**DESIGN AND CONSTRUCTION
OF HYDRAULIC FLUME AND BACKWATER EFFECTS
OF SEMI-CIRCULAR CONSTRICTIONS
IN A SMOOTH RECTANGULAR CHANNEL**

JANUARY 1960

NO. 3

**Joint
Highway
Research
Project**

by

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**PURDUE UNIVERSITY
LAFAYETTE INDIANA**

Progress Report No. 2

TO: K. E. Woods, Director
Joint Highway Research Project

January 21, 1960

FROM: H. L. Michael, Assistant Director
Joint Highway Research Project

File: 9-8-2
Project: C-36-62B

Attached is a progress report entitled, "Design and Construction of Hydraulic Flume and Backwater Effects of Semi-Circular Constrictions in a Smooth Rectangular Channel". This report has been prepared by Mr. H. J. Owen, graduate research assistant on our staff, under the direction of J. W. Delleur. Mr. Owen also utilized the report as his thesis in partial fulfillment for the requirement of the M.S.C.E. degree.

The material reported in this report is a summary of the progress that has occurred on the Hydraulics of Arch Bridges Project which is being conducted in cooperation with the Indiana State Highway Department and the U. S. Bureau of Public Roads. Copies of this report will also be distributed to the State Highway Department and the Bureau of Public Roads for their review and comments.

The report is presented to the Board for the record.

Respectfully submitted,

H. L. Michael

H. L. Michael, Secretary

HLM:lmc

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Design and Construction of Hydraulic Flume
and
Backwater Effects of Semi-Circular Constrictions in
A Smooth Rectangular Channel

by

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Joint Highway Research Project
Project No. C-36-626
File No. 9-8-2

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January 21, 1960

ACKNOWLEDGMENTS

The author wishes to acknowledge and thank Dr. J. W. Delleur whose guidance and help was invaluable. Appreciation must also be expressed by the author to his wife Pat. Without her help, the preparation of the manuscript would have been difficult.

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ABSTRACT

H. James Owen, M.S.C.E. Purdue University, January 1960. "Design and Construction of Hydraulic Flume and Backwater Effects of Semi-Circular Constrictions in a Smooth Rectangular Channel". Major Professor: Dr. J. W. Delleur.

The purpose of this study was to investigate the hydraulics of semi-circular constrictions in smooth rectangular channels. To this end a large part of this work consisted of the design and construction of a hydraulic flume 64 feet long and 5 feet wide. This study is part of a general program sponsored by the State Highway Department of Indiana and the U.S. Bureau of Public Roads at Purdue University on the Hydraulics of River Flow Under Arch Bridges.

INTRODUCTION

This project was initiated in the Hydraulics laboratory of Purdue University by the Indiana State Highway Department in cooperation with the U.S. Bureau of Public Roads, to study the backwater and regain phenomena associated with arched constrictions such as are presented by arch bridges. The problem was approached by both theoretical analysis and model study. The preliminary analysis of the general problem was made by Mr. S. T. Husain⁽¹⁾. Mr. A. A. Sooky⁽²⁾ derived exact and approximate equations for the two dimensional sharp edged constrictions, and carried out a preliminary testing program in a small flume. The present study reports on the design and construction of the flume for the main testing program. The design was begun in July, 1958 and construction was started in March, 1959. The flume and its appurtenances were constructed and operative in August, 1959. Also included is the testing of two dimensional semi-circular constrictions carried out from September 1959 to November 1959.

1. Superscripts refer to bibliography at end of text.

REVIEW OF THE LITERATURE

Lane⁽³⁾, in 1915 and 1916, performed several experiments on contracted openings of various types. The full series of tests was not completed but data was collected and evaluated on the following:

1. Sharp edged vertical contractions
2. A rounded edge vertical contraction
3. A short flume with rounded entrance
4. A short flume with sharp corner entrance
5. An expanding flume

The paper describes the apparatus and procedure utilized as well as the results obtained. Lane found that the Weisbach formula ($Q = C_D b \sqrt{2g} \left[\frac{2}{3} H^{3/2} + y H^{1/2} \right]^*$ where H is the total drop of the water in passing through the contraction including the change in the velocity head and y is the average downstream depth.) was most nearly in agreement with the experimental results obtained in the case of the sharp edged contractions.

Kindsvater and Carter⁽⁴⁾ attempted to correlate laboratory results with field data obtained by the USGS. A general equation was developed for discharge through a contracted opening: $Q = C_D b y \sqrt{2g(\Delta h - h_f + \alpha \frac{V^2}{2g})}$ where y

* See list of symbols in Appendix A

is the downstream depth. This expression contained a discharge coefficient which was determined for several cases. The base curves of the discharge coefficient in terms of the contraction ratios and length of the constriction were presented for a standard condition of square abutment and Froude number of 0.5. The base curves are supplemented by auxiliary curves modifying the coefficient due to:

1. Froude number different from standard
2. Upstream corner rounding
3. Eccentricity
4. Skew
5. Channel contraction
6. Chamfers of the abutments
7. Side depths
8. Abutment side slopes
9. Submergence
10. Bridge piers and piles

Tracy and Carter⁽⁵⁾ made a laboratory study in 1954 of the flow through contractions. The data was used to develop a set of base curves relating the backwater ratio ($h_1^* / \Delta h$) and the contraction ratio M for different values of roughness. These base curves were obtained using vertical-faced contractions with square edged abutments, and a Froude number of 0.5. Auxiliary graphs were presented to modify the result according to the geometry of the constriction if other than the basic geometry.

vallentine⁽⁶⁾ used vertical sharp edged constriction plates placed normal to the flow. The discharge through the

constriction was related to the upstream depth by an equation containing an experimental discharge coefficient: $Q = C_D b \sqrt{2g} y_1^{3/2}$. The variation of this discharge coefficient with the Froude number of the unconfined flow was given for several values of the contraction ratio.

Nagler⁽⁷⁾ did considerable experimental work with three dimensional models of bridge piers. Although he worked at an extremely small scale, several curves are presented relating the backwater to the geometry of the pier.

Husain⁽¹⁾ carried out two and three dimensional tests of arched openings on a small scale in preparation for a larger study of which the work reported here is a part. The problem was also approached by dimensional analysis. General profiles of the backwater curves were obtained and recommendations were made concerning the design of the future equipment.

Sooky⁽²⁾ carried on the small scale testing program begun by Husain and included two and three dimensional models with and without channel roughness. For the two dimensional case, equations relating the discharge to the geometry of the opening and the backwater height were developed. Curves were obtained from experimental data which relate the coefficient of discharge and the backwater height to the Froude number of the unconfined flow and the contraction ratio. These equations are presented in Appendix B.

DESIGN AND CONSTRUCTION OF THE APPARATUS

Testing Flume

Preliminary Computations

Before the design of the flume itself could proceed, it was necessary to determine whether the backwater and regain phenomena could be represented to a convenient and easily measurable scale in the space available in the hydraulics laboratory.

Several sets of arch bridge plans provided by the Indiana State Highway Department were analyzed for the values of backwater. The theory of varied flow and the equations and tables presented by Bakhmeteff⁽⁸⁾ and by the U.S. Bureau of Public Roads⁽⁹⁾ were used. It was only possible to make an approximate calculation of the backwater because of the unknown effect of an arch-type constriction. A sample calculation is presented for bridge S79 (Clay County, Indiana).

Assumption

- 1) Velocity through the bridge constriction
= 6 feet/second. This value was suggested by Mr. J. I. Perry, Chief Engineer, Indiana State Flood Control and Water Resources Commission, as an average flood flow velocity through typical bridges in Indiana.

From bridge plan

Waterway area through the bridge utilized by the
stream = 125 feet²

Slope of the stream bed = 0.0019 feet/foot

Flow = 750 cfs

The total waterway area under the arched constriction of the bridge was approximated by a trapezoid. (See figure 1 appendix C) The unconstricted area of the waterway upstream of the bridge was found to be 300 feet².

In order to find the percentage of constriction of the stream it was necessary to proportion the flow within the subsections of the original area.

The tabulation for the proportionment is shown below.

Sub Section	n	1.486/n	A feet ²	p feet	R feet
1	.05	29.8	45	12	3.75
2	.05	29.8	210	30	7
3	.05	29.8	<u>45</u>	12	3.75

$$A = 300$$

Sub Section	$R^{2/3}$	$AR^{2/3}$	K cfs	Q cfs
1	2.41	108.5	3,240	86
2	3.65	776.0	22,800	578
3	2.41	108.5	<u>3,240</u>	<u>86</u>

$$K = 28,280 \quad Q = 750$$

The proportionment was made on the basis of conveyance. The interface of sub sections was considered as a boundary.

The total waterway area through the bridge was 143.5 feet² located entirely in sub section 2. The loss in capacity based on relative conveyance was computed as follows.

$$\text{Area deducted section 2} = 66.5 \text{ feet}^2$$

(see figure 1 appendix C)

$$\text{Area total section 2} = 210.5 \text{ feet}^2$$

$$\text{Flow deducted section 2} =$$

$$(66.5/210) \times 578 = 182 \text{ cfs}$$

$$\text{Flow deducted section 1} = 86 \text{ cfs}$$

$$\text{Flow deducted section 3} = \underline{86} \text{ cfs}$$

$$\text{Total flow deducted} = 354 \text{ cfs}$$

The loss in capacity represented by this constriction is 354 cfs. The contraction ratio M then is $(354/750) \times 100$ or 47%.

The following computations are based on data taken from the U.S. Bureau of Public Roads report.⁽⁹⁾

Assuming the geometry of a 30° wing wall normal crossing, the backwater coefficient $K_b = 1.075$ for a contraction ratio M = 47%

$$v^2/2g = 36/64.4 = .56 \text{ feet}$$

$h_1^* = 1.075 (.56) = 0.602 \text{ feet}$. This is the maximum backwater superelevation.

Length of Backwater Profile. The backwater profile was computed using Bakhmeteff's⁽⁸⁾ function $\phi(\eta)$. The computations made on this basis neglect the regain of kinetic energy in an expanding stream. In order to use the tables of $\phi(\eta)$, the hydraulic exponent n characteristic of the channel upstream of the bridge was computed by use of the

equation $n = 2 \cot \alpha$. (α is shown in figure 2 appendix C)

$$y = 3 \text{ feet}$$

$$A = 90 + 18 = 108 \text{ feet}^2$$

$$p = 30 + 12 = 42 \text{ feet}^2$$

$$R = 108/42 = 2.57 \text{ feet}$$

$$C = 1.486/.05 (2.57)^{1/6} = 34.8$$

$$K = 108 \times 34.8 \times 1.6 = 6000$$

$$\text{Log } K = 3.78$$

$$\text{Log } y = 0.478$$

$$y_1 = 7 \text{ feet}$$

$$A_1 = 300 \text{ feet}^2$$

$$p_1 = 44 \text{ feet}$$

$$R_1 = 300/44 = 6.82 \text{ feet}$$

$$C_1 = 1.486/.05 \times (6.82)^{1/6} = 41.0$$

$$K_1 = 300 \times 41 \times 2.61 = 32,100$$

$$\text{Log } K_1 = 4.506$$

$$\text{Log } y_1 = 0.846$$

$$n = 2(4.506 - 3.78/0.486 - .478) = 2(.726/.368) = 3.95$$

y	η	ϕ	$\Delta\phi$	l	L
feet				feet	feet
7.602	1.086	0.660			
7.500	1.070	0.599	.061	628	628
7.400	1.058	0.544	.055	565	1193
7.300	1.041	0.446	.098	1010	2203
7.200	1.030	0.362	.084	865	3068
7.100	1.013	0.137	.255	2320	5388
7.050	1.008	0.004	.133	1370	6758
7.020	1.003	0.264	.278	2860	9618

Original normal depth = 7.00 feet

Backwater = 0.60 feet

Depth at maximum

backwater = 7.60 feet

Superelevation

decrease = 0.58 feet

This reach then includes $0.58/0.60 \times 100$ or 96% of the superelevation.

Complete information is lacking on the length of the regain curve.

It may be assumed that the angle of expansion of the stream downstream of the contraction may be approximated by the divergence angle of a submerged jet, and that the regain curve will be complete when the expansion has reached the full width of the flume. Albertson⁽¹⁰⁾ found a divergence angle on each side of the centerline of a free jet of 11 to 14 degrees. Henry⁽¹¹⁾ found the free boundary to diverge at approximately 7 degrees for the flow from a submerged sluice gate. An angle of 5° is estimated with this particular bridge in order to obtain an approximation by excess of the regain curve as follows. (See figure 3, appendix C).

Clear span of bridge at spring line = 30 feet.

Surface width of arch at maximum high water = 11 feet.

Average width of opening = $(30 + 11)/2 = 20.5$ feet.

Stream width = 46 feet

The regain to normal depth should therefore be complete in a length of 146 feet measured from the downstream side of the constriction. This approximate computation indicates that the total length of the backwater curve reach (within 0.02 feet of the normal depth) and the estimated regain curve plus the bridge is $9618 + 146 + 48 = 9812$ feet. It is now desired to find the required length of flume to represent to scale the totality of the regain curve and a

reasonable portion of the backwater curve.

The water flow available in the hydraulic laboratory was 2100 Gpm.

$$2100 \text{ Gpm} = 2100/449 = 4.7 \text{ cfs.}$$

Since $Q_m/Q_p = L_r^{5/2}$ the required scale ratio is found to be $4.7/750 = L_r^{5/2} = 0.00627$.

$$L_r = (.00627)^{2/5} = .135$$

$$\text{or } L_r = 1:7.4$$

The nearest convenient scale is 1:10. For this ratio, the superelevation = .06 feet = .72 inches. The usable length of the flume in the proposed space would equal 60 feet. For this ratio, 600 feet of the prototype stream could be represented. This would include the totality of the regain curve, the bridge model and some 400 feet of the prototype backwater curve.

From this and similar computations as well as small scale tests,⁽¹⁾ it appeared that a flume utilizing all of the easily available space in the laboratory that is a flume length of 64 feet but capable of extension would be satisfactory. The width of the flume was fixed at 5 feet. This was based on a consideration of the scale ratios and the space available.

The cross section of the flume was to be rectangular since this configuration lent itself well to both ease of construction and adaption.

Design Procedure

In order to test the flows under varying slope conditions, the flume was to be tiltable about one end.

Several methods are available to achieve slope control including:

1. Hydraulic Jacks
2. Screw Jacks
3. wedges
4. Cables and Winches

Screw Jacks were selected because of accuracy and ease of control, as well as permanence and appearance.

At the time the preliminary design was made, only an estimate could be made of the final weight of the flume and the water contained therein.

In order to keep the deflections due to the variable weight of water within the same order of magnitude of the smallest readings of the point gage for depth measurement, 0.1 mm, the flume bottom was designed of 1/4 inch steel plate supported at 2 foot intervals on channels. The channels in turn were to be supported by two or more main beams riding on the jacks. The side plates were designed of 1/4 inch steel plate supported by vertical angles resting on the channel members. A longitudinal horizontal angle mounted on the vertical angles served as a support for the guide rails. The guide rails, which serve as a reference plane from which measurements are based, were to be polished stainless steel to minimize corrosion and scale.

The ~~preliminary~~ design was based on a possible water depth of 2. feet. In the immediately proposed tests this will allow a freeboard of approximately 1 foot. However, deflection will be within the set limits for a loading of

2 feet of water which may occur at a later date.

A portion of the first design is presented.

Dead Load

1/4 inch plate

2 side plates 64 feet long x 2 feet wide

@ 20.4 lb/foot = 2620 lb.

1 bottom plate 64 feet long x 5 feet wide

@ 51.0 lb/foot = 3260 lb.

5880 lb.

Main Beam (18 I 54.7)

2 x 64 feet x 55 lb/feet = 7040 lbs.

Channels

33 x 5 feet x 8.2 lb/feet = 1360 lbs.

14,280 lbs.

Extras @ 30% 4,300 lbs.

total 18,580 lbs.

For design purposes this is an average load of

18,580 lbs/64 feet = 290 lbs/foot.

Live Load. At the maximum depth of 2 feet the volume of water contained is 10 feet³ per foot of length. This imposes a load of 10 feet³ x 62.4 lbs/foot³ = 624 lbs/foot. The total load per foot then is 290 pounds/foot + 624 lbs/foot = 914 lbs/foot.

The distance between supports was set as 20 feet. Since the exact nature of the main beam connections was unknown, the solution was made based on a simply supported condition.

Two alternatives were presented. The first was to use beams whose stiffness would make any deflections negligible

and the second was to use lighter beams and correct for the deflections by adjusting screws. The first alternative was chosen as the second would necessitate adjustment after each change in conditions such as slope, or water depth.

The beam first selected was an 18 I 54.7 which gave a calculated deflection of 0.00225 feet under the design loading. Contacts with the fabricator and erector were made at a later date and it was found that a 20 I 65.4 would be available at a cost less than that of the lighter beam. The use of the heavier beam was accepted and the design proceeded based on this beam. The deflection due to the variable water weight was approximately 0.002 feet for a depth of one foot.

It was felt that some form of transverse leveling was necessary. Adjustment bolts were an obvious solution but the location was yet to be selected. The first sketches incorporated adjustment bolts between the channel and the bottom plate. This produced an indeterminate situation with regard to flexure, inconvenient locations for adjustment, and high fabrication cost. The subsequent designs placed the adjustment bolts between the channels and the main beams which gave the bottom plate full support across its width at 2 foot intervals.

The bottom plate was designed slightly wider than the flume width. This permitted attaching an angle to hold the bottom edge of the vertical plate fixed. The upper edge of the vertical plate had nuts welded on at the two foot points. Studs were attached between the nuts and the

vertical angles to support the plate and provide an adjustment for its longitudinal alignment. The inside of the flume was finished with an epoxy resin applied with a hand roller. The flume construction is shown in figures 4 and 5. (Appendix C)

In operations of this kind, guide rails are generally fixed in the level position. They then may be used to control the slope. Since the rails were attached to the flume, direct slope measurements was not possible. Instead, differences between the surface of a standing body of water and the flume floor were used to measure the slope and calibrate a revolution indicator which served as the primary method of slope control.

Tailgate

Control over the depth was exercised by a gate mounted at the end of the flume. The gate was manually operated from the side of the flume. Figure 6 (appendix C) shows the gate. The gate was made in such a way that it could be used either as a sluice or as a weir. Throughout this first series of tests, the gate was used exclusively as a weir.

Forebay

The forebay, 8 feet wide and 10 feet long, was constructed of plywood and lined with sheet metal, and is shown in figure 10 of appendix C. The 3 inch and 6 inch pipes entered the rear of the forebay at the top. The 6 inch

line was centered and the 3 inch line was placed slightly off center. The diffusing mechanism for each supply line consisted of a tee and cross pipe of the same size as the line at the bottom of the forebay. The turbulence of the entering water was controlled by a 4 inch gravel baffle and three wire mesh screens of 13 mesh per inch. The transition section continuing into the bottom and side walls of the flume was made of quarter ellipses in the horizontal and vertical planes respectively with a ratio of major to minor axes of 1.5 to 1.0. The joint between the flume and the forebay was sealed with a flexible rubber gasket mounted so as not to interfere with the flow.

Slope Control Mechanism

The flume rests upon six screw jacks and a hinge. The hinge is located at the joint of the flume and the forebay. The jacks are similar in all respects with the exception of the gear ratio. The jacks are divided into three pairs with rates of raise of one, two, and three inches for 96 turns of the shaft. Since the hinge was a fixed point and the opposite end of the flume was the point of maximum movement, the jacks were arranged such that the pair nearest the hinge moved the least and the pair at the opposite end of the flume had largest displacement per revolution. This maintained the bottom of the flume as a plane while it was being raised and lowered.

The jacks on each side of the flume were driven by a common 1 inch shaft line connected at one end to a 90° miter gear. The miter gears on either side in turn were connected

to a single 60:1 ratio gear reducer. The power to operate all the jacks was supplied by a 1-1/2 horsepower, 1750 revolutions per minute, reversible, electric motor connected directly to the gear reducer. This provides a rate of vertical displacement at the downstream jack station of approximately 1 inch per minute.

The jacks were arranged in such a way that the downstream end of the flume may move from 12 inches below horizontal to 3 inches above horizontal, resulting in a maximum positive slope of 1/60 and a maximum adverse slope of 1/240. The motor was controlled by a raise, lower, and stop control switch. Safety switches were located both near the motor and near the control switch. It was necessary to unlock these before the control circuit could be completed. In addition, automatic limit switches were provided to prevent running the jacks beyond their limits. The general arrangement of jacks and gears is shown in figure 7.

In order to connect the ends of the jacks (which move in a vertical line) to the flume (which moves in an arc), it was necessary to use a pinned linkage. A photograph of the linkage is shown in figure 8. (Appendix C)

Due to the arc of the linkage, the relation between the rise of the flume and the revolutions turned by the jack shafts was not linear. Therefore, it was necessary to make a calibration of the slope rather than computing it.

At the time of erection, the jacks were leveled at .001 foot before the flume was erected. During the alignment procedure the jacks were raised or lowered individually as needed

to obtain a level base. The shaft couplings were then installed and no further individual movements were made.

Construction

Considerable time was spent in obtaining the requisitions, bids from several contractors and actual supervision of the erection of the flume. The foundations, jack piers, plumbing, and electrical controls were installed by Purdue University Physical Plant. The structural parts of the flume were built and assembled by a contractor, the flume adjustments, construction of the instrument carriage, installation of manometers and calibrations were done by the Research Assistant and student labor when needed.

PREPARATION FOR TESTING

Slope Calibration

After the flume was aligned and leveled a slope calibration was made by visually counting the revolutions of the slowest speed shaft and measuring the depth of a still pool of water at two points 50 feet apart. A steel tape was installed with station 0 at the upstream end and station 64 at the downstream end of the flume. The points chosen for slope measurement were stations 6 and 56. These points had previously given consistently good results when measurements were made of the distance between the rails and the flume floor. The calibration of slope versus revolutions appears in figure 9. (Appendix C) The apparent scatter of the points toward the downstream end of the flume is due to the magnification resulting from the logarithmic scale at that end.

In order to avoid the necessity of visually counting shaft revolutions to keep track of the slope, a revolution indicator was made and installed at jack station number 3. The lowering of the flume activated the pointer which both multiplied and reversed the motion. The tip of the pointer rode on a lucite strip mounted below the motor controls. A mark was scribed on the lucite strip at each revolution over the range 0 through 40 as well as at the test slopes. On the end of the shaft a circular lucite plate divided into ten

parts was mounted along with a fixed pointer. The slope desired was set by using the large indicator to the nearest revolution and setting the tenths of a revolution by using the small dial. This equipment was later replaced by a commercial revolution counter mounted at the same location. This counter read directly to a tenth of a revolution and the reading could be interpolated to one half of that. This device provides a slope control with an accuracy of ± 0.0000025 feet/feet. Figure 8 (Appendix C shows the counter)

Water Supply and Measurement System

The water available in the laboratory is recirculated through the system by two pumps rated at 300 Gpm and 2000 Gpm. The 3 inch discharge line from the 300 Gpm pump was connected to a new 3 inch line. This line contained a new 3 x 2.25 inch venturi accurate to 0.5% over the range from 30 Gpm to 300 Gpm. The line was installed using long sweep elbows to reduce head loss. A 60 inch differential manometer reading to 0.01 inch was connected to the venturi and filled with tetrabromoethane (Specific gravity 2.95) which gave a manometer deflection of 51 inches with a flow of 336 Gpm.

The 2000 Gpm pump was connected to an existing 6 inch line which was improved by the installation of long sweep elbows in place of tee's and short elbows. In addition, a new 6 x 4.176 inch venturi accurate to 0.5% over the range 200 Gpm to 2000 Gpm was installed preceded by a set of straightening vanes. A 30 inch differential manometer reading

to 0.01 inch was connected to the venturi and filled with mercury (Specific gravity 13.6). This manometer gave a deflection of 14.9 inches for a maximum flow of 1790 Gpm. In both cases the venturicis were fitted with air vents to insure proper measurements. After a portion of the tests had been made and the data evaluated, it was deemed necessary to improve the discharge measurements. The 60 inch manometer was attached to the 6 inch venturi and recalibrated using tetrabromoethane as the manometer fluid. This resulted in a larger manometric deflection improving the accuracy of the measurement. The 30 inch manometer was connected to the 3 inch venturi but there was no need to recalibrate the meter.

In order that the calibration of the venturi meters should have no error larger than that of the venturi meter, the scale to be used for the calibration was checked against standard weights by the Indiana State Board of Health, Division of Weights and Measures. The scale error was less than 0.2% or 2 pounds per 1000 pounds. For the purpose of calibrating of the venturi meters, branch lines led to a baffled concrete channel located above the weighing tank.

At the point immediately before the 3 inch and 6 inch lines entered the forebay, valves and valve bypasses were installed. The 6 inch line had a 2 inch bypass and valve and the 3 inch line was fitted with a 1 inch bypass and valve. The manometers were mounted in a position easily visible to the person adjusting the valves. The overall apparatus arrangement in the laboratory is shown in figure 10. (Appendix C)

Venturi Calibration

As soon as the essential piping was completed, calibration of the venturi meters was begun. The procedure was as follows. The flow was set using a valve located downstream of the venturi and a waiting period of approximately 5 minutes was allowed for the system to come to equilibrium under the new flow. The scales were preset to an arbitrary weight and the weighing tank valve closed. The manometer deflection was noted and the water diverted into the weighing tank. The scales were tripped and the timer manually started when the weight of water collected equalled the weight which had been preset on the scale beam. The scale weight was noted. This method of calibration avoids the errors of non instantaneous starting and stopping of flow but still does not correct or make allowances for the difference in the impact force of the water entering an empty tank as compared to a partially full tank. This error is of the magnitude of 1% which is compatible with accuracy of the remainder of the system. The intervals of calibration were selected so as to fall between one and two inches of deflection on the 60 inch manometer containing the lighter fluid and not to exceed 1 inch on the 30 inch manometer containing the mercury. The calibration curves are presented in figures 11, 12 and 13. (Appendix J)

MODEL TESTING

Model Construction

From results of the preliminary experiments,² (figure 14 Appendix C) it was found that the predominant variable was the contraction ratio and that the length of the model had little or no influence for Froude numbers less than 0.5. This range of Froude numbers corresponds to the case usually found in practice. It was therefore decided that the first series of tests would concentrate on two dimensional sharp edge semi-circular models with no skew.

The cost of machining mild steel plates to produce the constrictions was prohibitive. An estimate was obtained of \$125.00 per model. The frequency of handling and changing the constriction plates also necessitated a model that was light weight and still capable of sustaining hard use without damage. The models, as finally made, consisted of back up sheets of 1/2 inch exterior grade plywood faced with a sheet of 22 gauge galvanized iron and braced by a steel angle. The openings were cut out of the galvanized iron sheet accurate to 1/32 inch. The opening in the plywood backing had a radius of 1/2 inch greater than that used in the metal. The metal was bolted tightly against the wood. In the flume, the model was positioned with the metal face upstream. The construction of the models is shown in figure 15. (Appendix C)

Four models were made with contraction ratios of 0.3, 0.5, 0.7, and 0.9. At the slopes and flows tested, the model with $m = 0.3$ was submerged on a majority of the tests. The data presented in Appendix C is that collected for the other three models which are shown in figure 5. (Appendix C)

Free Surface Measurement

The position of the free surface was measured with an electric indicating point gage reading to the nearest 0.1 mm. The point gage was mounted on an aluminum and brass bar in such a way that the gage could traverse the width of the flume. This bar in turn was part of a carriage which rolled on the stainless steel rails along the flume. The carriage was rectangular and rolled on 4 wheels, two of which on one side of the carriage were grooved to provide alignment. The power supply was mounted at the back of the carriage. The operator rode on a second carriage which was on its own rails. Details concerning the instrument carriage can be seen in figure 16. (Appendix C)

The head of the point gage was comprised of two probes approximately $1/4$ inch in diameter. The probes were separated a distance of about $1-1/2$ inches. The rear electrode was 15 millimeters longer than the front, and served as the ground. The end of the front electrode was pointed and was adjusted to the water surface. In operation, the obstruction to the flow presented by the rear electrode caused a rise in the water surface of $v^2/2g$ against the electrode. The effect of this disturbance extended upstream to the measuring probe

and made it impossible to measure the true water surface. This effect was not a constant since the velocity head was different in each case.

In order to improve the performance of the instrument, the rear electrode was replaced by a small diameter copper wire. This wire presented a much smaller obstruction to the flow and the operation of the gage was not only simplified, but gave data that should be superior to that obtained with the former arrangement.

Uniform Flow Calibration

Preliminary tests were run to determine uniform flow conditions in the flume. The variables involved are: the discharge, the slope and the tailgate settings. For each flow and slope the tailgate setting was determined by trial and error until uniform depth was obtained along the largest possible reach of the flume. The models were located in such a way that the regain curve was complete within the uniform depth section. Effects of boundary layer growth were not considered, and the flow was considered uniform as long as the depth remained constant.

As the model tests progressed the uniform flow conditions were recorded in the auxiliary graphs of figures 20 and 21. (Appendix C)

The first was the normal depth versus the slope and the second was the height of the tailgate versus the slope. In both cases, the rate of flow was the parameter. In order to obtain data that would provide a complete coverage of the

range of Froude numbers investigated, the Froude number was first chosen and the normal depth computed in the following manner.

$$F_o = \frac{V}{\sqrt{gy_o}} = \frac{Q}{y_o B \sqrt{gy_o}}$$

or

$$y_o^3 = \frac{Q^2}{F_o^2 B \sqrt{g}}$$

from which

$$y_o = \left(\frac{Q^2}{F_o^2 B \sqrt{g}} \right)^{1/3}$$

After the desired normal depth was computed, the slope for a particular flow which would give this value could be determined from figure 20 (Appendix C). Then, entering figure 21 (Appendix C) with the slope and flow, the necessary gate setting to produce uniform flow could be determined.

Testing

After the slope and uniform flow calibrations were completed the testing was begun. The procedure used was to set the flow, which was the most difficult quantity to adjust, and let it remain constant while the slope, tail gate setting and models were changed. For each case of slope and flow, the tail gate height was set according to the previous uniform flow calibrations. In the majority of cases the conditions of uniform flow was obtained on the first trial and in every case no more than two trial gate settings were needed. Once the system had come to equilibrium and the

normal depth was obtained, a model was installed in the flume. For the first few runs, the complete profile was taken. It was observed that in the latter portion of the regain curve, the water surface fluctuated vertically as much as one centimeter from one minute to the next. This fluctuation did not occur rapidly but rather slowly. Testing was immediately suspended while the cause for this was determined. The first possible cause investigated was that of a variable inflow into the flume. However, observation of the backwater showed it to be very stable. This indicated that the flow into the flume was constant and the phenomena was due to the model or that portion of the flume beyond the model. Eddies were suspected as a possible cause of this phenomenon. In an effort to eliminate eddies, the gap between the metal facing and the plywood back up board was filled and beveled to produce an angle of 45° . This resulted in no appreciable change. The condition most suspect however, was channelization of the flow to one side of the flume with eddies on the opposite side. It seemed that an instability was developed by the constriction. The reason for this channelization of the flow is not known. Misalignment of the bottom was not a factor because it was level within 1 millimeter throughout the length of the flume. The tail gate was leveled to an accuracy of 1 millimeter and the models were installed, using a square to check the alignment both horizontally and vertically. It was discovered that in a given case with most of the flow on

one side, after artificially deflecting the flow to the other side, it would remain on that side. This suggested that the lack of symmetry was not a factor. One possible solution is the addition of roughness and increasing of slope to stabilize the flow. However, since these tests were to be specifically smooth boundary tests, this was not done and the affected portion of the regain section was neglected. For the remainder of the tests, only a short profile before and after the model was taken to locate the points of maximum and minimum depth. Figures 17, 18, and 19 (Appendix C) are photographs of the flume with model in place. Figure 18 shows the flow going to one side and figure 19 shows the flow centered.

Test Results

The test data and the calculated values of the discharge coefficient C_D , of the friction coefficient f , and of the Reynolds number are presented in table I. The ratio of the backwater depth to the normal depth y_1/y_0 is plotted versus the ratio of the velocity head to normal depth with the contraction ratio m as a parameter in figure 22. The ratio of the velocity head to the normal depth is a measure of the kineticity of the flow, it is exactly half of the kineticity as defined by Bakhmeteff⁽³⁾, or half the square of the Froude number. The discharge coefficient C_D calculated from equation 2 of Appendix B is plotted versus the ratio of the velocity head to the normal depth with the contraction ratio m as a parameter in figure 23.

The consistency of the data is well illustrated by the lack of scatter of the experimental points as plotted in figures 22 and 23. (Appendix C) These test results may be compared with the small scale tests for the contraction ratio of 0.5 which is common to both test series. Inspection of figure 14 (Appendix C) and figures 22 and 23 (Appendix C) show that the values are almost identical. For example, at a Froude number of 0.2, the value of the discharge coefficient C_D from the small scale tests (figure 14 Appendix C) is 0.38 and the superelevation ratio y_1/y_0 is 1.1. Compared to this, the large scale tests, (Figure 22 and 23 Appendix C) give C_D as 0.275 and y_1/y_0 as 1.119 for a kinematicity of 0.02 which corresponds to the Froude number of 0.2. At a Froude number of 0.4 the results of the small scale tests indicated C_D was equal to 0.53 and y_1/y_0 was 1.4. Similarly, at a corresponding kinematicity of 0.08, the large scale tests showed C_D to be 0.483 and y_1/y_0 to be 1.432. The reliability of the data may be better discussed in terms of the values of the friction coefficient and of the backwater ratio.

The Darcy-Weisbach friction factor and the Reynolds number for the uniform flow established before each test were calculated in table I as follows:

$$R = \frac{VR}{\nu}$$

$$f = \frac{89RS}{V^2}$$

The experimental friction factors were compared to the theoretical values obtained by adapting the Blasius and Prandtl formulas for flow in smooth pipes to the rectangular open channel.

The formulas for smooth pipe flow are:

$$\text{Blasius} \quad f = 0.3164 \left(\frac{VD}{\nu} \right)^{-1/4} \quad R < 10^5$$

$$\text{Prandtl-Von Karman} \quad \frac{1}{\sqrt{f}} = 2.0 \log \left(\frac{VD}{\nu} \sqrt{f} \right) - 0.8$$

Replacing D by 4R and simplifying, the equations become:

$$\text{Blasius} \quad f = 0.223 \left(\frac{VR}{\nu} \right)^{1/4}$$

$$\text{Prandtl-Von Karman} \quad \frac{1}{\sqrt{f}} = 2.0 \log \left(\frac{VR}{\nu} \sqrt{f} \right) + 0.40$$

Powell and Posey⁽¹²⁾, working with a triangular flume

found the formula governing their friction factor to be

$$\frac{1}{\sqrt{f}} = 2.074 \log (R \sqrt{f}) - 0.797 \quad \text{for}$$

tranquil flow in a smooth channel. The comparison between experimental values and the theoretical formulas is shown in figures 24 and 25. (Appendix C) In figure 24 the experimental values of the Darcy-Weisbach coefficient f are plotted versus the Reynolds number. Both f and R are as defined above based on the hydraulic radius. Also plotted on the same figure are the Blasius and Von-Karman relationship as well as the general range of experimental values obtained by Lansford and Robertson⁽¹³⁾ for smooth triangular channels.

In figure 25 (Appendix C) the friction coefficient f and the Reynolds number are based on the normal depth, assuming two dimensional flow. Although this assumption is not completely true for the depth to width ratio used in

the experiments this was done to compare the data with the experiments of Owen⁽¹⁴⁾ which were done in a glass channel.

In general, as shown in figure 24, (Appendix C) the data fall above the theoretical lines which are a lower limit for a perfectly smooth boundary. The average friction coefficient is $f = 0.021$ which corresponds to an absolute roughness of approximately $\epsilon = f (f) = 0.0025$ feet. This corresponds to the irregularities of the epoxy resin of the channel finish.

The percentage of probable error of the friction coefficient is calculated as follows. The calculations are made based on a flow of 3 cfs and a slope of 0.000100 foot/feet. The measurement of the flow during calibration had a possible error of 1% due to neglecting the impact in the weighing tank with different water depths, and the scales had a possible error of 0.2%. The possible error in the venturimeter was 0.5% and the observed manometer readings could have been in error by 0.02 inches. With the given flow, the manometer deflection was 52 inches of tetrabromoethane. This means that the possible error in reading the deflection value was approximately $0.02/54$ or 3.85%. Since the flow varies with the 0.5 power of the deflection, this would represent an error in the flow of 1.93%. Therefore, the total possible error in the flow is 3.63%. The slope was calibrated and set to less than one tenth revolution of the connecting shafts. A maximum error of one twentieth of a revolution is equal to 0.0000025 feet/feet. At the slope

chosen, this would produce an error of $0.0000025/.0001$ or 2.5%. The measurement of y_0 was made to 0.1 millimeter. An error of this magnitude with the value of y_0 found for this condition (21.70 cm) amounts to $0.01 \text{ cm}/21.70 \text{ cm} = 0.046\%$. The error in computing the wetted perimeter could be $2(0.01 + 0.5) \text{ cm}/192 \text{ cm} = 0.053\%$. Since $f = 8gRS/v^2$ or $8gSy_0^3/q^2p$, the error in f can be expressed as

$$\sqrt{2.5^2 + (3 \times 0.046)^2 + (2 \times 3.63)^2 + (0.053)^2} = \sqrt{58.97} = 7.68.$$

At flatter slopes or lower flows, this error would be even larger than calculated. Conversely, those tests at steeper slopes and higher flows should give the most nearly correct values of f . However, the points would not be expected to fall on the theoretical line since the materials used in the construction of the flume and the finish applied certainly caused some finite value of roughness.

A second check was made by determining the backwater ratio $h_1^*/\Delta h$ for each model test and comparing the values obtained to those presented by Tracy and Carter⁽⁶⁾ for rectangular constrictions. This comparison is shown in figure 26 (Appendix C) which shows that the backwater ratios are of the same order of magnitude although different as may be expected with different boundary geometries.

SUMMARY OF RESULTS AND CONCLUSIONS

Flume Design

The flume, as designed and built has proved adequate to carry out the proposed experiments. The entire apparatus necessary to carry out the testing program can be run conveniently by two men. The slope controls, including the jacks and motor, function well and changing slope takes less than 5 minutes.

Testing

Only the first series of tests, sharp edged, semi-circular constrictions, have been completed on a large scale so comparisons cannot be drawn as such. However, several things indicate the validity of the data.

1. Close agreement with the small scale tests.
2. Comparison with other investigators on:
 - a. Friction factor.
 - b. Mannings roughness
3. Adherence of the plotted data to a well defined pattern with little experimental scatter.

The conclusions possible so far are primarily drawn from figure 23 which relates the discharge coefficient and the kineticity of the flow, and figure 22 which shows the backwater in terms of kineticity. From the first, it can

be seen that above a value that corresponds with a Froude number of 0.5, the C_D value ceases to depend on the kineticity of the flow. The second, figure 22 shows the dependance of the backwater value on both the contraction ratio and the kineticity.

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APPENDIX A
NOTATIONS

NOTATIONS

<u>SYMBOL</u>	<u>UNIT</u>	<u>DEFINITION</u>
A	feet ²	Area of flow.
B	feet	Stream width at bridge site or flume width.
b	feet	Width of constriction opening equal to di- ameter of semi-circle.
C _D		Coefficient of discharge equation.
D	feet	Pipe diameter.
E		An infinite series of powers of y_o/r .
F		Froude number of flow. Subscript o refers to unconstricted stream.
f		Darcy-Weisbach friction factor.
g	feet/sec ²	Acceleration of gravity.
h ₁ [*]	feet	Superelevation of back- water above normal depth.
h _f	feet	Boundary friction loss.
Δ h	feet or centimeters	Elevation difference be- tween points of maximum

		and minimum depth.
K_b		Backwater head loss coefficient.
K	cfs	Conveyance of a channel or section of channel $= AC \sqrt{R}$.
l	feet	Length of reach in backwater computations.
L	feet	Accumulated length in backwater computation.
L_r		Scale of length.
M		Channel contraction ratio equal to $l-b/B$ (in review of literature).
m		Channel contraction ratio equal to b/B .
n	feet	Manning roughness or hydraulic exponent in $K^2 = \text{const. } y^n$.
p	feet	Wetted perimeter of stream or subsection.
Q	cfs	Total flow (subscripts m and p refer to model and prototype).
q	cfs	flow in a subsection (subscripts m and p

refer to small model
flume and to prototype
flume.)

R	feet	Hydraulic radius.
r	feet	Radius of semi-circular constriction.
IR		Reynolds number.
S	feet/feet	Slope of stream bed or flume.
V	feet/sec	Average velocity (sub- script o refers to un- constricted flow).
y	feet or centimeters	Depth of flow.
y_o	feet or centimeters	Normal depth of flow in unconstricted channel.
y_1	feet or centimeters	Maximum depth of flow upstream of constriction.
α		Slope of line on Con- veyance versus Depth graph.
β		Ratio of bottom slope to critical slope.
η		y/y_o
ϕ		Bakhmeteff backwater function.

APPENDIX B
EQUATIONS OF FLOW

DERIVATIONS OF EQUATIONS GOVERNING THE FLOW IN RECTANGULAR CHANNELS WITH SEMI-CIRCULAR CONSTRICTIONS

The equation for the discharge in rectangular channels with a sharp crested semi-circular constriction is obtained and is expressed in terms of an infinite series of powers of the ratio y_1/r . With reference to Figure 27, (Appendix C) Bernoulli theorem gives

$$V = C \sqrt{2gh'} = C \sqrt{2g(y_1 - h)}$$

The element of area is

$$dA = 2 \sqrt{r^2 - h^2} dh$$

and the discharge is thus

$$Q = \int V dA = \int_0^{y_1} C \sqrt{2g(y_1 - h)} \cdot 2 \sqrt{r^2 - h^2} dh \quad (1)$$

Expanding into a series, integrating term by term, and making use of the fact that $2r = b$:

$$Q = C_D \sqrt{2g} \frac{17}{24} y_1^{3/2} b \left[1 - 0.1294 \left(\frac{y_1}{r} \right)^2 - 0.0177 \left(\frac{y_1}{r} \right)^4 + \dots \right] \quad (2)$$

This may be written as

$$Q = C y_1^{3/2} b E \quad (3)$$

where

$$C = C_D \frac{17}{24} \sqrt{2g} \quad (4)$$

and

$$E = \left[1 - 0.1294 \left(\frac{y_1}{r} \right)^2 - 0.0177 \left(\frac{y_1}{r} \right)^4 + \dots \right] \quad (5)$$

The discharge in the rectangular flume is given by

$$Q = V_0 A_0 = F_0 \sqrt{g} B y_0^{3/2} \quad (6)$$

where

$$F_0 = \frac{V_0}{\sqrt{g y_0}} \quad (7)$$

is the Froude number of the undisturbed flow. Equating (2) and (6) and solving for the coefficient of discharge

$$C_D = \frac{12\sqrt{2}}{17} \cdot \frac{F_o}{mE} \left(\frac{y_o}{y_i} \right)^{3/2} \quad (8)$$

where

$$m = \frac{b}{B} \quad (9)$$

or

$$\frac{y}{y_o} = \left[\frac{12\sqrt{2}}{17} \cdot \frac{F_o}{mE C_D} \right]^{2/3}$$

APPENDIX C
FIGURES AND TABLES

IDEALIZED STREAM CROSS SECTION

FIGURE 1

FIGURE 2

FIGURE 3

FIGURE 4

FIGURE 5

FLUME AND MODELS

FIGURE 6

TAIL GATE CONSTRUCTION

PLAN VIEW OF JACKS AND GEARS

FIGURE 7

FIGURE 8

JACK DETAIL

FIGURE 9

APPARATUS ARRANGEMENT

FIGURE 10

FIGURE 11

FIGURE 12

FIGURE 13

FLOW IN RECTANGULAR CHANNELS WITH SEMI-CIRCULAR CONSTRICTIONS - COMPARISON OF TWO AND THREE DIMENSIONAL CASES

FIG. 14

FIGURE 15

FIGURE 16

INSTRUMENT CARRIAGE

FIGURE 17

MODEL IN PLACE

FIGURE 18

MODEL IN PLACE

FIGURE 19

MODEL IN PLACE

FIGURE 20

FIGURE 21

FIGURE 22

FIGURE 23

FIGURE 24

FIGURE 25

FIGURE 26

DEFINITION SKETCH

FIGURE 27

TABLE 1
OBSERVED AND CALCULATED DATA

Contraction Ratio	Slope ft/ft	γ_0 cm	γ_1 max cm	γ_1 min cm	R ft	$\frac{V_0}{V}$	Cd Appendix 2 Equation 2	f	n	$b_1/\Delta h$	$V^2/2g\Delta h$
0.5	0	23.78	24.53	22.97	.5934	25.147	.2087	—	—	—	.0033
		33.87	35.18	32.97	.7688	34.213	.1962	—	—	.48	.0041
		22.86	23.70	22.03	.5762	25.377	.2190	.0128	.0957	.55	.0048
	25	22.86	23.70	22.03	.5762	25.377	.2190	.0145	.1060	.49	.0048
		32.15	33.33	31.17	.7400	34.795	.2095	.0231	.1276	.55	.0060
		22.86	23.70	22.03	.5762	25.377	.2190	.0231	.1276	.49	.0060
	50	23.88	24.80	23.17	.5584	25.666	.2282	.0231	.1276	.49	.0060
		29.86	30.90	28.73	.5160	24.188	.2607	.0347	.1164	.48	.0091
		21.70	22.66	20.19	.5531	38.533	.3888	.0197	.1177	.56	.0158
	100	30.73	32.73	28.82	.7167	47.008	.2892	.0288	.1189	.51	.0099
		20.45	23.43	17.70	.5280	50.555	.4318	.0272	.1255	.52	.0407
		12.07	14.72	9.61	.3415	28.516	.4329	.0212	.1127	.52	.0397
0.7	0	15.94	19.45	12.14	.4339	40.959	.3807	.0208	.1161	.48	.0117
		13.06	13.06	13.06	.4975	52.860	.4860	.0191	.1138	.45	.0184
		18.77	22.25	18.77	.5160	40.959	.5026	.0197	.1166	.43	.0233
	25	9.25	13.07	5.40	.2704	29.459	.5026	.0197	.1166	.43	.0233
		12.38	13.07	7.60	.3490	42.615	.3888	.0202	.1107	.55	.0226
		13.24	21.45	4.04	.3696	56.270	.4685	.0138	.0920	.47	.0600
	50	13.24	23.93	23.46	.5934	25.147	.1506	—	.0920	.32	.0053
		33.97	34.18	33.57	.7688	34.213	.1357	—	.0957	.34	.0041
		21.46	41.76	40.53	.5762	25.377	.1385	.0128	.0957	.40	.0066
	100	32.15	32.15	31.72	.7400	34.795	.1564	.0148	.1060	.31	.0044
		32.15	32.15	31.72	.7400	34.795	.1564	.0148	.1060	.31	.0044
		40.14	40.41	39.63	.8600	43.180	.1446	.0231	.1276	.39	.0087
0.9	0	22.02	22.22	21.71	.5594	35.590	.1625	.0235	.1276	.41	.0075
		29.75	30.06	29.31	.7003	35.590	.1625	.0235	.1276	.41	.0075
		37.50	37.82	36.97	.8222	44.197	.1579	.0246	.1406	.38	.0054
	25	19.86	20.07	19.43	.5160	26.188	.1934	.0247	.1544	.33	.0091
		21.70	22.18	20.98	.5531	38.533	.2560	.0397	.1177	.40	.0158
		20.45	22.18	19.53	.5160	38.533	.2560	.0397	.1177	.40	.0158
	50	12.07	12.78	10.84	.3415	28.516	.3737	.0231	.1276	.37	.0097
		15.94	16.69	14.19	.4339	40.959	.3870	.0208	.1161	.35	.0419
		18.97	20.06	16.80	.4975	52.860	.4913	.0191	.1138	.33	.0904
	100	15.40	16.64	13.20	.4297	56.385	.4858	.0194	.1078	.40	.0848
		9.25	10.68	6.94	.2704	29.459	.3127	.0197	.1066	.38	.0236
		12.38	14.14	8.75	.3490	42.615	.4377	.0202	.1107	.33	.0600
0.9	0	33.97	33.97	33.62	.7400	34.795	.1564	.0148	.1060	.31	.0044
		22.86	22.86	22.86	.7400	34.795	.1564	.0148	.1060	.31	.0044
		40.14	40.14	39.97	.8600	43.180	.1446	.0231	.1276	.39	.0087
	25	22.02	22.06	21.92	.5594	35.590	.1625	.0235	.1276	.41	.0075
		29.75	29.78	29.65	.7003	35.590	.1625	.0235	.1276	.41	.0075
		37.50	37.57	37.37	.8222	44.197	.1579	.0246	.1406	.38	.0054
	50	21.70	21.78	21.57	.5531	38.533	.1984	.0247	.1544	.33	.0091
		30.73	30.78	30.51	.7167	47.008	.1597	.0397	.1177	.40	.0158
		20.45	22.18	19.53	.5160	38.533	.2560	.0397	.1177	.40	.0158
	100	12.07	12.78	10.84	.3415	28.516	.3737	.0231	.1276	.37	.0097
		15.94	16.69	14.19	.4339	40.959	.3870	.0208	.1161	.35	.0419
		18.97	18.99	18.82	.4975	52.860	.4959	.0191	.1138	.33	.0904