**STRESSES AND DEFLECTIONS IN
CONTINUOUSLY REINFORCED
CONCRETE PAVEMENTS**

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**PURDUE UNIVERSITY
LAFAYETTE INDIANA**

by

M.M. Miller, Jr.

FINAL REPORT

STRESSES AND DEFLECTIONS IN
CONTINUOUSLY REINFORCED CONCRETE PAVEMENTS

TO: K. B. Woods, Director
Joint Highway Research Project

January 30, 1958

FROM: H. L. Michael, Assistant Director

File: 5-11-13

Project: C-36-46M

Attached is a final report entitled, "Stresses and Deflections in Continuously Reinforced Concrete Pavements," by M. M. Miller, Jr. Mr. Miller also utilized this report as his thesis in partial fulfillment of the requirements for the M.S.C.E. degree. The research was performed under the direction of M. J. Gutzwiller and J. L. Waling.

The research reported in this study is an analytical study of the stresses and deflections that occur in continuously reinforced concrete pavements and is of particular interest in the area of pavement design.

Respectfully submitted,

Harold L. Michael

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File: 5-11-13
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The author wishes to express his gratitude for having been able to actively participate in the experimental projects at Purdue University which parallel this investigation. Such participation as well as the discussions with the investigators of these projects has provided the author with necessary background for his own investigation.

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LIST OF SYMBOLS

L	Total length of slab	inches
ℓ	Length of Individual Segments	inches
n	Total number of segments	
ϕ	Angle change at a crack	radians
P	Transverse load (Vertical)	lbs/inch
N	Longitudinal load (Horizontal)	lbs/inch
V_j	Shear at crack j	lbs/inch
M_j	Moment at crack j	inch-lbs/inch
k	Subgrade modulus	lbs/cubic inch
F_j	Subgrade force on a segment	lbs/inch
C	Assumed ratio M/ϕ	inch-lbs/inch/radian

ABSTRACT

Miller, Melton M. Jr. M.S.C.E., Purdue University, January 1958.

Stresses and Deflections in Continuously Reinforced Concrete Pavements.

Major Professor: Martin J. Gutzwiller.

This investigation is primarily concerned with the investigation of stresses in and deflections of continuously reinforced concrete pavements under the action of static transverse and longitudinal forces acting simultaneously. The pavement is assumed to be made up of a series of segments produced by equally spaced cracks, ℓ being the spacing between cracks. The segments are assumed to be straight between cracks, the angle changes, ϕ_n , in the slab being concentrated at the cracks. Further assumptions are:

- 1) The moment at a crack is some function of the angle change, i.e., $M/\phi = C$,
- 2) The subgrade modulus, k , is a constant throughout the full range of deflection, and
- 3) The deflection at a point some distance from the transverse load is equal to zero.

By considering the geometry of the deflected pavement and the equilibrium of each segment, a series of simultaneous equations may be written in terms of either the deflections at the cracks or the angle changes at the cracks. The equations expressed as functions of the angle changes are by far the simpler; therefore, this approach is used. Equations for deflections and shears at cracks are given.

Equations are written for total lengths of slab equal to 8ℓ , 12ℓ , 16ℓ , and 20ℓ . Combinations of two values of the parameters k , C , and N

were used with $P = 250 \text{ lbs./in.}$ and $\ell = 30 \text{ inches}$, giving a total of 24 different cases solved. In the solutions, the functional relationship between ϕ and M was assumed to be the constant C , therefore, the solutions obtained are approximate and are shown as illustrations of the method. Experimentation is in progress, at the time of this writing, to determine the functional relationship between ϕ and M .

The ultimate purpose of this investigation is to point the way for experiments leading to a rational method for the design of continuously reinforced concrete pavements. With accurate information as to the nature of the relationship between M and ϕ , a series of solutions may be worked out to cover any combination of slab thickness, percentage of steel, load, and subgrade condition.

HISTORICAL BACKGROUND

The theoretical analysis of a pavement slab, either reinforced or unreinforced, is a highly complex problem. If all of the possible physical characteristics of the loads, slab, and the support, i.e., the subgrade, could be idealized, the remaining problem of computing stresses and deflections is very complicated if at all possible. The fact that neither the loads, slab, nor the support is ideal produces the situation that engineers have been faced with since the paving of the first roadway. Many analyses of pavement slabs have been directed toward the development of a rational design of the pavement.

The first concrete pavements were constructed toward the end of the nineteenth century. From 1891 to 1893 concrete pavements were placed around the courthouse in Bellefontaine, Ohio. These pavements were unreinforced and stood up well as a durable wearing surface (1).* The use of concrete pavements developed quite rapidly through the early part of the twentieth century (2).

It was not until 1925, when H. M. Westergaard began his work on pavement slabs, that any rational analysis of a rigorous nature was done. A number of attempts at analysis had been made previous to 1925 but these yielded results which were quite inconsistent with what was known about the matter at the time. The most important of the analyses was the corner formula derived by Goldbeck (3). This analysis assumes the corner acts as a cantilever with no subgrade support. A later Westergaard analysis improves upon this formula for the computation of stresses in the region of a corner.

Westergaard's analysis (4) for plain concrete slabs involved the

* Numbers in parenthesis refer to item number in Bibliography.

computation of stresses at three critical points in the slab. The points were the corner, the edge, and the interior of the slab. In the analysis, he considered only the effects of wheel loads and neglected the effects of variations in temperature and other volume changes, the possible non-uniform thickness of the slab, the non-uniformity of the subgrade, and the dynamic characteristics of the slab and subgrade.

In 1927 Westergaard (5) presented an analysis of the deflections caused by temperature changes. This analysis derived formulas for stresses and deflections caused by a uniform change in temperature throughout the slab and by a temperature gradient throughout the slab. The stresses are worked out for points near the center of a broad, long slab, points near the edge of a broad, long slab, and points near the edge of a long slab of relatively narrow width. Tables are presented with various values of subgrade reaction and a temperature differential of 10° .

An experimental project designed to provide information on the rational design of pavements was initiated in 1930 by the Bureau of Public Roads at the Arlington Experiment Farm in Virginia (6). The stresses observed at interior points were not as great as those computed by Westergaard's equations. As a result Westergaard revised his analysis (7) in regard to his treatment of the subgrade reaction. Instead of assuming a linear relationship between deflection and subgrade reaction throughout the total area of the slab, he took into account the "bunching" of reaction forces under the point of application of load. This bunching of the reaction forces under the loads would reduce the deflection and stresses in this area. The result of the revision is a more accurate formula for the computation of stresses in the slab.

A number of simplifications and revisions of Westergaard's basic formulas as well as some empirical formulas were developed (8) in the years 1930 to 1939. These analyses did not offer a basically different approach than that of Westergaard's but they did contribute some knowledge of the action of unreinforced pavements.

Reinforcing concrete pavements with steel had come into use by 1931 (9). Joints had been reinforced for a number of years. The purpose of joint reinforcement was basically to provide load transfer across the joint. The principal use of longitudinal reinforcement was to prevent the formation of free edges in a slab where the slab was not thick enough to resist the imposed stresses. Whenever a transverse crack forms in an unreinforced pavement there is formed a free edge. In the center of the slab the pavement was not thick enough to resist the stresses which accompany a free edge; therefore, further breakdown of the slab was eminent. This problem was not as serious with pavements of uniform thickness as with the so-called balanced-design or thickened-edge pavements, but the formation of random transverse cracks was troublesome none-the-less.

Longitudinal reinforcement in conventionally reinforced pavements is generally designed to resist the stresses caused by a uniform change in temperature. The lowering of the temperature causes a shortening of the pavement which is partially resisted by the friction between the pavement and the subgrade. This resistance produces tension in the pavement which may crack the concrete. The reinforcement in the pavement takes up this tensile force and keeps the cracks from opening. Equations relating the required area of steel to the temperature drop, the length of uncracked pavement, the coefficient of thermal contraction,

the coefficient of subgrade friction and the allowable stress in the reinforcing steel have been derived and used in proportioning the reinforcement for pavement slabs. In this type of pavement design, joints are provided at intervals along the pavement. A possible spacing of contraction joints varies from 15 to 25 feet with expansion joints at intervals of 100 feet or less (9).

The design of joints and joint reinforcement is a problem which has also troubled pavement designers considerably. The inherent weakness of a joint has led to the logical consideration of a pavement in which no contraction joints are provided and expansion joints are provided only at locations where continuity of longitudinal reinforcement must be broken. Such a pavement provides ideal riding characteristics and eliminates the troublesome joints. However, the previous knowledge about the action of pavements is for the most part inapplicable and a new approach to the problem of the design of continuously reinforced concrete pavements is necessary.

The use of continuous reinforcement in long pavements does not prevent the formation of transverse cracks. In fact, the presence of the relatively heavy reinforcement causes the cracks to be more numerous than in an ordinary pavement. The fact that there are a greater number of cracks in the pavement is no problem as long as the cracks can be held tightly closed. On the few test roads of continuously reinforced pavements in existence this is apparently the case. Cracks are numerous but the cracks have not opened to a harmful extent in most instances. The results of the field experimental projects indicate that a properly designed continuously reinforced pavement will function adequately and perhaps more economically than conventional pavements (10).

The first statement of the principles concerning continuously reinforced concrete pavements was made in 1947 by W. R. Woolley (11). The basic ideas governing crack spacing and steel stress are set forth by Woolley. The previously mentioned field projects were designed to test some of his theories. The field tests have covered a wide range of actual field conditions but have been unsatisfactory in-so-far as having provided any real basic information which might be used in a comprehensive analysis of a continuously reinforced concrete pavement.

In 1955 Purdue University undertook the investigation of continuously reinforced concrete pavements on a laboratory basis. Specimens having the dimensions of 28 feet long by 3 feet wide by 8 inches deep were tested under the combined action of vertical and horizontal loads. The vertical loads simulated wheel loads while the horizontal loads simulated temperature and shrinkage forces. The slab was supported on a specially designed rubber base to represent the action of the subgrade. Slabs were tested using both deformed bar and welded wire reinforcement. The information obtained to date is presented in two theses (12, 13). The steel stress at cracks, the crack widths, and the deflections in the slab as functions of the combinations of loads are shown. These two theses are but the first phase of a continuing research and more complete reports will be prepared when the subsequent phases are completed.

At the same time the above research was being carried on at Purdue University, a theoretical research was sponsored by the Purdue University-State of Indiana Joint Highway Research Project. This theoretical research is the subject of the following discussion.

THEORETICAL DEVELOPMENT

This analysis of a continuously reinforced pavement with transverse cracks occurring at intervals involves four major assumptions, as follows:

- (a) the transverse cracks occur in some regular pattern, here assumed to be equally spaced,
- (b) the pavement is relatively straight between cracks,
- (c) the subgrade modulus is constant throughout the whole range of deflection, and
- (d) the deflection at a point some distance from a transverse load is zero.

The first assumption is reasonably correct in that a pavement of more than a few hundred feet in length may be considered infinite in length and the "middle" portion, which is completely anchored against movement, will have more or less equally spaced cracks. The second assumption is also quite correct. Observations on test specimens in the laboratory have shown that the slabs have very small curvatures between cracks. It would be expected that the cracks being relatively flexible compared to the pavement between cracks would account for most of the change in slope of the deflected slab. Assumptions (c) and (d) are well known assumptions used in most considerations of long beams on elastic foundations. In the following analysis of the slab an element one inch wide is considered.

A slab under the action of a transverse load will deflect until the sum of the subgrade reaction forces and the shearing forces on the ends of the slab equal the value of the load. Such a deflected configuration is assumed in Figure 1. The angle changes, ϕ_n , are shown in accordance

Typical Deflected Pavement

Figure 1

Forces on a Typical Segment

Figure 2

with assumption (b). Deflections at each of the cracks are written in terms of the angle changes as follows:

$$\begin{aligned}
 \Delta_1 &= \ell \phi_0 \\
 (1) \quad \Delta_2 &= \Delta_1 + \ell(\phi_0 + \phi_1) = \ell(2\phi_0 + \phi_1) \\
 \Delta_3 &= \Delta_2 + \ell(\phi_0 + \phi_1 + \phi_2) = \ell(3\phi_0 + 2\phi_1 + \phi_2) \\
 \Delta_n &= \ell(n\phi_0 + (n-1)\phi_1 + (n-2)\phi_2 + \dots + \phi_{n-1})
 \end{aligned}$$

The equations

$$(2) \quad \Delta_0 = 0 ; \quad \Delta_n = 0$$

satisfy the requirements of assumption (d), and provide a necessary relationship in the final solution.

The total force exerted on each segment by the subgrade is equal to k times the volume of the subgrade displaced by that segment. Such a typical segment is shown in Figure 2 (a). The volume of the subgrade displaced is equal to the average of the end deflections of the segment times the length ℓ . The resultant force is:

$$(3) \quad F_j = \ell/2 (\Delta_j + \Delta_{j+1})k$$

Figure 2(b) shows the complete force system acting on a typical segment. The equilibrium of each segment, the vertical equilibrium of the slab as a whole, and the conditions of equations (2) provide sufficient equations to solve for either the unknown deflections or the unknown angle changes. The unknown forces will be expressed in terms of the deflections and then equations (1) will be used to obtain the final form of the equations in terms of angle changes.

For simplicity the slab will be assumed to be loaded symmetrically, P being at the center line and a horizontal load N at each end. A certain number of segments, say eight, will be used for the purposes of definiteness. Because of the symmetry it is necessary only to consider half the

length, the right half being a mirror image of left half.

The equations for deflections are:

$$\begin{aligned}
 \Delta_0 &= 0 \\
 \Delta_1 &= \ell \phi_0 \\
 \Delta_2 &= \ell(2\phi_0 + \phi_1) \\
 (4) \quad \Delta_3 &= \ell(3\phi_0 + 2\phi_1 + \phi_2) \\
 \Delta_4 &= \ell(4\phi_0 + 3\phi_1 + 2\phi_2 + \phi_3) \\
 \Delta_8 &= \ell(8\phi_0 + 7\phi_1 + 6\phi_2 + 5\phi_3 + 4\phi_4 + 3\phi_5 + 2\phi_6 + \phi_7) \text{ or} \\
 \Delta_8 &= \ell(8\phi_0 + 8\phi_1 + 8\phi_2 + 8\phi_3 + 4\phi_4)
 \end{aligned}$$

The equations for the shears at each crack are:

$$\begin{aligned}
 V_0 &= V_0 \\
 V_1 &= V_0 + \ell k \Delta_1/2 \\
 (5) \quad V_2 &= V_0 + \ell k (\Delta_1 + \Delta_2/2) \\
 V_3 &= V_0 + \ell k (\Delta_1 + \Delta_2 + \Delta_3/2) \\
 V_4 &= V_0 + \ell k (\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4/2)
 \end{aligned}$$

The equilibrium of each segment taken individually yields the following equations:

$$\begin{aligned}
 M_1 + V_0 \ell + F_1 r_1 - M_0 - N \Delta_1 &= 0 \\
 (6) \quad M_2 + V_1 \ell + F_2 r_2 - M_1 - N(\Delta_2 - \Delta_1) &= 0 \\
 M_3 + V_2 \ell + F_3 r_3 - M_2 - N(\Delta_3 - \Delta_2) &= 0 \\
 M_4 + V_3 \ell + F_4 r_4 - M_3 - N(\Delta_4 - \Delta_3) &= 0
 \end{aligned}$$

The form of the final equations is somewhat simplified if the following approximation is made. By substituting $V_n = V'_n - \frac{N \Delta_n}{\ell}$ into equations (6) they become:

$$\begin{aligned}
 (6) \quad M_1 + V'_0 \ell + F_1 r_1 - M_0 - N \Delta_1 &= 0 \\
 M_2 + V'_1 \ell + F_2 r_2 - M_1 - N \Delta_2 &= 0
 \end{aligned}$$

$$M_3 + V_2' \ell + F_3 r_3 - M_2 - N \Delta_3 = 0$$

$$M_4 + V_3' \ell + F_4 r_4 - M_3 - N \Delta_4 = 0$$

The values for V_n may be calculated when the solution for the deflections is obtained.

In equation (5), setting $V_4 = P/2$ and solving for V_0 yields:

$$(7) \quad V_0 = P/2 - k (\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4/2)$$

Substitution of equations (7) and (4) into the remaining equations (5) and (6) yields the following expressions in terms of the ϕ 's alone:

$$\begin{aligned} (47/6 \ell^2 k + N + C/\ell) \phi_0 + (4.5 \ell^2 k - C/\ell) \phi_1 + \ell^2 k (2\phi_2 + 0.5\phi_3) &= P/2 \\ (41/6 \ell^2 k + 2N) \phi_0 + (13/3 \ell^2 k + N + C/\ell) \phi_1 + (2 \ell^2 k - C/\ell) \phi_2 + 0.5 \ell^2 k \phi_3 &= P/2 \\ (8) \quad (29/6 \ell^2 k + 3N) \phi_0 + (10/3 \ell^2 k + 2N) \phi_1 + (11/6 \ell^2 k + N + C/\ell) \phi_2 + \\ &\quad (0.5 \ell^2 k - C/\ell) \phi_3 = P/2 \\ (11/6 \ell^2 k + 4N) \phi_0 + (4/3 \ell^2 k + 3N) \phi_1 + (5/6 \ell^2 k + 2N) \phi_2 + (\ell^2 k/3 + N + \\ &\quad C/\ell) \phi_3 = P/2 \end{aligned}$$

In equation (4), setting $\Delta_8 = 0$ yields:

$$(9) \quad \phi_0 + \phi_1 + \phi_2 + \phi_3 + 0.5\phi_4 = 0$$

This equation along with equations (8) gives five equations with five unknowns which can be solved simultaneously.

In these equations the coefficients may be calculated after a choice is made of the values for the parameters ℓ , k , and N . The parameter C need not be a constant but might be some function of ϕ itself. In this instance the equations might be non-linear in terms of ϕ but would still yield a solution under normal physical conditions. The solutions discussed here are based on C equal to a constant for lack of better information.

The equations for the cases of 20, 16, and 12 segments of equal length are given in Appendix I.

Due to the inherent symmetry of an infinitely long pavement it is unnecessary to consider a case where P is not at the center crack. However, for finite slabs with $N = 0$, this possibility might be critical.

In this case a similar set of equations may be written involving all of the angle changes and deflections as well as two different end shears, V_0 and V_n . A more formal statement of the equilibrium and boundary conditions makes this problem clearer.

Consider a slab with n segments, and a load P at the j th point. Consider also as unknowns the deflections, $(n + 1)$ in number; and the two end shears, V_0 and V_n . This is a total of $(n + 3)$ unknowns. The equilibrium condition of each segment yields n equations. The two boundary conditions $\Delta_0 = 0$, $\Delta_n = 0$ yield two more equations. The last equation which is necessary is obtained by considering the shear condition at the point of load. The condition is that the numerical sum of the shears to the left and to the right must equal the applied loads or:

$$(10) \quad |V_j \text{ (right)}| + |V_j \text{ (left)}| = P$$

By substituting appropriately, the results obtained previously for the symmetrical case may be verified and similar equations may be obtained for the unsymmetrical case.

While the equations are quite simple when the crack spacing, ℓ , is considered a constant, they are not much more complicated if some other arrangement of cracks is assumed. With a given crack distribution, either symmetrical or unsymmetrical, it would be necessary only to express each interval as some multiple of a unit length and carry these multipliers along in the equations.

PRESENTATION OF RESULTS AND CONCLUSIONS

The equations which are listed in Appendix I were solved for the combinations of parameters shown in Table I. In each case P equals 250 pounds per inch and ℓ equals 30 inches.

Table I

Combination No.	C-inch-lbs/inch/rad.	N-lbs/inch	k-lbs/cubic inch
1	2.5×10^6	1000	150
2	2.5×10^6	1000	440
3	2.5×10^6	0	150
4	2.5×10^6	0	440
5	0	1000	150
6	0	1000	440

The above six combinations are used for each of the four slab lengths; namely 8 ℓ , 12 ℓ , 16 ℓ , and 20 ℓ , giving a total of 24 different solutions.

The curves in Figures 3 through 8 show the results of a slab of length 8 ℓ . Figures 9 and 10 show partial results for a slab of length 20 ℓ . In each of the figures are shown the deflection, shear, and moment diagrams.

A number of interesting observations can be made concerning the results of the computations. The most obvious is the reduction in maximum deflection and maximum moment with an increase in subgrade modulus k. However, the relationship between the maximum deflection, maximum moment and the subgrade modulus cannot be deduced without more computations being made.

Figure 6

$N = 0$
 $C = 2.5 \times 10^6$ inch-lbs/inch/radian
 $k = 440$ lbs/cubic inch
 $P = 250$ lbs/inch
 $\ell = 30$ inches

The presence of the horizontal load H , has a slight effect on the deflections but practically no effect on the moments. The effect of higher values of H on the deflections and moments might be more pronounced, but again to determine this effect will require more computations.

The general shape of the diagrams presented agrees very well with the exact solution for a continuous beam on an elastic foundation (14). The characteristic vanishing of the deflection, shear, and moment at points of increasing distance from the point of application of load is evident. The fact that the curves, in addition to being deflection, shear, and moment diagrams for the fixed position of load are also influence lines for deflection, shear, and moment at a point is useful for the consideration of more than one load.

Figures 9 and 10 are shown for comparison with Figures 3 and 5 respectively. The purpose for the comparison of these two cases is to show specifically a fact that is true generally; namely, that only a small number of crack intervals need be considered in the solution. The use of a number of crack intervals larger than say, 10 to 20, depending possibly on the length of the crack interval, will yield no additional information, although it will increase the precision slightly. The optimum number of crack intervals in relationship to the crack spacing is still to be worked out. It seems unreasonable that the consideration of more than twenty intervals would ever be necessary to obtain an accurate solution in the region of the load.

The curves in Figures 7 and 8 are for the case where $C = 0$. In this case the only interaction between the segments is shear transfer

across the crack. This case is likely to arise when, in a continuously reinforced pavement, the crack width or opening is so large as to prevent the two concrete faces from coming into contact and thus provide resistance to moment. Such a condition in an actual pavement is very near if not at the state of failure. It is of interest here since it is the extreme possibility of the moment-angle change relationship.

The unusual moment diagrams in Figures 7 and 8 are the moment diagrams for the individual segments when the moments at the cracks equal zero ($C = 0$). Each segment is somewhat like a simply supported beam loaded with a varying distributed load from the subgrade. The resulting moment diagrams are quite reasonable when considered in this manner.

RECOMMENDATIONS FOR FURTHER RESEARCH AND PRACTICAL APPLICATIONS OF RESULTS

Many questions must be answered before the method of analysis developed in this research can be applied to the design of continuously reinforced concrete pavements.

Some of the questions which must be answered are:

- (a) What is the moment-angle change relationship for a wide range of pavement design variables (thickness of concrete, position of reinforcement, percentage of reinforcement)?
- (b) What is the necessary number of segments to be considered for the necessary accuracy with a given crack spacing?
- (c) What is the effect of high values of N on the moments?
- (d) What is the effect of the plate action of each segment?
- (e) What is the effect of repeated loads on the pavement design? and ultimately
- (f) What is the optimum percentage and best position of steel for given loads and field conditions?

The first question is susceptible to laboratory investigation. Experimentation with relatively small specimens which contain only one crack will eliminate some of the variables which obscure the nature of the moment-angle change relationship. A specimen may be tested by the application at the crack of pure moment, pure shear, or any combination of moment and shear. Longitudinal loads to control crack openings may also be applied. Measurements of the relative angle changes between the two segments formed by the crack may be made. The relative vertical displacement of the segments may also be measured. By varying the per-

centage and position of the reinforcement as well as the loads on the crack complete information about crack behavior can be obtained. Such an investigation is now in progress at Purdue University.

The answers to questions (b) and (c) can be readily obtained when the moment-angle change relationship has been established. In this investigation the optimum number of crack intervals is 8 using a crack spacing of 30 inches, while it may be some other number for other values of ℓ . By the use of the Purdue University Digital Computer a large number of solutions may be easily worked out. The solutions may be programmed in such a manner as to provide the final values of moments, shears, deflections, and stresses. With such a computer available it is possible to consider in a relatively short time the large range of parameters necessary to answer these questions.

In this investigation the pavement is treated as a beam one inch wide. In the actual pavement the cracks divide the slab into a series of transverse strips. The action of these strips under the load must be investigated. For rather closely spaced cracks the strip acts somewhat as a beam, while the larger spacings of cracks yield segments which act more like plates. The effects of this plate action must be determined and appropriate modifications must be incorporated in the final design procedure.

The effect of repeated loads on the pavement design can be investigated in the laboratory. Repeated application of loads in the research outlined above would provide information on the fatigue characteristics of various designs. This information can be combined with traffic surveys for proposed highways in the final design of the pavement.

The answer to the last question is really the ultimate purpose of all of these investigations. What combinations of slab thickness, percentage of reinforcement, and position of reinforcement will support the given load in a given field condition? The existence of an answer to this question implies the existence of a design procedure which can account for all of the variables in the problem. It is obviously very difficult experimentally to account for each variable separately and then combine their effects to arrive at a design for each given set of conditions. Perhaps the best design procedure then, is one which utilizes the results of verified mathematical solutions. The results may be presented in table and chart form so that engineers may design safe and economical continuously reinforced concrete pavements. It is sincerely hoped that this research will help make this possibility a reality.

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A P P E N D I X

APPENDIX I

The general equations for the solution for the values of the angle changes in the cases of L equal to 12ℓ , 16ℓ , and 20ℓ respectively are as follows:

$$L = 12\ell$$

$$(107/6\ell^2k + N + C/\ell)\phi_0 + (12.5\ell^2k - C/\ell)\phi_1 + \ell^2k(8\phi_2 + 4.5\phi_3 + 2\phi_4 + 0.5\phi_5) = P/2$$

$$(101/6\ell^2k + 2N)\phi_0 + (37/3\ell^2k + N + C/\ell)\phi_1 + (8\ell^2k - C/\ell)\phi_2 + \ell^2k(4.5\phi_3 + 2\phi_4 + 0.5\phi_5) = P/2$$

$$(89/6\ell^2k + 3N)\phi_0 + (34/3\ell^2k + 2N)\phi_1 + (47/6\ell^2k + N + C/\ell)\phi_2 + (4.5\ell^2k - C/\ell)\phi_3 + \ell^2k(2\phi_4 + 0.5\phi_5) = P/2$$

$$(71/6\ell^2k + 4N)\phi_0 + (28/3\ell^2k + 3N)\phi_1 + (41/6\ell^2k + 2N)\phi_2 + (13/3\ell^2k + N + C/\ell)\phi_3 + (2\ell^2k - C/\ell)\phi_4 + 0.5\ell^2k\phi_5 = P/2$$

$$(47/6\ell^2k + 5N)\phi_0 + (19/3\ell^2k + 4N)\phi_1 + (29/6\ell^2k + 3N)\phi_2 + (10/3\ell^2k + 2N)\phi_3 + (11/6\ell^2k + N + C/\ell)\phi_4 + (0.5\ell^2k - C/\ell)\phi_5 = P/2$$

$$(17/6\ell^2k + 6N)\phi_0 + (7/3\ell^2k + 5N)\phi_1 + (11/6\ell^2k + 4N)\phi_2 + (4/3\ell^2k + 3N)\phi_3 + (5/6\ell^2k + 2N)\phi_4 + (0.5\ell^2k + N + C/\ell)\phi_5 = P/2$$

$$\phi_0 + \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + 0.5\phi_6 = 0$$

$$L = 16\ell$$

$$(191/6\ell^2k + N + C/\ell)\phi_0 + (24.5\ell^2k - C/\ell)\phi_1 + \ell^2k(18\phi_2 + 12.5\phi_3 + 8\phi_4 + 4.5\phi_5 + 2\phi_6 + 0.5\phi_7) = P/2$$

$$(185/6\ell^2k + 2N)\phi_0 + (73/3\ell^2k + N + C/\ell)\phi_1 + (13\ell^2k - C/\ell)\phi_2 + \ell^2k(12.5\phi_3 + 8\phi_4 + 4.5\phi_5 + 2\phi_6 + 0.5\phi_7) = P/2$$

$$(173/6\ell^2k + 3N)\phi_0 + (70/3\ell^2k + 2N)\phi_1 + 107/6\ell^2k + N + C/\ell)\phi_2 + (12.5\ell^2k - C/\ell)\phi_3 + \ell^2k(8\phi_4 + 4.5\phi_5 + 2\phi_6 + 0.5\phi_7) = P/2$$

$$\begin{aligned}
& (155/6\ell^2k + 4N)\phi_0 + (64/3\ell^2k + 3N)\phi_1 + (101/6\ell^2k + 2N)\phi_2 + (37/3\ell^2k + \\
& N + C/\ell)\phi_3 + (8\ell^2k - C/\ell)\phi_4 + \ell^2k(4.5\phi_5 + 2\phi_6 + 0.5\phi_7) = P/2 \\
& (141/6\ell^2k + 5N)\phi_0 + (55/3\ell^2k + 4N)\phi_1 + (89/6\ell^2k + 3N)\phi_2 + (34/3\ell^2k + 2N) \\
& \phi_3 + (47/6\ell^2k + N + C/\ell)\phi_4 + (4.5\ell^2k - C/\ell)\phi_5 + \ell^2k(2\phi_6 + 0.5\phi_7) \\
& = P/2
\end{aligned}$$

$$\begin{aligned}
& (101\ell^2k + 6N)\phi_0 + (43/3\ell^2k + 5N)\phi_1 + (71/6\ell^2k + 4N)\phi_2 + (28/3\ell^2k + 3N)\phi_3 \\
& + (41/6\ell^2k + 2N)\phi_4 + (13/3\ell^2k + N + C/\ell)\phi_5 + (2\ell^2k - C/\ell)\phi_6 + \\
& 0.5\ell^2k\phi_7 = P/2
\end{aligned}$$

$$\begin{aligned}
& (65/6\ell^2k + 7N)\phi_0 + (28/3\ell^2k + 6N)\phi_1 + (47/6\ell^2k + 5N)\phi_2 + (19/3\ell^2k + 4N) \\
& \phi_3 + (29/6\ell^2k + 3N)\phi_4 + (10/3\ell^2k + 2N)\phi_5 + (11/6\ell^2k + N + C/\ell) \\
& \phi_6 + (0.5\ell^2k - C/\ell)\phi_7 = P/2
\end{aligned}$$

$$\begin{aligned}
& (23/6\ell^2k + 8N)\phi_0 + (10/3\ell^2k + 7N)\phi_1 + (17/6\ell^2k + 6N)\phi_2 + (7/3\ell^2k + 5N) \\
& \phi_3 + (11/6\ell^2k + 4N)\phi_4 + (4/3\ell^2k + 3N)\phi_5 + (5/6\ell^2k + 2N)\phi_6 + \\
& (0.5\ell^2k + N + C/\ell)\phi_7 = P/2
\end{aligned}$$

$$\phi_0 + \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 + \phi_7 + 0.5\phi_8 = 0$$

$$L = 20\ell$$

$$\begin{aligned}
& (299/6\ell^2k + N + C/\ell)\phi_0 + (40.5\ell^2k - C/\ell)\phi_1 + \ell^2k(32\phi_2 + 24.5\phi_3 + 18\phi_4 + \\
& 12.5\phi_5 + 8\phi_6 + 4.5\phi_7 + 2\phi_8 + 0.5\phi_9) = P/2
\end{aligned}$$

$$\begin{aligned}
& (293/6\ell^2k + 2N)\phi_0 + (121/3\ell^2k + N + C/\ell)\phi_1 + (32\ell^2k - C/\ell)\phi_2 + \ell^2k \\
& (24.5\phi_3 + 18\phi_4 + 12.5\phi_5 + 8\phi_6 + 4.5\phi_7 + 2\phi_8 + 0.5\phi_9) = P/2
\end{aligned}$$

$$\begin{aligned}
& (281/6\ell^2k + 3N)\phi_0 + (118/3\ell^2k + 2N)\phi_1 + (191/6\ell^2k + N + C/\ell)\phi_2 + \\
& (21.5\ell^2k - C/\ell)\phi_3 + \ell^2k(18\phi_4 + 12.5\phi_5 + 8\phi_6 + 4.5\phi_7 + 2\phi_8 + \\
& 0.5\phi_9) = P/2
\end{aligned}$$

$$\begin{aligned}
& (263/6\ell^2k + 4N)\phi_0 + (112/3\ell^2k + 3N)\phi_1 + (185/6\ell^2k + 2N)\phi_2 + \\
& (73/3\ell^2k + N + C/\ell)\phi_3 + (18\ell^2k - C/\ell)\phi_4 + \ell^2k(12.5\phi_5 + 8\phi_6 + \\
& 4.5\phi_7 + 2\phi_8 + 0.5\phi_9) = P/2
\end{aligned}$$

$$\begin{aligned}
& (239/6\ell^2k + 5N)\phi_0 + (103/3\ell^2k + 4N)\phi_1 + (173/6\ell^2k + 3N)\phi_2 + \\
& (70/3\ell^2k + 2N)\phi_3 + (107/6\ell^2k + N + C/\ell)\phi_4 + (17.5\ell^2k - C/\ell)\phi_5 + \\
& \ell^2k(8\phi_6 + 4.5\phi_7 + 2\phi_8 + 0.5\phi_9) = P/2 \\
& (209/6\ell^2k + 6N)\phi_0 + (91/3\ell^2k + 5N)\phi_1 + (155/6\ell^2k + 4N)\phi_2 + (64/3\ell^2k + 3N)\phi_3 + \\
& (101/6\ell^2k + 2N)\phi_4 + 38/3\ell^2k + N + C/\ell)\phi_5 + (8\ell^2k - C/\ell)\phi_6 + \\
& \ell^2k(45\phi_7 + 2\phi_8 + 0.5\phi_9) = P/2 \\
& (173/6\ell^2k + 7N)\phi_0 + (76/3\ell^2k + 6N)\phi_1 + (141/6\ell^2k + 5N)\phi_2 + (55/3\ell^2k + 4N)\phi_3 + \\
& (89/6\ell^2k + 3N)\phi_4 + (34/3\ell^2k + 2N)\phi_5 + (47/6\ell^2k + N + C/\ell)\phi_6 + \\
& (4.5\ell^2k - C/\ell)\phi_7 + (2\phi_8 + 0.5\phi_9) = P/2 \\
& (131/6\ell^2k + 8N)\phi_0 + (58/3\ell^2k + 7N)\phi_1 + (101/6\ell^2k + 6N)\phi_2 + (43/3\ell^2k + 5N)\phi_3 + \\
& (71/6\ell^2k + 4N)\phi_4 + (28/3\ell^2k + 3N)\phi_5 + (41/6\ell^2k + 2N)\phi_6 + \\
& (13/3\ell^2k + N + C/\ell)\phi_7 + (2\ell^2k - C/\ell)\phi_8 + 0.5\ell^2k\phi_9 = P/2 \\
& (83/6\ell^2k + 9N)\phi_0 + (37/3\ell^2k + 8N)\phi_1 + (55/6\ell^2k + 7N)\phi_2 + (28/3\ell^2k + 6N)\phi_3 + \\
& (47/6\ell^2k + 5N)\phi_4 + (19/3\ell^2k + 4N)\phi_5 + (29/6\ell^2k + 3N)\phi_6 + \\
& (10/3\ell^2k + 2N)\phi_7 + (11/6\ell^2k + N + C/\ell)\phi_8 + (0.5\ell^2k - C/\ell)\phi_9 = P/2 \\
& (29/6\ell^2k + 10N)\phi_0 + (13/3\ell^2k + 9N)\phi_1 + (23/6\ell^2k + 8N)\phi_2 + (10/3\ell^2k + 7N)\phi_3 + \\
& (17/6\ell^2k + 6N)\phi_4 + (7/3\ell^2k + 5N)\phi_5 + (11/6\ell^2k + 4N)\phi_6 + \\
& (4/3\ell^2k + 3N)\phi_7 + (5/6\ell^2k + 2N)\phi_8 + (0.5\ell^2k + N + C/\ell)\phi_9 = P/2 \\
& \phi_0 + \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 + \phi_7 + \phi_8 + \phi_9 + 0.5\phi_{10} = 0
\end{aligned}$$