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Developed Mathematical Model of the Self-Acting Valves of the Reciprocating Compressor and Its Application for Tongue Valves

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"DEVELOPED MATHEMATICAL MODEL OF THE SELF-ACTING VALVES OF THE RECIPROCATING COMPRESSOR AND ITS APPLICATION FOR TONGUE VALVES "

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ABSTRACT

In the paper an extended mathematical model of compressor self-acting valves has been presented. The model provides a better possibility of a more precise analysis of the valve action, in particular collisions between the working plate and valve stop or seat. An elasto-plastic model of collision has been introduced. A method of the application of the mathematical description for both plate valve and valve with deformable working element has also been presented. The description of valve operation has been used on a simulation model of gas and refrigerating compressor. In the model the heat exchange in the cylinder and medium loss due to the leakage have been considered. The medium has been described individually by a selected equation of state of real gas. The dynamic effect of the installation has also been taken into account on the basis of the acoustic model. The model has been verified in experimental tests of small gas and refrigerating compressors with inlet tongue valves.

INTRODUCTION

Numerous investigations in the field of air, gas and refrigerating compressors carried out in our Department were the basis to work out a universal, one-dimensional model describing the processes occurring in the reciprocating compressor. The complex model describing thermodynamic gas in the compressor cylinder, taking into consideration the influence of installation have been worked out. The analysis showed that while creating the reciprocating compressor simulation model special attention should be paid to working valves dynamics. Hence in the mathematical description of the valves operation all the most important phenomena determining their work have been considered. The mathematical model has been worked out for plate valves most frequently used in a new types of compressors . The worked out model of valves, though one-dimensional can be successfully adopted for different type of valves with deformable working element such as tongue or channel valves which are frequently used in small or medium refrigerating compressors.

DESCRIPTION OF PROCESSES IN RECIPROCATING COMPRESSOR CYLINDER.

The mathematical model of valve action presented in this paper is an integral part of a complex model of the compressor, based on the I law of thermodynamics for the medium in the cylinder. The medium has been treated as real gas, and both the effect of leakage and heat exchange in the cylinder have been taken into account. After introducing differentiation with respect to crank angle φ one obtains the equation describing thermodynamic changes in the cylinder in the form:

$$\frac{dT}{d\varphi} = \frac{1}{m\,c_v}\frac{dQ}{d\varphi} - \frac{h}{m\,c_v}\frac{dm_t}{d\varphi} - \frac{h}{m\,c_v}\frac{dm_n}{d\varphi} + \frac{h_{cs}}{m\,c_v}\frac{dm_s}{d\varphi} - \frac{h}{m\,c_v}\frac{dm}{d\varphi} -$$

$$- \frac{T}{m\, c_v} \left(\frac{\partial p}{\partial T} \right)_v \frac{dV}{d\varphi} + \frac{vT}{m\, c_v} \left(\frac{\partial p}{\partial T} \right)_v \frac{dm}{d\varphi} \qquad (1)$$

In this equation the following notation has been used on T - temperature, p - pressure, v - specific volume, h - enthalpy, m - mass, h_{cs} - enthalpy of the medium entering the cylinder through suction valve, c_v - specific heat.

$\frac{dm_s}{d\varphi}$; $\frac{dm_t}{d\varphi}$; $\frac{dm_n}{d\varphi}$ - mass flow through suction, discharge valves and leakage, respectively

Equation (1) has been supplemented by the mass balance and kinematic relationship for compressor volume $V(\varphi)$. In our model the working medium can be treated as both ideal gas described by Clapeyron's equation and real gas describedby equation of state, adequate for the given medium. The refrigerant properties have been defined on the basis of Redlich-Kwong-Soave equation of state, introduced in the form of computer library.

The method of calculation of heat exchange rate in the cylinder has been based on the relations given in [7]. In the presented model this dependence has been made more complete by introducing, on the basis of global energy balance for cylinder wall, an iterative formula for calculating its mean temperature $T_{śc_i}$:

$$T_{śc_i} = \frac{\int_0^{2\pi} \frac{dQ}{d\varphi}^{i-1}(\varphi)d\varphi + 2\pi\ N_f}{2\pi\ A_z\ k_{ch}} + T_m \qquad (2)$$

where: index i denotes i-th crankshaft revolutism, N_f- power lost on friction in cylinder, A_z - area of outer surface of cylinder cooled by refrigerant, k_{ch}- substitute of overall heat transfer coefficient which does not consider convection coefficient from cylinder side.

The heat loss due to leakage in cylinder piston system has been described in a simple form experimentally confirmed given in [3]. In the program there has also been included the possibility of valve leakage simulation, which so far has been neglected.

Installation of the gas under compression is a wave-guide, which can be modeled as a system of series connected tubes of various diameters. The calculation of the run of changes in a pressure pulsation in this installation requires a solution of the partial differential equations which describe pulsating flow of gas with equivalent boundary conditions taken into account. The equations describing pulsating flow of gas in a straight pipeline of constant cross-section are considered below. These equations are derived by means of various simplifying assumptions on the basesof equations of movement, equation of continuity and equation of state Assuming that the gas density is constant and equal to its mean value with respect to time and linearizing nonlinear resistance expression, this equation can be presented in the following form:

$$\begin{cases} -\dfrac{\partial v}{\partial t} = \alpha \dfrac{\partial p}{\partial x} + \gamma v \\ -\dfrac{\partial p}{\partial t} = \beta \dfrac{\partial v}{\partial x} \end{cases} \tag{3}$$

where: $\alpha = \dfrac{A}{\rho_o}$, $\beta = \dfrac{\rho_o a^2}{A}$, $\gamma = \dfrac{\lambda w_o}{2D}$;

Symbol p denotes here variable pressure of gas, t - time, D - internal diameter of pipeline, a - sonic gas velocity, λ - friction losses factor, w_o - mean analytical gas velocity, A - cross-sectional area of the tube, ρ_o - mean gas density, v - variable volumetric velocity. The equations of non stationary flow have been solved on the basis of electro-acoustic analogy. This made it possible to solve this problem on the PC computer taking into consideration also multicylinder interaction problem.

GENERAL DYNAMIC MODEL OF COMPRESSOR SELF-ACTING VALVES

The movement of valve working plate with mass m (fig.1) can be described by equation:

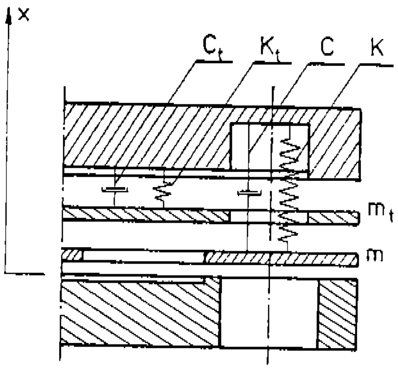

Figure 1. Phisical model of the self-acting valve.

$$\left[m + m_t H(x-x_t)\right] \ddot{x} + \left[C(\dot{x},x) + C_t(\dot{x},x)\ H(x-x_t)\right] + \left[K(x) + K_t(x)H(x-x_t)\right] =$$

$$= F_s - F_{10}\delta(x) + F_{l,1}\ \delta(x-h) + F_{lt}\delta(x-x_t) + S_0(R_0,\dot{x}_-)\delta(x) + S_-(R_1,\dot{x}_+)\delta(x-h) \tag{4}$$

In this equation the effect of dumping plate with mass m_t has been considered by introducing Heafiside function H(x), nonlinear force of elasticity K(x) and nonlinear force of dumping $C(x,\dot{x})$. Sticking of the working plate to seat F_{10}, valve stop F_{11} and dumping plate F_{1t} have also been taken into account. Moreover, collision impulse forces S_0 and S_1 depending on coefficient of restitution R_i and velocity before collision have been introduced. A two-parameter model of collision has been adopted. The flow force F_s on the valve plate has been determined on the basis of dependencies commonly adopted for the given type of valve, or on the ground of results of static tests.

Eq. (4) has been derived for plate valves but it is possible to extend this method for mathematical description of valve working plate dynamics. To make its application possible a method of calculating equivalent forces and concentrated masses has been worked out for this type of valves. As follows from both theoretical solutions [2] as well as experiments (Table I) the tonque valve plate vibrates when the valve is being opened with a frequency higher than the frequency of first harmonic of free vibrations. This means that the tonque valve plate should be treated as a vibrating system in which the vibration frequency is determined by cyclic rebounds of the valve plate tip against stop A method of calculating equivalent forces and concentrated masses is based on the assumption that the equivalent mass is concentrated at the seat geometric center and that the stream pressure is applied at the same point. Until contacting the valve stop the working plate behaves as a fixed rod loaded by a concentrated force It is assumed that the part of the rod off the point of force application is not deformed until it gets into contact with the stop. From the moment it contacts the stop, the plate is treated as a rod of length l fixed on one side, and on the other side support is rotational and slideable, loaded by a concentrated force (fig. 2).

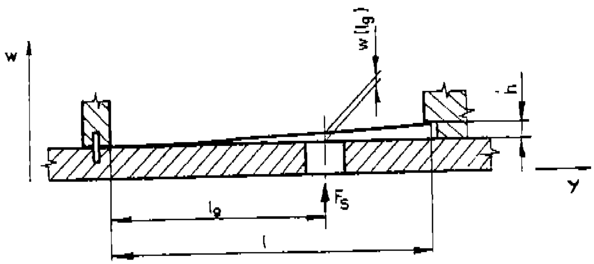

Figure 2. Scheme of a tongue valve plate

Depending on the relation between the stream pressure and elasticity, its tip may be torn away from the stop when the valve is opened, or it may remain in contact all the time. The values of forces induced by the plate elasticity vary in particular phases of its deformation. The numerical values of these forces can be defined experimentally or calculated on the basis of dependencies from the theory of elasticity. On the ground of this theory it is also possible to determine the lift at any point of the plate at any point of time.

As a result of elasticity force the valve plate, after reaching the maximal deflection, makes effort to return to its primary position. The height of displacement of the tip of the valve plate reaching the valve stop w(l) is equal to the maximal valve lift h and could be expressed by the formula:

$$h = w(l) = w(l_g) + w'(l_g)(l - l_g) \qquad (5)$$

in which the heigh of valve displacement at the point of flow force action is denoted as $w(l_g)$ and:

$$w(l_g) = \frac{F\, l_g^3}{3\, E \int_0^{l_g} \frac{J(y)}{y}\, dy} \qquad (6)$$

where: E - Young's modulus, J(y) - moment of inertia of valve plate in cross-section y, $w'(l_g)$-value of first derivative of function describing the plate shape as l_g point

Formula (5) makes it possible to calculate the displacement of the reduced at the concentrated mass at the moment the valve tip reaches the limiter.

The reduced mass has been calculated on the basis of conventional mechanical relation of the displacement equilibrium:

$$m_r = \frac{\sum_{i=1}^{n} m_i\, w(l_i)}{w(l_g)} \qquad (7)$$

In this equation m_i denotes the mass of tongue valve part with the displacement $w(l_i)$. The m_i values depends on the shape of the cross section in the corresponding fragment of valve plate The value of the reduced mass depends on the phase of valve movement. The calculated reduced values of mass m_r and displacement w(l) correspond with the values m, m_t, and x in equation (4).

To describe the state of medium flowing through the valve various equations of state can be applied. Most commonly the flowing medium is treated as ideal gas. The authors treat the medium flowing through the valve as real gas described by equation:

$$p = \frac{\sigma(p,T)\; R\; T}{V} \qquad (8)$$

the coefficient of compressibility σ being selected seperately for medium mean parameters for inlet or outlet valve. The flows through valves have been calculated on the basis of Costagliola's equations, derived with the assumption that the flow is steady. However to improve the model also the inertia of gas in the valve gap has been considered especially during opening and closing the valves. It has been decided to introduce the correction for gas inertia based on the assumption that the pressure difference on the valve gap changes in a linear way during the time that elementary mass of gas flows through the gap.

EXPERIMENTAL VERIFICATION OF THE SIMULATION PROGRAM WORKED OUT

The verification of the above presented mathematical model and the computer simulation program based on this model have been carried out on the basis of experimental investigations of two refrigerating compressors SAF-23 and SAF-5 of Polish make. Both compressors have been equipped with suction tongue valves (fig. 3) and discharge channel valves. The investigation has been carried out within a very broad range of variations of compressing ratios determined by standard temperatures of coolant vaporization and condensation. During the tests the pressure pulsations in the cylinder and valve plate displacement have been registered. The valve plate displacements have been recorded using our own construction capacitance transducer, coupled with the traditional measurement system consisting of the feeder, amplifiers and oscilloscope. The graph of plate displacements in the function of crank angle obtained on the oscilloscope screen have been photographed. In fig.4 the displacement of chosen point of the suction valve for both compressors obtained by measurements and simulation have been compared.

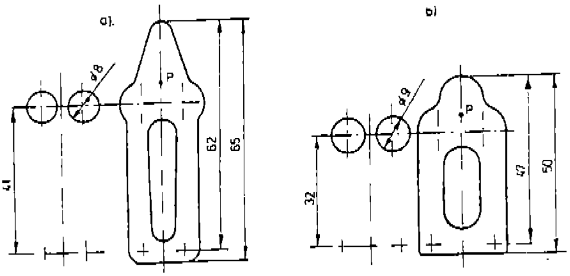

Figure 3. Shapes of suction valve tongue plates of
a)SAF-23 compressor, b) SAF-5 compressor

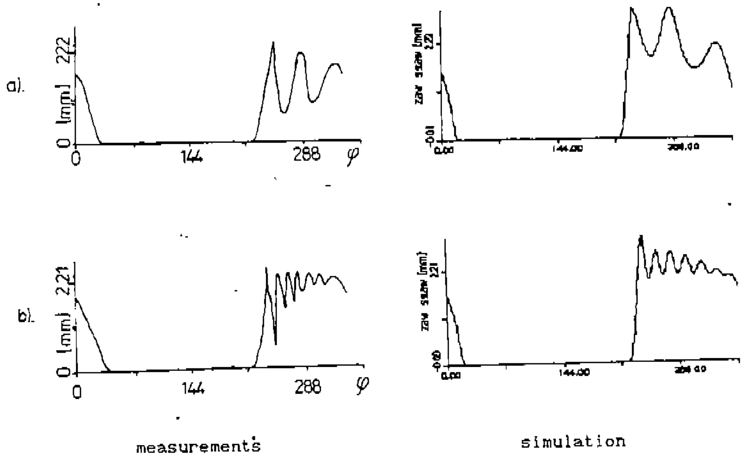

Figure 4. Displacement of chosen point of suction valve obtained by measurements and simulation. a) SAF-23. b) SAF-5.

In case of the SAF-23 compressor the used working medium used was air. This made it possible to define precisely the coefficients of medium flow through valves, by means of static tests, and besides it facilitated experimental registration of the valve plate displacement. In case of the SAF-5 compressor the investigations have been carried out using R-12 as a working medium, which allowed us to confirm the universality of the worked out model. Relatively simpler investigations of the SAF-23 compressor while pumping air, allowed us to carry out experiments over a broad range of working parameters. The tests have been made using valve plates with different thickness and the same shape. In fig.4a. the displacement of the valve plate for the recommended working parameters are shown. During compressor tests working in conditions significantly different from recommended ones the oscillations of a higher amplitude occurred. The valve plate worked with alternate collisions with valve seat and stop. The same kind of movement has been achieved by means of computer simulation.

Pressure pulsations in valve chambers have a fundamental influence on the valve work. That is why while comparing registered and calculated valve plate displacements also pressure changes in the cylinder and valve chambers should be compared.

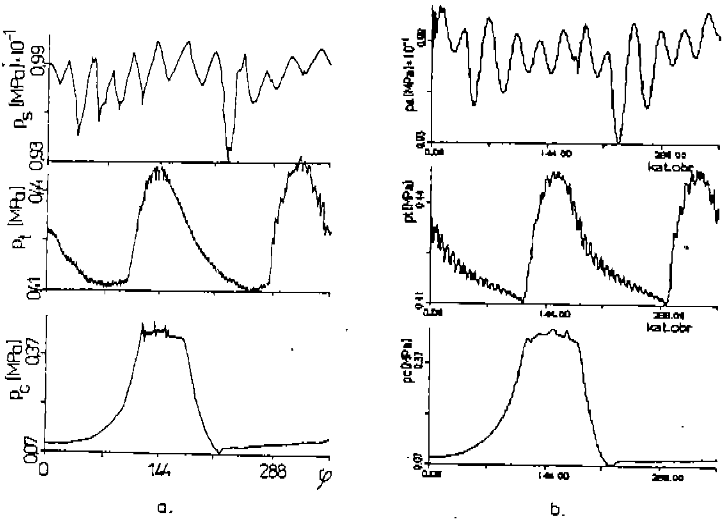

Figure 5. Pressure changes in the cylinder and valve chambers of SAF-23 compressor. a) measured b) calculated.

In fig.5 registered and calculated pressure pulsations in the cylinder and suction and discharge valve chambers for SAF-23 compressor are shown. They have been registered for the same working conditions as diagrams of suction valve displacements shown in fig.4a.

Interesting information could be provided by dynamic tests of the valve plates, carried out outside the compressor and especially its self-frequency oscillation tests. The results of test which have been carried out for suction valves of SAF-23 compressor are given in Tab.I.

TABLE I.

Oscillation frequency of the tongue valve.

Type of plate	Natural frequency of tongue valve [Hz]			Frequency of tongue valve during its work in compressor [Hz]	Remarks
mode	I	II	III		
"a"	45	290	320	----	without pin
	43	282	310	about 110	with pin
"b"	80	495	-	about 320	without pin

For the plate "a" from fig.3a the results of experiments for standard valve plate and the plate with pin, which is the inner screen of the capacitor, are shown. The oscillation frequency of the valve plate during its work in the compressor has been calculated on the basis of the experimental results. What needs attention is the fact that the frequency of plate vibration during its functioning in the valve is about 2.5 - 4 times higher than the original frequency of natural vibrations of this plate determined by means of an oscillator. It follows both from the change of the oscillation character from the moment that the plate reaches its limiter, as well as the effect of collision itself. This confirms the results of theoretical works done by the authors earlier on the effect of rebound from the limiter and seat on the frequency of working plate vibrations in the valve shown in /3/. Those results have been used for working out the presented model.

CONCLUSION

On the basis of the investigations it can be stated that the presented mathematical model describing the functioning of compressor automotive valves is universal in character. It can be easily adapted for description of functioning of various types of compressor valves. An example of such an adaptation for an automotive suction tongue valve of a small refrigerating compressor has been demonstrated in this paper. The presented model of valve plate displacements, in which the frequency of its vibrations is determined by the phenomenon of cyclic rebound of its tip from the limiter, is closer to reality than the clasical model which does not cover such rebounds. An advantage of our model is the fact that it makes it possible to estimate the impulse forces of the working plate colliding with the limiter and seat. The model, together with the description of the phenomena in the compressor cylinder and adjoining installation, allows an analysis of valves acting during compressor work with various cooling media. The properties of these coolants can be described by means of individually selected equations of state for real gas.

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