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# CALCULATING MOOEL AND EXPERIMENTAL INVESTIGATION OF GAS LEAKAGE

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## ABSTRACT

In this paper, a new mathematical model to calculate gas leakage in small clearance is established. In the model, viscous force and inertia force are considered at the same time. Theoratical analysis and experimental results show that the calcuating results coincide well with the experimental results.

## NOMENCLATURE

| | |
|---|---|
| $h$ | height of passage (m) |
| $h_{min}$ | the smallest height of passage (m) |
| $k$ | specific heat ratio |
| $P$ | pressure (Pa) |
| $p_1$ | pressure in high pressure cavity (Pa) |
| $p_2$ | pressure in low pressure cavity (Pa) |
| $R$ | universal gas constant (J/kg. k) |
| $Q$ | volume flow rate ($m^3$/h) |
| $r$ | piston radius (m) |
| $T$ | temperature (k) |
| $T_1$ | temperature in high pressure cavity (k) |
| $u$ | partial veloceity along x direction (m/s) |
| $w$ | partial velocity along z direction (m/s) |
| $\mu$ | kinetic viscosity (N. s/$m^2$) |
| $\rho$ | density of gas (kg/$m^3$) |
| $\xi$ | compressibility coefficient |

## INTRODUCTION

Gas leakage is an important factor affecting the performance of compressor. If more accurate equation is found to calculate leakage, we could do it better to predict compressor's performance.

In many cases, leak—passage is composed of two curved surface with same length. This kind of leaking passage is unfolded as figure 1. For calculating the gas leakage through this kind of passage, nozzle flow model was used in most of literatures[1,2,3,4,5]. The precondition of this model is that the effect of viscous friction on flow is neglected. In other literatures(6), it is suggested to use viscous flow model. In this kind of model, the effect of inertia force on flow is neglected. In fact, because gas is easy to compress and it leaks through small clearance, both viscous force and inertia force have the equivalent influence on gas leakage.

In this paper, a new mathematical model is presented in which viscous force and inertia force are considered at the same time. The calculating results are compared with those of experiments. The comparison shows that the new calculating model is more accurate than nozzel flow model or viscous flow model. The accuracy of calculation is improved greatly.

## FLOW FIELD ANALYSIS AND FUNDAMENTAL EQUATIONS

The leaking passage is shown in figure 1. When gas leaking occurs, the high pressure gas first flows into passage through section 1—1, then passes a small clearance channel in which section area is changing, and last flows out of passage through section 2—2. Because the width of passage is same and it is far bigger than height of passage, the flow field could be considered as uniform along y direction and be handled as two demension flow field. Navier—Stocks equation can be simplifed as following:

$$\rho(u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z})=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\{\mu[2\frac{\partial u}{\partial x}-\frac{2}{3}(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}]\}+\frac{\partial}{\partial z}[\mu(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z})] \qquad (1)$$

$$\rho(u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z})=-\frac{\partial p}{\partial z}+\frac{\partial}{\partial z}\{\mu[2\frac{\partial w}{\partial z}-\frac{2}{3}(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z})]\}+\frac{\partial}{\partial x}[\mu(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z})] \qquad (2)$$

Since the height of passage changes slightly, the pressure change in x direction can be considered far bigger than that along z direction, i. e. $\frac{\partial p}{\partial x}\gg\frac{\partial p}{\partial z}$. From numerical analysis, it can be known that each item in eq. (2) has the same order, so the change of gas momentum in z direction can be neglected. In addition, because the clearance is very small, the change of velocity u along z direction is far bigger than that along other directions, i. e. $\frac{\partial u}{\partial z}\gg\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial z}\gg\frac{\partial w}{\partial z}$, $\frac{\partial u}{\partial z}\gg\frac{\partial w}{\partial x}$. From numerical analysis as above, it can be deduced that $\frac{\partial^2 u}{\partial z^2}\gg\frac{\partial^2 w}{\partial z\partial x}$, $\frac{\partial^2 u}{\partial z^2}\gg\frac{\partial^2 u}{\partial x^2}$, so eq. (1) can be simplified as following:

$$\rho(u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z})=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial z}(\mu\frac{\partial u}{\partial z})$$

In order to simulate leaking mathematically, we put forward some assumptions:

1. Pressure is uniform in z direction on every section.

2. Flow process is an adiabatic process.

According to above analysis and assumptions, the following fundamental equations of gas leakage are gained:

momentum eq. $$\rho\left(u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right) \tag{3}$$

continuity eq. $$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial z}(\rho w)=0 \tag{4}$$

state eq. $$p=\xi R\rho T \tag{5}$$

process eq. $$p/\rho^{k}=C \tag{6}$$

## CALCULATING MODEL ESTABLISHING

On the basis of assumption (1), volumetric flowrate of gas through unit width is

$$Q=\int_0^h u\,dz \tag{7}$$

The viscosity of gas is a function of pressure and temperature: $\mu=\mu(p,T)$, i.e. $\frac{\partial\mu}{\partial z}=0$, so momentum eq. (3) is simplified as

$$\rho\left(u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial z^2} \tag{8}$$

In small clearance passage, the fluid film of gas is very thin, so inertia item in above equation can be averaged along z direction, we get

$$\rho\left(u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)=\frac{\rho}{h}\int_0^h\left(u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)dz$$

Taking above equation into eq. (8) and arranging it

$$\frac{\partial^2 u}{\partial z^2}=\frac{1}{\mu}\frac{\partial p}{\partial x}+\frac{\rho}{\mu h}\int_0^h\left(u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)dz \tag{9}$$

The right of above equation is a function of x only, i. e.

$$\frac{\partial^2 u}{\partial z^2}=f(x) \tag{10}$$

Utilizing boundary conditions

$$\begin{cases}u(x,0)=0\\ u(x,h)=0\end{cases}$$

Integrating above eq. twice

$$u=\frac{z(z-h)}{2}\cdot f(x) \tag{11}$$

Taking u into eq. (7)

$$Q=-\frac{h^3 f(x)}{12} \tag{12}$$

Obviously, in order to calculate leakage rate Q, the frist step is to find the expression of f(x). we calculate the partial derivative of x and z, aiming at eq. (11)

$$\frac{\partial u}{\partial x}=\frac{z(z-h)}{2}\frac{\partial f(x)}{\partial x}-\frac{zf(x)}{2}\frac{\partial h}{\partial x} \tag{13}$$

$$\frac{\partial u}{\partial z}=\frac{(2z-h)f(x)}{2} \tag{14}$$

Developing eq. (4)

$$\rho\frac{\partial u}{\partial x}+u\frac{\partial \rho}{\partial x}+\rho\frac{\partial w}{\partial z}=0 \tag{15}$$

Calculating partial derivative of x, aiming at eq. (6)

$$\frac{\partial p}{\partial x}=\frac{kp}{\rho}\frac{\partial \rho}{\partial x} \tag{16}$$

Taking eq. (6), eq. (16) into eq. (15), we get

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}+\frac{u}{kp}\frac{\partial p}{\partial x}=0$$

Taking eq. (13) into above equation

$$\frac{\partial w}{\partial z}=\frac{zf(x)}{2}\frac{\partial h}{\partial x}-\frac{z(z-h)}{2}\frac{\partial f(x)}{\partial x}-\frac{z(z-h)f(x)}{2kp}\frac{\partial p}{\partial x} \tag{17}$$

Integrating above eq. and using boundary condition: $w(x, o)=0$, we get

$$w=\frac{z^2f(x)}{4}\frac{\partial h}{\partial x}-\frac{1}{2}\left(\frac{z^3}{3}-\frac{z^2h}{2}\right)\left(\frac{\partial f(x)}{\partial x}+\frac{f(x)}{kP}\frac{\partial p}{\partial x}\right) \tag{18}$$

Taking above expressions of u, $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial z}$ and $w$, i. e. eq. (11), (13), (14), (18) into eq. (9), then integrating and arranging them

$$\frac{\partial^2 u}{\partial z^2}=\frac{\partial p}{\mu\partial x}+\frac{\rho h^3 f(x)}{\mu}\left(\frac{h}{60}\frac{\partial f(x)}{\partial x}+\frac{f(x)\partial h}{24\partial x}+\frac{hf(x)}{120kp}\frac{\partial p}{\partial x}\right) \tag{19}$$

Removing f(x) from eq. (5), (10), (12), (19) and arranging them, we get

$$\frac{\partial p}{\partial x}=\frac{\frac{12\mu\xi Q}{h^3}-\frac{6\xi pQ^2}{5RTh^3}\frac{\partial h}{\partial x}}{\frac{6\xi Q^2}{5kRTh^2}-1} \tag{20}$$

Eq. (20) is the mathematical model established in this paper, i. e. the differential relation of pressure change with leakage rate, pressure, temperature and geometrie parameters of the passage.

Using numerical method to solve above equation, we can get the value of leakage and pressure distribution in flow field under defined condition.

## EXPERIMENT

Experimental device scheme is shown in figure 2. The construction of experimental part is shown in figure 3, working media are R12 and R22 respectively. The frist step is to charge liquid freen into vessel 1 and heat it. The vessel pressure is kept at desired value by adjusting heat power. Next open the valve v1, high pressure gas goes into dry filter 3, water and oil are absorbed. Then gas is heated to a determined temperature by superheater 4. Finally, gas goes into part 5 where circumferential leaking occurs. In this paper, the effect of high pressure and smallest clearance on leakage is studied experimentally aiming at R12 and R22 respectively. In order to avoid the effect of surrounding temperature on the leakage, the whole experimemtal part is soaked in a constant—temperature—water sink. In order to ensure that leaking gas goes only through clearance of radius direction, we sealed other passages where gas may leak.

## COMPARISON AND DISCUSSION ABOUT CALCULATING RESULTS AND EXPERIMENTAL RESULTS.

In order to compare under the same conditions, the calculating results of three models and the results of experiments are all shown in figure 4 ,5. We named model A for that established in this paper, model B for nozzle flow model and model C for viscous model. Obviously, the calculating results of model A is the best comparing with the experimental results.

From figure 4, it can be known that when pressure difference between high pressure cavity and low pressure cavity is small, the calculating results of model A and C are similar and accuration. In this case, the change of gas velocity in leaking fluid field is small, the inertia force in this time is small and can be neglected. On the contrary, the effect of inertia foree is bigger than that of viscous force, the calculating results of model A and B are more accurate than that of model C.

From figure 5, we can see that when clearance is small, the calculating results of model A and C are similar to experimental results, but when clearance is big, the results of model C are not so good. The reason is that when clearance is small, the divalent derivative of gas partial velocity u (along x direction) is very big in the direction of passage height, so viscous item is bigger than inertia item, the latter can be neglected. Under the condition of big clearance, the conclusion is opposite. When the clearance is in middle size, the results of model A and C all are not so idea.

On the whole, because model A contains the effect of viscous force and inertia force at the same time, its calculating results is the smallest and the best comparing with experimental results. While the pressure difference and clearance is small, the effect of viscous force is dominant, the intention of curve A is similar with that of curve C. As the pressure difference and the clearance is big, the effect of inertia force is dominant, the trend of curve A is similar with that of curve B. But when pressure difference and clearance is in middle value, the effect of viscous force and inertia force are almost same, the trend of curve A is between that of curve B and C.

## CONCLUSIONS

1. Comparing with the nozzle flow model and viscous flow model, the new calculating model which considers viscous force and inertia force at the same time is great developed. The accuracy of its calculating results is improved obviously. Both the nozzle flow model and viscous flow model have positive deviation.

2. When clearance and pressure difference are small, the effect of viscous force on

leaking flow is bigger than that of inertia force, on the contrary, the effect of inertia force is dominant.

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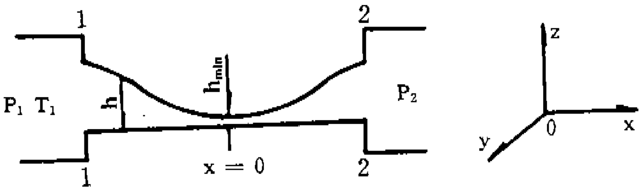

Figure 1. Simplified leaking passage

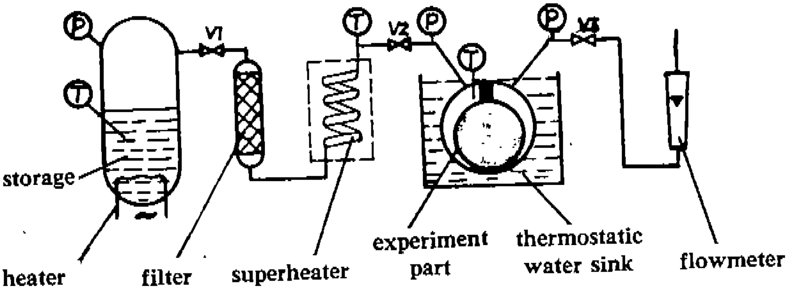

Figure 2. Schematic diagram of experimental set.

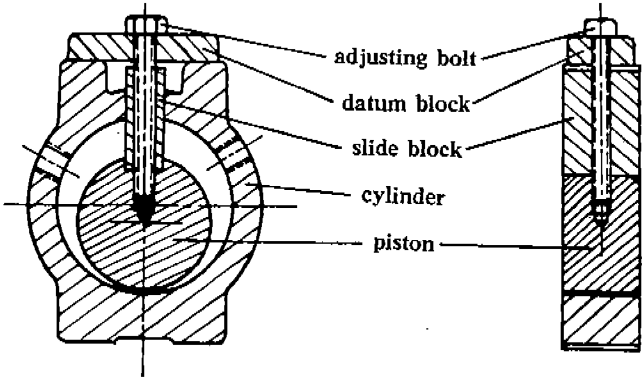

Figure 3. Schematie diagram of the construction of experimental part

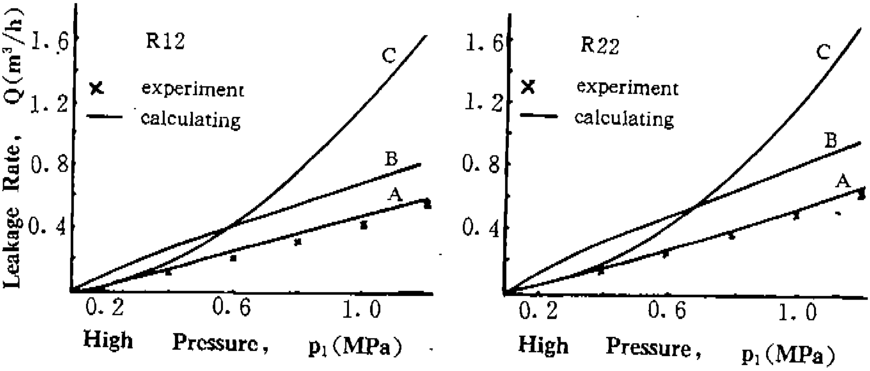

Figure 4. Effect of high pressure on leakage rate

$p_2 = 0.098MPa$ $h_{min} = 10\mu m$ $T_1 = 323k$

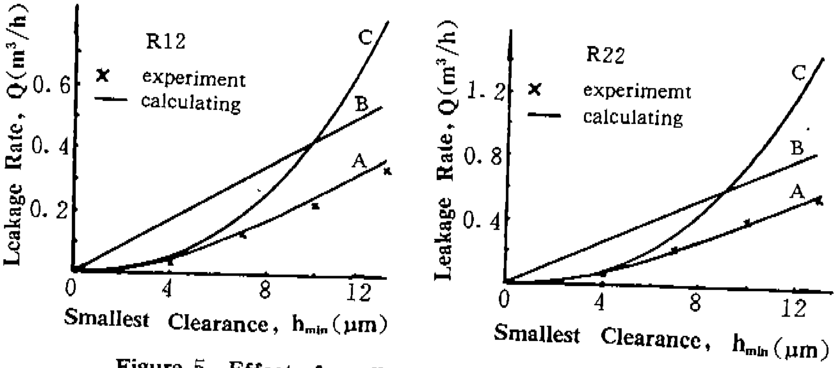

Figure 5. Effect of smallest clearance on leakage rate

$p_2 = 0.098MPa$. $T_1 = 323k$