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ANALYSIS OF LEAKAGE FLOW THROUGH CLEARANCE ON ROTOR FACE IN VANE COMPRESSORS

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ABSTRACT

Performance of vane compressors used for automotive air conditioners is affected greatly by leakage which flows through a clearance between a rotor face and a sideplate. We analyze an instantaneous leakage flow and a time averaged one through the clearance on the rotor face under pressure boundary conditions changing periodically with rotation of the rotor. The flow field on the rotor face at each moment can be evaluated under a quasi-steady condition with an instantaneous pressure distribution on the rotor circumference. On the other hand, the average flow field on the rotor face must be analyzed under an averaged boundary condition which is an average of the pressure changing periodically on the rotor circumference.

NOMENCLATURE

A_k, B_k : Fourier coefficients
a_0, b_0 : Fourier coefficients
h : clearance on rotor face
n : approximate order of Fourier series
P : pressure
P_b : oil supply pressure ($= P_1$)
r : radius
t : time
v : velocity
z : ordinate of clearance height
θ : angular position on rotor face from axial seal
μ : coefficient of viscosity of oil
ν : coefficient of kinematic viscosity of oil
ρ : density of oil
ϕ : rotational angle of rotor
ω : angular velocity of rotor

Subscripts

b : boundary between gas and oil regions
r : radial direction
θ : tangential direction
1 : inner circumference of rotor
2 : outer circumference of rotor

Superscript

$\bar{\ }$: averaged value in height direction

INTRODUCTION

In recent years, a vane compressor is used for an automotive air conditioners by taking advantages of its small size and its light weight. Leakage which flows through a clearance between a rotor face and a sideplate greatly affects performance of the compressor /(1)/. There have been some studies which analyzed distributions of pressure and velocity on the rotor face based on the Navier-Stokes equation by applying the perturbation technique /(2)/, the Finite Element Method /(3)/ or the Finite Difference Method /(4)/. But in these studies, the flow field was analyzed under some special boundary pressure condition on the rotor circumference, and discussion about a practical leakage flow occurring under the periodic changing boundary pressure condition on the rotor circumference was insufficient.

In this study, we analyze an instantaneous leakage flow at each moment through the clearance on the rotor face based on the Navier-Stokes equation by applying the Fourier series analysis /(2)/, and examine an appropriate manner of a decision of the boundary pressure condition on the rotor circumference to analyze an averaged leakage flow.

THEORETICAL ANALYSIS

Pressure Distribution

Figure 1 shows a schematic view of the vane compressor and a modeling of the flow field between the rotor face and the sideplate. Lubricating oil is supplied from an oil groove on the sideplate to the rotor face. The flow field of leakage flow through the clearance on the rotor face is modeled as that between a stationary disc and a rotating disc on which circumference the pressure changes periodically with the rotor rotation. The leakage flow in this flow field is governed by the Navier-Stokes equations and an equation of continuity, and these equations are expressed as follows since the clearance is very small by comparison with the flow field.

$$\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\frac{\partial^2 v_r}{\partial z^2} \tag{1}$$

$$\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta} + \nu\frac{\partial^2 v_\theta}{\partial z^2} \tag{2}$$

$$\partial(r v_r)/\partial r + \partial v_\theta/\partial \theta = 0 \tag{3}$$

In the above Navier-Stokes equations, an inertial term, a centrifugal term and a coriolis term are assumed to be negligible since the clearance on the rotor face is very small. By integrating equations (1) and (2) respectively and substituting boundary conditions on both the stationary and the rotational discs into those equations, radial and tangential velocities are derived as follows.

$$v_r = (z^2 - hz)(\partial P/\partial r)/(2\mu) \tag{4}$$

$$v_\theta = (z^2 - hz)(\partial P/\partial \theta)/(2\mu r) + zr\omega/h \tag{5}$$

Substituting equations (4) and (5) into equation (3), we obtain the Laplace equation for pressure.

$$\partial(r\partial P/\partial r)/\partial r + (\partial^2 P/\partial \theta^2)/r = 0 \tag{6}$$

The Laplace equation for what is called the Dirichlet problem is solved by applying the Fourier series /(2)/. The pressure on an inner circumference is assumed to be constant (=P_b) and that on an outer circumference of the rotor is expressed as follows.

$$P_2 = A_0 + \sum_{k=1}^{n}(A_k\cos k\theta + B_k\sin k\theta) \tag{7}$$

Where, coefficients in equation (7) are derived by discrete Fourier approximation using sampled data of the pressure on the rotor circumference. By using these boundary pressure distributions, the pressure distribution on the rotor face is derived as follows.

$$P = a_0 + b_0\log r + \sum_{k=1}^{n}(A_k\cos k\theta + B_k\sin k\theta)\frac{(r/r_1)^k - (r_1/r)^k}{(r_2/r_1)^k - (r_1/r_2)^k} \tag{8}$$

where

$$a_0 = (A_0\log r_1 - P_b\log r_2)/\log(r_1/r_2)$$

$$b_0 = (P_b - A_0)/\log(r_1/r_2)$$

Velocity Distribution

In this study, we will express the velocity distribution with an averaged one in height direction. The radial and tangential average velocities are derived as follows by integrating equations (4) and (5) respectively.

$$\overline{v_r} = -(\partial P/\partial r)h^2/(12\mu) \tag{9}$$

$$\overline{v_\theta} = -(\partial P/\partial \theta)h^2/(12\mu r) + r\omega/2 \tag{10}$$

In the above equations, pressure gradients are given by differentiating equation (8). In the Fourier analysis, in general, it is not appropriate to use the gradient obtained by differentiating a function expressed by the Fourier series, and we will discuss this problem at the later section.

Gas Flow Region

In the neighborhood of an axial seal, refrigerant gas as a working fluid flows through the clearance on the rotor face from a high pressure compression chamber into a suction chamber, and the leakage greatly affects the performance of the compressor. In a practical use, the gas flow region on the rotor face near the axial seal can be solved sufficiently under a flow field with a single phase of oil by approximation with a group of streamlines which have the velocity of inner direction on the rotor circumference /(5)/. In the present study, we analyze the average gas flow region using the average velocity distribution. A boundary line between the gas and the oil regions is obtained by solving the following equations applying the Runge-Kutta method with an initial position where the gas begins to enter the clearance on the rotor face.

$$\left.\begin{aligned} dr_b/dt &= \overline{v_r} \\ d\theta_b/dt &= \overline{v_\theta}/r \end{aligned}\right\} \tag{11}$$

Boundary Pressure Condition

Since the rotor circumference faces the compression chambers, the pressure on the rotor circumference changes periodically with the rotor rotation. Table 1 shows specifications of the compressor studied here and analytical conditions, and Figure 2 shows volume and pressure changes of the compression chamber leading a vane located at rotational angle of ϕ.

Taking into account of periodic pressure change with the rotor rotation, we analyzed the leakage flow through the clearance on the rotor face under the following three different boundary pressure conditions on the rotor circumference. The first boundary condition is an instantaneous pressure distribution corresponding to the rotor rotation (denoted as Boundary condition 1). The pressure distribution on the rotor circumference changes periodically every $2\pi/3$ rad, since the compressor studied here has three vanes. Figure 3 shows the instantaneous pressure distribution on the rotor circumference corresponding to the rotational angle of 0, $\pi/6$, $\pi/3$ and $\pi/2$ rad. In this figure, the pressure distribution at positions of the vane and the axial seal is assumed to be linear.

The second one is an average pressure distribution that the pressure at each angular position on the rotor circumference is equal to the time average of the pressure in the compression chamber which is facing that position. It is shown in Figure 4 with a solid line (Boundary condition 2).

The last one is a linear pressure distribution from the suction side to the discharge side which is employed in the past studies /(2)-(4)/, and shown in Figure 4 with a broken line (Boundary condition 3). The analytical results corresponding to the each boundary conditions are compared with one another.

RESULTS AND DISCUSSION

Accuracy of Pressure Gradient

At first, we discuss accuracy of the pressure gradient obtained by differentiating the function expressed by the Fourier series. As an example, we calculate the tangential pressure gradient on the rotor circumference by differentiating equation (7) when the rotational angle of the rotor is $\pi/2$ rad, and compare it with an analytical pressure gradient. Figure 5 shows the analytical pressure gradient and the pressure gradients which are obtained by differentiating equation (7) whose number of sampling points is 360 and approximate order is 180 and 120 respectively. As shown in these figures, the pressure gradients obtained by differentiating equation (7) fluctuate at the position where the pressure gradient changes abruptly. The fluctuation of the pressure gradient depends on the number of sampling points and the approximate order of the Fourier series, and they must be selected carefully. But an error of the velocity caused by the error of the pressure gradient is much smaller than the second term on the right hand of equation (10), and this error disappears when we integrate the velocity to estimate the leakage flow rate. The velocity distribution, therefore, can be solved sufficiently by differentiating the Fourier series in a practical use. Once the pressure and velocity distributions are expressed by the Fourier series, it is useful that the center of pressure, flow rate and shearing force of oil for an arbitrary rotational angle of the rotor can be expressed by the Fourier series. In the following analysis, we employ the Fourier series whose number of sampling points is 360 and the approximate order is 120.

Instantaneous Flow Fields

Figure 6(a)-(d) show the instantaneous pressure and velocity distributions on the rotor face under the boundary pressure condition 1 (rotational angle $\phi = 0, \pi/6, \pi/3$ and $\pi/2$ rad). In these figures, mark '∇' indicates the position of the axial seal (θ = 0 rad), and mark ']' indicates the position of the vane at each moment. As shown in these figures, lubricating oil flows from the inside to the outside of the rotor for the most part, and high pressure refrigerant gas passes through the clearance on the rotor face into the suction side in the neighborhood of the axial seal.

The pressure and velocity distributions shown in Figure 6 are steady flow fields under each boundary conditions shown in Figure 3, and these can be regarded as the instantaneous flow field only when the inertial force is as small as negligible. Figure 7 shows magnitude of the inertial force in radial direction against the viscous force about fluid element existing at middle height of the clearance on the rotor circumference when the rotational angle of the rotor is $\pi/2$ rad. Figure 7(a) is radial velocity at the middle height of the clearance on the rotor circumference, (b) is magnitude of unsteady term of inertial force which is the first term on the left hand of equation (1), and (c) is magnitude of convective term of inertial force which is the second and the third terms on the left hand of equation (1). These figures show that the radial velocity and the inertial force have sharp peaks at the positions of the vane ($\theta = 7\pi/6, 11\pi/6$ rad) and the axial seal (θ = 0 rad). However, even when the rotational speed and the clearance increase, portions where the inertial force is big are local and have little influence on whole flow field and whole flow rate. The flow field on the rotor face at each moment, therefore, can be evaluated under the quasi-steady condition with the instantaneous pressure distribution on the rotor circumference.

Average Flow Fields

Figure 8 and 9 show the average pressure and velocity distributions corresponding to the second and the third boundary conditions respectively. In the case of boundary condition 2, the lubricating oil flows from the inside to the outside of the rotor at most of the rotor face. On the other hand, the oil which flows from the outside to the inside of the rotor increases under boundary condition 3 since the high pressure region on the rotor circumference is larger than that of the boundary condition 2. The flow rate of the leakage oil which flows out from the clearance on the rotor face estimated by integrating the radial velocity on the rotor circumference is 3.40 cm^3/s under the boundary condition 2 and 1.16 cm^3/s under the boundary condition 3. The average leakage flow rate estimated as the average of the instantaneous flow rate at each moment is equal to that under boundary condition 2.

Figure 10 shows the gas flow regions on the rotor face estimated by using the average velocity distributions under the boundary conditions 2 and 3 respectively. The gas flow regions which have great influence on the performance of the compressor are quite different from each other. Thus when we analyze the average flow field on the rotor face, it is appropriate to use the boundary condition that the pressure at each angular position on the rotor circumference is equal to the time average of the pressure in the compression chamber which is facing that position.

CONCLUSIONS

Based on the Navier-Stokes equation, we analyzed the instantaneous and time averaged leakage fields on the rotor face clearance by applying the Fourier series analysis under the more realistic boundary pressure conditions which changes periodically with the rotor rotation. The results are summarized as follows.

(1) In the practical use, once the number of sampling points and the approximate order of the Fourier series are selected adequately, the velocity distribution can be solved sufficiently by applying the differentiation of the Fourier series.

(2) The flow field on the rotor face at each moment can be evaluated under the quasi-steady condition with the instantaneous boundary pressure distribution on the rotor circumference because of less influence of the inertial force than that of the viscous force.

(3) The average flow field on the rotor face must be analyzed under the boundary condition that the pressure at each angular position on the rotor circumference is equal to the time average of the pressure in the compression chamber which is facing that position.

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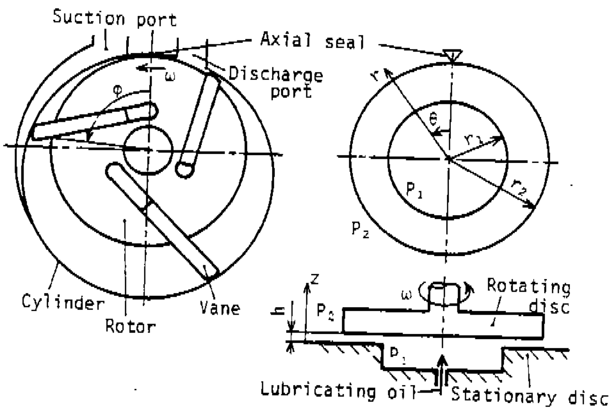

Figure 1 Schematic view of vane compressor and analytical model

Table 1 Specifications of compressor and analytical condition

Number of vane		3
Cylinder radius	r_c	36.3 mm
Rotor radius	r_2	28.8 mm
Oil groove radius	r_1	20.0 mm
Gap height	h	30.0 μm
Suction pressure	P_s	0.309 MPa
Discharge pressure	P_d	1.52 MPa
Oil supply pressure	P_b	1.06 MPa
Oil viscosity	μ	0.005 Pa·s
Rotational speed	N	2000 rpm

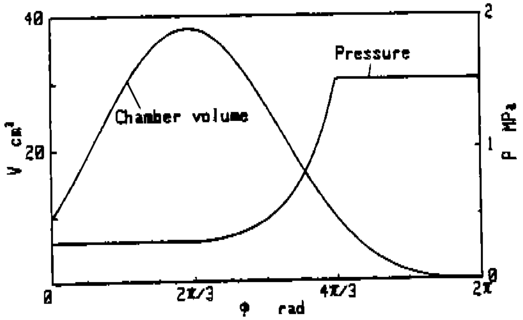

Figure 2 Volume and Pressure changes of compression chamber

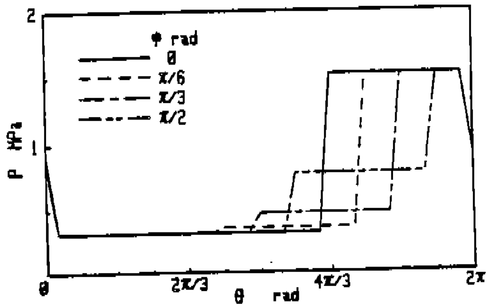

Figure 3 Boundary pressure condition 1 on rotor circumference

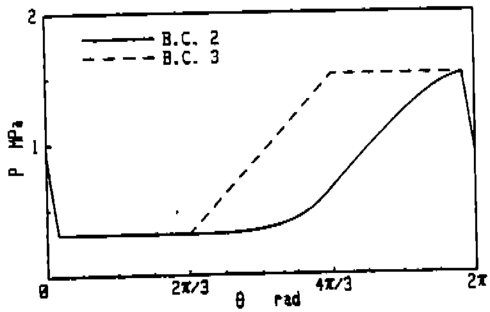

Figure 4 Boundary pressure condition 2 and 3 on rotor circumference

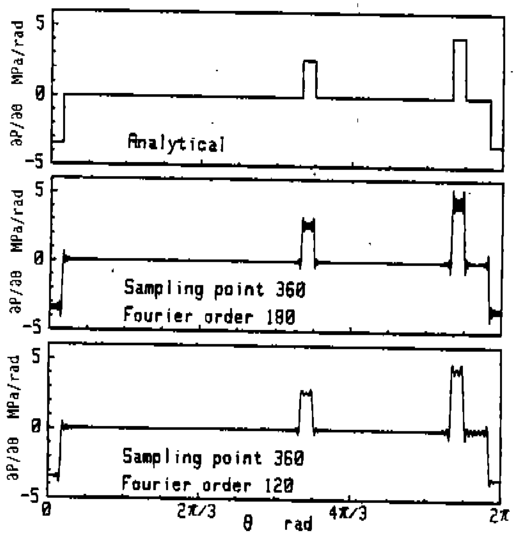

Figure 5 Pressure gradients obtained by differentiating Fourier series ($\phi = \pi/2$ rad)

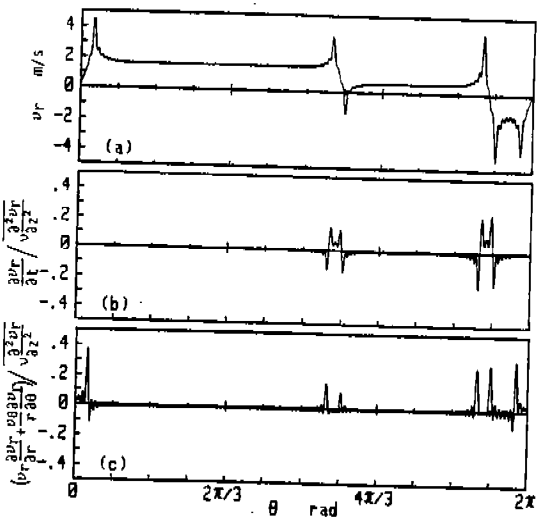

Figure 7 Magnitude of inertial force against viscous force ($\phi = \pi/2$ rad, $z = h/2$)

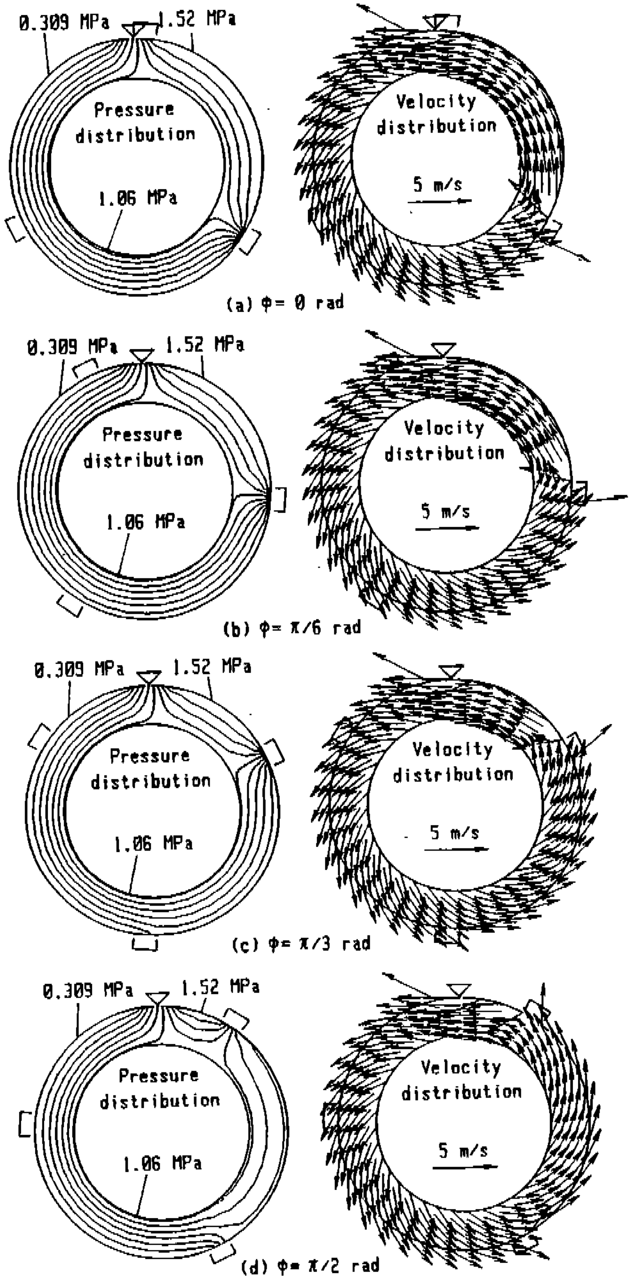

Figure 6 Instantaneous distributions of pressure and velocity under boundary condition 1

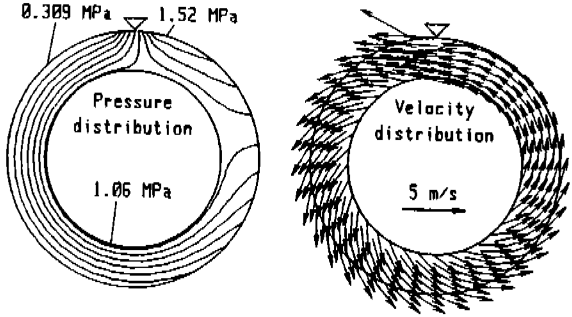

Figure 8 Average distributions of pressure and velocity under boundary condition 2

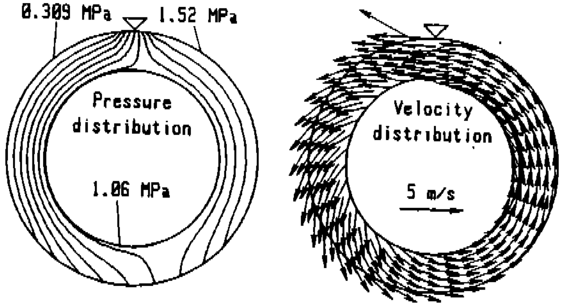

Figure 9 Average distributions of pressure and velocity under boundary condition 3

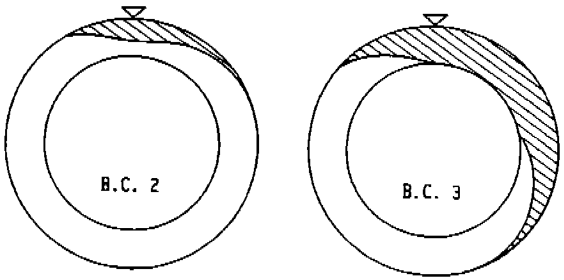

Figure 10 Gas flow region