \Pr\{\mathbf{N}^t < \mathbf{x}\} = 1 , \tag{2.4}$$

then the process is called *substable* by Loynes (1962), *tight* by Breiman (1968), and *bounded in probability* by Meyn and Tweedie (1992). Otherwise, the system is unstable (for more details see Loynes (1962), Walrand (1988) and Brandt *et al.* (1990)).

The relationship between stability and substability is, of course, that a stable sequence is necessary substable, and a substable sequence is stable if the distribution function tends to a limit (cf. Loynes (1962)). For example, if $\mathbf{N}^t$ is an aperiodic and irreducible Markov chain defined on a countable state space, then substability is *equivalent* to stability, since a limiting distribution exists (it may be degenerate) for any such Markov chains (cf. Meyn and Tweedie (1992)). This might not be true for a Markov chain defined on a general space, but very mild conditions are needed even in this case (cf. Meyn and Tweedie (1992)).

A rigorous proof of our main result is presented in the next section. Below, we present an informal overview of our approach. First of all, we construct a modified ALOHA system as follows. Let $\mathcal{P} = (\mathcal{S},\mathcal{U})$ be a partition of $\mathcal{M}$ such that users in $\mathcal{S} \neq \mathcal{M}$ work exactly in the same manner as in the original ALOHA model, while users in $\mathcal{U}$ *persistently* attempt to send packets even if their buffers are empty (e.g., dummy packets). We call users in $\mathcal{U}$ *persistent* (or jamming), and users in $\mathcal{S}$ *nonpersistent*. Note that a system consisting of users in $\mathcal{S}$ forms a smaller copy of the original ALOHA system with slightly new probabilities of transmissions.

Furthermore, it is easy to see that the system with persistent and nonpersistent users, stochastically dominates the queue lengths process in the original ALOHA system (cf. Tsybakov and Mikhailov (1979) and Szpankowski (1986)). Therefore, proving stability of such a dominant system – that is, the one under the partition $(\mathcal{S},\mathcal{U})$ – suffices for stability of the original system. To accomplish this, we prove stability conditions for users in $\mathcal{S}$ by mathematical induction (since the cardinality of $\mathcal{S}$ is assumed to be smaller than M and $\mathcal{S}$ is a smaller copy of the original system). Stability of a nonpersistent user is established by applying Loynes' stability criteria (Loynes (1962)) for a general $G|G|1$ queue with stationary inputs. Finally, the stability region for the whole ALOHA system is a *union* of stability regions found for every partition $\mathcal{P}$.

To be more precise, let for a given partition $\mathcal{P} = (\mathcal{S},\mathcal{U})$ of $\mathcal{M}$ such that $\mathcal{S} \neq \mathcal{M}$, the process $\overline{\mathbf{N}}_{\mathcal{P}}^t = (\overline{\mathbf{N}}_{\mathcal{S}}^t, \overline{\mathbf{N}}_{\mathcal{U}}^t)$ denote the queue lengths in the modified system where $\overline{\mathbf{N}}_{\mathcal{S}}^t$ (resp. $\overline{\mathbf{N}}_{\mathcal{U}}^t$) represents the queue lengths in $\mathcal{S}$ (resp. in $\mathcal{U}$). As indicated above, $\overline{\mathbf{N}}_{\mathcal{P}}^t$ dominates the original queueing process $\mathbf{N}^t$, that is,

$$\mathbf{N}^t \leq_{st} \overline{\mathbf{N}}_{\mathcal{P}}^t \tag{2.5}$$

provided $\mathbf{N}^0 = \overline{\mathbf{N}}_{\mathcal{P}}^0$. Note that by our construction, the process $\overline{\mathbf{N}}_{\mathcal{S}}^t$ is an $|\mathcal{S}|$-dimensional Markov chain that mimics the behavior of the ALOHA system. Clearly, by mathematical induction $\overline{\mathbf{N}}_{\mathcal{S}}^t$ is stable under the same type of stability condition as the original system $\mathbf{N}^t$, which are assumed to hold. But $\overline{\mathbf{N}}_{\mathcal{S}}^t$ is a Markov chain, hence its stability implies the existence of a stationary and ergodic version of this Markov chain. We further assume that $\mathbf{N}_{\mathcal{S}}^t$ is stationary and ergodic Markov chain under the same stability conditions as for $\mathbf{N}^t$

but adopted to the set $\mathcal{S}$.

Let $Y_j^t(\mathcal{S})$ be the output process from the jth queue in the dominant ALOHA system represented by $(\overline{\mathbf{N}}_{\mathcal{S}}^t, \overline{\mathbf{N}}_{\mathcal{U}}^t)$. For a given partition $\mathcal{P} = (\mathcal{S}, \mathcal{U})$, we also denote by $P_{succ}^j(\mathcal{S})$ the probability of a successful transmission from the jth user in the dominant system $\overline{\mathbf{N}}_{\mathcal{P}}^t = (\overline{\mathbf{N}}_{\mathcal{S}}^t, \overline{\mathbf{N}}_{\mathcal{U}}^t)$. Clearly, we have $P_{succ}^j(\mathcal{S}) = EY_j^t(\mathcal{S})$ provided $Y_j^t(\mathcal{S})$ is a stationary sequence, and this is assumed to hold as a simple consequence of our assumption regarding stationarity of $\overline{\mathbf{N}}_{\mathcal{S}}^t$.

Let also $\Theta_{\mathcal{S}}$ be a set of all zero-one $|\mathcal{S}|$-tuples, that is,

$$\Theta_{\mathcal{S}} = \{\mathbf{z} : \mathbf{z} = (z_1, \ldots, z_{|\mathcal{S}|}),\ z_j \in \{0,1\},\ j \in \mathcal{S}\} . \tag{2.6}$$

We shall write $\mathbf{z}_{\mathcal{S}} \in \Theta_{\mathcal{S}}$ to denote an element of such a set. It is easy now to see that in the dominant system, the probability of success $P_{succ}^j(\mathcal{S})$ becomes (cf. Tsybakov and Mikhailov (1979), Szpankowski (1986))

$$P_{succ}^j(\mathcal{S}) = r_j \prod_{k \in \mathcal{U}-\{j\}} (1 - r_k) \sum_{\mathbf{z}_{\mathcal{S}} \in \Theta_{\mathcal{S}}} \Pr\{\chi(\overline{\mathbf{N}}_{\mathcal{S}}^0) = \mathbf{z}_{\mathcal{S}}\} \prod_{k \in \mathcal{S}-\{j\}} (1 - r_k)^{z_k} , \tag{2.7}$$

where $k \in \mathcal{A} - \{j\}$ (in the above $\mathcal{A}$ is either $\mathcal{S}$ or $\mathcal{U}$) means that k belongs to $\mathcal{A}$ and it is *not* equal to j (even if $j \notin \mathcal{A}$). In the above, $\chi(\overline{\mathbf{N}}_{\mathcal{S}}^0)$ represents an $|\mathcal{S}|$-dimensional vector whose jth component is either zero or one depending whether the jth queue is empty or not.

The next theorem is our main finding, and it provides sufficient and necessary condition for stability of the buffered ALOHA system. It is proved in Section 3.

Theorem 1. *The buffered ALOHA system is stable iff**[1] $\lambda \in \mathcal{R}$ *where*

$$\mathcal{R} = \bigcup_{k=1}^{M} \{\lambda = (\lambda_1, \ldots, \lambda_M) :\ \lambda_j < P_{succ}^j(\mathcal{M}_k),\ \ \textit{for all}\ \ j \in \mathcal{M}\} , \tag{2.8}$$

where $\mathcal{M}_k = \mathcal{M} - \{k\}$ *and* $P_{succ}^j(\mathcal{M}_k)$ *is defined in (2.7) for the partition* $\mathcal{P} = (\mathcal{M}_k, \{k\})$, *that is,* $\mathcal{S} = \mathcal{M}_k$ *and* $\mathcal{U} = \{k\}$. ■

Remark. Note that according to Theorem 1 only partitions $\mathcal{P}_k = (\mathcal{M}_k, \{k\})$ contribute to the stability region $\mathcal{R}$. In fact, in Section 3 we prove that $\bigcup_{k=1}^{M} \mathcal{R}_{\mathcal{M}_k} = \bigcup_{\mathcal{S} \subset \mathcal{M}} \mathcal{R}_{\mathcal{S}}$ where $\mathcal{R}_{\mathcal{S}}$ is the stability region for the partition $(\mathcal{S}, \mathcal{U})$ (hence, $\mathcal{R}_{\mathcal{M}_k}$ is the stability region for $\mathcal{P}_k$, and by (2.8) $\mathcal{R}_{\mathcal{M}_k} = \{\lambda :\ \lambda_j < P_{succ}^j(\mathcal{M}_k),\ j \in \mathcal{M}\}$).

The stability region $\mathcal{R}$ in Theorem 1 has a quite complicated form, and it can*not* be simplified except for the symmetric case. Note that to compute $\mathcal{R}$, one needs an explicit

[1] By iff* we mean *if and only if* with possible exception of the boundary points of $\mathcal{R}$. In fact, our technique usually does not give an ultimate answer to the stability for the boundary points of $\mathcal{R}$.

formula for the probability of success (2.7) which depends on the behavior of $(M-1)$-dimensional processes $\mathbf{N}^{l}_{\mathcal{M}_k}$. This is consistent with some general results of Malyshev and Mensikov (1981). Below, we illustrate how to apply Theorem 1 to get explicit stability regions for $M=2$ and $M=3$ users, and for the symmetric ALOHA system. We also derive from (2.8) various bounds for the stability region. In fact, all known bounds for $\mathcal{R}$ can be obtained from our condition (2.8).

The stability region for the ALOHA system (and several others) can be simplified if one considers the so called *envelope* of the stability regions which is defined as the set of all $\lambda=(\lambda_1,\ldots,\lambda_M)$ such that *there exists* a vector of transmission probabilities $(r_1,\ldots,r_M)$ for which the ALOHA system is stable. Tsybakov and Mikhailov (1979) conjectured that the envelope $\mathcal{E}$ of the stability region for the ALOHA can be characterized as follows

$$\mathcal{E}=\{\lambda=(\lambda_1,\ldots,\lambda_M):\ \lambda_j<r_j\prod_{k\neq j}(1-r_k)\ ,\ 0\le r_j\le 1 \text{ for all } j\in\mathcal{M}\}\ .$$

This result was proved for a *very simple input process* by Anantharam (1991). However, for general ALOHA it is still an open problem. We believe that our Theorem 1 can be used to settle this conjecture, but we do not tackled this in the current paper.

2.3 Special Cases and Bounds

Provided Theorem 1 is proved, we apply it to establish stability regions for $M=2$ and $M=3$, and the symmetric ALOHA system. We also discuss some bounds on $\mathcal{R}$.

We start with $M=2$, and we consider separately two partitions $\mathcal{P}_1=(\mathcal{M}_1,\{1\})$ and $\mathcal{P}_2=(\mathcal{M}_2,\{2\})$ where $\mathcal{M}_1=\{2\}$ and $\mathcal{M}_2=\{1\}$. Let $\mathcal{R}=R_1\cup\mathcal{R}_2$ where $\mathcal{R}_i$ is the stability region for the partition $(\mathcal{M}_i,\{i\})$ with $i=1,2$. We discuss in details only the construction of $\mathcal{R}_1$. We have $\mathcal{S}=\{2\}$ and $\mathcal{U}=\{1\}$, that is, the first user persistently jams the second one. According to (2.7) we have

$$\begin{aligned}P^1_{succ}(\mathcal{M}_1) &= r_1(\Pr\{\overline{N}^0_2=0\}+(1-r_2)\Pr\{\overline{N}^0_2\ge 1\})\\ P^2_{succ}(\mathcal{M}_1) &= r_2(1-r_1)\ .\end{aligned}$$

Noting that for $\lambda_1<P^1_{succ}(\mathcal{M}_1)$ and $\lambda_2<P^2_{succ}(\mathcal{M}_1)$ the probability $\Pr\{\overline{N}^0_2\ge 1\}=\lambda_2/(r_2(1-r_1))$, we obtain

$$\mathcal{R}_1=\{(\lambda_1,\lambda_2):\ \lambda_1<r_1\left(1-\frac{\lambda_2}{1-r_1}\right)\text{ and }\lambda_2<r_2(1-r_1)\}\ .$$

In a similar manner, considering $\mathcal{M}_2$ we obtain

$$\mathcal{R}_2=\{(\lambda_1,\lambda_2):\ \lambda_1<r_1(1-r_2)\text{ and }\lambda_2<r_2\left(1-\frac{\lambda_1}{1-r_2}\right)\}\ .$$

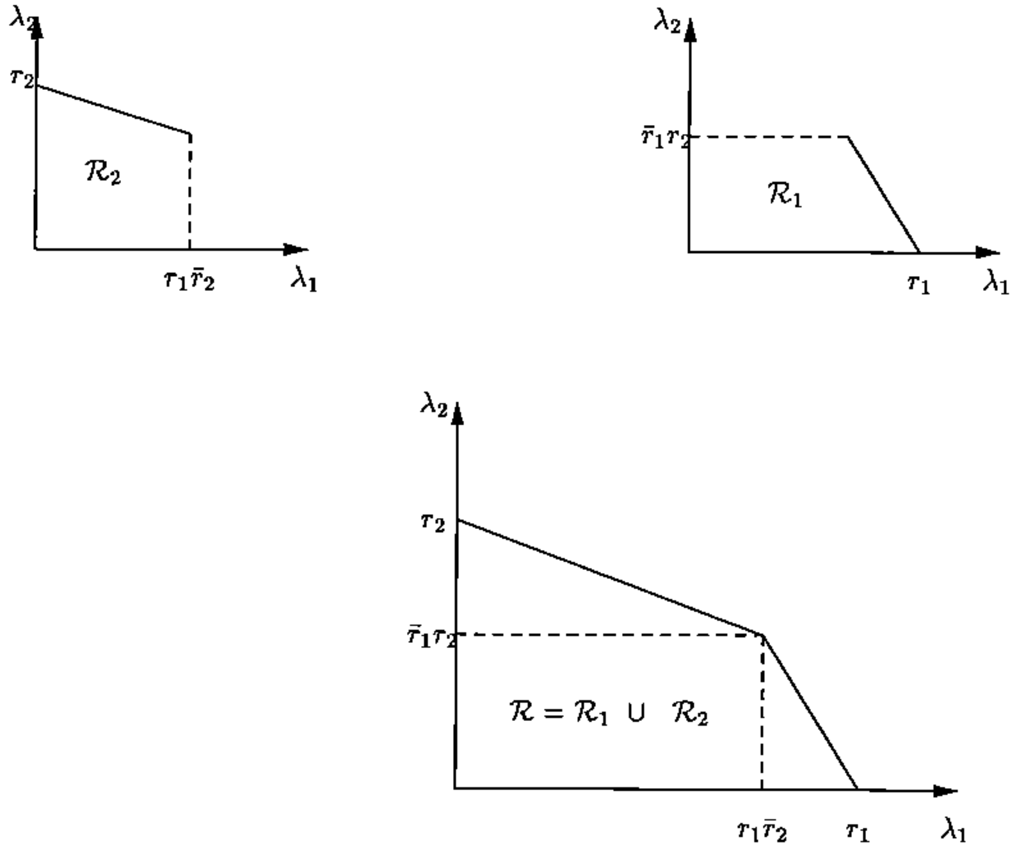

Figure 1: Stability region $\mathcal{R}$ for $M = 2$ as a union of stability regions $\mathcal{R}_1$ and $\mathcal{R}_2$.

The stability region $\mathcal{R}$ is the sum of $\mathcal{R}_1$ and $\mathcal{R}_2$ and it is illustrated in Figure 1. In passing, we note that the case $r_1 + r_2 < 1$ is different from the case $r_1 + r_2 > 1$ since the former stability region $\mathcal{R}$ is a convex set while the latter not.

Putting everything together, we summarize the stability result for $M = 2$ in the corollary below. This result also follows from an extension of the Malyshev's criteria due to Fayolle (1989) and Rosenkrantz (1989) under only very slightly more restrictive assumptions regarding the arrival process. Another arguments have been used by Rao and Ephremides [24] to establish this result for a simple Bernoulli arrival process.

Corollary 2. *For $M = 2$, the buffered ALOHA system is stable for all $(\lambda_1, \lambda_2) \in \mathcal{R}$ such that*

$$\mathcal{R} = \{\lambda_1 < r_1(1 - \lambda_2/\bar{r}_1) \text{ and } \lambda_2 < \bar{r}_1 r_2\} \cup \{\lambda_1 < r_1\bar{r}_2 \text{ and } \lambda_2 < r_2(1 - \lambda_1/\bar{r}_2)\}\ , \quad (2.9a)$$

and the system is unstable for $(\lambda_1, \lambda_2) \in \overline{\mathcal{R}}$ *where*

$$\overline{\mathcal{R}} = \{\lambda_1 > r_1(1 - \lambda_2/\bar{r}_1) \text{ or } \lambda_2 > \bar{r}_1 r_2\} \cap \{\lambda_1 > r_1\bar{r}_2 \text{ or } \lambda_2 > r_2(1 - \lambda_1/\bar{r}_2)\} , \qquad (2.9b)$$

where $\bar{r}_j = 1 - r_j$. ■

Now, we consider the case $M = 3$ which is more intricate, and to the best of our knowledge not tackled before. We investigate three partitions, namely $\mathcal{P}_1 = (\mathcal{M}_1, \{1\})$, $\mathcal{P}_2 = (\mathcal{M}_2, \{2\})$ and $\mathcal{P}_3 = (\mathcal{M}_3, \{3\})$ where $\mathcal{M}_1 = \{2,3\}$, $\mathcal{M}_2 = \{1,3\}$ and $\mathcal{M}_3 = \{1,2\}$. Only the first partition will be discussed in details. As before, the stability region $\mathcal{R}$ is the sum of three regions $\mathcal{R}_1$, $\mathcal{R}_2$ and $\mathcal{R}_3$ each corresponding to $\mathcal{M}_1$, $\mathcal{M}_2$ and $\mathcal{M}_3$, respectively.

We now consider $\mathcal{R}_1$. In the corresponding dominant system, the first user only contributes to jamming and it is never empty. Note that such a system can be viewed as a two-user ALOHA system with an additional user who jams the other users. As expected, we need probabilities of empty/nonempty in this two-dimensional ALOHA model. Such an analysis was done by Nain (1985). Before we plunge into the stability investigation of this case, we first briefly summarize some of Nain's results adopted to our setting.

Let $F_1(x, y)$ denote the generating function of (N_2^t, N_3^t) with the first user being a jamming one (i.e., it is never empty). Then, with a minor modification, it is proved in Nain (1985) (see also Szpankowski (1986)) that

$$\lambda_2 = \bar{r}_1 r_2 \bar{r}_3 (1 - F_1(0,1)) + \bar{r}_1 r_2 r_3 (F_1(1,0) - F_1(0,0)) , \qquad (2.10a)$$

$$\lambda_3 = \bar{r}_1 \bar{r}_2 r_3 (1 - F_1(1,0)) + \bar{r}_1 r_2 r_3 (F_1(0,1) - F_1(0,0)) , \qquad (2.10b)$$

where, as before, $\bar{r}_i = 1 - r_i$. Moreover, for $z_2, z_3 \in \{0,1\}$ we use the following simplified notation $P_1(z_2, z_3) = \Pr\{\chi(\overline{N}_2) = z_2, \chi(\overline{N}_3) = z_3\}$ with the first user being a persistent one. Note that the above probabilities are related to the generating function $F_1(\cdot,\cdot)$ as follows $P_1(1,0) = F_1(1,0) - F_1(0,0)$, $P_1(0,1) = F_1(0,1) - F_1(0,0)$, $P_1(1,1) = 1 - F_1(0,1) - F_1(1,0) + F_1(0,0)$, and $P_1(0,0) = F_1(0,0)$. From Nain (1985) we have

$$P_1(0,0) = \left(1 - \frac{\lambda_2}{\bar{r}_3\bar{r}_1} - \frac{\lambda_3}{r_3\bar{r}_1}\right) \exp\left(\frac{\gamma(1)}{2\pi i}\right) \int_{|t|=1} \frac{\log g(t)}{t(t-\gamma(1))} dt \qquad (2.11a)$$

or

$$P_1(0,0) = \left(1 - \frac{\lambda_2}{r_2\bar{r}_1} - \frac{\lambda_3}{\bar{r}_2\bar{r}_1}\right) \exp\left(\frac{\gamma(1)}{2\pi i}\right) \int_{|t|=1} \frac{\log g_1(t)}{t(t-\gamma(1))} dt \qquad (2.11b)$$

depending whether $P_1(0,0)$ is computed from $F_1(0,y)$ or $F_1(x,0)$. We note that $\gamma(1)$ and $g(t)$ depend of the input rates λ_2 and λ_3 ! Moreover, the probabilities $\Pr\{N_2 \geq 1, N_3 = 0\} = F_1(1,0) - F_1(0,0)$ and $\Pr\{N_2 = 0, N_3 \geq 1\} = F_1(0,1) - F_1(0,0)$ needed in our stability analysis, are given in Nain (1985) page 58. For example, $F_1(1,0)$ corresponding to (2.11a) becomes

$$\begin{aligned} F_1(1,0) &= \left(1-\frac{\lambda_2}{\bar{r}_3\bar{r}_1}-\frac{\lambda_3}{r_3\bar{r}_1}\right)\left(\frac{1-r_3\bar{r}_1}{1-r_3\bar{r}_1-r_2\bar{r}_1}\right)\int_{|t|=1}\frac{\log g(t)}{((t-\gamma(1))^2}dt \\ &- \frac{r_2\bar{r}_1}{1-r_2\bar{r}_1-r_3\bar{r}_1}P_1(0,0) \end{aligned}$$

The region of validity of (2.11) is defined in Nain (1985). In (2.11), $\gamma(x)\,|_{x=1}$ is the inverse of a conformal mapping of a unit circle onto a curve L_x defined in Nain (1985) (p. 54 and Lemma 4.1). The functions $g(t)$ and $g_1(t)$ are defined in Nain (1985), too.

Now we are ready to present the stability region for $M=3$ ALOHA system. Using (2.7) for the partition $(\mathcal{M}_1, \{1\})$, we obtain

$$P^1_{succ}(\mathcal{M}_1) = r_1\left(P_1(0,0)+P_1(1,0)(1-r_2)+P_1(0,1)(1-r_3)+P_1(1,1)(1-r_2)(1-r_3)\right) ,$$

$$P^2_{succ}(\mathcal{M}_1) = r_2(1-r_1)(1-(1-F_1(1,0))r_3) ,$$

$$P^3_{succ}(\mathcal{M}_1) = r_3(1-r_1)(1-(1-F_1(0,1))r_2) ,$$

where the probabilities $P_1(z_2,z_3)$ must be computed according to (2.10)-(2.12). For example, using (2.10) we can show that

$$P^{(1)}_{succ}(\mathcal{M}_1) = r_1\left\{1-\frac{\lambda_2\bar{r}_2/\bar{r}_1+\lambda_3\bar{r}_3/\bar{r}_1+r_2r_3(P_1(0,0)-1)}{1-r_2-r_3}\right\} . \tag{2.12}$$

In a similar manner, we can express $P^2_{succ}(\mathcal{M}_1)$ and $P^3_{succ}(\mathcal{M}_1)$ in terms of $P_1(0,0)$.

In summary, we obtain the following corollary.

Corollary 3. *The buffered ALOHA system is stable iff* $(\lambda_1,\lambda_2,\lambda_3)\in R=\bigcup_{i=1}^3\mathcal{R}_i$ *where*

$$\begin{aligned} \mathcal{R}_1 &= \{\lambda_1 < P^1_{succ}(\mathcal{M}_1),\ \lambda_2 < r_2\bar{r}_1(1-(1-F_1(1,0))r_3),\ \lambda_3 < r_3\bar{r}_1(1-(1-F_1(0,1))r_2)\} \\ \mathcal{R}_2 &= \{\lambda_1 < r_1\bar{r}_2(1-(1-F_2(1,0))r_3),\ \lambda_2 < P^2_{succ}(\mathcal{M}_2),\ \lambda_3 < r_3\bar{r}_2(1-(1-F_2(0,1))r_1)\} \\ \mathcal{R}_3 &= \{\lambda_2 < r_1\bar{r}_3(1-(1-F_3(1,0))r_2),\ \lambda_2 < r_2\bar{r}_3(1-(1-F_3(0,1))r_1),\ \lambda_3 < P^3_{succ}(\mathcal{M}_3)\} \end{aligned}$$

where the appropriate probabilities above are computed from the Nain's (1985) model as discussed above (cf. (2.10)-(2.12)) with some obvious modifications. ■

Stability region $\mathcal{R}$ for $M=3$ is shown in our Figure 2. Note that the following points belong to the boundary of the stability region: $\omega=(\lambda_1,\lambda_2,\lambda_3)=(r_1\bar{r}_2\bar{r}_3,\bar{r}_1r_2\bar{r}_3,\bar{r}_1\bar{r}_2r_3)$, $A=(r_1\bar{r}_2,\bar{r}_1r_2,0)$, and $B=(r_1,0,0)$, $C=(r_1\bar{r}_3,0,\bar{r}_1r_3)$, $D=(0,0,r_3)$, $E=(0,r_2\bar{r}_3,\bar{r}_2r_3)$ and $F=(0,r_2,0)$. In passing, we stress the fact that the probability of success $P^j_{succ}(\mathcal{M}_k)$ *does* depend explicitly on the probability $P_1(0,0)$, which is a *nonlinear* function of the input

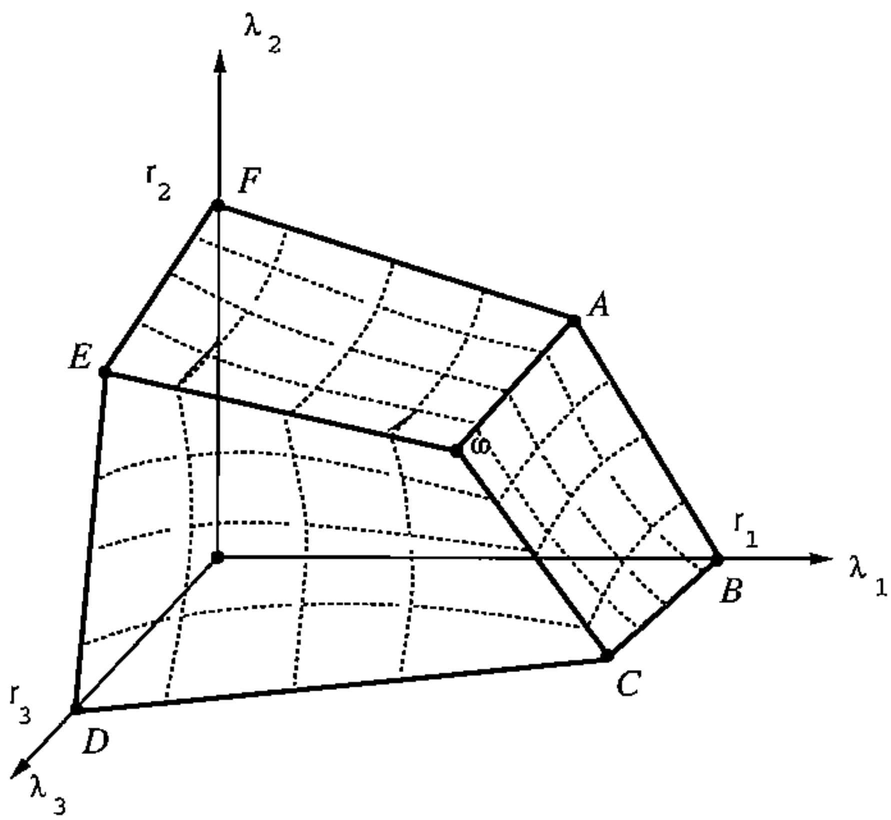

Figure 2: Stability region for $M = 3$ users in the slotted ALOHA system.

rates. This implies that the boundaries of the stability region for the ALOHA system for $M \geq 3$ are *not* linear functions of $(\lambda_1, \lambda_2, \lambda_3)$. Moreover, (2.11) indicates that there is no simple explicit formulae for the stability region for $M > 3$.

A generalization of the above to $M > 3$ is even more harder, since we need to estimate the probability of empty/nonempty buffers in three and higher dimensional ALOHA systems, which is not available to us. Nevertheless for $M > 3$, some bounds are easy to obtain from Theorem 1. For example, the bounds derived by Tsybakov and Mikhailov (1979) and Szpankowski (1988) directly follow from (2.7) and (2.8). Indeed, since $\prod_{k \in \mathcal{M}_j}(1 - r_k)^{z_k} \geq \prod_{k \in \mathcal{M}_j}(1 - r_k)$, one immediately proves from (2.7) that $\lambda_j < r_j \prod_{k \in \mathcal{M}_j}(1 - r_k)$ for $j \in \mathcal{M}$ is sufficient for stability of the ALOHA system. On the other hand, since $\prod_{k \in \mathcal{M}_j}(1 - r_k)^{z_k} \leq 1$ we prove that $\lambda \geq r_j$ for some $j \in \mathcal{M}$, is sufficient for instability of the ALOHA system.

The above simple bounds can be used to establish sufficient and necessary conditions for stability of the symmetric ALOHA system. We prove the following result which was already known to Tsybakov and Mikhailov (1979) (see also Szpankowski (1988)).

Corollary 4. *Let $r_j = r$ and $\lambda_j = \lambda$ for all $j \in \mathcal{M}$. Then, such a symmetric ALOHA system is stable if and only if the following holds*

$$\lambda < r(1 - r)^{M-1} . \tag{2.14}$$

Proof. This directly follows from Theorem 1 since in the symmetric case we really deal with a one dimensional problem. The stability region in this case is the intersection of the line $\lambda_1 = \lambda_2 = \cdots = \lambda_M = \lambda$ with the region $\mathcal{R}$ as defined in (2.8) for $r_1 = r_2 = \cdots = r_M = r$. Even simpler proof can be obtained, by noting that (2.14) is a direct consequence of our upper bound $\lambda_j < r_j \prod_{k \in \mathcal{M}_j}(1 - r_k)$ for $j \in \mathcal{M}$ just derived above. Setting in this inequality the symmetric model assumptions, we obtain (2.14). From Tsybakov and Mikhailov (1979), and Szpankowski (1988) (see Szpankowski (1986) for a simple proof) we also know that (2.14) is necessary for the stability. ∎

To obtain more sophisticated bounds for the stability region $\mathcal{R}$, we need a tighter estimate for the probability $\Pr\{\chi(\overline{\mathbf{N}}^t_{\mathcal{S}}) = \mathbf{z}\}$ in (2.7). Let us mention here one possibility (for a more sophisticated approach see Szpankowski (1988), and Rao and Ephremides (1989)). We first note that the probability $P^j_{succ}(\mathcal{S})$ defined in (2.7) for a given partition $(\mathcal{S}, \mathcal{U})$, can be alternatively expressed as

$$P^j_{succ}(\mathcal{S}) = r_j \left(1 - \sum_{k=1}^{|\mathcal{S}|} \sum_{(i_1, \ldots, i_k) \in \mathcal{S}} (-1)^k r_{i_1}, \ldots, r_{i_k} \Pr\{\overline{N}^t_{i_1} \geq 1, \ldots, \overline{N}^t_{i_k} \geq 1\} \right) . \tag{2.15}$$

This follows directly from the fact that $\sum_{z\in\Theta_S}\Pr\{\chi(\overline{\mathbf{N}}_S^t = z\} = 1$, and some algebraic manipulations. But, $\Pr\{\overline{N}_{i_1}^t \geq 1,\ldots,\overline{N}_{i_k}^t \geq 1\} \leq \Pr\{\overline{N}_{i_\ell}^t \geq 1\}$ for some $1 \leq \ell \leq k$. The latter probability we can estimate as follows

$$\frac{\lambda_\ell}{r_\ell} \leq \Pr\{\overline{N}_{i_\ell}^t \geq 1\} \leq \frac{\lambda_\ell}{r_\ell \prod_{k=1,k\neq\ell}(1-r_k)} . \tag{2.16}$$

To obtain the left-hand side of (2.16), we assume that all users except the jth are always empty, while in the right-hand size of (2.16) we postulate that all users except the jth are always nonempty. Using (2.15) and the above, upper and lower bounds on $P_{succ}^j(\mathcal{S})$ can be obtained, whence via Theorem 1 also bounds on the stability region of the ALOHA system.

Finally, the most sophisticated bound suggested by Rao and Ephremides (1989) (and the best up-to-date for not-very-asymmetric ALOHA system) follows from our Theorem 1, too. In this case, however, the estimate of the above probabilities must be more careful, and therefore more lengthy. In fact, this bound extends the idea of Tsybakov and Mikhailov (1979) by analyzing more terms in (2.7). More precisely, all probabilities $\Pr\{\chi(\overline{\mathbf{N}}_S^t) = \mathbf{z}\}$ in (2.7) are skillfully bounded by a one dimensional probability $\Pr\{\overline{N}_k^t \geq 1\}$ where $k \in \mathcal{S}$. The interested reader is referred to the original paper by Rao and Ephremides (1989). Other bounds suggested in Szpankowski (1988) that have been derived from the Lyapanov function approach, also follow from our Theorem 1 after some tedious algebraic manipulations.

3. ANALYSIS: AN ALTERNATIVE APPROACH

In this section, we present our approach to compute stability conditions for multiqueue systems. We concentrate on proving Theorem 1, however, generality of the approach will be apparent from our discussion. We first prove sufficient conditions (cf. Section 3.1), and then we concentrate on necessary conditions (cf. Section 3.2) for stability. In the proof below, we shall use two general results, namely the so called *isolation lemma* (cf. Lemma 5) and Loynes' scheme (Loynes (1962)) adopted to our situation (cf. Lemma 6).

We start with the isolation lemma which states that for substability of a multidimensional process $\mathbf{N}^t$ (not necessary Markovian) one requires substability of its components. More precisely, we prove the following.

Lemma 5. (i) *If for all $j \in \mathcal{M}$ the one dimensional processes N_j^t are substable, then the M-dimensional process $\mathbf{N}^t = (N_1^t, N_2^t, \ldots, N_M^t)$ is substable.*

(ii) *If for some j, say j^*, $N_{j^*}^t$ is unstable, then $\mathbf{N}^t$ is also unstable.*

(iii) *If $\mathbf{N}^t$ is a Markov chain defined on a countable state space, then the stability of N_j^t for all $j \in \mathcal{M}$ implies stability of the multidimensional Markov chain $\mathbf{N}^t$.*

Proof. We first prove part (i). Since each component of the process $\mathbf{N}^t$ is substable, then by definition (2.4) for all $j \in \mathcal{M}$ we have $\lim_{x_j\to\infty} \lim_{t\to\infty} \sup \Pr\{N_j^t > x_j\} = 0$. But

$$1 \geq \lim_{x\to\infty} \liminf_{n\to\infty} \Pr\{N_j^n \leq x_j, \text{ for } \quad j = 1, 2, \ldots, M\}$$

$$\geq 1 - \sum_{j=1}^{M} \lim_{x_j\to\infty} \limsup_{n\to\infty} \Pr\{N_j^n > x_j\} = 1 .$$

Thus, $\lim_{x\to\infty} \lim_{t\to\infty} \inf \Pr\{\mathbf{N}^t < \mathbf{x}\} = 1$, and $\mathbf{N}^t$ is substable by (2.4). If $\mathbf{N}^t$ is a Markov chain defined on a countable state space, then substability implies stability since such a Markov chain always converges to a random variable, which might be dishonest (cf. Asmussen (1987)). This proves (iii). For results concerning Markov chains defined on more general spaces the reader is referred to Meyn and Tweedie (1992).

For part (ii) we notice that instability of N_{j*}^t implies $\lim_{x_{j*}\to\infty} \lim_{t\to\infty} \inf \Pr\{N_{j*}^t < x_{j*}\} < 1$. Hence,

$$\lim_{x_{j*}\to\infty} \lim_{t\to\infty} \inf \Pr\{\mathbf{N}^t < \mathbf{x}\} \leq \lim_{x_{j*}\to\infty} \lim_{t\to\infty} \inf \Pr\{N_{j*}^t < x_{j*}\} < 1$$

which proves Lemma 5. ■

Our second general result is a simple consequence of the Loynes' scheme (Loynes (1962)). It provides stability criteria for a general single queue as described in (2.1), that is,

$$N_j^{t+1} = (N_j^t - Y_j^t)^+ + X_j^t \tag{3.1}$$

with stationary input sequences. We prove the following lemma.

Lemma 6. (Loynes 1962). *Let the pair (X_j^t, Y_j^t) be a strictly stationary and ergodic process. We denote by $\lambda_j = EX_j$ and EY_j the corresponding mean values of X_j^t and Y_j^t respectively. Then the following holds*

(i) *if $\lambda_j < EY_j$, then the queue length N_j^t satisfying (3.1) is stable in the sense of definition (2.3),*

(ii) *if $\lambda_j > EY_j$, then the queue is unstable,* and $\lim_{t\to\infty} N_j^t = \infty$ *(a.s.).*

Proof. We reduce the problem to the Loynes' scheme. Note that by setting $W_j^{t+1} = N_j^{t+1} - X_j^t$ and $U_j^t = X_j^{t-1} - Y_j^t$, we obtain the Loynes' equation (cf. Loynes (1962)), namely

$$W_j^{t+1} = (W_j^t + U_j^t)^+ .$$

Hence, by Loynes (1962) N_j^t is stable for $EU_j^t < 0$ and unstable for $EU_j^t > 0$. To prove the second part of (ii), we note that $W_j^{t+1} \geq W_j^0 + \sum_{k=0}^t U_j^t$, and by Birkhoff's *Individual Ergodic Theorem* (Breiman (1968)) we have $\sum_{k=0}^t U_j^t \to \infty$ (a.s.) provided $EU_j^t > 0$. Note that $EU^t = 0$ may lead to stable or unstable queue. For more detailed analysis of this case see Loynes (1962) and Walrand (1988). ■

3.1 Sufficient Conditions

We use mathematical induction to establish sufficient condition for stability of the Markov chain $\mathbf{N}^t$ describing the ALOHA system. This part of the proof resembles the idea already used by Georgiadis and Szpankowski (1992).

For $M = 1$ the proof is easy. It suffices to note that the average drift becomes $E\{N^{t+1} - N^t | N^t \geq 1\} = \lambda - EY^t$. Hence, the proof follows from the Lyapunov function method (cf. Tweedie (1976), Szpankowski (1990)).

Now we assume that Theorem 1 is true for $M - 1$ and we prove that it can be extended to M queues. The main idea is to consider a modified ALOHA system in which the set of users $\mathcal{M}$ is partition into $\mathcal{P} = (\mathcal{S}, \mathcal{U})$ with $\mathcal{S} \neq \mathcal{M}$ where $\mathcal{S}$ is a copy of the ALOHA model that mimics its behavior, while users in $\mathcal{U}$ persistently jam users in $\mathcal{S}$ (i.e., a user in $\mathcal{U}$ attempts to send a dummy packet even if empty). For a more detailed description of the modified system, the reader is referred to Section 2. From Tsybakov and Mikhailov (1979), and Szpankowski (1986) it is known that the queue lengths process $\overline{\mathbf{N}}_{\mathcal{P}}^t = (\overline{\mathbf{N}}_{\mathcal{S}}^t, \overline{\mathbf{N}}_{\mathcal{U}}^t)$ in such a modified system stochastically dominates the original queue length $\mathbf{N}^t$, that is, (2.5) holds.

Hereafter, we concentrate on proving stability condition for the dominant system represented by $\overline{\mathbf{N}}_{\mathcal{P}}^t = (\overline{\mathbf{N}}_{\mathcal{S}}^t, \overline{\mathbf{N}}_{\mathcal{U}}^t)$. We first consider users in $\mathcal{S}$, and establish stability condition for $\overline{\mathbf{N}}_{\mathcal{S}}^t$. Note that the set of users restricted to $\mathcal{S}$ is a *smaller* copy of the ALOHA model, and users in $\mathcal{U}$ contribute only to jamming. Therefore, we can say that a user $j \in \mathcal{S}$ attempts to transmit a packet in the modified system with a new probability of transmission equal to $r_j \prod_{k \in \mathcal{U} - \{j\}} (1 - r_k)$, hence the probability of success $P_{succ}^j(\mathcal{S})$ is given by (2.7) for $j \in \mathcal{S}$. Let $\lambda_{\mathcal{S}} = (\lambda_{i_1}, \ldots, \lambda_{i_{|\mathcal{S}|}})$ where $i_j \in \mathcal{S}$ for $1 \leq i_j \leq |\mathcal{S}|$. Since $|\mathcal{S}| < M$, Theorem 1 holds for $\mathcal{S}$ by mathematical induction arguments. In terms of Theorem 1, this means that $\overline{\mathbf{N}}_{\mathcal{S}}^t$ is stable for $\lambda_{\mathcal{S}} \in \mathcal{R}_{\mathcal{S}}^1$ where

$$\mathcal{R}_{\mathcal{S}}^1 = \bigcup_{\ell=1}^{|\mathcal{S}|} \{\lambda_{\mathcal{S}} : \ \lambda_j < P_{succ}^j(\mathcal{S}_\ell) \ \text{ for all } \ j \in \mathcal{S}\} , \tag{3.2}$$

and $\mathcal{S}_\ell = \mathcal{S} - \{\ell\}$ according to the notation introduced in Theorem 1. In words, we consider a partition $(\mathcal{S}_\ell, \{\ell\})$ of $\mathcal{S}$ such that the ℓth user is a persisting one. For such a partition, we conclude that $\lambda_j < P^j_{succ}(\mathcal{S}_\ell)$ for all $j \in \mathcal{S}$ is sufficient for stability of $\overline{\mathbf{N}}^t_{\mathcal{S}_\ell}$. Hence, the Markov chain $\overline{\mathbf{N}}^t_{\mathcal{S}}$ is stable if there exists a partition $(\mathcal{S}_\ell, \{\ell\})$ for which $\overline{\mathbf{N}}^t_{\mathcal{S}_\ell}$ is stable. This leads to (3.2).

We now provide stability condition for a persistent queue $j \in \mathcal{U}$. The idea is to apply Lemma 6 to an isolated persistent queue $j \in \mathcal{U}$. For this we need to establish stationarity and ergodicity of the output process $Y^t_j(\mathcal{S})$.

We proceed as follows. Let $\lambda_{\mathcal{S}} \in \mathcal{R}^1_{\mathcal{S}}$ (cf. (3.2)), and consider a persistent user, say $j \in \mathcal{U}$. The Markov chain $\overline{\mathbf{N}}^t_{\mathcal{S}}$ is stable by the mathematical induction. More precisely, this Markov chain is ergodic, hence there exists a unique honest stationary distribution π for this process. But, $(\overline{\mathbf{N}}^t_{\mathcal{S}}, Y^{t+1}_j(\mathcal{S}))$ is another $|\mathcal{S}| + 1$-dimensional Markov chain. It is easy to see that this chain is irreducible and aperiodic, too. Moreover, $Y^t_j(\mathcal{S})$ is bounded from the above by one, hence $(\overline{\mathbf{N}}^t_{\mathcal{S}}, Y^{t+1}_j(\mathcal{S}))$ is substable. By Lemma 5(iii), this implies that the $|\mathcal{S}| + 1$-dimensional Markov chain is stable, and also ergodic. Let its stationary distribution be denoted by $\bar{\pi}$.

Now we construct a stationary version of the process $(\overline{\mathbf{N}}^t_{\mathcal{S}}, Y^{t+1}_j(\mathcal{S}))$ by starting it with the initial distribution $\bar{\pi}$. In fact, it is enough to assume that $\overline{\mathbf{N}}^t_{\mathcal{S}}$ starts with the distribution π, that is, $\overline{\mathbf{N}}^0_{\mathcal{S}}$ is distributed according to π. This will imply that the process $Y^t_j(\mathcal{S})$ for $j \in \mathcal{U}$ is stationary and ergodic for such an initial distribution. Then, by Lemma 6 the jth persistent queue is stable if $\lambda_j < EY^t_j(\mathcal{S}) = P^j_{succ}(\mathcal{S})$ where $P^j_{succ}(\mathcal{S})$ is given by (2.7). This is true for every persistent queue $j \in \mathcal{U}$, and hence by Lemma 5 the process $\overline{\mathbf{N}}^t_{\mathcal{U}}$ is stable for $\lambda \in \mathcal{R}^2_{\mathcal{S}}$ where

$$\mathcal{R}^2_{\mathcal{S}} = \{\lambda_{\mathcal{U}} : \; \lambda_j < P^j_{succ}(\mathcal{S}) \;\; \text{for all} \;\; j \in \mathcal{U}\} \;, \tag{3.3}$$

and the Markov chain $\overline{\mathbf{N}}^t_{\mathcal{P}} = (\overline{\mathbf{N}}^t_{\mathcal{S}}, \overline{\mathbf{N}}^t_{\mathcal{U}})$ is stable for $\lambda \in \mathcal{R}'_{\mathcal{S}} = \mathcal{R}^1_{\mathcal{S}} \cup \mathcal{R}^2_{\mathcal{S}}$.

So far, we have established stability condition for a *given* partition $\mathcal{P} = (\mathcal{S}, \mathcal{U})$ of $\mathcal{M}$. Clearly, the dominant system represented by $\overline{\mathbf{N}}^t$ is stable if *there exists* a partition $\mathcal{P}$ such that $\overline{\mathbf{N}}^t_{\mathcal{P}}$ is stable. Therefore, the stability region for $\overline{\mathbf{N}}^t$ becomes $\mathcal{R}' = \bigcup_{\mathcal{S} \subset \mathcal{M}} \mathcal{R}'_{\mathcal{S}}$ where the sum is over all subsets of $\mathcal{M}$ such that $\mathcal{S} \neq \mathcal{M}$.

We prove now that $\mathcal{R}' = \mathcal{R}$ where $\mathcal{R}$ is defined in (2.8) of Theorem 1, that is, $\mathcal{R} = \bigcup_{k=1}^{M} \mathcal{R}_{\mathcal{M}_k}$ where

$$\mathcal{R}_{\mathcal{M}_k} = \{\lambda : \; \lambda_j < P^j_{succ}(\mathcal{M}_k) \quad \text{for all} \quad j \in \mathcal{M}\} \; . \tag{3.4}$$

First, however, we simplify the expression for $\mathcal{R}'$. Note that $\bigcup_{\mathcal{S} \subset \mathcal{M}} \mathcal{R}'_{\mathcal{S}} = \bigcup_{k=1}^{M} \mathcal{R}'_{\mathcal{M}_k}$ where by (3.2) and (3.3)

$$\mathcal{R}'_{\mathcal{M}_k} = \bigcup_{\substack{\ell=1 \\ \ell \neq k}}^{M} \{\lambda : \ \lambda_j < P^j_{succ}(\mathcal{M}_{k,\ell}) \ \text{ for all } \ j \in \mathcal{M} - \{k\} \ \text{ and } \ \lambda_k < P^k_{succ}(\mathcal{M}_k)\} \ , \quad (3.5)$$

with $\mathcal{M}_{k,\ell} = \mathcal{M} - \{k, l\}$. This shows that the only partitions that contribute to the stability region are of the form $(\mathcal{M}_k, \{k\})$. Indeed, it is a simple consequence of $(1-r_k)^{z_k} \geq (1-r_k)$ where $z_k \in \{0, 1\}$, and the following monotonicity property of the probability of success $P^j_{succ}(\mathcal{S})$ (cf. Tsybakov and Mikhailov (1979), and Szpankowski (1986))

$$\mathcal{S}' \subset \mathcal{S} \quad \Longrightarrow \quad P^j_{succ}(\mathcal{S}') \leq P^j_{succ}(\mathcal{S}) \ .$$

We now prove $\mathcal{R}_{\mathcal{M}_k} = \mathcal{R}'_{\mathcal{M}_k}$ which suffices for $\mathcal{R} = \mathcal{R}'$. Clearly, $P^j_{succ}(\mathcal{M}_{k,\ell}) \leq P^j_{succ}(\mathcal{M}_{j,\ell})$ (cf. Tsybakov and Mikhailov (1979), Szpankowski (1986) and (1988)). Indeed, this is true since under the partition $(\mathcal{M}_{k,\ell}, \{k, \ell\})$ the kth user is never empty, hence its queue length can be made always larger that the the queue length at user k under the partition $(\mathcal{M}_{j,\ell}, \{j, \ell\})$. Therefore, we can simply $\mathcal{R}'_{\mathcal{M}_k}$ as

$$\mathcal{R}'_{\mathcal{M}_k} = \{\lambda : \ \lambda_j < P^j_{succ}(\mathcal{M}_{k,j}) \ \text{ for all } \ j \in \mathcal{M} - \{k\} \ \text{ and } \ \lambda_k < P^k_{succ}(\mathcal{M}_k)\} \ .$$

Now it suffices to show that $P^j_{succ}(\mathcal{M}_{k,j}) = P^j_{succ}(\mathcal{M}_k)$. But this is easy. Consider the definition (2.7) of the probability of success, and proceed as follows

$$\begin{aligned}
P^j_{succ}(\mathcal{M}_{k,j}) &= r_j(1-r_k) \sum_{\mathbf{z}_{\mathcal{M}_{k,j}}} \Pr\{\mathbf{N}_{\mathcal{M}_{k,j}} = \mathbf{z}_{\mathcal{M}_{k,j}}\} \prod_{\ell \in \mathcal{M}_k} (1-r_\ell)^{z_\ell} \\
&= r_j(1-r_k) \sum_{\mathbf{z}_{\mathcal{M}_k}} \Pr\{\mathbf{N}_{\mathcal{M}_k} = \mathbf{z}_{\mathcal{M}_k}\} \prod_{\ell \in \mathcal{M}_k} (1-r_\ell)^{z_\ell} (\Pr\{N_j = 0\} + \Pr\{N_j \geq 1\}) \\
&= P^j_{succ}(\mathcal{M}_k)
\end{aligned}$$

This shows $\mathcal{R} = \mathcal{R}'$, and completes the proof of Theorem 1.

3.2 Necessary Conditions

In this section we prove necessary conditions for stability of the ALOHA system. More precisely, we establish the following result, which is the "necessary part" of Theorem 1.

Theorem 1A. *If $\lambda \in \overline{\mathcal{R}} = \bigcap_{k=1}^{M} \overline{\mathcal{R}}_k$ where*

$$\overline{\mathcal{R}}_k = \{\lambda = (\lambda_1, \ldots, \lambda_M) : \ \lambda_j > P^j_{succ}(\mathcal{M}_k) \quad \text{for some} \ \ j \in \mathcal{M}\} \ , \qquad (3.6)$$

then the Markov chain $\mathbf{N}^t$ is unstable, that is, there exists a queue, say j one, such that $N^t_j \to \infty$ (a.s.). ■

We carry out the proof of Theorem 1A in three steps. The main idea behind the proof is to show that:

(i) if (3.6) holds, then the kth queue under the partition $(\mathcal{M}_k, \{k\})$ is unstable *in the dominant system* $\overline{\mathbf{N}}^t_{\mathcal{P}}$ defined above,

(ii) the last assertion can be extended to the original system (i.e., still only for queue k);

(iii) we can generalize (ii) above to other queues in the original system.

We discuss these three steps in sequel.

Step 1. *Instability of the dominant system*

Let us consider the dominant system under the partition $\mathcal{P} = (\mathcal{M}_k, \{k\})$. Define a subset $\overline{\mathcal{R}}_k(k)$ of the instability region $\overline{\mathcal{R}}_k$ as

$$\overline{\mathcal{R}}_k(k) = \{\lambda = (\lambda_1, \ldots, \lambda_M) : \lambda_k > P^k_{succ}(\mathcal{M}_k) \text{ and } \lambda_j < P^j_{succ}(\mathcal{M}_k) \text{ for all } j \in \mathcal{M} - \{k\}\}\,. \tag{3.7}$$

The arrival process to a persistent queue (e.g., the kth one) does not effect the stability condition for nonpersistent queues represented by the Markov chain $\overline{\mathbf{N}}^t_{\mathcal{S}}$. Hence, for $\lambda \in \overline{\mathcal{R}}_k(k)$ the Markov chain $\overline{\mathbf{N}}^t_{\mathcal{S}}$ is *stable*, as proved in Section 3.1. Using the same construction as above, we can assure stationarity and ergodicity of $\overline{\mathbf{N}}^t_{\mathcal{S}}$ as well as the output process $Y^t_k(\mathcal{M}_k)$. Therefore, the stationarity and ergodicity assumptions of Loynes' scheme hold, and we can apply Lemma 6 to the kth queue. In particular, part (ii) implies that this queue is unstable if $\lambda_k > EY^t_k(\mathcal{M}_k) = P^k_{succ}(\mathcal{M}_k)$, that is, $\overline{N}^t_k \to \infty$ (a.s.) for $\lambda \in \overline{\mathcal{R}}_k(k)$.

Step 2. *Instability of the kth queue in the original system*

We now prove that for $\lambda \in \overline{\mathcal{R}}_k(k)$ the kth queue is also unstable in the *original* system. We shall use an *anti-coupling* argument to show that with high probability the dominant system and the original system are *indistinguishable* for $\lambda \in \overline{\mathcal{R}}_k(k)$, that is, the kth queue becomes empty only finitely many times in the original system.

Consider the dominant system $\overline{\mathbf{N}}^t_{\mathcal{P}}$ where $\mathcal{P} = (\mathcal{M}_k, \{k\})$. From Step 1, we know that for $\lambda \in \overline{\mathcal{R}}_k(k)$ the kth queue length in the dominant system tends to infinity (a.s.). Therefore, the kth queue in the dominant system returns to zero only finitely many times. Consider the last return to zero, and denote such a time by $L \geq 0$. Clearly, $\Pr\{L < \infty\} = 1$. Now, at $t = -L$ we start running the dominant system, and at $t = 0$ the kth queue is empty for the last time (a.s.). Define $\mathbf{N}^1 = \overline{\mathbf{N}}^1_{\mathcal{P}}$, that is, the original system starts with the initial queue lengths being equal to the queue length in the dominant system at $t = 1$. Define $\tau = \min\{t > 0 : N^t_k = \overline{N}^t_k = 0\}$. Observe that $\Pr\{\tau = \infty\} = 1$.

Note that for $0 < t \leq \tau$ the original system and the dominant one are identical, that is, $\mathbf{N}^t = \overline{\mathbf{N}}^t_{\mathcal{P}}$ for all $0 < t \leq \tau$. But, by our construction $\Pr\{\tau = \infty\} = 1$ for $\lambda \in \overline{\mathcal{R}}_k(k)$. Therefore, with high probability the kth queue in the *original* system tends to infinity, that is, $N^t_k \to \infty$. This proves that the kth queue in the original system is also unstable for $\lambda \in \overline{\mathcal{R}}_k(k)$.

Step 3. *Extension of instability to the whole set* $\overline{\mathcal{R}}$.

We shall prove now that if $\lambda \in \overline{\mathcal{R}}_k$, where $\overline{\mathcal{R}}_k$ is defined in (3.6), then the original system, hence the Markov chain $\mathbf{N}^t$, is unstable.

Consider another queue, say $\ell \neq k$ in the original system. Consider first only the region $\overline{\mathcal{R}}_k(k)$, and increase the the input rate λ_ℓ (beyond $P^\ell_{succ}(\mathcal{M}_k)$). It is known (cf. Tsybakov and Mikhailov (1979), and Szpankowski (1986)) that such an increase will only lead to the increase of the kth queue length. Therefore, at least for one queue, namely the kth one, we have $N^t_k \to \infty$ (a.s.), as needed for the instability of $\mathbf{N}^t$, by Lemma 5(ii). This completes the proof of Theorem 1A.

4. FURTHER RESULTS AND CONCLUSION

The technique adopted in the proof of Theorem 1 is clearly not limited to the ALOHA system. It is, however, restricted to multidimensional *queueing systems* since the real engine of our proof is the Loynes' scheme which allows for the treatment of non-Markovian subsystems (of persistent queues). Several other multiqueue models can be treated in a similar manner. In Georgiadis and Szpankowski (1992) we used this technique to prove another long standing open problem in the stability analysis, namely stability of the token passing ring.

To generalize our scheme, we repeat main ingredients of the proof of Theorem 1. As the first step, we partition the set of users $\mathcal{M}$ into nonpersistent users $\mathcal{S}$ and persistent ones $\mathcal{U}$. Users in $\mathcal{S}$ form a smaller copy of the original system. Persistent users constantly jam users in $\mathcal{S}$ by attempting to send dummy packets even when empty. We denote by $\overline{\mathbf{N}}^t_{\mathcal{P}}$ the queueing process representing this modified system. For our technique to work, we need the following two properties:

(P1) *Monotonicity Condition*

The queue lengths in all users increase whenever dummy messages are sent by a persistent user. More precisely, if $\overline{\mathbf{N}}^t$ is the queue lengths in our modified system (with persistent users), and $\mathbf{N}^t$ is the queue length in the original system, then $\mathbf{N}^t \leq_{st} \overline{\mathbf{N}}^t$.

(P2) *Stationary Version of* $\mathcal{S}$

The set of users $\mathcal{S}$ is a smaller copy of the original system, and by the induction assumption we can assume it is in a stable mode. We request that the set of stable nonpersistent users $\mathcal{S}$ has a stationary ergodic version such that the departure process from $\mathcal{S}$ that enters a nonpersistent queue $k \in \mathcal{U}$ is stationary and ergodic. Then, Loynes' scheme can be applied to establish stability of a nonpersistent queue.

If the above two properties are satisfied, then we can carry out our analysis, and establish sufficient and necessary condition for stability. We illustrate it again, on another multiqueue systems with buffered users and different multiaccess protocol. We leave details of the proof to an interested reader who should follow our footsteps from Section 3.

We shall investigate a buffered multiaccess system with conflict resolution algorithm. The system works in a manner similar to ALOHA except that it adopts another multiaccess protocol, namely the so called *blocked conflict resolution algorithm* (blocked CRA) of Capetanakis (1979), and Tsybakov and Mikhailov (1978). To the best of our knowledge, the stability region of such a system was not obtained before. The closest problem was tackled by Paterakis *et al.* (1987) who established stability conditions for the *symmetric* system with *exhaustive* queueing discipline. Such an analysis, as discussed in Section 2, is considerable simpler from the stability viewpoint (i.e., it is really a one dimensional stability problem).

The system works in the following manner. If there is a collision, then all users *not* involved in it are blocked, and are not allowed to transmit until the current collision is resolved. The collision is solved by a divide-and-conquer algorithm, that is, all users involved in a collision flip a coin and only those who flipped "tails" are allowed to transmit in the next slot. This process is continued until all users in the current collision are successfully transmitted. The quality of such a systems depends on the length L^t of the conflict resolution session (CRS), where t is a nonnegative integer representing the beginning of the tth CRS. More details can be found in Capetanakis (1979), Tsybakov and Mikhailov (1978), and Szpankowski (1987).

It is easy to verify that properties (P1) and (P2) hold for the above system. Therefore, we should be able to derive sufficient and necessary conditions for its stability. As in the ALOHA system, we partition the set of users $\mathcal{M}$ into $(\mathcal{S},\mathcal{U})$ where $\mathcal{S} \neq \mathcal{M}$ is a smaller copy of the original system, while users in $\mathcal{U}$ *persistently* try to transmit packets even when their buffers are empty. After some thoughts, one should be able to reproduce our previous analysis to establish the following result.

Theorem 7. *The buffered multiaccess system with a blocked conflict resolution algorithm is stable iff $\lambda \in \mathcal{R}$ where*

$$\mathcal{R} = \bigcup_{k=1}^{M} \{\lambda = (\lambda_1, \ldots, \lambda_M) : \ \lambda_j EL^0(\mathcal{M}_k) < 1 \quad \textit{for all} \ \ j \in \mathcal{M}\} \tag{4.1}$$

where, as before, $\mathcal{M}_k = \mathcal{M} - \{k\}$ and $EL^t(\mathcal{M}_k)$ denotes the average session length under the partition $(\mathcal{M}_k, \{k\})$. ∎

Evaluation of the average session length EL^t depends on the *conditional* average session length EL_m^t (which is a function of the algorithm used to solve the conflict), and the content of buffers. More precisely, EL_m^t denote the average conflict resolution session when the initial conflict is of multiplicity m (i.e., there are m nonempty buffers). Let also π_m denote the probability that there are exactly m nonempty buffers at the beginning of a CRS. Note that the stationary distribution π_m exists even for unstable systems. Naturally, $EL^t = \sum_{m=1}^{M} \pi_m EL_m^t$. Moreover, if Z^t denotes a random variable representing the number of nonempty buffers at the beginning of a CRS, then we have by the conservation law (cf. Miyazawa (1985) for rigorous treatment)

$$EL^t \sum_{i=1}^{M} \lambda_i = \sum_{m=1}^{M} \pi_m m = EZ^t \ . \tag{4.2}$$

The above can be used to re-write (4.1) in terms of $EZ^t(\mathcal{M}_k)$.

As in the ALOHA case, no explicit formula for the average $EL^t(\mathcal{M}_k)$ exists. However, in some cases we can obtain exact stability conditions or bounds on the stability region.

We consider first two users case, $M = 2$ with Capetanakis (1979) algorithm to solve conflicts. Two partitions must be investigated, namely $\mathcal{P}_1 = (\mathcal{M}_1, \{1\})$ and $\mathcal{P}_2 = (\mathcal{M}_2, \{2\})$. For the second partition, in the stationary regime of $\overline{\mathbf{N}}_S^t$ we obtain

$$EL(\mathcal{M}_2) = (1 - \lambda_1 EL(\mathcal{M}_2)) + \lambda_1 EL(\mathcal{M}_2)\ell_2 \tag{4.3}$$

where $\ell_2 = EL_2(\mathcal{M}_2)$ is the conditional CRS length when the multiplicity of the conflict is two. For a fair coin model, we know that $\ell_2 = 5$ (cf. Tsybakov and Mikhailov (1978), Szpankowski (1987)). Therefore, $EL(\mathcal{M}_2) = 1/(1 - 4\lambda_1)$. In summary, we can prove the following result.

Corollary 8. *The buffered CRA system with $M = 2$ users is stable if and only if $(\lambda_1, \lambda_2) \in \mathcal{R}$ such that*

$$\begin{aligned} \mathcal{R} &= \{\lambda_1 < 1/\ell_2 = 0.2 \ \ \textit{and} \ \ \ \lambda_2 < 1 - 4\lambda_1\} \cup \{\lambda_1 < 1 - 4\lambda_2 \quad \textit{and} \quad \lambda_2 < 1/\ell_2 = 0.2\} \\ &= \{\lambda : \ \lambda_2 + 4\lambda_1 < 1 \quad \textit{or} \quad \lambda_1 + 4\lambda_2 < 1\} \end{aligned}$$

where the Markov chain $\mathbf{N}^t$ *is imbedded at the beginning of CRS.* ∎

We can also present a simple bound for the stability region. We use the fact that the conditional CRS, EL^t_m, is asymptotically a linear function of m (cf. Capetanakis (979) and Szpankowski (1987)). In fact, we know that $EL_m < \alpha m + \beta$, where $\alpha = 2.8867$ and $\beta = 1.2336$. Then, using (4.2), after some algebraic manipulation, we obtain the following.

Corollary 9. *The buffered CRA system is stable if* $\lambda \in \mathcal{R}_1$, *where*

$$\mathcal{R}_1 = \bigcup_{k=1}^{M} \{\lambda = (\lambda_1, \ldots, \lambda_M) : \ \lambda_j < \frac{1 - \alpha \sum_{i \neq k} \lambda_i}{\alpha + \beta} \quad \textit{for all} \quad j \in \mathcal{M}\} \tag{4.4}$$

where the constants α *and* β *are given above.* ∎

Finally, it should be mentioned that our technique can be extended to multiply channels (i.e., multiserver case) when several packets are sent simultaneously. This even works for infinite number of servers (cf. Georgiadis and Szpankowski (1992)). For such multidimensional models Malyshev's criteria do not work since homogenity property is not preserved. It might be interesting to see whether our experiences can be used to extend Malyshev's criteria.

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