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REMARKS ON THE CALCULATION OF RADIATED SOUND FROM COMPRESSOR SHELL SIDE WALLS USING EQUIVALENT CYLINDERS

by

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ABSTRACT

In this paper, it is proposed to approximate compressor shells of non-circular cross section by equivalent circular cylindrical shells. The sound radiation from a point source on a circular cylinder of infinite length is reviewed and the solution is applied to the sound radiation calculation of a finite length circular cylindrical shell, integrating all the point sources on the surface. The sound radiation space is simplified by introducing an infinite cylindrical baffle. Sound radiation fields are presented for typical shell modes. It is also pointed out how these single mode results can be superimposed to calculate sound fields due to realistic compressor shell vibrations.

INTRODUCTION

This paper introduces the calculation of radiated sound from compressor shell side wall by equivalent cylinders. The paper by D.T. Laird and K. Cohen [1] showed the sound radiation field from a source on a rigid cylinder of infinite length. These results are repeated and verified for the horizontal amplitude pattern of a rectangular source on an infinite cylindrical baffle, by using similar calculations. In addition, the sound radiation calculation of a finite length circular cylindrical shell is presented, using the same technique which was applied to the piston source. The sound radiation of each vibration mode of a cylindrical shell will be calculated as example. If more than one mode are exited, the total sound radiation field can be calculated by superimposing the contributions of all modes.

RADIATION FROM AN INFINITELY LONG CYLINDER

The acoustic wave equation in cylindrical coordinates r,ϕ,z can be expressed as

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} + \frac{1}{r^2}\frac{\partial^2 P}{\partial \phi^2} + \frac{\partial^2 P}{\partial z^2} + k^2 P = 0 . \tag{1}$$

If the velocity of the cylinder wall is assumed to be

$$U(z,\phi,t) = U_n \cos n\phi \ \cos k_m z \, e^{j\omega t}, \tag{2}$$

the acoustic field can be expressed as the series

$$P(r,z,\phi) = \sum_{m,n} A_{mn} R_{mn}(r) \cos k_m z \cos n\phi . \tag{3}$$

Both the acoustic pressure field and the velocity field have to satisfy the boundary condition at the cylinder wall, which is

$$\left.\frac{\partial P}{\partial r}\right|_{r=a} = -\rho_o \frac{\partial u_r}{\partial t} = -j\omega\rho_o u_r . \tag{4}$$

Using equation (3), equation (1) becomes

$$\frac{\partial^2 R_{mn}(r)}{\partial r^2} + \frac{1}{r}\frac{\partial R_{mn}(r)}{\partial r} + [\, k^2 - k_m^2 - \left(\frac{n}{r}\right)^2 R_{mn}(r) \,] = 0 . \tag{5}$$

The solution of equation (5) is a linear combination of the Bessel functions of the first and second kind.

$$R_{mn}(r) = A\, J_n\, [(k^2 - k_m^2)^{1/2}\, r] + B\, Y_n\, [(k^2 - k_m^2)^{1/2}\, r]. \qquad (6)$$

The ratio of the constant B/A can be determined by noting that the radiated field must tend to be a plane wave form in the far field. For outgoing waves in the far field, the mode in r direction is

$$R(r) = \bar{A}\, e^{ikr}, \qquad (7)$$

or

$$R(r) = \bar{A}\,(\cos kr + i \sin kr), \text{ as } r \rightarrow \infty. \qquad (8)$$

Using the large-argument asymptotic limit of Bessel functions,

$$\lim_{n \rightarrow \infty} J_n(x) = \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{(2n+1)\pi}{4}\right), \qquad (9)$$

$$\lim_{n \rightarrow \infty} Y_n(x) = \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{(2n+1)\pi}{4}\right), \qquad (10)$$

$$\lim_{kr \rightarrow \infty} R(r) = \sqrt{\frac{2}{\pi kr}}\, A\left[\cos\left(kr - \frac{(2n+1)\pi}{4}\right) + \frac{B}{A} \sin\left(kr - \frac{(2n+1)\pi}{4}\right)\right]. \qquad (11)$$

Comparing equation (7) and (10), the coefficient B/A is set to i. Finally, the solution of equation (5) is therefore,

$$R_{mn}(r) = J_n\, [(k^2 - k_m^2)^{1/2}\, r] + i\, Y_n\, [(k^2 - k_m^2)^{1/2}\, r], \qquad (12)$$

or

$$R_{mn}(r) = H_n^{(1)}\, [(k^2 - k_m^2)^{1/2}\, r], \qquad (13)$$

where $H_n^{(1)}$ is the Hankel function of the first kind. Combining equations (2), (3) and (4),

$$-j\omega\rho_o U_n \cos n\phi \cos k_m z\, e^{i\omega t} = e^{i\omega t} A_{mn}\, ((k^2 - k_m^2)^{1/2}\, H_n'^{(1)}\, [\,(k^2 - k_m^2)^{1/2}\, a\,] \cos n\phi \cos k_m z\ . \qquad (14)$$

Therefore,

$$A_{mn} = \frac{-j\omega\rho_o U_n}{(k^2 - k_m^2)^{1/2}\, H_n'^{(1)}\, [(k^2 - k_m^2)^{1/2} a]}\ , \qquad (15)$$

where the prime of the Henkel function of the first kind denotes differentiation with respect to the argument of the function which is $[(k^2 - k_m^2)^{1/2} a]$. Substituting equations (13) and (15) into equation (3) goes, finally, the sound pressure

$$P(r,z,\phi) = \sum_{m,n} \frac{-j\omega\rho_o U_n H'^{(1)}_n [(k^2 - k_m^2)^{1/2} r]}{(k^2 - k_m^2)^{1/2} \; H'^{(1)}_n [(k^2 - k_m^2)^{1/2} a]} \cos n\phi \cos k_m z \; . \quad (16)$$

RADIATION FROM A RECTANGULAR PISTON ON A RIGID CYLINDER OF INFINITE LENGTH

The radiation from an infinite cylinder was explained in the far field. In this chapter, the radiation from a piston on a rigid infinite cylinder is presented by using a similar approach. The basic geometry of a rectangular piston on an infinite cylinder is shown in Figure 1. The velocity field of the rectangular piston is

$$U = U_o \; , \; \text{at } (-\alpha < \phi < \alpha, \; -L < z < L) \; , \; \text{and } U = 0 \; , \; \text{elsewhere} \; . \quad (17)$$

This velocity field can be expanded by Fourier transforming it in ϕ and z direction. In ϕ direction,

$$u_\phi(\phi) = \frac{a_o}{2} + \sum_n a_n \cos n\phi + b_n \sin n\phi, \quad (18)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} u(f) \cos n\phi \, d\phi = \frac{1}{\pi} \int_{-\alpha}^{\alpha} U_o \cos n\phi \, d\phi = \frac{2U_o \sin n\alpha}{n\pi} \; . \quad (19)$$

Also,

$$a_o = \frac{2\alpha}{\pi}, b_n = 0 \; . \quad (20, 21)$$

Therefore,

$$u_\phi(\phi) = \sum_n a_n \cos n\phi, \quad (22)$$

where $a_o = (\alpha/\pi)$, $a_n = (2U_o \sin n\alpha) / (n\pi)$, $n= 1,2,\cdots$. In z direction,

$$U_z(z) = \int_{-\infty}^{\infty} a(k_z) \, e^{ik_z z} \, dk_z, \quad (23)$$

where

$$a(k_z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U_z(z) \, e^{-ik_z z} \, dk_z = \int_{-L}^{L} U_o \, e^{-ik_z z} dk_z = \frac{1}{2\pi} U_o \frac{2\sin k_z L}{k_z} \; . \quad (24)$$

Therefore,

$$U_z(z) = \int_{-\infty}^{\infty} \frac{1}{\pi} U_o \frac{\sin k_z L}{k_z} e^{ik_z z} \, dk_z \; , \quad (25)$$

or

$$U_z(z) = U_o \int_{-\infty}^{\infty} F(k_z) \, e^{ik_z z} \, dk_z \; , \quad (26)$$

where $F(k_z) = (\sin k_z L)/(\pi k_z)$.

The velocity field is found to be

$$U = \sum_n a_n \cos n\phi \int_{-\infty}^{\infty} F(k_z)\, e^{ik_z z}\, dk_z \,. \tag{27}$$

The acoustic field can be expressed using separation of variables : $P(r,\phi,z) = R(r)\, G(\phi)\, Z(z)$. From equation (1),

$$\frac{\partial^2 R(r)}{\partial r^2} + \frac{1}{r}\frac{\partial R(r)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 R(r)}{\partial \phi^2} + \frac{\partial^2 R(r)}{\partial z^2} + k^2 R(r) = 0 \,. \tag{28}$$

The solution is $R_n(r) = H_n^{(1)}\,[(k^2 - k_z^2)^{1/2}\, r]$, which is the same as equation (13). The Pressure is expressed as follows :

$$P(r,z,\phi) = \sum_n \cos n\phi \int_{-\infty}^{\infty} A_n(k_z)\, H_n^{(1)}\,[k_r\, r]\, e^{ik_z z}\, dk_z, \tag{29}$$

where $k_r = (k^2 - k_z^2)^{1/2}$. Using boundary condition at the cylinder wall,

$$A_n(k_z) = \frac{i\rho\omega U_0 a_n F(k_z)}{k_r\, H'^{(1)}_n(k_r a)} \,. \tag{30}$$

Finally, the pressure field is found to be

$$P(r,z,\phi) = \sum_n i\rho\omega U_0 a_n \cos n\phi \int_{-\infty}^{\infty} \frac{F(k_z)\, H_n^{(1)}\,[k_r\, r]}{k_r\, H'^{(1)}_n(k_r a)}\, e^{ik_z z}\, dk_z \,. \tag{31}$$

Now, to calculate equation (27) numerically, the integral has to be solved. The stationary-phase approximation to the far field of the cylindrical radiator is used to evaluate the integration. To use this numerical technique, cylindrical coordinates need to be transformed to spherical coordinates, which is also shown in Figure 1. Using the relations $r = R \sin n\theta$, $z = R \cos n\theta$, we obtain

$$P(R,\phi,\theta) = \sum_n i\rho\omega U_0 a_n \cos n\phi \int_{-\infty}^{\infty} \frac{F(k_z)\, H_n^{(1)}\,[k_r\, r]}{k_r\, H'^{(1)}_n(k_r a)}\, e^{ik_z z}\, dk_z \,. \tag{32}$$

Using the large argument asymptotic expression for the Hankel function,

$$H_n^{(1)}(x) = \sqrt{\frac{2}{\pi x}}\,(-i)^n \exp(ix - i\tfrac{\pi}{4}) \,, \quad x >> n^2+1, \tag{33}$$

we obtain for the pressure field,

$$P(R,\phi,\theta) = \sum_n i\rho\omega U_0 a_n \cos n\phi \exp[-i(n+\frac{1}{2})\frac{\pi}{2}]\sqrt{\frac{2}{\pi\sin\theta}} \int_{-\infty}^{\infty} \frac{F(k_z)\exp[iR[k_r\sin\theta + k_z\cos\theta]}{k_r^{3/2} \quad H'^{(1)}_n(k_r a)} dk_z . \quad (34)$$

For the sake of a simpler expression, we set

$$I = \int_{-\infty}^{\infty} \Phi(k_z)\exp[i\Psi(k_z)]\, dk_z, \quad (35)$$

where

$$\Phi(k_z) = \frac{F(k_z)}{k_r^{3/2} \, H'^{(1)}_n(k_r a)} dk_z \,, \; \Psi(k_z) = R[k_r\sin\theta + k_z\cos\theta]. \quad (36)$$

The stationary phase technique is based on the fact that the main contribution over the integration range where $\Phi(k_z)$ varies slowly with k_z while the phase $\Psi(k_z)$ fluctuates rapidly, is relatively small. This is because of a cancellation effect between neighboring regions which have opposite phase and nearly equal amplitude. Since we have

$$\frac{\partial^2\Phi(k_z)}{\partial k_z^2} = \frac{-R}{k\sin^2\theta} < 0 \,, \quad (37)$$

We obtain

$$I = \sqrt{2\pi}\, \frac{\Phi(k_z)\exp[-\frac{\pi}{4}i + i\Psi(k_z)]}{\left(\frac{\partial^2\Phi(k_z)}{\partial k_z^2}\right)^{1/2}} \,, \quad (38)$$

and

$$k_z = k\cos\theta \,. \quad (39)$$

The pressure field is, thus, ready to be calculated numerically :

$$P(R,\phi,\theta) \approx 2\rho\frac{\omega}{k}U_0 \frac{\exp(i(kR-\omega t))}{R} \frac{F(k\cos\theta)}{\sin\theta} \sum_{n=0}^{\infty} \frac{a_n \exp(-i(\frac{n\pi}{2}))}{H'^{(1)}_n(ka\sin\theta)} \cos n\phi, \quad (40)$$

where

$$F(k\cos\theta) = \frac{\sin(kL\cos\theta)}{k\pi\cos\theta},\; a_0 = \frac{\alpha}{\pi},\; a_m = \frac{2\sin m\alpha}{m\pi} \,. \quad (41,42,43)$$

RESULTS FOR POINT SOURCE RADIATION

Figure 2 shows the horizontal pressure amplitude pattern for a rectangular source with ka=14, α=3.7 and θ=90°, which was already presented in [1]. The vertical axis is shown in the [dB] scale which is

$$SPL\,[dB] = 20\log\frac{P_e}{P_{ref}} \,, \quad (44)$$

where

$$P_{ref} = \text{Pressure at } \phi = 0,\ Pe = \frac{|P|}{\sqrt{2}} . \quad (45)$$

Since the radiation effect will be in phase at $\phi=180^o$, a hump is expected to be present at $\phi=180^o$. The rectangular piston source results of Figure 3 are shown in the actual dB scale which employs $P_{ref} = 20 \times 10^{-6}$ Pa .

RESULTS FOR SOUND RADIATION FROM A SIMPLY SUPPORTED CIRCULAR CYLINDRICAL SHELL(COMPRESSOR) IN AN INFINITELY LONG BAFFLE

Assuming a simply supported cylindrical shell, the velocity is

$$U(\phi,z) = Uo \sum_{m,n} \sin \frac{m\pi z}{2L} \cos n\phi. \quad (46)$$

The velocity modes in ϕ and z direction can be transformed in the same way as in the case of a rectangular piston source. For even modes of the velocity modes, when m=2q+1, q=0,1,2,···,

$$F(k_z) = \frac{1}{k_q^2 - k_z^2} \frac{1}{\pi} k_q(-1)^q \cos k_z L, \quad (47)$$

where $k_q = (0.5+q)\pi / L$. For odd modes,

$$F(k_z) = \frac{1}{k_q^2 - k_z^2} \frac{1}{\pi} k_q(-1)^q \sin k_z L, \quad (48)$$

where $k_q=(q\pi / L)$ Since we know $F(k_z)$, we can calculate the pressure field from equation (35). Figure 4 shows the natural frequencies of a simply supported cylindrical shell, calculated by using a formula given in [4] :

$$\omega_{mn} = \frac{1}{2} \sqrt{\frac{\left(\frac{m\pi a}{L}\right)^4}{\left[\left(\frac{m\pi a}{L}\right)^2 + n^2\right]^2} + \frac{\left(\frac{h}{a}\right)^2}{12(1-\mu^2)} \left[\left(\frac{m\pi a}{L}\right)^2 + n^2\right]^2} \sqrt{\frac{E}{\rho}} , \quad (49)$$

where a=0.07m, h=0.0032m, L=0.2m, m=0.3, E=20.6 x 104 N/mm^2 and r=7.85 x 10-9 Nsec^2/mm^4. The modes where (m,n)=(1,3), (2,4) and (3,5) have the lowest natural frequencies at each m=1,2 and 3.

Figure 5 shows the vibrations and the sound radiation patterns of each mode in ϕ direction when (m,n) = (1,1), (1,2), (1,3) and (1,4). The sound radiation patterns also have nodal lines at the same ϕ as the vibration patterns have nodal lines. These nodal lines come from the wave cancellation at that region. The vibrations and the sound radiation patterns of each mode in z direction are shown in Figure 6, when (m,n)=(1,3), (2,4) and (3,5), which correspond to the lowest natural frequencies of the simply supported cylinder at each m. The nodal lines in the radiation pattern can be seen in odd modes of the vibration pattern because the waves are canceled out. The sound pressure level at the fundamental natural frequency, where (m,n)=(1,3), would be expected to be higher in a real compressor than the sound pressure levels at the other natural frequencies where (m,n)=(2,4) and (3,5).

SOUND RADIATION PATTERN BY THE SUPERPOSITION OF EACH MODE

If we know the modal participation in the vibration of a compressor shell, then the whole sound radiation pattern can be calculated by superimposing all the sound radiation pattern of each mode. Since every single mode will effect the sound radiation field, the radiation patterns will have irregular shapes.

Figure 7 shows the sound radiation pattern in the ϕ direction if modes (m,n) = (1,2) and (1,3) dominate the vibrations. Figure 8 shows the sound radiation pattern in the z direction if modes (m,n)=(1,3) and (2,4) dominate.

CONCLUSION

The sound radiation pattern of a rectangular piston source on an infinite cylinder was reviewed. Using the same theory, the radiation field from a finite cylindrical radiator was solved. And the radiation pattern from each mode was shown.

The actual noise radiation field can be approximated by considering all the modes effects. If we know the velocity field of a cylindrical shell, then the pressure expression in the far field now can be calculated. In a practical sense, if a point pressure source is exciting the compressor shell, the velocity of the shell can be calculated by solving the equation of motion of the forced shell vibration. This case resembles to some extend the excitation of the compressor shell by a vibrating shock loop or by distributed gas pulsations. Once the velocity field is known, the pressure field can be calculated by using the technique which was shown in this paper.

ACKNOWLEDGEMENT

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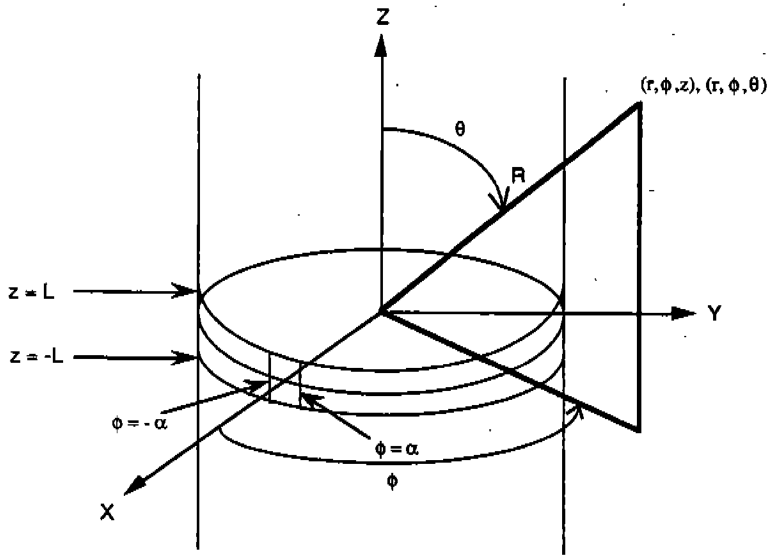


Figure 1 Basic geometry with a rectangular piston on an infinite cylinder

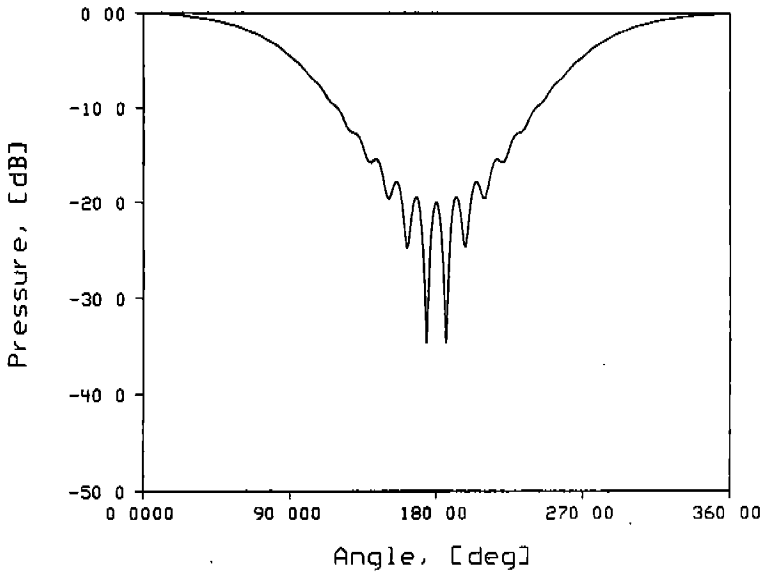


Figure 2 **Point source result for the case presented in reference [1].**

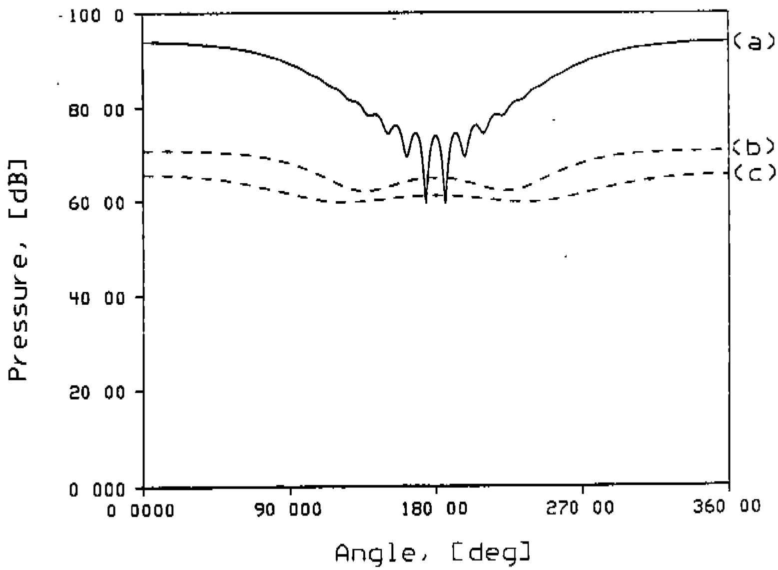


Figure 3 **Point source results for compressor domain at different angular locations θ : (a) θ=90°, (b) θ=60°, (c) θ=30°.**

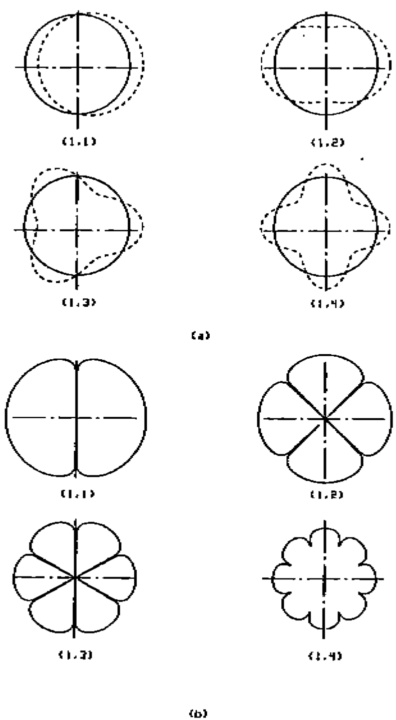


Figure 5 Top view of compressor shell : (a) Vibration modes identified by (m,n) numbers and (b) sound radiation associated with these modes.

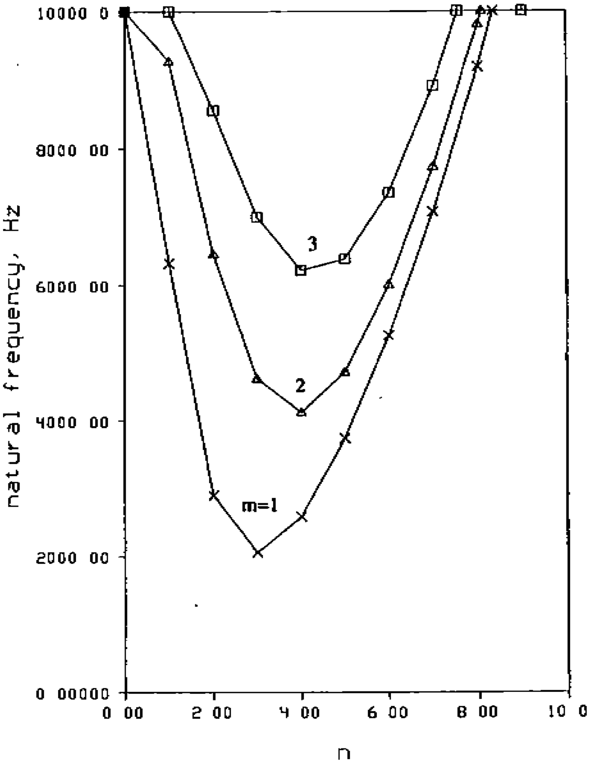
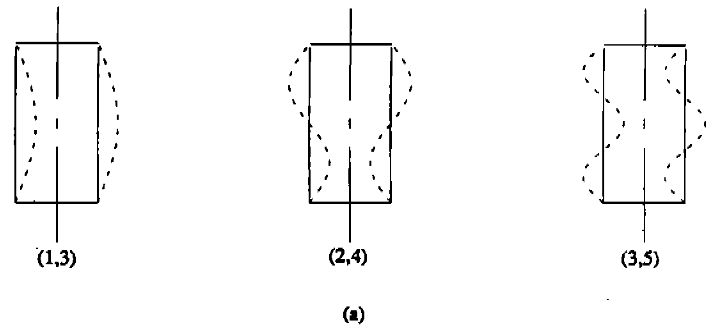


Figure 4 Natural frequencies of a typical compressor shell



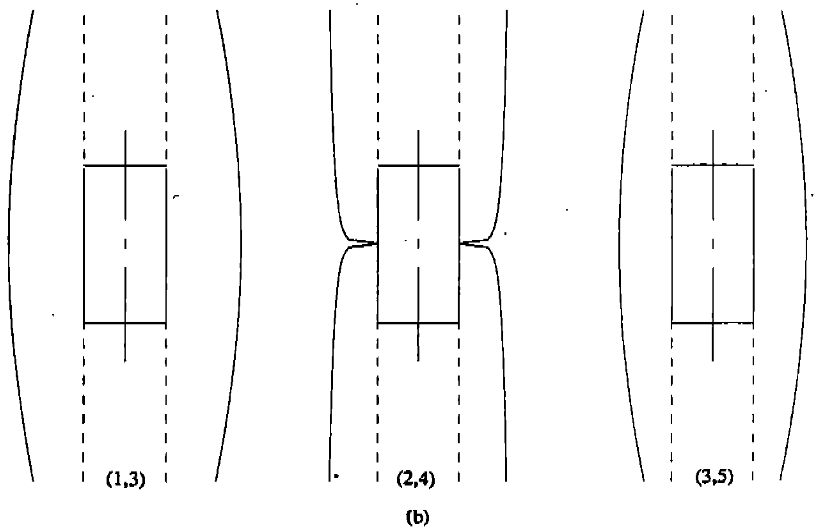


Figure 6 Side view of compressor shell : (a) Vibration modes identified by (m,n) numbers and (b) sound radiation associated with these modes.

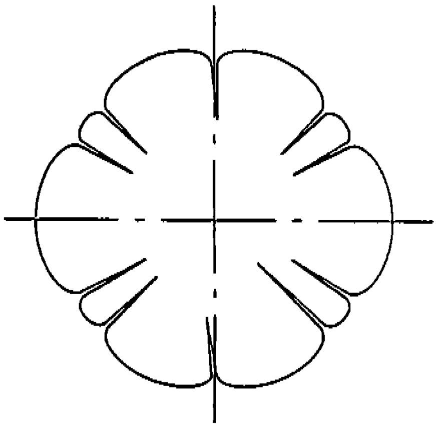


Figure 7 Sound radiation in top view if modes (m,n)=(1,2) and (1,3) dominate.

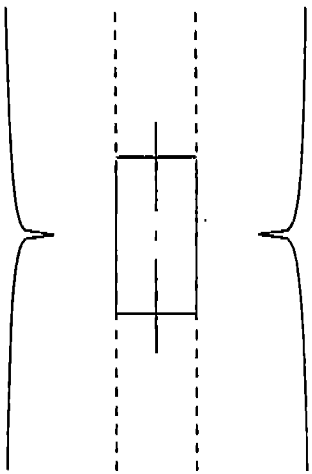


Figure 8 Sound radiation in side view if modes (m,n)=(1,3) and (2,4) dominate.