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Abstract

The stability condition for the standard token passing ring is "known" since the seminal paper of Kuehn in 1979. However, this condition was derived without formal proof, and the proof seems to be of considerable interest to research community. In fact, Watson observed that in the performance evaluation of token passing rings "*it is convenient to derive stability conditions ... (without proof)*". Our intention is to fill this gap, and provide a formal proof of the sufficient stability condition for the token passing ring. In this paper we present the case when the arrival process to each queue is Poisson but service times and switchover times are generally distributed. We only consider ℓ -limited (and exhaustive) service discipline for each station. We also discuss some generalizations, and indicate that our technique can be successfully applied to the stability analysis of several other distributed multiqueue systems.

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1. INTRODUCTION

Distributed multiqueue systems which share a single scarce resource (i.e., server) such as a communication channel or a processor, have received a considerable amount of attention in the recent literature. Important examples of such distributed multiqueue systems are local area networks (e.g., ALOHA system, Ethernet, token passing ring, FDDI ring, etc.), multiprocessor systems, distributed computations, distributed data base, and so forth. Of special interest is the token passing ring (cf. [8], [21], [22]) due to a number of reasons. In particular, it appears that determination of sound measures of performance for such a system, under realistic assumptions such as asymmetric traffic, finite or infinite buffers, non-exhaustive service and general input are fairly difficult to obtain, as can be witnessed from the literature ([2], [3], [7], [9], [22]). For example, it is known that obtaining the distribution of the number of messages queued in each station is a formidable open problem, as is the problem of obtaining the waiting time distribution. Surprisingly enough, the stability condition for the token passing ring was *heuristically* predicted by Kuehn [8] in 1979, and then reproduced with some minor changes in many other papers (e.g., [5]). But . . . , Watson [25] observed that in the performance evaluation of token passing rings "*it is convenient to derive stability conditions ... (without proof)*". In fact, no formal proof of the stability condition for the token passing ring was published (for some preliminary results see [20] and [18]). Our intention is to fill this gap, and provide formal proof for the stability condition of the token passing ring.

There is a version of the token passing ring, one on which the original token passing LAN was defined, which is particularly formidable for the analysis, and therefore it will be of our prime interest. This is the problem of *nonexhaustive* service on an *asymmetric* system, where at most ℓ_i messages are transmitted by station $i \in \mathcal{M} = \{1, \dots, M\}$ each time the i th station acquires the free token. Following the literature we call such a system ℓ -limited token passing ring. It must be stressed that up-to-date no exact analysis of such a system exists except for two stations system with $\ell_i = 1$ for $i = 1, 2$ (cf. [2]). Nevertheless, even without such an explicit analysis we present in this paper a rigorous proof for the stability conditions of such a system with Poisson arrivals, and general service and switchover times.

Stability is of considerable importance to the engineering and scientific communities. It is a fundamental issue in the design of any distributed system since only stable systems can work in practice. Hereafter, by stability we understand an ability of a system, in particular the token passing ring, to keep the queue lengths or waiting times in a bounded region. Sometimes, a more restricted definition is adopted, namely the existence of unique limiting

distribution of the queue length is required. For irreducible and aperiodic Markov chains these two definitions coincide with the *ergodicity* of the Markov chain (cf. [10], [24]).

Despite a vigorous research in the area of stability over last twenty years (cf. [23], [18], [24]), very few computable stability criteria are known for multidimensional processes, in particular multidimensional Markov chains. The most popular approach through the *Lyapunov test function* (cf. [23]) did not succeed in the past to provide general computable criteria for multidimensional Markov chains. However, due to pioneering work of Malyshev [11], continued by Mensikov [13], and Malyshev and Mensikov [12] some progress has been made in obtaining stability condition for a class of two-dimensional and three-dimensional Markov chains. Recently, weaker stability criteria for two dimensional chains have been presented by Fayolle [4] and Rozenkrantz [16]. Unfortunately, these conditions are still difficult to apply in practice for higher dimensional processes (see [6] for an application of this to a multidimensional ALOHA system). A more practical approach to stability of multidimensional Markov chains was discussed in Szpankowski [17] (for more details see survey [18]).

Our approach to the stability of token passing rings follows the idea suggested in Szpankowski [19], and differs significantly from the standard methodology of the test function (cf. [23]). It resembles, however, the general idea of stability criteria proposed by Malyshev and Mensikov [12]. Our approach is based on a simple idea of stochastic dominance technique, and application of Loynes [10] stability criteria for an isolated queue. We use the stochastic dominance to verify technical stationarity requirements in Loynes' criteria. We shall indicate that this approach is *not* restricted to ℓ -limited token passing rings, and stability of several other distributed systems can be assessed by this methodology (cf. [19] for the stability of the ALOHA system).

In the rest of this paper, we will consider the gated version of the ℓ -limited policy, i.e., the customers that are allowed to be served at queue i are only those that are present at the instant of token arrival at that queue. This is done for ease of exposition. The methodology presented in this paper can be easily extended to cover the case when customers arriving at a queue while it is being served can also be transmitted. We now summarize our main results. We shall analyze the token passing ring with Poisson arrivals with parameter λ_i for the i th station, general distribution of service times $\{S_i^k\}_{i=1}^M$ and switchover times $\{U_i^k\}_{i=1}^M$. Our first result (cf. Theorem 4) establishes a stochastic dominance among token passing rings, and it can be used to establish some bounds on the performance evaluation of the system (cf. [9]). We use this result to prove our main result regarding stability of the system, which can be formulated as follows (see also Theorem 7).

PROPOSITION. Consider a token passing ring consisting of M stations with ℓ_i limited service schedule for the i th station, and Poisson arrivals. Then the system is stable if

$$\lambda_j < \frac{\ell_j}{u_0}(1 - \rho_0) \quad \text{for all } j \in \mathcal{M} = \{1, 2, \dots, M\}$$

where $u_0 = \sum_{i=1}^M EU_i$ is the average total switchover time, and $\rho_0 = \sum_{i=1}^M \rho_i$ with $\rho_i = \lambda_i s_i$ and $s_i = ES_i$ being the average service time at the i th station. ■

Note that the above stability criteria are really represented in terms of a set of *linear* inequalities with respect to input rates λ_i for $i \in \mathcal{M}$. Figure 1 shows the stability region for $M = 3$.

The paper is organized as follows. In the next section we present our preliminary results that are of their own interests for the performance evaluation of the token passing ring. In particular, we find Markovian representations of the system, prove a crucial stochastic dominance relationship, and establish some Wald's type formulas. Finally, in Section 3 we present our main construction that leads to the proof of the above Proposition. We also suggest several generalizations of our main results.

2. PRELIMINARY RESULTS

In this section we present several results that are required to establish our main finding regarding the stability of the token passing ring. These results are of their own interests, and can be used to obtain some estimates for the performance evaluation of the system. In the sequel, we list our main assumptions, prove Markovian character of an imbedded queueing process, show two simple Wald's type identities, and finally establish a stochastic dominance relationship.

We start with a precise definition of our stochastic model. We shall adopt the following assumptions.

- (A1) There are M stations (queues) on a loop, each having infinite capacity buffer.
- (A2) Maximum number of customers served during the token visit at the i th queue is limited to $\ell_i < \infty$. Only customers that are present at the instant of token arrival can be served. This assumption will be relaxed later, to include the case $\ell_i = \infty$ (see Corollary 9).
- (A3) Arrival process $\{A_i^t\}_{i=1}^M$, $t \in [0, \infty)$, to the i th queue is a Poisson process with parameter λ_i . Here, A_i^t is the number of arrivals at queue i up to time t . The arrival process at a queue is independent of the arrival processes to other queues.

Figure 1: Stability region for the token passing ring with $M = 3$ users

- (A4) Service time process $\{S_i^k\}_{i=1}^M$ at queue i is i.i.d. with $s_i = ES_i$. The service time process at a queue is independent of the arrival processes at all queues and independent of the service time processes at other queues.
- (A5) The switchover time between i and $i + 1 \bmod M$ queue $\{U_i^k\}_{i=1}^M$ is i.i.d. with the average total switchover time equal to $u_0 = \sum_{i=1}^M EU_i$, and the process is independent of the arrival and the service time processes. To avoid unnecessary complications we assume that $P(U_i^n > 0) = 1$, $i = 1, \dots, M$.

Now we are ready to present a Markovian description of the system. We need a little bit of notation. By (A1), the token visits stations in a cyclic order. Let n denote the n th visit of the token to any queue. Then, $k_n = \lfloor (n-1)/M \rfloor$ denotes the cycle number in which the n th visits occurs (we start counting cycles from zero and assume that the token starts from queue 1). Note that the queue visited at the n th visit is just $J_n = n - Mk_n$. Let also T_n be the time instant of the n th visit of the token to any queue. Define an M -dimensional process $\tilde{N}^n = (\tilde{N}_1^n, \dots, \tilde{N}_M^n)$, where $\tilde{N}_i^n$ as the number of customers in queue i at time T_n . In addition, by $\mathcal{N}_i^n$ we mean the total number of customers served from queue i up to time T_n . Theorem below proves that $\tilde{N}^n$ is a Markov chain.

Theorem 1. *The process $\tilde{N}^n$ is a nonhomogeneous Markov chain.*

Proof. Let L_i^n be the number of customers served from queue J_n at the n th visit of the token. According to (A2) $L_{J_n}^n = \min\{\tilde{N}_{J_n}^n, \ell_{J_n}\}$ and $L_i^n = 0$ for $i \neq J_n$. The time B_n that elapses between n th and $n + 1$ st visit of the token to any queue is

$$B_n = \sum_{j=1}^{L_{J_n}^n} S_{J_n}^{\mathcal{N}_{J_n}^n + j} + U_{J_n}^{k_n}, \quad (1)$$

and the number of arrivals X_i^n to queue i between the n th and $n + 1$ st visits are

$$X_i^n = A_i^{T_n + B_n} - A_i^{T_n}. \quad (2)$$

Finally, the following recursions hold for the queue size in the i th station

$$\begin{aligned} \tilde{N}_i^{n+1} &= \tilde{N}_i^n + X_i^n & \text{if } i \neq J_n \\ \tilde{N}_i^{n+1} &= [\tilde{N}_{J_n}^n - \ell_{J_n}]^+ + X_i^n & \text{if } i = J_n, \end{aligned} \quad (3)$$

where $[x]^+ = \max\{x, 0\}$. The transmission policy depends only on the number of customers in the queues at time T_n which in turn depends on the service times of the customers served up to time T_n , as well as the interarrival times and the switchover times up to time T_n .

It follows from assumptions (A3)-(A5), that the processes $S_i^{N_i^n+j}$, $j = 1, \dots, A_i^{T_n+t} - A_i^{T_n}$, $t \in [0, \infty)$, and the random variable $U_{j_n}^{k_n}$, are independent of $\tilde{N}^m$, $1 \leq m \leq n$. From the above discussion we conclude that $\tilde{N}^{n+1}$ is of the form $\tilde{N}^{n+1} = f(\tilde{N}^n, Y^n)$ for some (measurable) function $f(\cdot)$ where Y^n is composed of the processes $S_i^{N_i^n+j}$, $A_i^{T_n+t} - A_i^{T_n}$ and $U_{j_n}^{k_n}$. Therefore, $\tilde{N}^n$ is a Markov chain (see, for example, page 34 of [15]). ■

There are other Markovian descriptions of the system. For example, define $N_j^n(i)$ to be the number of customers at queue j when the token visits queue i for the n th time. Then, the process $N^n(i) = (N_1^n(i), \dots, N_M^n(i))$ can be deduced from $\tilde{N}^n$ since $N^n(i) = \tilde{N}^{(n-1)M+i}$. This implies that for a fixed i , $N^n(i)$ is a Markov chain too. In fact, repeating the arguments of the proof of Theorem 1 it can be seen that $N^n(i)$ is a homogeneous Markov chain. It is easily verified also that the chain is irreducible and aperiodic. Hence, we have the following.

Corollary 2. *The process $N^n(i)$ of the queue lengths registered by the token when it visits (reference) queue i , is a homogeneous, irreducible and aperiodic Markov chain. ■*

We will need some Wald's type relationships between the average number of customers served per token visit and the average cycle time. Let L_i^n be the number of customers served by queue i during the n th visit of the token. Let also C_i^n be the cycle length, that is, the length of time between the n th and $n+1$ st visits of the token to the reference queue i . By EL_i and EC we denote the limiting averages of L_i^n and C_i^n , if they exist (it will be seen that the limiting average of the cycle length C_i^n does not depend on the reference queue i). The following result is proved by an extension of the method used in [21, page 9]. For completeness, and since we will need some of the steps of the proof in later sections, we provide the proof here.

Theorem 3. *Let the Markov chain $N^n(i)$ be positive recurrent (ergodic). Then,*

$$EL_j = \lambda_j EC, \quad j \in \mathcal{M} \quad (4)$$

and

$$EC = \frac{u_0}{1 - \sum_{j=1}^M \rho_j}, \quad (5)$$

where u_0 is the total average switchover time (cf. assumption (A5)) and $\rho_i = \lambda_i s_i$ is the utilization coefficient for the i th queue.

Proof. Without loss of generality, let $i = 1$. By the assumption, $N^n(1)$ is an ergodic Markov chain. Note that $N^n(1)$ has a natural regeneration structure, namely when all

queues are empty, that is, when the process returns to zero state $0 = (0, 0, \dots, 0)$. Assume $N^1(1) = 0$, and $K^1 = 1$. Define

$$K^{n+1} = \min\{m > K^n : N^m(1) = 0\},$$

and $R^n = K^{n+1} - K^n$. We shall also use $R = R^1$ as the length of a regeneration cycle. Due to the ergodicity of $N^n(1)$ we have $ER < \infty$. The sequences C_1^n, L_j^n , $n = 1, \dots$ are regenerative with respect to R^n . Since it easily seen that R^n is aperiodic, from Asmussen [1, Chapter V] Corollary 1.5 and Theorem 3.1, we have that

$$\lim_{n \rightarrow \infty} \frac{\sum_{k=1}^n L_j^k}{n} = \frac{E\left(\sum_{k=1}^R L_j^k\right)}{ER} \quad a.s. \quad \lim_{n \rightarrow \infty} \frac{\sum_{k=1}^n C_1^k}{n} = \frac{E\left(\sum_{k=1}^R C_1^k\right)}{ER}. \quad (6)$$

Moreover, L_j^n and C_1^n converge in distribution to L_j and C_1 such that

$$EL_j = \frac{E\left(\sum_{k=1}^R L_j^k\right)}{ER}, \quad EC_1 = \frac{E\left(\sum_{k=1}^R C_1^k\right)}{ER}. \quad (7)$$

Now we are in position to prove (4) and (5). Note first, that $\sum_{k=1}^R L_j^k \leq R\ell_j$ and since $ER < \infty$, we also have that $E\left(\sum_{k=1}^R L_j^k\right) < \infty$. Observe next that in the interval $[0, \sum_{k=1}^R C_1^k)$ *all the arriving customers from all queues must be served*. If A_j is the number of arrivals to queue j in the interval $[0, \sum_{k=1}^R C_1^k)$, then $EA_j = E\left(\sum_{k=1}^R L_j^k\right)$, and due to the Poisson assumption (A3) we also have $EA_j = \lambda_j E\left(\sum_{k=1}^R C_1^k\right)$. Therefore,

$$E\left(\sum_{k=1}^R L_j^k\right) = \lambda_j E\left(\sum_{k=1}^R C_1^k\right) < \infty. \quad (8)$$

The above, and (7) lead to $EL_j = \lambda_j EC_1$, which completes the proof of (4).

To prove (5) we note that the cycle length C_1^n is

$$C_1^n = U^n + \sum_{j=1}^M \sum_{m=0}^{L_j^n} S_j^{m+\sum_{k=1}^{n-1} L_j^k}, \quad (9)$$

where $U^n = \sum_{j=1}^M U_j^n$. Summing the above over first R visits of the token, taking the expectation of it, and using (8) one obtains the following

$$E\left(\sum_{n=1}^R C_1^n\right) = u_0 \cdot ER + \sum_{j=1}^M \lambda_j s_j E\left(\sum_{n=1}^R C_1^n\right). \quad (10)$$

Since by (8) $E\left(\sum_{n=1}^R C_1^n\right) < \infty$, using (7) we obtain from the above $EC_1 = EC = u_0/(1 - \sum_{i=1}^M \rho_i)$ as needed for (5). ■

The next result is our main finding in this section, and it is of prime importance for our stability analysis. Before we plunge into technical details, we first give a brief overview of our approach. In the process of estimating stability we need to build several dominant systems of the original token passing ring. For example, when we study stability of an isolated station, say the j th one, we partition all other stations into a class $\mathcal{S}$ of *nonpersistent* queues and a class $\mathcal{U}$ of *persistent* queues. A nonpersistent queue serves customers in the normal way as in the original token passing ring. A persistent queue, however, always sends the *maximum* allowable number of customers, that is, ℓ_i for $i \in \mathcal{U}$, by sending if necessary 'dummy' customers. A question is whether such a new system dominates the original token passing ring in some sense. If the answer is *yes*, then by proving stability of the dominant system we establish stability of the original token passing ring.

To attack the above problem, we first consider a simple example. Take a single $G|G|1$ queue. It is known (cf. Whitt [26]) that the increase in the service times (in a stochastic sense) will lead to the increase in the queue length and waiting time. What happens if we consider a $G|G|1$ queue with vacations $\{V^k\}$? Unfortunately, the increase of the vacation times does not lead – in general – to the increase of the queue length and waiting time. We will show in Theorem 4, however, that under the Poisson assumption of the arrival processes, an increase in the vacation times implies increase (in a stochastic sense) of the queue sizes seen by the token at the instants it visits the queues. The Poisson assumption is crucial to this result. To see this, consider the following example below.

EXAMPLE. *Counter-example.*

Consider a single queue with gated service. Assume that the interarrival times, vacation times and service times are constant. Let customers arrive every 5 seconds and have service times of 5 seconds. Consider two versions of the same system: one with vacations of 4 seconds and the other with vacations of 6 seconds. Assume that the system is initially empty. Then, it is easy to see that the queue size at the instant of the fourth visit of the token to the queue is 2 when the vacations are 4 seconds and 1 when the vacations are 6 seconds.

We state the next result in a general form, since it can be useful in the performance evaluation of other service disciplines. Specifically, in the terminology of [9], we consider the class of 'monotonic', 'contractive' policies. This amounts to replacing assumption (A2) with the following more general one.

(A2') Let $f_i(n)$ be the number of customers served from queue i when there are n queued

messages at the instant of token arrival at queue i . We assume that $f_i(n)$ is a *non-decreasing* function of the number of customers in the i th queue. In addition, the following relation holds.

$$f_i(n_1) - f_i(n_2) \leq n_1 - n_2 \quad \text{if} \quad n_1 > n_2. \quad (11)$$

Now we are ready to formulate our result. Consider two token passing rings, say θ and Θ . Both satisfy assumptions (A1)-(A5) with (A2) replaced by the weaker assumption (A2'). The system θ represents our original token passing ring. The system Θ differs only in the switchover times, namely, we assume that the switchover time for Θ is replaced by $\{\Delta_i^k + U_i^k\}_{i=1}^M$ for $k = 0, 1, \dots$. We assume that for every $i \in \mathcal{M}$ and every $k \geq 0$ we have $\Delta_i^k \geq 0$. We make the following assumption for the process Δ_i^k .

- (A6) The random variable Δ_i^k is independent of the service times, switchover times and the Poisson increments of the arrival processes to all stations after time $T_{Mk+(i+1)} - U_i^k$ (see Fig.2).

Theorem 4. *Let $\tilde{N}^n(\theta)$ and $\tilde{N}^n(\Theta)$ denote the queue lengths in both systems. Then, under the above assumptions, and under the condition that the token starts from the same queue, say queue number one, and with the same number of initial customers in both systems, the following holds*

$$\tilde{N}^n(\theta) \leq_{st} \tilde{N}^n(\Theta), \quad (12)$$

where $\leq_{st}$ means stochastically smaller.

Proof. To avoid cumbersome notation we present the proof only for $M = 2$ users. The proof can be easily extended to any number of users.

We define some new variables. For a system θ let T_n^θ and D_n^θ denote the instances of the n th visit and the n th token departure from any queue respectively. As before, J_n^θ denotes the queue number visited at the n th visit of the token. Finally, $L_i^n(\theta)$ denotes the number of customers served from queue i at the n th visit of the token. Clearly, for our two station system $L_1^n(\theta) = 0$ for n even, and $L_2^n(\theta) = 0$ for n odd. In a similar manner we define respective quantities in the Θ system.

We assume that S_i^k are assigned upon the beginning of the service. Since the service policies we are considering do not depend on the knowledge of the service requirements of the customers, this assumption does not change the distributions of the processes involved, hence also stochastic dominance property of the systems. Under this modification, we show

Figure 2: Illustration to the proof of Theorem 4

how to construct from the system Θ a token passing ring $\bar{\theta}$, which is stochastically equivalent to the system θ and for which we have that

$$\tilde{N}^n(\bar{\theta}) \leq \tilde{N}^n(\Theta). \quad (13)$$

Figure 2 should help to understand our construction. Assume $\tilde{N}_i^1(\bar{\theta}) = \tilde{N}_i^1(\Theta)$ for $i = 1, 2$. Now, we assign to the $\tilde{N}_1^1(\bar{\theta})$ customers of $\bar{\theta}$, the service times S_i^k exactly as in Θ . The same functions $f_i(n)$, $i = 1, 2$ are used in both systems. Therefore, the decision to switch to queue 2 will occur at the same time, namely $D_1^{\bar{\theta}} = D_1^{\Theta}$. The switchover time for $\bar{\theta}$ becomes now U_1^1 , and of course $T_2^{\bar{\theta}} \leq T_2^{\Theta}$ since $\Delta_1^1 \geq 0$ (see Fig. 2).

The arrivals in the system $\bar{\theta}$ in $[D_1^{\bar{\theta}}, T_2^{\bar{\theta}})$ are now assumed be identical to the arrivals in $[D_1^{\Theta} + \Delta_1^1, T_2^{\Theta})$ in Θ system. Therefore clearly $\tilde{N}_i^2(\bar{\theta}) \leq \tilde{N}_i^2(\Theta)$ for $i = 1, 2$. We continue our construction, and at time $T_2^{\bar{\theta}}$ service times are assigned from S_2^k in the same order as in Θ . Also, the arrivals to system $\bar{\theta}$ in $[T_2^{\bar{\theta}}, T_2^{\bar{\theta}} + S_2^1 + \dots + S_2^{L_2^1(\bar{\theta})})$ are taken to be identical to the arrivals in $[T_2^{\Theta}, T_2^{\Theta} + S_2^1 + \dots + S_2^{L_2^1(\bar{\theta})})$. Note that this can be done since by (A2') $L_2^1(\bar{\theta}) \leq L_2^1(\Theta)$. Observe also that $T_2^{\bar{\theta}} + S_2^1 + \dots + S_2^{L_2^1(\bar{\theta})} = D_2^{\bar{\theta}}$ (Fig. 2).

To complete the description of the system $\bar{\theta}$ we have to specify the arrivals in $[D_2^{\bar{\theta}}, D_2^{\bar{\theta}} + U_2^1)$. These are taken to be exactly the arrivals in $[D_2^{\Theta} + \Delta_2^1, D_2^{\Theta} + \Delta_2^1 + U_2^1)$ in the dominant system Θ (see Fig. 2). Note from the construction that

$$\tilde{N}_1^3(\bar{\theta}) = \tilde{N}_1^2(\bar{\theta}) + A_{[T_2^{\bar{\theta}}, T_3^{\bar{\theta}}]} \leq \tilde{N}_1^2(\Theta) + A_{[T_2^{\Theta}, T_3^{\Theta}]} = \tilde{N}_1^3(\Theta)$$

and also by (11),

$$\tilde{N}_2^3(\bar{\theta}) = \tilde{N}_2^2(\bar{\theta}) - L_2^2(\bar{\theta}) \leq \tilde{N}_2^2(\Theta) - L_2^2(\Theta) = \tilde{N}_2^3(\Theta) .$$

We can now repeat exactly the same procedure to construct $\bar{\theta}$ in the interval $[T_n^{\bar{\theta}}, T_{n+1}^{\bar{\theta}})$, $n \geq 3$, in the same manner as it was constructed in the interval $[T_2, T_3)$. By construction the service times and switchover times of system $\bar{\theta}$ are identically distributed to the corresponding variables of system θ and are independent of the interarrival process. In addition, assumption (A6) and the fact that the servicing policy is nonanticipative assures that the times $T_{n+1}^{\bar{\theta}} - U_{J_n}^{k_n}$ are stopping times for the Poisson arrival processes to all stations. The independence of the increments of the Poisson process implies now, that the constructed interarrival process in system $\bar{\theta}$ is Poisson with rate λ_i for queue i . Moreover, by construction (13) holds. Since $\bar{\theta}$ is stochastically equivalent to θ , we have that the distribution of $\tilde{N}^n(\theta)$ is identical to the distribution of $\tilde{N}^n(\bar{\theta})$. This completes the proof of Theorem 4. ■

3. MAIN RESULTS

In this section we present a proof of our proposition from the Introduction. However, before we plunge into technical details an overview of our stability approach is discussed. We shall argue that our idea is novel (cf. [19]), and can be successfully used to establish stability of some other distributed systems (see Szpankowski [19], [18] for applications to the ALOHA system and coupled-processors system).

Our approach is based on three simple observations. *At first*, we note that a multi-dimensional process is stable if and only if its components are stable. More precisely, if $N^n = (N_1^n, \dots, N_M^n)$ is a stochastic process – not necessary a Markov chain – then say the process is stable if the distribution of N^n as $n \rightarrow \infty$ *exists* and the distribution is *honest*. In other words, N^n is stable if for $\mathbf{x} \in \mathcal{R}^M$, where $\mathcal{R}$ is the set of real numbers the following holds for all points of continuity of $F(\mathbf{x})$ below.

$$\lim_{n \rightarrow \infty} \Pr\{N^n \leq \mathbf{x}\} = F(\mathbf{x}) \quad \text{and} \quad \lim_{\mathbf{x} \rightarrow \infty} F(\mathbf{x}) = 1 \quad (14)$$

where $F(\mathbf{x})$ is the limiting distribution function, and by $\mathbf{x} \rightarrow \infty$ we understand that $x_j \rightarrow \infty$ for all $j \in \mathcal{M} = \{1, \dots, M\}$. If a weaker condition holds, namely,

$$\lim_{\mathbf{x} \rightarrow \infty} \lim_{n \rightarrow \infty} \inf \Pr\{N^n \leq \mathbf{x}\} = 1 , \quad (15)$$

then the process is called *substable* [10] or *tight* or *bounded in probability* sense. Otherwise, the system is unstable (for more details see Loynes [10]). The isolation property mentioned above can be formally presented as follows.

Lemma 5. (i) *If for all $j \in \mathcal{M}$ the one dimensional processes N_j^n are stable (substable), then the M -dimensional process $N^n = (N_1^n, N_2^n, \dots, N_M^n)$ is substable.*

(ii) *If for some j , say j^* , $N_{j^*}^n$ is unstable, then N^n is also unstable.*

Proof. See Szpankowski [18] and [19]. ■

Secondly, to obtain stability conditions for a single isolated station in the token passing ring, we apply the technique of Loynes [10] who proved that a single $G|G|1$ queue is stable if the input rate is smaller than the average service time provided that service times and interarrival times are jointly stationary and ergodic. To verify a technical stationarity condition in Loynes' criteria we apply the stochastic dominance result of Theorem 4. More precisely, we partition the set of queues, $\mathcal{M}$, into a set $\mathcal{S}$ of nonpersistent queues and into a set $\mathcal{U}$ of persistent queues as was described in section 2. By Theorem 4 the new system stochastically dominates the original one, and by proving stability of it, we clearly establish stability conditions for the original token passing ring. We use induction to establish stability conditions for the nonpersistent queues in the new system, while the stability condition for a persistent queue is established by using Loynes' criteria.

To fulfill the above plan, we start by showing a result that will be useful in proving the condition for a persistent queue in the dominant system. More formally, we consider a single queue that always services ℓ (dummy in necessary) customers, even if there are less than ℓ customers in the queue when the token (server) arrives. The server is of walking type, and after servicing the ℓ customers it goes for a vacation. It is assumed that the cycle time C^n represents a *stationary and ergodic sequence* with mean EC (no independence is required). The arrival process A^t to this queue is a Poisson process with parameter λ , independent of the process of cycle times. Let N^n represent the queue length at the beginning of the n th cycle, and by X^n we denote the number of customers arrived during the n th cycle. Note that since the processes C^n and A^t are independent and A^t is Poisson, the process X^n is a stationary and ergodic sequence with mean $EX = \lambda EC$. Then, the queue length satisfies the following recurrence

$$N^{n+1} = \max\{N^n + X^n - \ell, X^n\}. \quad (16)$$

We prove the following stability result.

Lemma 6. *Consider the queueing system just described. If $\lambda EC < \ell$, then the queue is stable in the sense of definition (14).*

Proof. We apply Loynes' scheme to prove the lemma. We may assume without loss of generality that X^n is a two-sided stationary process, that is it is defined for $-\infty < n < \infty$. Note next, that the recursion (16) is such that the RHS of it represents a nondecreasing and left continuous (in N^n) function. Therefore, by Lemma 1 in Loynes [10] we conclude that there exists a stationary sequence $\mathcal{N}^k$ such that N^n converges weakly (in distribution) to $\mathcal{N}^k$. Now, we need to find out when $\mathcal{N}^k$ is honest. Recursion (16) is not quite the same as the one treated by Loynes, however we can use similar arguments as follows.

By telescoping the recurrence (16) we immediately obtain

$$N^n = \max_{1 \leq r \leq n-1} \left\{ X^r + \sum_{k=r+1}^{n-1} \bar{X}^k \right\}, \quad (17)$$

where $\bar{X}^k = X^k - \ell$, provided $N^0 = 0$. Using the stationarity of X^n and arguing as in Loynes [10] we have that $\mathcal{N}^k$ is honest if and only if

$$\sup_{r \geq 1} \left\{ X^{-r} + \sum_{k=1}^{r-1} \bar{X}^{-k} \right\} < \infty,$$

The last condition can be equivalently rewritten as

$$\limsup_r \left\{ X^{-r} + \sum_{k=1}^{r-1} \bar{X}^{-k} \right\} < \infty. \quad (18)$$

Now, we proceed as follows

$$\begin{aligned} X^{-r} + \sum_{k=1}^{r-1} \bar{X}^{-k} &= (r-1) \left(\frac{X^{-r}}{r-1} + \frac{\sum_{k=1}^{r-1} \bar{X}^{-k}}{r-1} \right) \\ &= (r-1) \left(\frac{\ell}{r-1} + \frac{\sum_{k=1}^{r-1} \bar{X}^{-k}}{r} \cdot \frac{r}{r-1} \right). \end{aligned} \quad (19)$$

But as discussed above, $EX^n = \lambda EC$. Therefore, under the hypothesis of the Lemma $E\bar{X}^n = \lambda EC - \ell < 0$, and then by Ergodic Theorem $\sum_{k=1}^r \bar{X}^{-k}/r \rightarrow \lambda EC - \ell < 0$ as $r \rightarrow \infty$. Since $\ell/(r-1) \rightarrow 0$, we immediately show (18), and this completes the proof. ■

Now we are ready to prove our main result described already in Proposition of the Introduction. We repeat it below for the reader's convenience. The proof uses the idea presented in the overview above, however due to technical reasons we carry it out formally through the mathematical induction.

Theorem 7. *The Markov chain $N^n(j)$ representing the queue lengths in the token passing ring when it visits queue $j \in \mathcal{M}$ is ergodic if*

$$\lambda_j < \frac{\ell_j}{u_0}(1 - \rho_0) \quad \text{for all } i \in \mathcal{M} \quad (20)$$

where $\rho_0 = \sum_{j=1}^M \rho_j$.

Proof We use mathematical induction. For $M = 1$ the proof is simple. In this case, since the switchover times are independent of the service times and the interarrival times, the one dimensional process $N^n(1)$ is a Markov chain satisfying (16). By the Lyapunov test function method (cf. [18], [23]) we have $E\{N^{n+1} - N^n \mid N^n = k \geq \ell\} = \lambda(\ell s_1 + u_0) - \ell = \lambda u_0 - \ell(1 - \rho_0)$, where $\rho_0 = \lambda s_1$. Note that this drift is negative when $\lambda < \ell(1 - \rho_0)/u_0$ as needed.

Now we assume that the theorem is true for $M - 1$ and prove that it can be extended to the M queue case. Let $(\mathcal{U}, \mathcal{S})$ be a partition such that $\mathcal{U} \neq \emptyset$, of the set $\mathcal{M}$ of M queues into *persistent* and *nonpersistent* queues. Assume for a moment that $\mathcal{S} \neq \emptyset$. Note that the cardinality $|\mathcal{S}|$ of $\mathcal{S}$ is not larger than $M - 1$. Let $\bar{N}^n(i) = \{\bar{N}_1^n(i), \dots, \bar{N}_M^n(i)\}$ be the queue lengths when the token visits the i th queue for the n th time in the $(\mathcal{U}, \mathcal{S})$ system in which persistent queues $\mathcal{U}$ send dummy packets as discussed above. Observe that the modified system differs from the original token ring system only in the switchover time from a persistent queue to the successor of that queue in the ring. Specifically, if $i \in \mathcal{U}$, then the switchover times become,

$$\bar{U}_i^k = \Delta_i^k + U_i^k,$$

where Δ_i^k is the time needed to service the dummy messages at node i (if any). Since the queue length at the n th visit of the token to queue i is independent of future arrivals or future service times, and the service times of the dummy messages are independent of the rest of the processes in the system, it is clear that Δ_i^k satisfies condition (A6) of Section 2. Therefore, according to Theorem 4, if $N^1(1) = \bar{N}^1(1)$, then

$$N^n(j) \leq_{st} \bar{N}^n(j), \quad \text{for all } n, \quad j \in \mathcal{M}. \quad (21)$$

Note now that the queues in $\mathcal{S}$ constitute a token passing ring with $|\mathcal{S}|$ stations satisfying conditions (A1)-(A5) of Section 2, whose operation is independent of the interarrival processes in the persistent queues. The total average switchover time $\bar{u}_0$ to this ring is equal to

$$\bar{u}_0 = u_0 + \sum_{i \in \mathcal{U}} \ell_i s_i.$$

Let the queue lengths in such a system be denoted as $\{\bar{N}_S^n(i)\}_{i \in \mathcal{S}}$. Clearly, $\bar{N}_S^n$ is a Markov chain, and since $|\mathcal{S}| \leq M - 1$ we can apply the induction hypothesis. Hence, for $i \in \mathcal{S}$, $\bar{N}_S^n$ is ergodic if

$$\lambda_i < \frac{\ell_i}{u_0 + \sum_{i \in \mathcal{U}} \ell_i s_i} \left(1 - \sum_{i \in \mathcal{S}} \rho_i \right) \quad i \in \mathcal{S}. \quad (22)$$

Assume now that (22) holds, and consider a queue in $\mathcal{S}$, say queue 1 and let $C_S^n(1)$ be the process of cycle lengths (successive visits to queue 1). The processes $N_S^n(1)$, $C_S^n(1)$ are regenerative with respect to the renewal process of the successive visits of the process $N_S^n(1)$ to state 0 (see the proof of Theorem 3). Since $N_S^n(1)$ is ergodic, the renewal process has finite mean, and therefore, we can construct a strictly stationary version of $(N_S^n(1), C_S^n(1))$ [14]. Consider now that the original and the dominant system $(\mathcal{U}, \mathcal{S})$ are defined on a probability space on which the processes $(N_S^n(1), C_S^n(1))$ are strictly stationary and ergodic. We define for $i \in \mathcal{U}$, $\bar{N}_i^1(1) = 0$; also, $N^1(1) = \bar{N}^1(1)$. Let $T^n(i)$ be the time of the n th visit of the token to queue i . Let also $X_1^n(i)$, $X_2^n(i)$, $i \in \mathcal{S}$ be the number of arrivals to queue i in the interval $[T^n(1), T^n(i))$, $[T^n(i), T^{n+1}(1))$ respectively, and $X^n(i) = X_1^n(i) + X_2^n(i)$. The process $\bar{N}_i^n(1)$, $i \in \mathcal{U}$ satisfies the relations,

$$\bar{N}_i^{n+1}(1) = \max(\bar{N}_i^n(1) + X_1^n(i) - l, 0) + X_2^n(i) \leq \max(\bar{N}_i^n(1) + X^n(i) - l, X^n(i)). \quad (23)$$

Define now the process M_i^n as follows. $M_i^1 = 0$, and

$$M_i^{n+1} = \max(M_i^n + X^n(i) - l, X^n(i)), \quad n = 1, 2, \dots \quad (24)$$

By definition, $X^n(i)$ is the number of independent Poisson arrivals at queue i in the cycle $C_S^n(1)$, and since the sequence $C_S^n(1)$ is strictly stationary, Lemma 6 applies to the process M_i^n . We conclude that the process M_i^n is stable if

$$\lambda_i < \frac{\ell_i}{E\bar{C}_S^1(1)} = \frac{\ell_i}{u_0 + \sum_{i \in \mathcal{U}} \ell_i s_i} \left(1 - \sum_{i \in \mathcal{S}} \rho_i\right) \quad i \in \mathcal{U}, \quad (25)$$

where the equality in (25) follows from the fact that by Theorem 3,

$$E\bar{C}_S^1(1) = \frac{\bar{u}_0}{1 - \sum_{i \in \mathcal{S}} \rho_i} = \frac{u_0 + \sum_{j \in \mathcal{U}} \ell_j s_j}{1 - \sum_{j \in \mathcal{S}} \rho_j}. \quad (26)$$

Since $M_i^1 = \bar{N}_i^1(1) = 0$, using (23), (24) it follows by an inductive argument that

$$\bar{N}_i^n(1) \leq M_i^n, \quad n = 1, 2, \dots, \quad i \in \mathcal{U}$$

and therefore, the process $\bar{N}_i^n(1)$, $i \in \mathcal{U}$ is substable provided that (25) holds. Since the process $\bar{N}_i^n(1)$, $i \in \mathcal{S}$ is stable by construction, it follows from Lemma 5(i) and (21) that the irreducible, aperiodic Markov chain $N^n(1)$ is substable and therefore, ergodic. Finally, the ergodicity of $N^n(1)$ implies ergodicity of $N^n(j)$, $j = 1, \dots, M$. To see this note that $N^n(j)$ is regenerative with respect to the renewal process of the successive visits of the

process $N^n(1)$ to state 0. Since, by the ergodicity of $N^n(1)$, this renewal process has finite mean, the steady state distribution of $N^n(j)$ exists.

Putting everything together, from (22) and (25) we finally have that the Markov chain $N^n(j)$ is ergodic for every $j \in \mathcal{M}$ if

$$\lambda_i < \frac{\ell_i}{u_0 + \sum_{i \in \mathcal{U}} \ell_i s_i} \left(1 - \sum_{i \in \mathcal{S}} \rho_i \right) \quad i \in \mathcal{M} . \quad (27)$$

It is easy to see that (27) holds also in the case $\mathcal{S} = \emptyset$, since in this case the cycles are i.i.d. random variables. Since (27) holds for every partition $\mathcal{P} = (\mathcal{S}, \mathcal{U})$ of the set $\mathcal{M}$ such that $\mathcal{S} \neq \mathcal{M}$, we conclude that the sufficient condition for stability of the system is

$$\mathcal{R} = \bigcup_{\mathcal{S} \subset \mathcal{M}} \mathcal{R}_{\mathcal{S}} , \quad (28)$$

where

$$\mathcal{R}_{\mathcal{S}} = \{ \lambda = (\lambda_1, \dots, \lambda_M) : \text{condition (27) holds} \} . \quad (29)$$

Finally, to complete the proof we need to show that

$$\bigcup_{\mathcal{S} \subset \mathcal{M}} \mathcal{R}_{\mathcal{S}} = \{ \lambda = (\lambda_1, \dots, \lambda_M) : \lambda_i < \frac{\ell_i}{u_0} (1 - \sum_{i=1}^M \rho_i) \quad i \in \mathcal{M} \} . \quad (30)$$

This requires only algebraic manipulations, and it is delayed till the Appendix. This completes the proof of Theorem 7. ■

We can use Theorem 7 to establish some other stability results. We concentrate here on two problems. First, Theorem 7 can be extended to the process of queue lengths at arbitrary time instants, that is the process $\tilde{N}(t) = (\tilde{N}_1(t), \dots, \tilde{N}_M(t))$, where $\tilde{N}_i(t)$ is the queue length at queue i at time t . The second extension deals with the *gated-unlimited service* discipline (and also *exhaustive discipline*) in which we set $\ell_i \rightarrow \infty$ in assumption (A2).

Let us first consider the stability of $\tilde{N}(t)$. Assume that $N^n(1)$ is ergodic. Using the notation of the proof of Theorem 3, we have from (8) that $E \left(\sum_{k=1}^R C_1^k \right) < \infty$, i.e., the renewal process $\tilde{C}^n$ of the *length of time* between two successive returns to state 0 of the process $N^n(1)$ has finite expectation. Since the interarrival times are exponential, this renewal process is non-lattice. Since $\tilde{N}(t)$ is regenerative with respect to $\tilde{C}^n$, we conclude that

Corollary 8. *The process $\tilde{N}(t)$ is stable if (20) holds. ■*

The next result extends assumption (A2) to gated-unlimited service disciplines. In fact, our methodology allows to establish rigorously stability conditions for many other service disciplines such as *Bernoulli*, *random geometric*, *time limited* and so forth (cf. [9], [22]). This needs, however, extensions to our results, and it will be reported in a forthcoming paper. Here, we only concentrate on the extension to the gated-unlimited service discipline. Assume that a subset $\mathcal{M}_\infty$ of the queues employs the gated-unlimited service discipline and let $\mathbf{N}_\infty^n(i)$ be the vector of queue sizes at all stations when the token visits the i th queue for the n th time. Then we have the following

Corollary 9. For $i \in \mathcal{M}$, the Markov chain $\mathbf{N}_\infty^n(i)$ is ergodic if $\rho_0 = \sum_{j=1}^M \rho_j < 1$ and

$$\lambda_j < \frac{\ell_j}{u_0}(1 - \rho_0), \quad j \in \mathcal{M} - \mathcal{M}_\infty. \quad (31)$$

Proof. For arbitrary $l > 0$, let $\mathbf{N}_l^n(i)$ be the queue sizes when the queues in $\mathcal{M}_\infty$ are employing the ℓ -limited policy with threshold l . As in Theorem 3, define for $1 \leq l \leq \infty$, $\mathbf{N}_l^1(1) = \mathbf{0}$, $K_l^1 = 0$,

$$K_l^{n+1} = \min\{m > K_l^n : \mathbf{N}_l^m(1) = \mathbf{0}\}$$

and $R_l^n = K_l^{n+1} - K_l^n$. Using the Poisson assumption of the interarrival process and techniques similar to those used in the proof of Theorem 4 it can be shown that $R_\infty^1 \leq_{st} R_l^1$, $l = 1, 2, \dots$. Therefore, for any l , if $\mathbf{N}_l^n(1)$ is ergodic, so is $\mathbf{N}_\infty^n(1)$. Since $\rho_0 < 1$ we can pick l large enough so that

$$\lambda_j < \frac{l}{u_0}(1 - \rho_0), \quad j \in \mathcal{M}_\infty.$$

This, together with (31) implies that the Markov chain $\mathbf{N}_l^n(1)$ is ergodic, which in turn implies that $\mathbf{N}_\infty^n(1)$ is ergodic. ■

Note that Corollary 9 can be easily extended to the case where a subset of the nodes employs the *exhaustive* discipline. Indeed, in this case the proof of Corollary 9 can be repeated verbatim with the only exception that now $\mathcal{M}_\infty$ is interpreted as the set of nodes employing the exhaustive discipline.

APPENDIX

We prove (30). Let $\mathcal{M}_i = \mathcal{M} - \{i\}$, and denote the RHS of (30) as $\tilde{\mathcal{R}}$, that is,

$$\tilde{\mathcal{R}} = \{\lambda = (\lambda_1, \dots, \lambda_M) : \lambda_i < \frac{\ell_i}{u_0}(1 - \sum_{i=1}^M \rho_i) \quad i \in \mathcal{M}\}. \quad (A1)$$

We will prove that

$$\bigcup_{i=1}^M \mathcal{R}_{\mathcal{M}_i} = \tilde{\mathcal{R}}. \quad (A2)$$

At first, we show that for every $i \in \mathcal{M}$ we have $\mathcal{R}_{\mathcal{M}_i} \subset \tilde{\mathcal{R}}$. Let $\lambda \in \mathcal{R}_{\mathcal{M}_i}$. Then, by (27)

$$\lambda_i < \frac{\ell_i}{u_0 + \ell_i s_i} \left(1 - \sum_{k \neq i} \rho_k \right)$$

and this is equivalent to

$$\lambda_i < \frac{\ell_i}{u_0} (1 - \rho_0). \quad (A3)$$

But (A3) implies that $u_0 \rho_i \leq \ell_i s_i (1 - \sum_{i=1}^M \rho_i)$. Using this, after some algebra, we conclude that the following also holds

$$\frac{(1 - \sum_{k \neq i} \rho_k)}{u_0 + \ell_i s_i} \leq \frac{(1 - \sum_{k=1}^M \rho_k)}{u_0}. \quad (A4)$$

Therefore, for every $j \in \mathcal{M}_i$ we have

$$\lambda_j < \frac{\ell_j (1 - \sum_{k \neq i} \rho_k)}{u_0 + \ell_i s_i} \leq \frac{\ell_j (1 - \sum_{k=1}^M \rho_k)}{u_0}, \quad (A5)$$

and (A3), (A5) imply $\mathcal{R}_{\mathcal{M}_i} \subset \tilde{\mathcal{R}}$, as needed.

Now we prove that $\bigcup_{i=1}^M \mathcal{R}_{\mathcal{M}_i} \supset \tilde{\mathcal{R}}$. Note that

$$\mathcal{R}_{\mathcal{M}_i} = \{ \lambda \in R^M : \rho_j < \frac{\ell_j s_j (1 - \sum_{k \neq j} \rho_k)}{u_0 + \ell_j s_j}, \quad j = 1, \dots, M \}. \quad (A6)$$

Let $\lambda \in \tilde{\mathcal{R}}$, and let k be such that $\rho_k / (\ell_k s_k) \geq \rho_j / (\ell_j s_j)$ for all $j \in \mathcal{M}$. Then, $\lambda \in \tilde{\mathcal{R}}$ implies $\rho_j u_0 < \ell_j s_j (1 - \rho_0) + \rho_k \ell_j s_j - \rho_j \ell_k s_k$, $j = 1, \dots, M$, and this leads to $\lambda \in \mathcal{R}_{\mathcal{M}_k}$, as required. This proves (A2).

We also need to show that $\bigcup_{S \subset \mathcal{M}} \mathcal{R}_S = \bigcup_{i=1}^M \mathcal{R}_{\mathcal{M}_i}$. This can be proved by applying similar algebraic manipulations as above. Details are left for the reader.

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