1992

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APPLICATION OF FOUR POLE PARAMETERS FOR GAS PULSATION ANALYSIS OF MULTI-CYLINDER COMPRESSORS WITH SYMMETRICALLY ARRANGED GAS CAVITIES

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ABSTRACT

Analysis of a single input and single output acoustic system can be done very efficiently by utilizing the concept of four pole parameters. Although there are more than one inputs to the system, gas pulsation analysis of multi-cylinder compressors can be done by using four pole parameters if their gas cavities are arranged symmetrically about the cylinders and the phase angles between pistons are identical. Acoustic couplings between cavities and sources are able to be decoupled by considering symmetric and anti-symmetric modes separately which allows to formulate equivalent four poles of the system. It is shown that one can fully utilize the advantage of four pole parameters.

INTRODUCTION

Four pole parameters are a very useful concept for efficient analysis of a composite acoustic system because of its computational efficiency and simplicity in formulation of the acoustic equation. Therefore, they are extremely useful to be used with design programs or simulation programs. Application examples of four pole parameters for the performance analysis of compressors are found in reference [1]. Good basic discussions on the concept and derivation of four poles of some basic acoustic elements are found in reference [2]. In reference [3], a general method is discussed on how to formulate four poles of continuous systems from pressure response solutions using natural mode superposition method. The result was also applied to the system with multiply connected cavities[4].

One limitation of the four pole method is that it cannot be used for the systems with more than one input because the concept is defined for single input single output (SISO hereafter) systems. Therefore, at least to the knowledge of the author, four poles have not been used to solve the system acoustic equation of multi-cylinder compressors. Gas pulsation analysis of the multiple cylinder was first treated in reference [5] using the Helmholtz resonator approach. The method results in a matrix equation of the same form as the vibration problem of a lumped parameter system with multi-degree of freedoms. In fact, vibration analogy was used to solve multi-cylinder gas pulsation problems in reference [6]. The eigenfunction superposition method can be used to solve the equation, however formulation and computation of the equation can be quite involved as the system becomes complex.

In this paper, it is shown that four pole parameters can be used for the analysis of the multi-cylinder compressor if *the gas manifold is arranged symmetrically with respect to each cylinder and the piston has the same phasing between each other*, for which we use simply *symmetric gas manifold* from now on. This condition is not considered to limit the application value of the method seriously because gas manifolds of most of multi-cylinder compressors or engines would be designed in such a way. The basic idea and formulation method are presented using the double cylinder compressor as an example. Then the idea is extended to compressors with multiple cylinders. A brief discussion is made on how to utilize the method for the analysis of overall gas manifold system.

FORMULATION OF EQUIVALENT FOUR POLE PARAMETERS

1. Double Cylinder Compressor

Figure 1 shows a part of the gas manifold of a compressor with two cylinders. The two identical cylinder head volumes V1 and V2 are connected to a common collector volume Vc by two identical pipes p1 and p2. If the phase between two pistons is 180 degrees, the flow inputs to the head cavities V1 and V2 are $Q_i e^{jn\omega t}$ and $(-1)^n Q_i e^{jn\omega t}$, where n=1,2,3..., and ω is the motor speed. In general, the output flow from the collector volume may enter to another acoustic system, such as an anechoic pipe or an expansion type muffler.

Figure 2 illustrates the basic idea to be used. Because of the symmetry of the system, the inputs corresponding to an even n will excite only the symmetric modes (Fig.2-a), while the inputs corresponding to an odd n will excite only antisymmetric modes (Fig.2-b). Therefore, it is possible to consider symmetric and antisymmetric inputs separately. Let us define the equivalent four pole equation of the system in Figure 1 in the same way as we do for regular SISO systems.

$$\begin{Bmatrix} Q_i \\ P_i \end{Bmatrix} e^{j\omega nt} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} Q_e \\ P_e \end{Bmatrix} e^{j\omega nt} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} 1 \\ Z_e \end{Bmatrix} Q_e e^{j\omega nt} \tag{1}$$

where, A, B, C, and D are equivalent four poles of the system, $Z_e = P_e / Q_e$ is the acoustic impedance at the system output point e, which depends on the acoustic system to be attached to, and P_i, P_e are pressure amplitudes at the system input and output points. Unlike ordinary four poles, they define the relationship between one of the two input pairs of the system and the output pair, which is why they are named *equivalent* four poles.

(1) Symmetric Modes

First, let us look at the common collector volume Vc subjected to the symmetric inputs (n = 2,4,6..) shown in Figure 3-a. The pressure at a point r in the volume can be represented as,

$$P(r) = Q_1 f_1(r) + Q_1 f_2(r) - Q_e f_e(r) \tag{2}$$

where, $f_i(r)$ is the pressure response of the cavity at the point r to the harmonic flow input of unit magnitude at the point r_i . Pressure responses may be obtained by any experimental, analytical or numerical methods. In reference [3], pressure responses were obtained by using natural mode superposition method using analytical natural modes. Let us use $f_{ij} = f_i(r_j)$ for short notation, the first subscript meaning the input point and the second subscript meaning the response point. From the reciprocity, we also can utilize the fact that $f_{ij} = f_{ji}$. The pressure at the output point e is

$$P_e = Q_1 (f_{1e} + f_{2e}) - Q_e f_{ee} = 2 Q_1 f_{1e} - Q_e f_{ee} \tag{3}$$

where the fact that $f_{1e} = f_{2e}$ was used from the symmetry. The input point pressure P_1 is

$$P_1 = Q_1 (f_{11} + f_{21}) - Q_e f_{e1} \tag{4}$$

By rearrange equation (3), we obtain

$$Q_1 = (f_{ee} / 2 f_{1e}) Q_e + (1/2 f_{1e}) P_e \tag{5}$$

Substituting equation (5) to equation(4),

$$P_1 = \left[(f_{ee} / 2 f_{1e})(f_{11} + f_{21}) - f_{e1} \right] Q_e + \left[(f_{11} + f_{21}) / 2 f_{1e} \right] P_e \tag{6}$$

Equations (5) and (6) defines the equivalent four pole relationship of the cavity 3. Therefore equivalent four poles of the collector volume with two symmetric inputs and a single output are

$$A_e = f_{ee} / 2f_{1e} \tag{7}$$

$$B_e = 1 / 2f_{1e} \tag{8}$$

$$C_c = (f_{ee} / 2f_{1e})(f_{11} + f_{21}) - f_{e1} \tag{9}$$

$$D_c = (f_{11} + f_{21}) / 2f_{1e} \tag{10}$$

The overall equivalent four poles of the system in Figure 1 for symmetric inputs are now obtained by utilizing the cascading property of four poles.

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_{v1} & B_{v1} \\ C_{v1} & D_{v1} \end{bmatrix} \begin{bmatrix} A_{p1} & B_{p1} \\ C_{p1} & D_{p1} \end{bmatrix} \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} \tag{11}$$

The first two matrices are regular four pole matrices of the volume V1 and the pipe p1. Equations of four poles of various acoustic elements can be found in reference [6,7]. If all the acoustic elements in the system are small enough to be treated as lumped parameter elements, the equation represents the half of the system as shown in Figure 2-c.

(2) Antisymmetric Modes

If we look at the antisymmetric input cases (n=1,3,5...) shown in Figure 2-b, we know that $Q_e = 0,\ P_e \neq 0$ from the geometry. Therefore, we don't have to consider these modes for the analysis of the system beyond the collector volume. The flow and pressure after the collector volume will have only the even harmonics(n=2,4,6,...).

In many cases, it is necessary to know the pressure or flow before the collector volume. For example, the pressure in V1 is the back pressure of the valve which is necessary to be known for an accurate compressor performance simulation. From Figure 3-b, the pressure at point 1 is,

$$P_1 = Q_1 f_{11} - Q_1 f_{21} = Q_1 (f_{11} - f_{21}) \tag{12}$$

Therefore, the input point impedance at the pint 1 of the collector volume becomes,

$$Z_1 = P_1 / Q_1 = f_{11} - f_{21} \tag{13}$$

The four pole relationship between the system input point and the input point of the collector volume is,

$$\begin{Bmatrix} Q_i \\ P_i \end{Bmatrix} = \begin{bmatrix} A_{v1} & B_{v1} \\ C_{v1} & D_{v1} \end{bmatrix} \begin{bmatrix} A_{p1} & B_{p1} \\ C_{p1} & D_{p1} \end{bmatrix} \begin{Bmatrix} 1 \\ Z_1 \end{Bmatrix} Q_1 = \begin{bmatrix} A_T & B_T \\ C_T & D_T \end{bmatrix} \begin{Bmatrix} 1 \\ Z_1 \end{Bmatrix} Q_1 \tag{14}$$

Equation (14) can be used to calculate the head cavity pressure as it will be shown later. From equation (12), it is also known that $Z_1 = 0$ if the collector is relatively small, therefore the pressure inside is nearly uniform. Furthermore, if all acoustic elements are small, the equation represents a Helmholtz resonator in Figure 2-d as expected. It should be noted that *large* or *small* is a relative term compared with the shortest wave length of interest.

2. Multiple-Cylinder Compressor

(1) Symmetric Modes

The previous discussion can be extended to the case of a multi-cylinder compressor. Figure 4 shows a compressor with three symmetrically arranged cylinders. From similar considerations as before, only symmetric modes contribute to the gas pulsations of the system beyond the collector volume. Notice that the modes corresponding to n=3,6,9,..are considered symmetric for the three cylinder case. For the m cylinder case, four poles of the collector volume have to be modified from equations (7),(8),(9) and (10) as follows.

$$A_c = f_{ee} / m f_{1e} \tag{15}$$

$$B_e = 1 / m f_{1e} \tag{16}$$

$$C_c = (f_{ee} / m f_{1e}) (\sum_{k=1}^{m} f_{kl}) - f_{el} \tag{17}$$

$$D_c = (\sum_{k=1}^{m} f_{kl}) / m f_1 e \tag{18}$$

(2) Antisymmetric Modes

Input point impedance of the collector volume can be obtained in a similar way as double cylinder cases. Let us look at the collector volume of the symmetric three cylinder compressor when n=1,4,5,.. as shown in Figure 5. The pressure in the volume is

$$P_1 = Q_1 + Q_2 f_{21} + Q_3 f_3 \tag{19}$$

Because $Q_2 = Q_1 e^{j2n\pi/3}$ and $Q_3 = Q_1 e^{j4n\pi/3}$ and $f_{21} = f_{31}$ from geometry, equation (19) becomes

$$P_1 = Q_1 (f_{11} - f_{21}) \tag{20}$$

One can notice that this is exactly the same equation as equation (12) of the double cylinder case. It can be shown that equation (12) has the same form for all antisymmetric modes of any multi-cylinder compressors with symmetric manifold. Therefore equation (13) and (14) can be used without modification for the calculation of pressure or flow before the collector volume.

APPLICATION TO THE SYSTEM ANALYSIS

1. Compressor Performance Simulation.

Gas pulsations in the compressor head cavity should be calculated to simulate compressor performance accurately because the pulsating pressure in the cavity is what the piston is working against, therefore high amplitude gas pulsations can cause performance losses, undesirable noise problems and sometimes even valve failures[2].

From the four pole equation, harmonic amplitude of the pressure in the cavity is,

$$P_i = (C + ZD) / (A + ZB) Q_i \tag{21}$$

where, a different set of four poles and impedance Z should be used for the symmetric and antisymmetric modes. For example, in the case of the double cylinder compressor in Figure 1, equation (11) and the system impedance Z_e should be used for the symmetric modes, while equation (14) and the impedance Z_1 in equation (13) should be used for antisymmetric modes.

Q_i is the amplitude of the nth harmonic input volume flow through the compressor valve and it can be obtained by the Fourier Transform of the time series of the valve flow calculated from the previous iteration of simulation[1]. The time domain pulsating pressure in the head cavity can be obtained by adding the average pressure in the cavity p_a to the inverse Fourier Transform of the pressure amplitudes P_i .

$$p(t) = p_a + \sum_{n=1}^{N} P_i e^{jn\omega t} \tag{22}$$

Therefore, gas pulsation analysis of the multi-cylinder compressor can be handled by virtually the same procedure as the single cylinder case once equivalent four poles are obtained. Actual transform and inverse transform can be implemented extremely fast if the FFT and Inverse FFT algorithms are used.

2. Muffler Design

If a muffler is to be designed after the collector volume, one needs to consider only symmetric modes because the antisymmetric modes do not contribute to the unsteady flow beyond the common collector volume. Suppose that we want to design an expansion type muffler to be attached to the system in Figure 1, therefore that we want to obtain transfer functions before and after the muffler is used. If the muffler exit is subjected to pressure release condition, impedance Z_e in equation (1) is zero. Then, the transfer function is obtained as,

$$TF(n\omega) = Q_e / Q_i = 1/A \tag{23}$$

where, A is the equivalent pole A of the overall system of the symmetric modes and n = m, 2m, 3m,.. and m is the number of cylinders. The transfer function of the system when the muffler is attached can be easily obtained from the new four pole obtained by multiplying one more four pole matrix representing the muffler to the system matrix without it. It also has to be noted that the transfer function defined by equation (23) is the ratio of one of the input flows to the output flow of the system.

Analysis to use a side branch type muffler can also be handled easily following to the procedure in reference [1]. Since the side branch muffler has a band filter characteristics, it will be particularly efficient for the noise control of the symmetric multi-cylinder compressors because unsteady flow harmonics are separated by much larger frequency intervals from each other compared with single cylinder compressors with the same motor speed. For example, if the compressor has four cylinders with 60 Hz motor, then the gas pulsation components beyond the head cavity would have frequencies of 240 Hz, 480 Hz, ..., instead of the harmonics at every 60 Hz of the single compressor. Therefore, it would be relatively easy to design a side branch muffler to filter out a few undesirable noise components without amplifying other frequency components.

SUMMARY

It was shown that how the concept of four pole parameters can be utilized for the analysis of multi-cylinder compressors with symmetric gas manifold. A concept of equivalent four poles was introduced to formulate the system four pole equation by considering the responses of the system to symmetric inputs and antisymmetric inputs separately. The method enables us to utilize the full advantage of the four pole parameters such as simple formulation procedure and efficient

computation in solving the acoustic equation of multi-cylinder compressors. Multicylinder compressors often have relatively large cavities and long pipes, therefore the method has another advantage because it is easier to handle continuous acoustic systems using four poles compared with other approaches.

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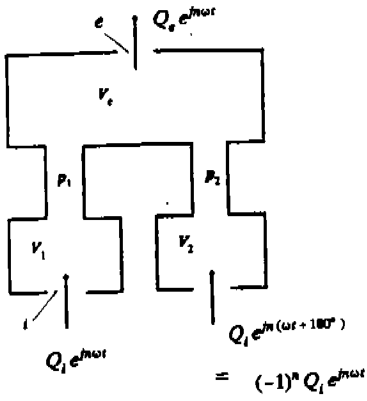
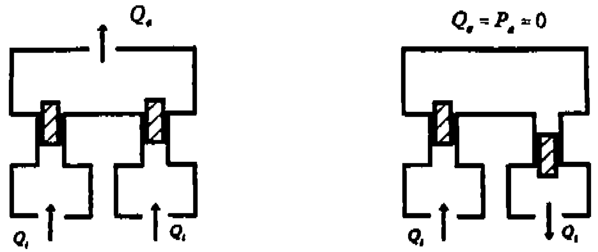
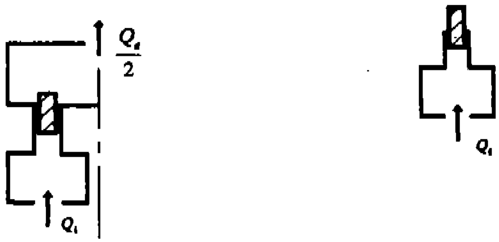


Figure 1 Double-cylinder compressor with symmetric gas manifold



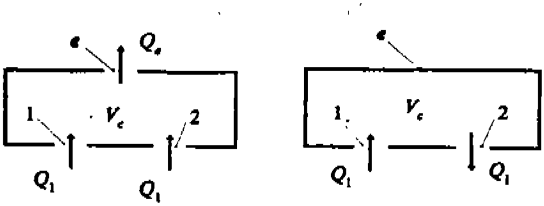
(a) Symmetric modes(n=2,4,6,..) (b) Anti-symmetric modes (n=1,3,5,..)



(c) Symmetric modes (d) Anti-symmetric modes

Equivalent systems of the lumped parameter system

Figure 2 Symmetric and anti-symmetric modes of the double-cylinder compressor



(a) Symmetric mode input (b) Anti-symmetric mode input

Figure 3 Collector volume

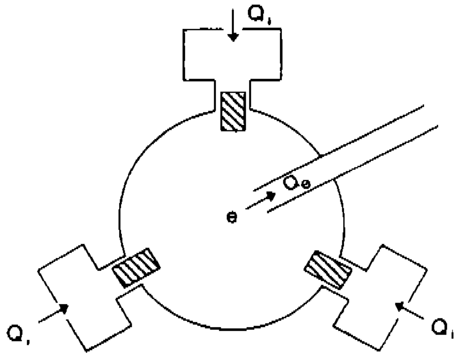


Figure 4 Symmetric mode of the three-cylinder compressor

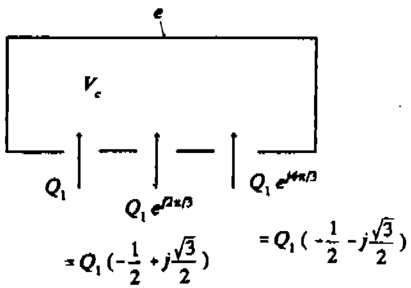


Figure 5 Collector volume with anti-symmetric inputs ; three-cylinder compressor