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The Role of Estimating the Stiffness of Rolling Element Bearings, in the Analysis of Semi-Hermetic, Twin-Screw Compressors

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THE ROLE OF ESTIMATING THE STIFFNESS OF ROLLING ELEMENT BEARINGS, IN THE ANALYSIS OF SEMI-HERMETIC,TWIN-SCREW COMPRESSORS.

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INTRODUCTION

The clear trends in rotating machinery design and construction are towards lighter and stiffer machines, and faster rotational speeds. Most machines suffer from some degree of mass imbalance, which is a major source of rotating forces. At high speeds these forces increase dramatically, and may excite resonances of the rotating machinery and the housing of the machine, causing those parts of the machine to vibrate. A stiff housing tends to absorb little of the vibration energy and therefore would act as a sound emitter, thus justifying the common opinion which regards rotating machines as potential noise and vibration generators.

The Twin-Screw Compressor

The compressors discussed in this paper typically consist of a male and female screw rotors, parallel to each other in close proximity, and supported by rolling element bearings. The female rotor is driven by the male through the screw lobe so that they are accurately synchronised. Both screw rotors are loaded radially and axially by forces generated from the gas flow. For better efficiency twin-screw compressors are constructed with tight clearance between the two screws. This design requires that excessive vibration of the rotors be avoided because it may lead to mutual hammering of the screws and consequently noise and accelerated wear. Therefore it may prove profitable to engage rotor dynamics analysis methods early in the design stage to predict the performance of the rotors. Such methods are able to foresee pitfalls, to simulate alternative solutions and to evaluate them prior to the actual manufacture of a test model. In this way the time and costs of correcting deficiencies may be dramatically reduced.

Two types of twin-screw compressors will be addressed here, with somewhat different assemblies:

(a) 'Open Shaft' - The rotors system is assembled into a massive housing with a flexible coupling to the drive motor on the protruding end of the male rotor. So the male rotor is driven by a power unit external to the compressor.

(b) 'Semi-Hermetic' - An electric motor is integrated into the compressor in such a way that the electro-rotor is mounted on the screw rotor. This is mostly an overhang part of the rotor, and therefore undergoes greater bending deflection than type (a).

Rotor Dynamics Performance

Each rotor of the twin-screw compressor orbits (whirls) in addition to rotating about its undeformed centre-line. The whirl orbit of a perfectly isotropic rotor in a vertical position would be a circle, otherwise it can take any form [1][1]. When the rotor whirls at the speed of rotation it is called 'synchronous' whirl, and otherwise 'asynchronous' whirl. Synchronous forward whirl is mostly a kinematic phenomenon caused by the forces generated by mass imbalance. Synchronous reverse , and asynchronous whirl are dynamic phenomena analogous to beam vibration. Thus dynamic whirl is associated with a form of bending of the rotor, referred to as a 'natural whirl mode shape', in short 'whirl mode', seen by a stationary observer as a vibrating beam. The frequency [Hz] at which the rotor whirls in a specific mode shape is named the 'natural whirl frequency' with reference to that mode, or in short the 'whirl frequency'. Sometimes the whirl frequency is called the 'whirl speed' and is then in [r/min].

[1]The numbers in square brackets refer to the List of References.

Every machine possesses an infinite number of whirl modes of the various bending shapes and their corresponding frequencies. The mode shapes and their natural whirl frequencies depend on the dynamic properties (mass and stiffness distribution) of the individual rotor. These properties depend in turn on the manufacturing tolerances of all the rotor components (e.g. mounting fits, bearing location,), which may vary slightly from one compressor to another. The variations, however small, lead to natural whirl frequencies which differ from those predicted for the nominal design, hence may differ from one compressor to another.

Vibration Excitation

It should be pointed out, however, that a rotor vibrates in a natural mode of whirl only if it is excited to resonate, i.e. in the presence of a forcing function with a frequency in close proximity to that mode. The most common case of excitation are the radial forces arising from mass imbalance of the rotors. As the speed of rotation of the rotor [r/min] approaches one of its natural whirl speeds (in [r/min]' = [Hz] x 60) these forces excite the mode into resonance. The speed at which a mode is excited is called a 'critical speed', and the ratio of the whirl speed to the rotational speed is called the 'vibration order' (in short 'order') so that excitation by mass imbalance has order #1. A shaft with asymmetric cross-section caused by a keyway may give rise to excitation of order #2. Thus a keyway may excite a natural whirl mode at a speed of rotation which is half the whirl speed of that mode. This concept will be substantiated and further discussed later in the text with the help of practical examples.

AN APPROACH TO THE ANALYSIS

Rotor dynamics analysis is normally motivated by the desire to avoid the risk of operating at a critical speed within the rated speed range of the compressor. Such analysis can be used to simulate the operating conditions and predict the dynamic performance of each of the rotors, and to evaluate the effect of design changes. The focal point of the analysis is the analytical model representing those features of the real compressor of interest in this analysis. In general these are the mass and stiffness distributions, and the boundary conditions, i.e. the supports of the rotor.
The analysis presented herein deals only with the male screw rotor of both twin-screw compressor types: the open-shaft and the semi-hermetic.

The Rotor Dynamics Models

The rotor dynamics model of the Open Shaft screw rotor described below closely resembles many twin-screw compressor designs. This type is introduced to facilitate a discussion of the analytical model verification against experimental data, further in the text.

A Cartesian coordinate system is defined with the Z-axis along the undeformed centre-line of the screw rotor, and the Y-axis in the direction opposite to gravity (Fig. 1). The rotor has a rotational symmetry (also the screw approximately), so that the model consists of a series of cylinders of various lengths and diameters [2]. It is assumed to be elastic radially and rigid axially[2]. The screw segment of the rotor is modeled with a diameter representing its bending stiffness. Since it is smaller than the screw diameter, the rest of its mass is modeled as a point mass. The remaining parts of the screw rotor which do not add to its stiffness are also lumped as point masses (denoted m) at relevant locations on the rotor. The rotor is supported by five rolling-element bearings modeled as springs (denoted k) as shown in the stick model. They are arranged so that the angular-contact ball bearings (ACBB) k_1, k_2, and k_3, carry exclusively thrust load, while the cylindrical roller bearings (CRB) k_4 and k_5 support only radial loads.

[2]The reasons for the assumption of a rigid body in the axial sense are as follows:

a) The rotors in these compressors are mostly robust, they have a relatively large diameter as compared to the short length.
b) The natural frequency of the flexural axial mode of vibration would be well above the range of interest of the analysis.

The inner rings (IR) of the four bearings at the left end of the rotor are rigidly clamped axially onto the shaft, thereby contributing to its bending stiffness, and are therefore considered a solid part of it.

In the analysis the axial Z-axis, and the radial X-axis and Y-axis are all assumed decoupled[3] from each other. Thus the model consists of three, mutually independent sub-systems:

(a) An elastic rotor with 2-Degrees-of-Freedom in the X-Z plane, supported by the spring constant of the CRBs in the X-direction;
(b) A similar rotor in the Y-Z plane, supported by the orthogonal spring constants of the same bearings in the Y-direction;
(c) A mass-spring sub-system with Single-Degree-of-Freedom in the Z-direction and the axial stiffnesses of the ACBBs.

Bearing Stiffness Computation

The stiffness of a rolling-element bearing arises from the resistance of the material of the mating elements to elastic deformation at the contact [3]. The factors controlling it are grouped as:

a) Internal Geometry - Dimensions of the elements and the clearance, contact angle, and the number of rolling elements;
b) Rotor Assembly - Mounting fits, shaft stiffness, mounting resilience, and the force or displacement of the axial or radial preloading;
c) Operating Conditions - Shaft speed, the applied loading component acting on each bearing, and the azimuth orientation of the rolling elements relative to the load vector.

The bearings' stiffnesses are computed with a three-dimensional quasi-static[4] analytical model of the complete rotor, in the steady-state. The rotor is assumed to operate at a constant speed and loading by the gas forces. The computations yield a 5X5 square symmetric matrix for each bearing (Table 1.) corresponding to 3 translational and 2 rotational degrees of freedom.

Table 1. Bearing stiffness matrix[5]

k_x	k_{xy}	k_{xz}	k_{xa}	k_{xb}
	k_y	k_{yz}	k_{ya}	k_{yb}
		k_z	k_{za}	k_{zb}
			k_a	k_{ab}
				k_b

The elements on the main diagonal named the 'pure elements' describe the stiffness in a given coordinate, i.e. the response to a force or moment in the same coordinate. So k_x is the stiffness in the X-direction due to a force acting in that direction. The remaining elements, called the 'coupling elements' describe the coupling between pairs of elements on the main diagonal.

[3]It is recognised that an error is introduced into the analysis through decoupling the sub-systems. This however, is of minor importance in weakly coupled systems (there are no disks), while it greatly simplifies the analysis.

[4]The term 'quasi-static' implies that the model accounts for the centrifugal forces acting on the orbiting rolling-elements but is otherwise a static model.

[5]The lower minor of the matrix is symmetric with the upper one, and therefore it need not be displayed.

In the matrix below k_{xa} is the coupling stiffness between a force acting in the X-direction and the resulting rotation about the X-axis. In the scheme of the decoupled system for rotor dynamics analysis, sub-systems (a) and (b) draw each a 2X2 square symmetric matrix from the matrix of the complete model, and sub-system (c) the pure axial stiffness term. The decoupled system is given in matrix form,

Table 2. Bearing stiffness matrix of the decoupled system

k_x	k_{xb}			
k_{bx}	k_b			
		k_y	k_{ya}	
		k_{ay}	k_a	
				k_z

A rotor of rotational symmetry where $k_x = k_y$ and $k_a = k_b$ for all the rotor supports[6] is regarded an isotropic rotor. In such a case, presumably, the respective coupling elements of sub-systems (a) and (b) are equal and these sub-systems become identical.

<u>Rotor Dynamics Analysis</u>

The rotor geometry and the stick model adequately describe the screw rotor for the purpose of dynamic analysis. Hence the bearing stiffnesses are computed prior to the rotor dynamics analysis to provide the necessary input data. Since in the quasi-static model the stiffness depends on the operating conditions of both load and rotor speed, these calculations are repeated for each combination of these conditions. The models used in the computations also account for mounting fits and resilience, and assembly errors. However the present stiffness computations and rotor dynamics analysis were conducted with the nominal data, without considerations of the manufacturing tolerances and possible assembly errors.

The bearing arrangement of the male screw rotor described above is designed so that the ball bearings carry exclusively the thrust load. They contribute, therefore, only to the axial stiffness of the rotor. The radial loading is supported by the CRBs which have radial clearance, hence their stiffnesses are expected to be anisotropic. This can be observed from the computation results tabulated below, comparing the two radial stiffness components of a bearing.

Table 2. Computed CRB stiffness [N/mm]

Bearing #4		Bearing #5	
k_x	k_y	k_x	k_y
543417	356512	400142	393084

[6] The support is defined to include the entire chain of springs from the shaft centre-line to ground.

The rotor dynamics analysis was used to predict the whirl modes and the associated whirl frequencies of the screw rotor within the frequency range 0-1000 Hz, tabulated below.

Table 3. Natural whirl frequencies [Hz]

Mode #0	Mode #1		Mode #2	
Z-axis	X-Z	Y-Z	X-Z	Y-Z
951	507	504	939	872

Evidently each of the radial whirl modes occurs twice, once in the X-Z plane and once in the Y-Z plane at different whirl frequencies, as could be expected for an anisotropic rotor. The results of the modal analysis are presented in the form of mode shapes of these modes (Fig. 2). The whirl frequencies of modes #1 are nearly identical and their mode shapes are similar, so that mode #1X only is shown in the diagram. The difference between the shape of modes #2X and #2Y is most pronounced at the left end of the rotor.

This may seem as though the rotor undergoes relatively larger radial bending deflections in the Y-direction than in the X-direction, at that end of the rotor. It should be noted, however, that different mode shapes have been normalised to their maximum deflection.

Model verification

Clearly it is strongly recommended to verify the analytical model before using it to evaluate the rotor dynamics performance of a compressor design. In the case under discussion the compressor has been operational so that some test data were available for comparison with the analysis. The signal was recorded from an accelerometer attached to the compressor successively at two locations,

1) On the housing, facing the male screw close to bearing #4, and pointing radially;
2) On the housing of bearing #5 close to the drive end, pointing axially.

The test data of the radial accelerations are presented in a diagram of the Frequency Response Function (Fig. 3). The curve consists of many spikes which seem to be super-imposed on a 'ridge' of a peak-amplitude plot shown by a dashed line. Further examination of the diagram reveals that the spikes are integer multiples of the rotational speed of the rotor, and their side-bands. Thus they can be regarded as a kinematic phenomenon depending mainly on the speed. The 'ridge' represents the dynamic characteristics of the compressor, including the screw rotors, as 'seen' by the accelerometer. Three of the computed whirl frequencies of Table 3 are depicted on the diagram by the vertical lines, and seem to support that hypothesis.
However assessment of the validity of the analytical model requires a more detailed examination of its characteristics. The main factors to be considered in interpreting the test data are outlined below:

1) Vibration excitation - For a screw rotor to vibrate in a natural mode of whirl, the mode must be excited to resonate. That occurs when the frequency of excitation, e.g. gear mesh, and the natural whirl frequency nearly coincide. This concept is presented in a Campbell diagram (Fig. 4) in the form of a critical speed map where the excitation frequencies are denoted by their orders. The horizontal lines represent the natural whirl modes, denoted by their number and coordinate. The frequency response function (Fig.3) can be thought of as a cross-section of the Campbell diagram at the operating speed 3550 r/min. It can be observed that modes #0 and #2X are excited by order #16, and mode #2Y by order #15, all at that speed.
2) Whirl mode shapes - The radial vibration forces are transmitted to the housing and sensor through the two CRBs #4 and #5 (Fig. 1). They are positioned at low response points relative to mode #1, and high response relative to modes #2.

The above observations indicate good agreement between the test data and the model.

DYNAMIC ANALYSIS

The Semi-Hermetic Rotor Model

The semi-hermetic compressor is similar to the open-shaft compressor and is modeled in much the same way (Fig. 5). From a rotor dynamics viewpoint, this rotor differs from the former type mainly by the electro-rotor mass carried on an overhung part of the rotor [4]. The structure of this mass and its mounting lead to the conclusion that it does not contribute to the bending stiffness of the rotor. Therefore the mass is lumped on the shaft at three points: m_2, m_3 and m_4, as displayed by the stick model.

Rotor Dynamics Analysis

The natural whirl frequencies and the associated mode shapes were computed for the male screw of the semi-hermetic compressor, at the rotor speed 3550 r/min. The frequencies are tabulated below,

Table 4. Natural whirl frequencies [Hz]

Mode #0	Mode #1		Mode #2		Mode #3	
Z-axis	X-Z	Y-Z	X-Z	Y-Z	X-Z	Y-Z
726	147	146	735	709	805	785

The modal survey is presented in a diagram (Fig. 6) of the mode shapes, and the rotor model for reference. The effect of the electro-rotor mass can be observed by comparing the modal survey of the open-shaft with that of the semi-hermetic rotors. For all the modes shown the latter type displays the maximum bending deflection at the drive end, as would be expected. Such mode shapes run the risk of the electro-rotor contacting the stator when operating close to a critical speed.

Critical Speed Problem

The rotor dynamics analysis has pointed at the risk that the whirl frequency of modes #3X and #3Y occur in close proximity to the operating speed of the male screw rotor 3550 r/min. That prediction is related to the event that excitation order #13.5 may be present in the compressor. The data obtained by computation were used to construct the critical speed map (Fig. 7). It can be observed that the horizontal lines representing these modes intersect with the line of order #13.5. That gives rise to the risk of critical speeds excitation at 3490 r/min (#3Y) and 3578 r/min (#3X).
Observations of this kind in the development phase of the compressor are normally followed by searching for ways and means to eliminate that risk. This involves introducing design modifications which lead to re-distribute the mass and stiffness, thereby shifting the whirl frequencies. The variety of options for design solutions is only limited by the requirements of the compressor process, or by the imagination of the engineers. One of the options, seemingly simple to implement, is examined here from the rotor dynamics viewpoint. In this modified design an attempt is undertaken at redistributing the radial stiffness of the rotor by shifting bearing k_3 towards the electro-rotor. The effect of the design change was examined through rotor dynamics analysis. The analysis indicates that both modes were shifted away from the operating speed, with mode #3Y further down to 3464 r/min and mode #3X higher up at 3649 r/min. Evidently now the critical speed is caused by order #14.4 exciting mode #3X. This is shown on the diagram (Fig. 7) with the shifted whirl modes marked in brackets.

DISCUSSION

The Role of the Bearing Stiffness

The compressor was described in terms of the analytical model of the screw rotor for the purpose of rotor dynamics analysis. The bearing stiffness was shown to affect the whirl modes and frequencies of the rotor significantly.

It was demonstrated that, for a given screw rotor, both the spring constant and its location determine the whirl frequency. The importance of the bearing' stiffness is, however, relative to the whirl mode shape for the following reasoning,

a) Rigid-body modes - For a rotor which may be assumed rigid (see footnote 2) the only source of elasticity is provided by the bearings. Hence the bearing stiffnesses completely dominate these whirl modes and their frequencies.

b) Flexural modes - The influence of the bearing stiffness depends here on the bending stiffness of the rotor relative to the bearings, and on the bearing positions relative to the mode shape. It can be shown that the rotor stiffness is connected to the bearing stiffnesses in series. The nature of such a combination is that the softer one dominates.
Thus in the case of the twin-screw compressors with fairly stiff screw rotors, the bearing stiffness predominates.
The reference to the mode shape can best be explained with the help of the modal survey (Fig. 2). It can be seen that mode #1 displays hardly any bending deflection at the location of bearing #4. Hence this bearing has a negligible effect on the shape and frequency of mode #1, while mode #2 is strongly influenced by it. Comparison of modes #2X and #2Y shows that bearing #4 is the main source of the rotor anisotropy.

Estimating Bearing Stiffness

The analysis presented in sections 2 and 3 was, as mentioned, conducted with respect to the nominal manufacturing and assembly specifications of all the rotor components. Moreover, both manufacturing tolerances and assembly errors may influence the performance of the compressor considerably. In this section. we examine the effect of some of the common mounting errors of bearings on estimating their stiffness and consequently on predicting critical speeds.

Axial Preloading

In general ball bearings are preloaded axially by the constant force of a spring mechanism, or by displacing one bearing, of a pair, relative to the other. Since the stiffness of these bearings depends strongly on the spin speed of the rotor it cannot be determined by static measurement. It is, therefore, necessary to rely on indirect estimates using measurements and computations, and finally verify the rotor dynamics model by comparison to test data. The effect of an erroneous estimate of the force of axial preloading on the whirl frequency was examined analytically. Results are presented in a Campbell Diagram for an error of 12.5% either below or above the nominal value (Fig. 8). The graphs demonstrate that the error of estimating the whirl speed diminishes with the speed[7], but still remains within a band of 4% at the operating speed.

Radial Clearance

The stiffness of Cylindrical Roller Bearings (CRB) depends on the internal radial clearance, in particular the orthogonal stiffness (k_y). Some of the causes of reduced clearance in the assembly of the compressor are an oversize shaft or an undersize bore of the bearing housing. Reducing the radial clearance of CRBs leads to a lower load on the individual rollers, thereby stiffer contact between the rollers and the raceways.

Eventually the bearing is stiffer and the radial natural frequency becomes higher. The extent of the effect of this assembly error can be illustrated with the help of a computed example. The radial clearance of two different CRBs with average pitch diameter of about 60 mm was reduced by 0.1 mm, and the increase of the bearing stiffness was computed.

[7]The reader would arrive at a similar observation from Fig. 4 of [3].

The results are tabulated below as the percentage change of the bearing stiffnesses from those with the nominal radial clearance.

Table 5. Computed increase [%] of CRB stiffness for reduction in radial clearance of 0.1 mm.

Bearing #4		Bearing #5	
k_x	k_y	k_x	k_y
9.7	24.6	17.4	19.6

The table shows that an assembly error of this kind may result in a significant error in predicting the natural whirl frequencies of the compressor.

Bearing Misalignment

One of the common reasons for this particular mounting error lies in the spring used for applying the constant force of axial preloading to ball bearings. These bearings in the twin-screw compressors discussed in the paper, were arranged to carry pure thrust loads. In this configuration the load is equally distributed among all the rolling elements, and the axial bearing stiffness is a maximum. For a misaligned bearing the load is no longer evenly distributed so that the bearing stiffness drops, and consequently also the natural whirl frequency. The model of the open-shaft compressor was used to illustrate the pronounced effect that may follow from a minor degree of misalignment. In that design (Fig.1) the back-up bearing #1 with 40 mm outer diameter was misaligned by tilting its outer ring an amount of 0.2 mm. This led to a lower natural frequency for the axial rigid-body mode, as could be expected. The simulation results for that model at half the applied load are tabulated below, with the whirl frequencies at the nominal mounting condition for reference.

Table 6. The effect of assembly errors on the axial whirl frequencies [Hz].

Condition	Male rotor speed [r/min]		
	10	2500	3550
Nominal	879	875	856
Misalignment	807	780	769

Also here, as for the other assembly errors, that error would lead to a substantial divergence of the test data from the computed data.

CONCLUSIONS

An approach has been described to rotor dynamics analysis of twin-screw compressors equipped with rolling-element bearings. It was shown with the help of case studies that the stiffness of the bearings can considerably influence the rotor dynamics behaviour of the screw rotors. A verified analytical model with correctly estimated bearing stiffness may be used to advantage to predict the performance of the compressor. That activity may help to reduce the risk of operating under resonant conditions and hence also to reduce noise and vibration levels.

However, the stiffness of rolling-element bearings can not be measured directly because it depends on the operating conditions and on possible assembly errors. One way of estimating bearing stiffness is to extract it from rotor dynamics analysis [5] by comparison with test data and with the knowledge of the bearing properties. Clearly, the effect of errors in the assembly must be allowed for in such estimates.

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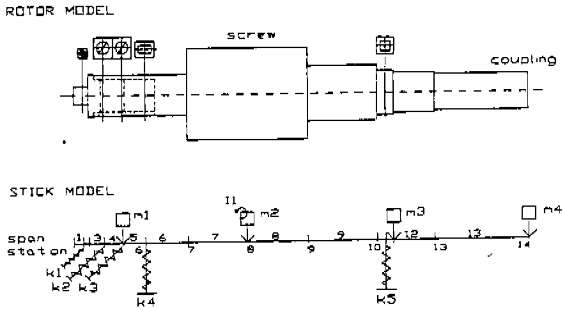

Fig 1 The male screw rotor of the twin-screw, open-shaft compressor modeled for rotor dynamics analysis

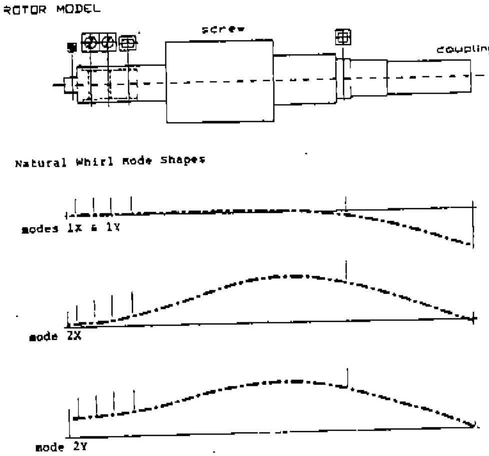

Fig. 2 Modal survey of the bending modes of the male screw of the twin-screw, open-shaft compressor using the analytical model.

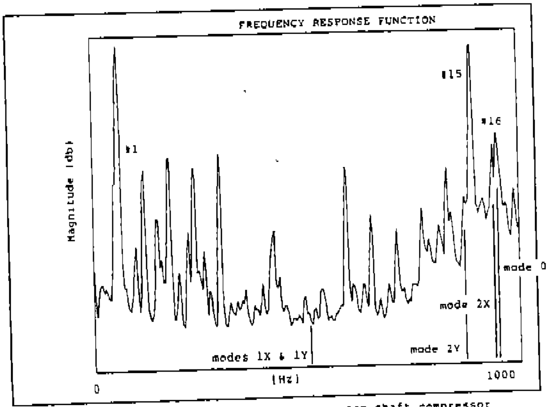

Fig 3 Radial vibration of the twin-screw, open-shaft compressor recorded during test at full load, 3550 r/min. Vertical lines indicate predicted whirl modes and frequencies. Courtesy Svenska Rotor Maskiner (SRM), Stockholm, Sweden.

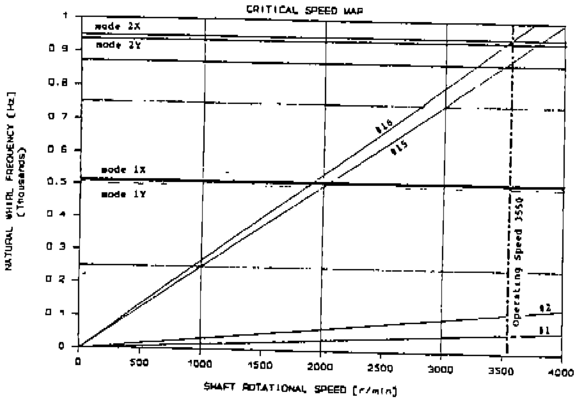

Fig. 4 The risk of critical speeds of the twin-screw, open-shaft compressor. Diagram constructed from computed data.

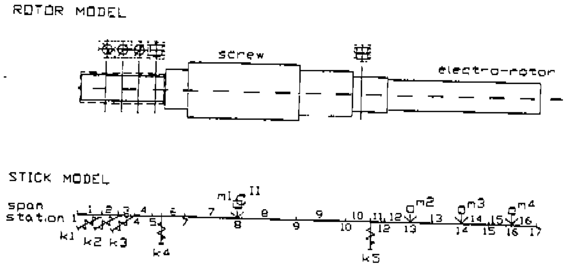

Fig. 5 The male screw rotor of the twin-screw, semi-hermetic compressor modeled for rotor dynamics analysis

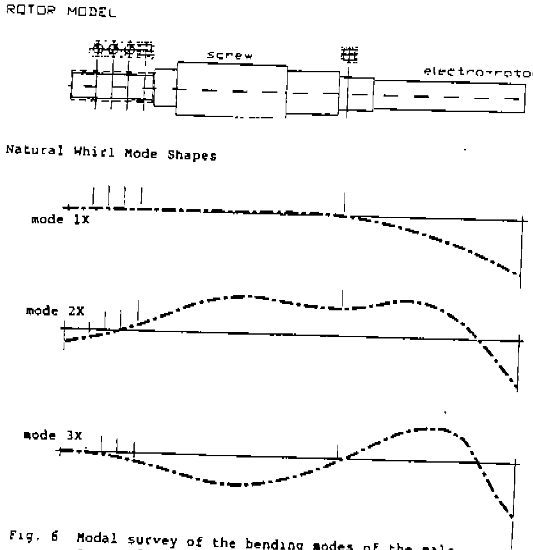

Fig. 6 Modal survey of the bending modes of the male screw of the twin-screw, semi-hermetic compressor using the analytical model.

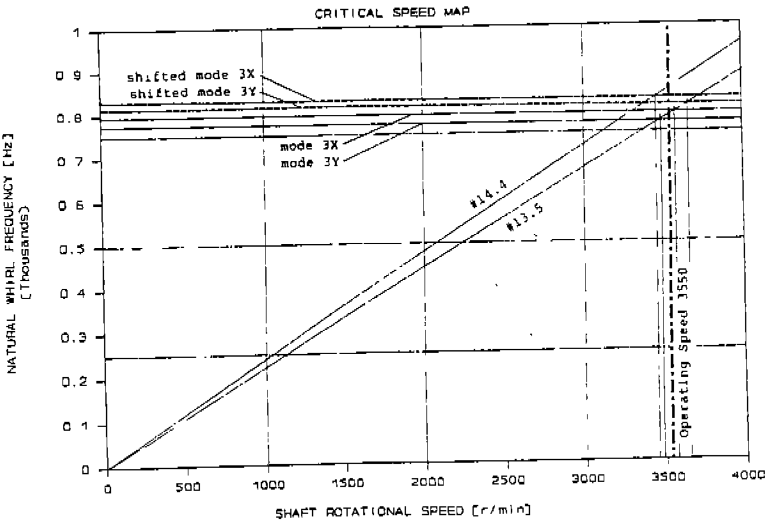

Fig. 7 The risk of critical speeds of the twin-screw, semi-hermetic compressor. Diagram constructed from computed data.

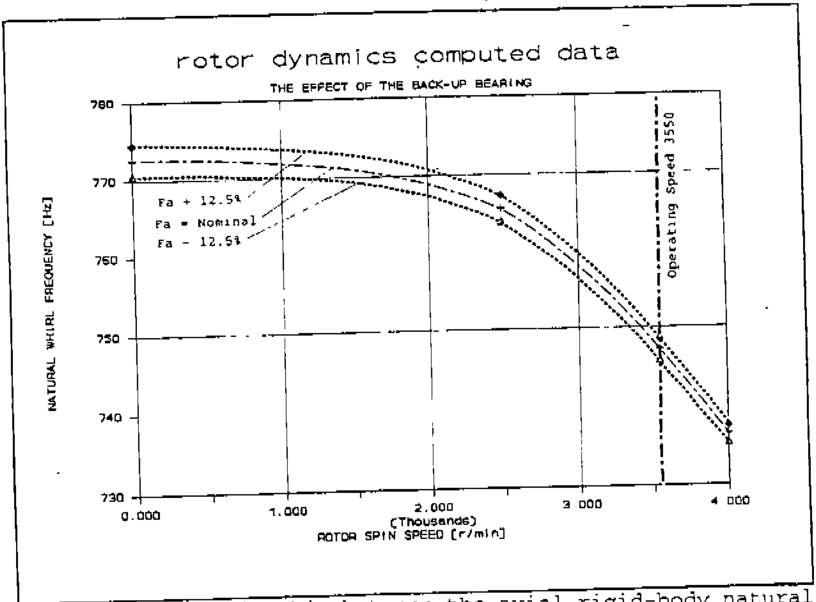

Fig. 8 The relationship between the axial rigid-body natural frequency and the spin speed, as function of the axial preloading of the back-up bearing.