4-2016

Optimization of wireless power networks for biomedical applications

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For the degree of Master of Science in Biomedical Engineering

Is approved by the final examining committee:

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Approved by Major Professor(s): Pedro Irazoqui

Approved by: Pedro Irazoqui  4/7/2016

Head of the Departmental Graduate Program  Date
OPTIMIZATION OF WIRELESS POWER NETWORKS FOR BIOMEDICAL APPLICATIONS

A Thesis
Submitted to the Faculty
of
Purdue University
by
Kyle A. Thackston

In Partial Fulfillment of the
Requirements for the Degree
of
Master of Science in Biomedical Engineering

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West Lafayette, Indiana
For my parents and brother.
ACKNOWLEDGEMENTS

I would like to thank my advisor Professor Pedro P. Irazoqui, for challenging me as a researcher and providing me opportunities to create new theory. I would like to thank my committee members Professor Dimitrios Peroulis and Professor Hugh Lee for their advice and time in reviewing this work. I would like to thank Jack Williams for his stimulating discussions on electromagnetics, Michael Sparapany for his discussion and general griping on the art of numerical optimization, and the rest of my lab members for their support. I would like to acknowledge Professor William Crossley of Purdue University for giving me access to the base genetic algorithm code which I found invaluable for this work; this code was originally developed by Andrew Potvin of Mathworks, Inc., and edited over the years by Professor Crossley and his graduate students. Most of all, I would like to thank Henry Mei for his invaluable mentorship during my time at the Center for Implantable Devices thus far.

“On two occasions I have been asked, 'Pray, Mr. Babbage, if you put into the machine wrong figures, will the right answers come out?' I am not able rightly to apprehend the kind of confusion of ideas that could provoke such a question.”

- Charles Babbage
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ABSTRACT


Successful physiological integration of electronics will open the doors to new methods of treatment and diagnoses. One of the key challenges of this integration is designing devices as small as possible while still maintaining high functionality, such as bio-signal recording, processing, telemetry, and stimulation. Wireless power transfer (WPT) can help shrink a device’s footprint by removing the need for bulky batteries. While many modalities of WPT exist for biomedical applications, the optimal power transfer efficiency (PTE) is seldom achieved due to improper impedance matching. Existing methods for determining the optimal impedance matching conditions tend to be application specific and make assumptions incompatible with biomedical applications.

In this work, I present a new formulation of the generalized coupling matrix, a tool typically used for filter synthesis, as a method for optimization of WPT networks. This impedance matching synthesis method can account for non-ideal resonators, weak couplings, complex loads, mixed couplings, and arbitrary sized WPT networks. Moreover, I present a hybrid optimization strategy that combines a genetic algorithm with SQP to generate numerical solutions for optimal impedance matching and user designed power splitting. I demonstrate the validity of the model, as well as the versatility by applying the optimization to both inductor coil and resonant cavity modalities of
WPT. This tool shows utility for rapid design of WPT networks and for dynamic tuning control methods.
CHAPTER 1. INTRODUCTION

1.1 Modern Healthcare: The Movement towards Device Based Treatment

While modern medicine has improved amazingly over the decades, electrical devices are beginning to offer an alternative to drug based treatment. For conditions such as epilepsy, depression, or chronic pain, drug based treatment requires a lifetime of costly medicine and often introduces harmful side effects to the patient.

Implanted electrical devices offer solutions to a myriad of health conditions. Nervous stimulators have been demonstrated to treat epilepsy, depression, chronic pain, and gastrointestinal issues [1] [2]. Wireless recording can also enable integration of prosthetics for amputees [3]. Fully implanted glucose sensors would increase the quality of life for diabetics, avoiding the need for transdermal continuous glucose monitoring or painful finger pricking [4] [5]. Furthermore, implanted bio-data recording would greatly aid diagnostic efforts.

1.2 Challenges of Physiological Integration of Electrical Devices

Operating an electrical device seamlessly alongside the body’s functions is no small feat. From a device perspective, the inside of the human body is a salt water environment; advanced packaging must allow devices to exist for extended periods of time without breaking down. Power consumption is a key constraint as well -- long term implantation requires a long term powering solution without harming the body by
introducing dangerous chemicals, radiation, or high levels of heat. On top of these issues, engineers must design devices as small as possible to fit easily into the body.

Current battery technology does not provide the energy density needed to power many types of long term devices in a compact form factor [6]. The best lithium ion batteries can power a pacemaker, a relatively low power implant, for approximately 20 years [7] [8]. Devices with nervous stimulation and telemetry capabilities use much more power over their lifetime. Moreover, batteries typically take up to 50% of a typical pacemaker’s volume, limiting miniaturization. Radioisotope batteries, devices which harvest energy from a thermal gradient caused by radioactive isotope degradation, were common for powering early pacemakers and actually had superior lifetimes to most batteries before lithium ion technology became widespread [9]. These devices lost popularity due to fear of radiation leak on the patient and governments’ development of nuclear isotope regulation. Just as well, these batteries also lend themselves poorly to miniaturization.

Two alternatives to energy storage are energy harvesting and wireless power transfer. Many modalities of passive energy harvesting have been investigated, ranging from bioreactors, thermocouples, RF energy harvesters, and kinetic energy harvesters [10] [11] [12]. Although attractive in theory, energy harvesting typically provides very low power output (<1 mW), often unreliably. For these reasons, directed wireless power transfer (WPT) has become an active research area for the design of powered biomedical implants. Wireless power transfer is roughly defined as the intentional transmission of electrical energy without the use of conventional conductors, such as power cables [13]. In a biomedical setting, this means implanted devices can be recharged wirelessly,
allowing smaller batteries and preventing additional surgeries. Alternatively, patients could power their devices continuously, removing the need for energy storage entirely.

Not without its challenges, WPT requires some receive structure and circuitry to convert transmitted energy into a stable supply for devices. The power must also be transmitted efficiently enough to successfully power the device, but not interfere destructively with biological tissue. As most WPT modalities use electromagnetic (EM) fields or waves, engineers typically attempt to stay below the specific absorption rate (SAR) for human tissue [14]. In cases such as animal research or dynamic device powering in humans, power transfer must also be robust enough to afford a range of motion for the user, often requiring total or partial omnidirectionality [15]. This can be either inherent in the structure or achieved with dynamic tuning. Miniaturization of receive structures, optimization of power transfer efficiency (PTE), and robust, omnidirectional powering are the primary research areas for biomedical wireless power transfer.

1.3 Summary of Wireless Power Solutions for Biomedical Devices

We can categorize biomedical WPT into three areas: near-field, far-field, and the recently proposed “mid-field” [16] [17]. A notable exception to this schema is ultrasound WPT, which transfers power through human tissue acoustically [18]. While this is certainly an interesting topic, analysis of ultrasound WPT systems is less developed than their electromagnetic counterparts. Ultrasound systems will not be considered for this work.
For the purposes of wireless power transfer, we define the near field as a region where standing electric or magnetic fields are the primary mode of transferring energy, as opposed to the far field where energy is carried in a propagating wave. Traditionally, far field power transfer is associated with higher frequency. The “mid-field” is a newly proposed frequency regime in which propagating waves are made to resonate in tissue via evanescent wave coupling, typically around 1.2 – 1.6 GHz [16]. Further research into this modality may result in a reclassification into near or far field, but for our purposes the mid-field is novel enough to consider separately.

1.3.1 Far-Field Powering

Because of the poor prospects of inductive charging, a large amount of early work on WPT revolved around far-field powering. Energy is contained in a propagating wave and can be transmitted over larger distances due to the inverse square law of radiated power intensity [19]. Electrical engineers specializing in communication systems have a variety of methods for designing miniaturized, omnidirectional antennas for far-field, which lend themselves well for translation to WPT applications. Indeed, far field energy transfer is perhaps the most promising method for commercial WPT, such as powering cellphones and other household appliances as conveniently as one would connect to WiFi.
Figure 1: Diagram representing far-field power transfer. Power is transmitted from an antenna source and propagates. Some of the power is reflected or absorbed by the tissue boundary, attenuating the power transmitted to the receive antenna structure. As with all described methods of WPT, AC power is converted to DC by a rectifier circuit before being delivered to the load.

Despite these advantages, far-field systems can seldom achieve the requisite efficiency for biomedical applications without exceeding SAR limitations. This is due to tissue’s inherent attenuation of propagating EM waves. Much of the propagating energy is reflected off the tissue boundary or absorbed directly and converted into heat, which can be unsafe for the tissue. This leaves little power to reach the receive antenna, thus making this modality suitable only for low power devices such as RFID tags. For systems with large power requirements or systems located deep under tissue, far-field is often unsuitable [20].

1.3.2 Near-Field Powering

Near-field powering can be divided into methods that transmit power via magnetic or electric fields. While electric near-field power transfer has been
demonstrated for biomedical systems, magnetic induction charging is far more popular, largely due to the magnetic transparency of human tissue at low frequencies [21]. Researchers have investigated inductive systems with much greater intensity since Kurs et al demonstrated wireless power transfer over large air gaps using a new method they coined as “magnetic resonance coupling” or MRC in 2007 [22]. Since then, it has been shown that MRC is optimal inductive power transfer, equivalent to load matching [23] [24] [25].

![Diagram representing near-field power transfer](image)

**Figure 2**: Diagram representing near-field power transfer. The AC excitation to a coil gives rise to an AC magnetic field, which induces a current in the receive coil via Faraday’s law of induction.

Inductive powering schemes have shown high PTEs compared to far field powering, but suffer from the inherently directional power transfer. Miniaturization is also often limited by low Q factors and the low coupling introduced by size mismatched coils. Inductive coupling has been demonstrated in commercial biomedical applications such as cochlear implants and can transfer large amounts of energy more safely than far-field, but applications are mostly size limited.
1.3.3 Mid-field Powering

Finally, recent work at Stanford University has produced mid-field powering, which relies on using a frequency that resonates with tissue and allows energy to be channeled into the body. This research has produced an animal testing environment using extremely miniaturized coils as well as a human body application to power a pacemaker [26] [27]. Although analysis comparing the physical phenomena of mid-field powering to far or near field is lacking, one key difference seems to be accounting for material properties of tissue and exploiting them, instead of operating at a frequency at which we can safely ignore those properties. Another is the use of evanescent wave coupling to result in propagating waves that channel energy into the body.

Figure 3: Diagram representing mid-field power transfer. A specifically designed current source generates evanescent waves, which propagate into tissue, delivering power to extremely small receive structures inside the body.

Mid-field powering may become more commercially widespread, but power transfer efficiencies are still very low for demonstrated systems (>0.1%) [16]. This makes midfield powering schemes useful for powering very small, deeply implanted devices,
but this method has not yet in been exploited in such a way to enable large power transfer.

1.4 Theory for Modeling WPT Systems

Wireless power transfer is inherently application specific; there is no silver bullet for solving WPT design problems. Each of the modalities described above has advantages and disadvantages. Every design scenario must be evaluated individually to determine what mode of WPT will best suit a particular project. This work does not present a new mode of WPT, but instead a versatile means of analyzing and optimizing WPT systems.

Kurs et al originally used coupled mode theory (CMT) to analyze and optimize their MRC system [22]. Physicists tend to prefer using CMT for WPT analysis; for engineers CMT is typically reserved for optoelectronics [28]. While CMT can accurately model WPT systems, the analysis does not lend itself well to modeling larger networks (i.e. systems with more than one source and one load). Comparative analysis by Kiani et. al. has shown that CMT is actually less accurate than circuit theory in cases of strong couplings or low Q factors, the latter being common in biomedical applications [29]. This study, however, relied on a circuit theory coined “reflected load theory” which used unnecessarily bulky analysis of WPT systems involving second order differential equations (as opposed to the first order differential equations used in CMT). This reflected load theory was also not used to model arbitrary WPT networks.

Because of the limitations and complications introduced by the aforementioned models, I will use generalized coupling matrix to model resonator systems in a manner
similar to band-pass filter (BPF) design. Unlike CMT, this method uses commonly defined quantities in electrical engineering, such as Q factors and electromagnetic coupling. It will be shown that the coupling matrix results in simpler, more powerful optimization of WPT systems and is easily extendable to arbitrary networks.

Originally, [23] used the generalized coupling matrix to model wireless power transfer systems. This mathematical model had previously been used exclusively for filter design. Later, work by [30] exploited the generalized coupling matrix to generate analytical solutions to optimal impedance matching for WPT systems. However, this work was limited to the simplest two resonator case (one source and one load), and could not accommodate for complex load matching. Furthermore, the theory did not explore discrepancies between electric and magnetic coupling.

In this work, I present a novel formulation of the generalized coupling matrix that allows modeling of complex WPT networks and impedance matching for complex loads. I explore the theory as it applies to electric and magnetic coupling and demonstrate successful modeling of an electrically and magnetically coupled system (resonant cavity enabled WPT). Finally, I demonstrate numerical optimization as a tool for calculating optimal impedance matching conditions for situations when analytical optimization as demonstrated by Mei is infeasible due to the complexity of the resultant equations. The numerical optimization also demonstrates utility for enforcing user designed power splitting in WPT networks.
CHAPTER 2. REPRESENTATION OF WIRELESS POWER TRANSFER THROUGH THE COUPLING MATRIX AND EXTENSION TO ARBITRARY NETWORKS

2.1 Representation of WPT with the Generalized Coupling Matrix

Traditional MRC was physically realized using two coils and two helical resonators. The coupling between the two helices afforded a large air gap, and the remaining mutual inductances served as impedance matching, in a manner similar to transformers. Replacing mutual inductance couplings with the appropriate T-network creates a structure similar to resonantly coupled bandpass filters (BPFs) [31]. Indeed, these mutual inductances can be modeled as impedance inverters (also called K inverters), a common structure in filter design which can be realized in many topologies using lumped elements.
2.1.1 Deriving the Generalized Coupling Matrix for Series Resonators

The coupling matrix, first introduced by Atia and Williams, is a useful construct for modeling complicated resonantly coupled bandpass filters [32]. Starting from the most general circuit representation of two coupled resonators (see figure 5), I will show that normalizing the impedance matrix will lead to a convenient tool for extracting S parameters, which gives us PTE.
Figure 5: General form of two magnetically coupled series LC resonators with K inverters (external coupling) attaching source and load

Where: 

\[ Z_S = R_S + jX_S, \quad Z_L = R_L + jX_L, \quad \text{and} \quad \frac{1}{L_1C_1} = \frac{1}{L_2C_2} = \omega_0^2. \]

In figure 5, \( L_{ij} \) represents mutual inductance between inductors \( L_i \) and \( L_j \), \( R_i \) is the series resistance of the \( i \)th resonator, and \( X_i \) is the additional frequency invariant reactance (FIR) detuning the \( i \)th resonator. Note that this model is based on a series resonator. This entails a series inductance and capacitance resonating at \( \omega_0 \), a series resistance representing loss, and series FIR representing detuning. Intentional detuning of the resonator can allow for impedance matching with complex loads. Coupling between a port and resonator is modeled as a K inverter, which serves the roll of impedance matching. The coupling between the two resonators is defined as a magnetic coupling (the case for electrical coupling will be derived later). Each port will have its own complex impedance.

This circuit lends itself to simple modeling via an impedance matrix [33].

\[
[Z] * [I] = [V] \quad (2.1)
\]
Where $[Z]$ is a 4x4 impedance matrix shown in figure 6. This allows us to model the WPT system as a network of electromagnetically resonators, a useful abstraction for modeling large networks.

$$
\begin{bmatrix}
\times \sqrt{R_S}^{-1} & \times \sqrt{\omega_0 L_1}^{-1} & \times \sqrt{\omega_0 L_2}^{-1} & \times \sqrt{R_L}^{-1} \\
\downarrow & \downarrow & \downarrow & \downarrow \\
X_S & jK_{S1} & 0 & 0 \\
jK_{S1} & R_1 + j\omega L_1 + \frac{1}{j\omega C_1} + jX_1 & jL_{12} & 0 \\
0 & jL_{12} & R_2 + j\omega L_2 + \frac{1}{j\omega C_2} + jX_2 & jK_{2L} \\
0 & 0 & jK_{2L} & Z_L
\end{bmatrix}
$$

Figure 6: Impedance Matrix of the circuit in figure 5. Arrows show row and column operations to create normalized variables.

Note that in figure 6: $Z_S = R_S + jX_S$, $Z_L = R_L + jX_L$, and $\frac{1}{L_1 C_1} = \frac{1}{L_2 C_2} = \omega_0^2$.

As shown in figure 6, the impedance matrix can be normalized using standard elementary row operations. This transforms the matrix into a collection of non-dimensionalized terms. We introduce these terms below in table 1 [31] [33]:
Table 1: Normalized variables used in representing the impedance matrix shown in fig. 6

<table>
<thead>
<tr>
<th>( k_{12} = \frac{L_{12}}{\sqrt{L_1L_2}} )</th>
<th>Inter-resonator coupling coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_{0i} = \frac{\omega L_i}{R_i} )</td>
<td>Unloaded Q factor of resonator ( i ) for ( i = 1, 2 )</td>
</tr>
<tr>
<td>( D_i = \frac{X_i}{\omega L_i} )</td>
<td>Detuning factor of resonator ( i ) for ( i = 1, 2 )</td>
</tr>
<tr>
<td>( E_{ij} = \frac{K_{ij}}{\sqrt{\omega_0 L_i R_j}} )</td>
<td>External Coupling between resonator ( i ) and port ( j ) for ( i = 1, 2 ) and ( j = S, L )</td>
</tr>
<tr>
<td>( x_j = \frac{X_j}{R_j} )</td>
<td>Normalized Reactance of port ( j ) for ( j = S, L )</td>
</tr>
<tr>
<td>( \Omega = \left( \frac{\omega - \omega_0}{\omega} \right) )</td>
<td>Normalized frequency variable</td>
</tr>
</tbody>
</table>

This normalized impedance matrix will be referred as \([\tilde{Z}]\). Substituting in these variables allows us to make the following decomposition of the matrix seen below:

\[
[\tilde{Z}] = j \begin{bmatrix}
 x_S & E_{S1} & 0 & 0 \\
 E_{S1} & D_1 - \frac{j}{Q_{01}} & k_{12} & 0 \\
 0 & k_{12} & D_2 - \frac{j}{Q_{02}} & E_{2L} \\
 0 & 0 & E_{2L} & x_L \\
\end{bmatrix}
\begin{bmatrix}
 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 \\
\end{bmatrix}
+ j\Omega
\begin{bmatrix}
 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(2.2)

\[
[\tilde{Z}] = j[M] + [q] + j\Omega[U]
\]

(2.3)

\[
[A] = -j[\tilde{Z}] = [M] - j[q] + \Omega[U]
\]

(2.4)

Where the matrix \([A]\) is built from the coupling matrix \([M]\), the source matrix \([q]\), and the resonator matrix \([U]\). It can be shown that from inverting the \([A]\) matrix, the scattering parameters can be found [31].
\[ S_{21} = -2j[A]_{14}^{-1} \] (2.5)
\[ S_{11} = 1 + 2j[A]_{11}^{-1} \] (2.6)
\[ PTE = |S_{21}|^2 \] (2.7)

Therefore, the coupling matrix allows us to predict the scattering parameters of a two resonator, two port system using only measurable lumped circuit parameters. For WPT applications, one would optimize PTE using the external coupling and detuning as the design variables.

2.1.2 Deriving the Generalized Coupling Matrix for Parallel Resonators

In filter theory, resonantly coupled BPFs are typically modeled as either series resonators (shown in figure 5) or shunt resonators (shown in figure 7). Shunt resonators are more typically evaluated via the Y-parameters instead of the Z-parameters [31].

Because in certain WPT applications modeling systems via shunt resonators is more convenient (see chapter 4 resonant cavity), I will show construction of the generalized coupling matrix for two ports and two resonators for the case of parallel resonators.

![Diagram of two electrically coupled shunt LC resonators with J inverters (external coupling) attaching source and load](image)

Figure 7: General form of two electrically coupled shunt LC resonators with J inverters (external coupling) attaching source and load
The key changes between the circuit model in figure 5 and the one in figure 7 are thus: source and load impedance are changed into admittance, impedance inverters (K-inverters) are changed to admittance inverters (J-inverters), the frequency invariant reactance becomes a susceptance, resistance is represented by a conductance, and the LC pair becomes a parallel resonator instead of series. Predictably, this circuit is more easily modeled by an admittance matrix.

\[ [Y] \ast [V] = [I] \]  

(2.8)

The same normalization to the admittance matrix \([Y]\) can be made as in figure 6; this is shown in figure 8.

\[
\begin{align*}
\times \sqrt{G_S^{-1}} & \quad \times \sqrt{\omega_0 C_1^{-1}} & \quad \times \sqrt{\omega_0 C_2^{-1}} & \quad \times \sqrt{G_L^{-1}} \\
\downarrow & \quad \downarrow & \quad \downarrow & \quad \downarrow \\
Y_S & \quad jj S_1 & \quad 0 & \quad 0 \\
jj S_1 & \quad G_1 + j \omega C_1 + \frac{1}{j \omega L_1} + j B_1 & \quad -j C_{12} & \quad 0 \\
jj S_1 & \quad 0 & \quad -j C_{12} & \quad G_2 + j \omega C_2 + \frac{1}{j \omega L_2} + j B_2 & \quad jj_{2L} \\
jj S_1 & \quad 0 & \quad 0 & \quad jj_{2L} & \quad Y_L \\
\end{align*}
\]

Figure 8: Admittance Matrix of the circuit in figure 7. Arrows show row and column operations to create normalized variables.

Note that in figure 8: \(Y_S = G_S + j B_S, Y_L = G_L + j B_L\), and \(\frac{1}{L_1 C_1} = \frac{1}{L_2 C_2} = \omega_0^2\).

Note that the mutual inductance \(L_{12}\) has been replaced with a mutual capacitance \(C_{12}\). This is typically associated with electrical coupling. But as shown in 2.3.1, these “coupling coefficients” can be used interchangeably at the resonant frequency for most practical applications. The normalization in figure 8 results in exactly the same matrix.
(save for replacing $x$ with $b$ for aesthetics), although the normalized variables take on different definitions as shown below in table 2.

Table 2: Normalized variables used in representing the impedance matrix shown in fig. 7

<table>
<thead>
<tr>
<th>$k_{12}$</th>
<th>$\frac{C_{12}}{\sqrt{C_1C_2}}$</th>
<th>Inter-resonator coupling coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{0i}$</td>
<td>$\frac{\omega_0 C_i}{G_i}$</td>
<td>Unloaded Q factor of resonator $i$ for $i = 1,2$</td>
</tr>
<tr>
<td>$D_i$</td>
<td>$\frac{B_i}{\omega_0 C_i}$</td>
<td>Detuning factor of resonator $i$ for $i = 1,2$</td>
</tr>
<tr>
<td>$E_{ij}$</td>
<td>$\frac{J_{ij}}{\sqrt{\omega_0 C_i G_{pj}}}$</td>
<td>External Coupling between resonator $i$ and port $j$ for $i = 1,2$ and $j = S, L$</td>
</tr>
<tr>
<td>$b_j$</td>
<td>$\frac{B_{pj}}{G_{pj}}$</td>
<td>Normalized susceptance of port $j$ for $j = S, L$</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>$\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$</td>
<td>Normalized frequency variable</td>
</tr>
</tbody>
</table>

Physically, all of these normalized variables represent the same quantities as their series counterparts (i.e. the Q factor of a resonator is the same whether modeled as shunt or series). The equations for decomposing the normalized $[Y]$ matrix and extracting the S-parameters from the $[A]$ matrix are identical to the series case in equations 2.5 and 2.6. In this sense, the generalized coupling matrix is very useful for modeling coupled resonators regardless of the circuit representation of these resonators. For this reason, filter designers tend to model connections of ports and resonators using the following abstract coupled resonator representation:
This abstract representation will become very useful later for modeling more complicated WPT networks.

2.2 Controlling the Asynchronous Tuning to Perform Complex Impedance Matching

While the normalized reactance terms $x_S$ and $x_L$ allow one to account for complex impedances, adjusting the external couplings is not sufficient for achieving optimal power transfer efficiency. This can be explained by observing figure 10:

The input impedance $Z_{in}$ can be defined using the standard relationship for impedance inverters:

$$Z_L = R_L + jX_L$$
\[ Z_{in} = \frac{K^2}{Z_L} \]  

(2.9)

Where \( Z_{in} \) is the impedance seen looking into the inverter, \( K \) is the characteristic impedance of the inverter, and \( Z_L \) is the load impedance seen on the other side of the inverter. So for any complex \( Z_L \), we observe a complex \( Z_{in} \). The imaginary portion of \( Z_{in} \) will “detune” the resonator, causing it to resonate at a different frequency than the other resonators, which will result in total PTE loss. In order to “retune” the resonator, we need to add a reactive element that will balance the imaginary portion of \( Z_{in} \), which is given by equation 2.10.

\[
\text{Im}(Z_{in}) = \text{Im}\left(\frac{K^2}{Z_L}\right) = \text{Im}\left(\frac{K^2}{R_L + jX_L}\right) = -\frac{K^2X_L}{R_L^2 + X_L^2} 
\]  

(2.10)

To “retune” the resonator, an extra series reactance \( X_s \) must be added to balance this new detuning.

\[
\text{Im}(Z_{in}) + X_s = 0 \quad (2.11)
\]

\[ X_s = \frac{K^2X}{R^2 + X^2} \quad (2.12) \]

Recall we defined the normalized variables for detuning factors and external couplings previously in equ 2.13 and 2.14. This allows us to simplify and solve for the detuning:

\[
\frac{X_s}{\omega L} = \frac{K^2X_L}{\omega L(R_L^2 + X_L^2)} = \frac{K^2}{\omega L} \ast \frac{R_LX_L}{R_L^2 + X_L^2} 
\]  

(2.13)

\[
D = \frac{E^2R_LX_L}{|Z|^2} 
\]  

(2.14)

Providing an analytical solution to the retuning of a resonator for a given complex load. This minimizes the number of design variables later when performing numerical
optimization. For completeness, it can also be shown that the asynchronous detuning parameter can be optimized for complex loads in similar manner as in equation 2.15.

\[ D = \frac{E^2 G_L B_L}{|Y|^2} \]  

(2.15)

2.3 A Short Note on Electric and Magnetic Coupling

In making this work, I noticed discrepancies within the literature in defining coupling. Historically, the definition of “coupling coefficient” changes to fit the needs of the designer. For example, in the Radio Engineer’s Handbook by Frederick Terman, the capacitive coupling coefficient is defined [34]:

\[ k_C = \frac{\sqrt{C_1 C_2}}{C_m} \]  

(2.16)

Which is clearly different to our definition in table 2. However, Terman defines the inductive coupling coefficient identically to what we defined in table 1 from Hong and Lancaster [31]. Typically, there is more agreement on the definition of magnetic coupling as the phenomena is much more common, appearing in filter design, transformers, and power electronics.

The reason for the discrepancy in capacitive coupling is because Terman decided to remain consistent with what the coupling coefficient meant in terms of impedances in a T-network, a useful definition for a good amount of network analysis. Hong and Lancaster, however, wanted the coupling coefficient to be meaningful in the generalized coupling matrix, which transcends differences between series and parallel resonators. In
this sense, it does not matter whether the inter-resonator coupling is electric, magnetic, or mixed.

Here, the reader may experience some confusion; electric and magnetic coupling are different physical phenomenon after all. In cases where a network is not consistent in its couplings, how can it be acceptable to treat magnetic and electric couplings as equivalent? Hong and Lancaster deal thoroughly with the discrepancies in resonators with mixed couplings (i.e. two resonators that are magnetically and electrically coupled). But surprisingly, the literature does not provide a satisfactory explanation for a network that contains multiple resonators with different couplings. The key to the BPF abstraction is that magnetic couplings are easily transformed into K-inverters and electric couplings are easily transformed into J-inverters. In figure 11, I demonstrate the dual case: how magnetic couplings can be made into J-inverters and electric couplings into K-inverters. I also show that for low couplings, magnetic and electric coupling coefficients can be used interchangeably.
Capacitive coupling between capacitors $C_1$ and $C_2$ shown in figure 11a is most easily modeled as a $J$-inverter between the two capacitors, as shown in figure 11e. Similarly, inductive coupling between inductors $L_1$ and $L_2$ is easily transformed into a $K$-inverter between the two inductors (figure 11d). The circuits in figure 11b and 11a can be described by equations 2.17, 2.18 and 2.19, 2.20 respectively.

$$V_1 = j\omega L_1 I_1 + j\omega L_{21} I_2$$  \hspace{1cm} (2.17)

$$V_2 = j\omega L_2 I_2 + j\omega L_{21} I_1$$  \hspace{1cm} (2.18)

$$I_1 = j\omega C_1 V_1 - j\omega C_{21} V_2$$  \hspace{1cm} (2.19)

$$I_2 = j\omega C_2 V_2 - j\omega C_{21} V_1$$  \hspace{1cm} (2.20)
By solving the networks in 11c and 11f to have similar voltage and current equations, we find equivalent expressions of \( L_j, C_k, L'_1, L'_2, C'_1, \) and \( C'_2 \) in terms of \( L_1, L_2, L_m, C_1, C_2, \) and \( C_m. \)

\[
L'_1 = \frac{L_1 L_2 - L_m^2}{L_2} \quad \text{(2.21)} \\
C'_1 = \frac{C_1 C_2 + C_m^2}{C_2} \quad \text{(2.24)} \\
L'_2 = \frac{L_1 L_2 - L_m^2}{L_1} \quad \text{(2.22)} \\
C'_2 = \frac{C_1 C_2 + C_m^2}{C_1} \quad \text{(2.25)} \\
L_j = \frac{L_1 L_2 - L_m^2}{L_m} \quad \text{(2.23)} \\
C_k = \frac{C_1 C_2 + C_m^2}{C_m} \quad \text{(2.26)}
\]

Substituting in the relationships for electric and magnetic coupling given by Hong (equations 1 and 2),

\[
k_M = \frac{L_m}{\sqrt{L_1 L_2}} \quad \text{(2.27)}
\]

\[
k_E = \frac{C_m}{\sqrt{C_1 C_2}} \quad \text{(2.28)}
\]

We simplify for the lumped elements solved in equations 2.21-2.26:

\[
L_j = \frac{(1 - k_M^2)\sqrt{L_1 L_2}}{k_M} \quad \text{(2.29)}
\]

\[
L'_n = L_n (1 - k_E^2) \text{ for } n = 1,2 \quad \text{(2.30)}
\]

\[
C_k = \frac{(1 + k_E^2)}{k_E} \sqrt{C_1 C_2} \quad \text{(2.31)}
\]

\[
C'_n = C_n (1 + k_E^2) \text{ for } n = 1,2 \quad \text{(2.32)}
\]

Finally, we can substitute these equations into the characteristic impedance and admittance of the pi and T network.
These equations demonstrate an important conclusion. For very small couplings \((k \ll 1)\),
the expressions for the inverters at the resonance frequency are the same whether the
coupling is electric or magnetic. It is only when the couplings become very strong that we
would ever observe the error in making this assumption, something that rarely occurs in
biomedical applications detailed in this work. If that did occur, equations 2.33 and 2.34
could be used to remove the error.

### 2.4 Realizing Inverters using Lumped Elements

As I have demonstrated the coupling matrix both for series and shunt resonators, I
will explore physically realizing both J and K inverters for WPT applications. In figure
12 (a) and (b), we see traditional representations of K and J inverters through negative
capacitances [35].
Clearly, the topologies shown in figure 12b are not easily realizable, relying on negative capacitances. When one wishes to terminate a resonantly coupled BPF with lumped elements, the inverter structures shown in 12c are used [36]. These still include negative capacitances, but they are absorbed into the resonant capacitance of the series or shunt resonator. These topologies are also functions of the complex load impedance, unlike ideal inverters. As a suitable citation for synthesizing these end inverters could not be found in the literature, I will perform the derivation below.

To derive the lumped capacitance values to realize these inverters, we set the relationship relating the input impedance (or admittance) of the network equal to what the equation that a K or J inverter would enforce. By solving for the real an imaginary parts
of these equations, we can solve the system for the two capacitances. The equations outlining this process are shown below:

\[ Z_L = R + jX \] \hspace{1cm} (2.35) \hspace{1cm} \[ Y_L = G + jB \] \hspace{1cm} (2.40)

\[ |Z_L| = \sqrt{R^2 + X^2} \] \hspace{1cm} (2.36) \hspace{1cm} \[ |Y_L| = \sqrt{G^2 + B^2} \] \hspace{1cm} (2.41)

\[ Z_{in} = \frac{K^2}{Z_L} = \frac{1}{j\omega C_s} + \left( j\omega C_p + \frac{1}{Z_L} \right)^{-1} \] \hspace{1cm} (2.37) \hspace{1cm} \[ Y_{in} = \frac{J^2}{Y_L} = j\omega C_p + \left( \frac{1}{j\omega C_s} + \frac{1}{Y_L} \right)^{-1} \] \hspace{1cm} (2.42)

\[ C_p = \frac{KX + \sqrt{|Z_L|^4 - K^2 R^2}}{\omega K |Z_L|^2} \] \hspace{1cm} (2.38) \hspace{1cm} \[ C_s = \frac{J(BJ + \sqrt{|Y_L|^4 - G^2 J^2})}{\omega |Y_L|^2} \] \hspace{1cm} (2.43)

\[ -C_s = \frac{|Z_L|^2}{\omega K \left( \sqrt{|Z_L|^4 - K^2 R^2} - KX \right)} \] \hspace{1cm} (2.39) \hspace{1cm} \[ -C_p = \frac{J(BJ + \sqrt{|Y_L|^4 - G^2 J^2})}{\omega |Y_L|^2} \] \hspace{1cm} (2.44)

As can be seen, the capacitance values are functions of the complex impedance, the operating frequency, and the desired characteristic impedance or admittance.

### 2.5 Expanding the Generalized Coupling Matrix to Account for Arbitrary Sized Networks

While the matrix derived above is a very useful tool, it requires modifications to deal with arbitrary networks. Next, I will analyze the case with \( n \) resonators and \( m \) ports (once multiple ports exist, the distinction between “source” and “load” becomes unnecessary; from now on both will be referred to as ports). This multi-port matrix is similar to that derived for specific filter applications, best described by Skaik [37]. The new arbitrary coupling matrix \([M]\) is size \((n+m \times n+m)\) and given by three different matrices:

\[
[M] = \begin{bmatrix} [P] & [E]^T \\ [E] & [R] \end{bmatrix}
\] \hspace{1cm} (2.45)
The matrix $[P]$ will be referred to as the port matrix, and represents the complex loads of each port on the diagonal. Every other entry is zero.

$$
[P] = \begin{bmatrix}
    x_1 & \cdots & 0 \\
    \vdots & \ddots & \vdots \\
    0 & \cdots & x_m
\end{bmatrix}
$$

Each port has complex impedance given by $Z_p = R_p + jX_p$. Because there are $m$ ports, $[P]$ is a $m \times m$ sized matrix.

The matrix $[R]$ is defined by the properties of the resonators and their couplings. The diagonal entries quality factors and asynchronous tuning of each resonator. The off-diagonal entries represent how each resonator couples with one another.

$$
[R] = \begin{bmatrix}
    D_1 - \frac{j}{Q_{01}} & k_{12} & \cdots & k_{1,n-1} & k_{1n} \\
    k_{12} & D_2 - \frac{j}{Q_{02}} & \cdots & k_{2,n-1} & k_{2n} \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    k_{1,n-1} & k_{2,n-1} & \cdots & D_{n-1} - \frac{j}{Q_{0n-1}} & k_{n-1,n} \\
    k_{1n} & k_{2n} & \cdots & k_{n-1,n} & D_n - \frac{j}{Q_{0n}}
\end{bmatrix}
$$

The matrix $[R]$ is size $n \times n$. Finally, the external coupling matrix $[E]$ is made out of external coupling coefficients.

$$
[E] = \begin{bmatrix}
    E_{11} & \cdots & E_{1m} \\
    \vdots & \ddots & \vdots \\
    E_{n1} & \cdots & E_{nm}
\end{bmatrix}
$$

Where $E_{ij}$ represents the external coupling between resonator $i$ and port $j$. This external coupling is usually controlled by the designer. The matrix $[E]$ is size $n \times m$.

From the normalized impedance matrix, the $S$ matrix can be calculated.

$$
S_{ij} = -2j[A]_{ij}^{-1}
$$
\[ S_{ii} = 1 + 2j[A]^{-1}_{ii} \quad (2.50) \]

From this formulation, we can extract the entire \( m \times m \) scattering matrix from measureable properties of the network. This is a novel use of the multi-port coupling matrix to characterize WPT network. The literature traditionally refers to coupling matrices representing \( n \) resonators as \( n+2 \) matrices. The “+2” refers to the extra entry for the source and load. As I am dealing with multiple sources and loads, I will refer to our formulation as the \( n+m \) matrix. By reformulating our matrix for WPT from the ground up, I remove extraneous terms such as the Fractional Bandwidth (FBW), which previous works misinterpreted for WPT.

For small networks, one can derive symbolic expressions for scattering matrix and optimize PTE analytically. But for networks exceeding the simple two-port two-resonator system, this quickly becomes impractical. In chapter 3, I will discuss formulation for constructing an optimization problem from the \( n+m \) coupling matrix. This will provide a tool allowing one to optimize any arbitrary network of WPT resonators.
CHAPTER 3. NUMERICAL OPTIMIZATION OF THE $N+M$ COUPLING MATRIX

3.1 Formalization of the Optimization Problem

Any optimization problem is described in terms of three things: the objective function, the design variables, and the constraint functions. The objective function calculates the “cost” of the design. This is the value we attempt to minimize. For multi-objective optimization problems, there are multiple objective functions. The objective function and the constraint function are functions of the design variables; these are the variables we can control to minimize our objective function. Lastly, the constraint functions represent boundaries of our problem not described in the objective function. These can fall into inequality (the constraint function must be below a certain limit) or equality constraints (the constraint function must equal a certain value) [38]. Depending on the optimization method, constraint functions are handled in many different ways.

3.1.1 Formalization of the Objective Function

For any given application in WPT, we typically desire to optimize PTE from sources to loads. Given the $n+m$ matrix formulation described in chapter 2, we can now describe the entire behavior of the network in terms of the S-parameters. The non-diagonal entries of the S matrix will denote the transfer function between ports. This part of the optimization is the most dependent on specific applications; for our purposes I will demonstrate optimization of one source to multiple loads. The design variables are the
external couplings from ports to their respective resonators. Other designs would require
the user to redesign the objective function based on their needs.

Because of our analysis in section 2.2, I do not need to include the detuning
variable $D$ into our design variables, as I can explicitly state the optimal value of $D$ in
terms of $E$ and other known parameters. This reduces the number of design variables and
therefore optimization complexity. To maximize PTE, we will sum the magnitude of the
transfer functions of all three loads at the resonant frequency:

\[ \text{Maximize: } f(x) = \sum_{j=1}^{n} |S_{j1}(\omega_0)|^2 \]

\[ x = [E_{11}, E_{12} \ldots E_{nm}]^T \]

\[ x \geq 0 \]

In words, we are optimizing of the sum of the transfer functions from one source to $n$
loads by adjusting the existing positive valued external couplings.

3.1.2 Formalization of the Constraint Functions

To physically realize the system, there are two constraints we must observe
sequentially. It should be noted that the constraints described are entirely imposed by the
topology of our inverters. For different realization of K or J inverters, different
constraints would be necessary. As an example, consider a high frequency system where
a quarter wave transformer could be used to represent the inverter, where the constraints
would be imposed by physical dimensions [35]. Traditionally, inequality constraint
functions are denoted by $g(x)$ where $x$ is a column vector containing all design variables.
The constraint function is not violated if $g(x) \leq 0$. Equality constraint functions are
denoted by $h(x)$ and are not violated only when $h(x) = 0$. 
3.1.2.1 Inequality Constraints

To realize our system, the capacitances to realize the inverters must be real valued. Based on the capacitor equations given section 2.4, this gives us one set of constraint functions (one function for every port) that constrains the term under the square root to be positive (equations 2.38, 2.39 and 2.43, 2.44). Second, the combination of the resonator, detuning, and inverter capacitor must be positive to be realizable. This gives us the second set of constraints. The second set of constraints, however, cannot be evaluated unless the first constraint is satisfied; one cannot determine if the sum of the capacitance is positive if it’s a complex quantity. Therefore, each constraint function will be built using conditional statements. Equations 3.2 and 3.3 show the constraint functions for the series and shunt resonator case respectively.

\[
g_j(x) = \begin{cases} 
\frac{k_{ij}(x) R_p}{|Z_p|^2} - 1 & \text{if } g_j(x) \geq 0 \\
-10^9 \left( \frac{1}{c_j} - \omega_0 X_j(x) + \frac{1}{c_{ij}(x)} \right)^{-1} & \text{otherwise} 
\end{cases}
\]  

(3.2)

OR:

\[
g_j(x) = \begin{cases} 
\frac{j_{ij}(x) g_p}{|Y_p|^2} - 1 & \text{if } g_j(x) \geq 0 \\
-10^9 \left( c_j + \frac{B(x)}{\omega_0} + C_{pj}(x) \right) & \text{otherwise} 
\end{cases}
\]  

(3.3)

3.1.2.2 Equality Constraints: Power Splitting via the Optimization Process

It follows that for large WPT networks the designer will often have situations where there are different devices in the network with different power consumption. In this case, it would be convenient for the designer to incorporate a power splitting constraint on the optimization of the network. Methods for power splitting have been
demonstrated earlier for simple systems, but no one provided a solution to achieving power splitting constraints for arbitrary networks [24]. Here I demonstrate a means for numerically enforcing power splitting constraints while still achieving optimal power transfer efficiency.

Power splitting requires that the power from one device to another be a fixed ratio. In optimization terms, this describes an equality constraint. As an example, consider the following network shown in figure 13.

We will consider port 1 as the source. If we wish to achieve twice as much power delivery to port 2 as ports 3 and 4, then we must enforce the following expressions.

\[ h_1(x) = \frac{1}{2}|S_{21}|^2 - |S_{31}|^2 = 0 \]  \hspace{1cm} (3.4)

\[ h_2(x) = \frac{1}{2}|S_{21}|^2 - |S_{41}|^2 = 0 \]  \hspace{1cm} (3.5)

\[ h_3(x) = |S_{41}|^2 - |S_{31}|^2 = 0 \]  \hspace{1cm} (3.6)

These equations do not lend themselves, however, to construction of penalty functions, which we will discuss later. Therefore, we will reform these as functions that are minimized when the equality constraints are met.
Clearly these functions can never be negative, so the best the optimizer can do is force the functions to be zero, thus enforcing the equality constraint.

3.2 The Genetic Algorithm as a Tool for WPT Optimization

From the previous sections we can formally state our optimization thusly:

Maximize: \( f(x) = \sum_{j=1}^{n} |S_{j1}(\omega_0)|^2 \), \( x = [E_{11}, E_{12} \ldots E_{nm}]^T \), \( 0 \leq x \)

Subject to:

\[
g_j(x) = \begin{cases} \frac{K_{ij}(x)R_{pj}}{|Z_{pj}|} - 1 & \text{if } g_j(x) \leq 0 \\ -10^9 \left( \frac{1}{c_j} - \omega X_j(x) + \frac{1}{c_s j(x)} \right)^{-1} & \text{otherwise} \end{cases} 
\]

OR:

\[
g_j(x) = \begin{cases} \frac{I_{ij}(x)C_{pj}}{|Y_{pj}|} - 1 & \text{if } g_j(x) \leq 0 \\ -10^9 \left( C_j + \frac{B(x)}{\omega_0} + C_{pj}(x) \right) & \text{otherwise} \end{cases} 
\]

Where clearly the equality constraints have been cast in a rough form that might change based on the needs of the designer.
Please note that although we are maximizing PTE, I will refer to our optimization as minimization and solutions as global minima. I apologize for any confusion, but it is traditional to formulate optimization problems as minimizations, and that is how most existing programs are designed. This is achieved by multiplying the objective function by -1.

As we are planning for our algorithm to optimize WPT networks of arbitrary complexity, we have a high likelihood of encountering a solution landscape with local minima, as has been observed in analogous high order filter problems [37]. Moreover, our objective function includes absolute values of complex values, and our constraint functions are piecewise. These functions are difficult to assign analytical or even numerical gradients, which many common optimization problems require.

Accordingly, I use a global, non-smooth solver to ensure that the algorithm is not falling into a local minimum and to traverse our non-smooth solution landscape. Examples of global, non-smooth solvers include but are not limited to the Nelder-Mead simplex, simulated annealing methods, and genetic algorithms [39]. Genetic algorithms are convenient because they require no initial guess at a solution, converge well in the area of a global solution, and do not require construction of continuous, differentiable, or explicit functions [40].

Genetic algorithms (GAs) solve problems by loosely mimicking natural selection. In a non-continuous GA, the solution space is discretized so that every relevant outcome of the solution space can be represented by binary strings. Each element in the string can be considered a gene from the particular individual. Populations are generated
stochastically and grow according to their fitness, thus converging on global solutions [41].

Key to any GA are three essential operators:

**Selection**: Individual solutions (often referred to as “chromosomes”) are selected to reproduce based on their fitness.

**Crossover**: Two or more chromosomes are recombined to create the next generation, which has traits of the parent chromosomes.

**Mutation**: Some of the bits making up the chromosome are randomly flipped with some small probability.

A standard GA starts with a randomly generated population with \( n \) chromosomes, each containing \( l \) bits. The algorithm evaluates the fitness of every chromosome in the population. Now the selection phase occurs and parent chromosomes are selected based on their fitness. New offspring are generated between the parents in the crossover phase and the offspring undergo mutation. This is repeated until enough chromosomes have been generated to completely replace the old generation. A GA’s stopping criteria is typically based on a maximum amount of generations and/or a convergence criterion. For this work, I will use the convergence criteria called the bit-string affinity (BSA), which measures the homogeneity of a population [42].

\[
BSA = \left( 2 \sum_{i=1}^{l} \left| \frac{\sum_{j=1}^{N} b_{ij}}{N} - 0.5 \right| \right)
\]  

(3.10)

Where again, \( N \) is the population size, \( l \) is the number of bits in each chromosome, and \( b_{ij} \) is the bit value of the \( j \)th bit of the \( i \)th individual. The BSA ranges from 0 to 1, where 0 means no chromosome in the population shares any bits with one another, and 1
meaning there is no diversity in the population. There exist a countless variety of GAs with varying degrees of complexity and no attempt will be made here to cover the extent of the subject. More on genetic algorithms in theory can be found in [40] and in [41] for application.

3.2.1 Accommodating Penalties using the GA

Unlike other optimization methods, the genetic algorithm cannot produce a feasible solution (one that does not violate the constraints) directly. In order to accommodate constraints, the objective function must be combined with a penalty function to produce the desired results. This subset of optimization often falls under an umbrella term “Indirect Methods” because the constraints are handled indirectly. A detailed description of the use of indirect methods can be found in [43].

The sum of the objective function and the penalty functions is called the pseudo-objective function (see equation 3.11).

\[
\varphi(x) = f(x) + r_p \sum_{j=1}^{P} P_j(x)
\]  

(3.11)

Where \( f(x) \) is the objective function, \( r_p \) is scalar weight to the penalty, and \( P_j(x) \) is the penalty function of the \( j \)th constraint. Penalty constraints are typically some function of the equality and inequality constraints. Because the GA does not require differentiable functions, we will use the extended step linear penalty function (see equation 3.12).

\[
P_j(x) = \begin{cases} 
0 & \text{if } g_j(x) \leq 0 \\
 c_j * \left( 1 + g_j(x) \right) & \text{else} 
\end{cases}
\]  

(3.12)

Where \( g(x) \) can be replaced with \( h(x) \) as necessary and \( c_j \) is a constant that changes between constraints to add appropriate weighting. While there are many methods for
making penalty functions in the literature, the key concept is penalizing the objective function when constraints are unsatisfied, thus encouraging the algorithm to find feasible solutions.

3.2.2 General GA Parameters

The genetic algorithm contains many parameters that will affect the rate of convergence, the accuracy, and the run time of the algorithm. First, we consider the parameters affecting variable encoding. Each design variable in the GA needs lower and upper bounds. Here we are fortunate our coupling matrix method generated normalized variables, resulting in similar magnitudes of external couplings for vastly different systems. For all test cases run in this work, a lower bound of zero and an upper bound of ten is used for the external couplings. Each variable is also encoded by a certain number of bits. The more bits used to encode a variable, the more resolution is obtained; run time however will be increased as each chromosome is now larger and will take longer to compute. Resolution is given by equation 3.13.

\[ r_i = \frac{x_i^U - x_i^L}{2^{b_i} + 1} \]  (3.13)

Where \( x_i^U \), \( x_i^L \), and \( b_i \) is the upper bound, lower bound and number of bits used for the design variable \( x_i \) respectively. Another parameter is the size of the population in each generation. Too small a population and it is possible to fall into a local minimum, but too large and computation time becomes excessive. For this work, we use a conventional rule of thumb formula to determine the population size (equation 3.14).

\[ N_{pop} = 4l \]  (3.14)
Where \( l \) is the number of bits used to encode all design variables. Finally, we set the mutation rate, which corresponds to the percentage of all bits that will be flipped in each generation. From another conventional rule of thumb, we use equation 3.15.

\[
P_{mut} = \frac{l + 1}{2N_{pop}l}
\]  

(3.15)

3.2.3 Secondary Optimization Using Sequential Quadratic Programming

One tradeoff of the GA is its accuracy. While the GA excels at finding the neighborhood of the global solution, GAs often have trouble zeroing in on the exact solution due to discrete binary representation of the design variables. For this reason, our algorithm will be a “hybrid method” using both the global GA solver with a secondary constrained optimization method. The output of the GA will be the initial solution of the constrained optimization. For our algorithm, we will use sequential quadratic programming (SQP), the most popular method for constrained optimization problems. Unlike the GA, the SQP algorithm will handle our constraints directly to ensure a feasible solution. See figure 14 for a flowchart describing the entire optimization method.
3.3 Validation of Computational Algorithm

To test the algorithm, a theoretical five-resonator, four-port system was devised (see figure 13). Physically, this would represent a single transmitter coupled to a relay that is coupled to three identical devices. We will analyze this for the series resonator case and explain the system in terms of impedances and inductances. Note that every resonator has an associated detuning \( D \), quality factor \( Q \), and inductance \( L \). Every port has an associated complex impedance \( Z = R + jX \). See table 3 for all values relevant to this WPT network.
Table 3: Summary of the component values representing the network in figure 13.

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>k₁₂</td>
<td>0.1</td>
</tr>
<tr>
<td>k₂₃, k₂₄, k₂₅</td>
<td>0.05</td>
</tr>
<tr>
<td>Q₁</td>
<td>300</td>
</tr>
<tr>
<td>Q₂</td>
<td>500</td>
</tr>
<tr>
<td>Q₃, Q₄, Q₅</td>
<td>100</td>
</tr>
<tr>
<td>L₁</td>
<td>500 nH</td>
</tr>
<tr>
<td>L₂</td>
<td>1000 nH</td>
</tr>
<tr>
<td>L₃, L₄, L₅</td>
<td>100 nH</td>
</tr>
<tr>
<td>Z₁</td>
<td>50 Ohms</td>
</tr>
<tr>
<td>Z₂, Z₃, Z₄</td>
<td>10 + 2j Ohms</td>
</tr>
<tr>
<td>ω₀</td>
<td>2π * 10 MHz</td>
</tr>
</tbody>
</table>

Because the GA is an inherently stochastic method, the algorithm was run three times, with the results recorded in table 3. Because each \( x_{star} \) does not lie on the constraint boundaries (see \( g(x_{star}) \)), the output of every run of the GA was made the initial guess for an unconstrained optimization using the MATLAB command \( \text{fmincon}(*) \) which used a numerical gradient SQP optimization to “fine tune” each solution. If the GA is given enough generations and fine enough resolution, it will typically converge on the correct minimum as the hybrid method, but the hybrid method performs significantly faster.
Table 4: Results of GA output and SQP output (using GA output as initial condition)

<table>
<thead>
<tr>
<th>Genetic Algorithm</th>
<th>Secondary Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{\text{gen}} )</td>
<td>( F_{\text{Eval}} )</td>
</tr>
<tr>
<td>Run 1</td>
<td>389</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Run 2</td>
<td>256</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Run 3</td>
<td>700</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notice how many function calls are required to run the GA. By comparison, the SQP algorithm requires on the order of > 1000 function calls, making its run time negligible compared to the GA. This allows the hybrid method to often achieve the same degree of accuracy as a more expensive GA more efficiently.

3.3.1) Comparison with Optimization in Circuit Simulation

To validate the matrix model and optimization method, the resonator network shown in figure 1 was created in the SPICE simulator ADS (see figure 16).
Advanced Design System (ADS®) is a particularly powerful microwave circuit simulator produced by Agilent with a built-in optimization GUI. This automated toolbox was used to optimize the same objective function as the GA (ADS optimized the capacitor values directly, instead of the external coupling values). A comparison of the results can be seen in table 4.

Table 5: Comparison of ADS® Optimization Cockpit and Custom Optimization Algorithm

<table>
<thead>
<tr>
<th></th>
<th>This Work</th>
<th>ADS Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{p1}$</td>
<td>675.69 pF</td>
<td>678.63 pF</td>
</tr>
<tr>
<td>$C_{s1}$</td>
<td>1311.05 pF</td>
<td>1305.52 pF</td>
</tr>
<tr>
<td>$C_{p2}, C_{p3}, C_{p4}$</td>
<td>4208.63 pF</td>
<td>4259.67 pF</td>
</tr>
<tr>
<td>$C_{s2}, C_{s3}, C_{s4}$</td>
<td>5790.61 pF</td>
<td>5720.57 pF</td>
</tr>
<tr>
<td>$\text{abs}(S_{21})^2 + \text{abs}(S_{31})^2 + \text{abs}(S_{41})^2$</td>
<td>0.8910</td>
<td>0.8911</td>
</tr>
</tbody>
</table>
The results of table 5 indicate the GA alone is useful for finding the neighborhood of the
global solution, but not great at zeroing in on the exact optimum. This justifies the use of
a hybrid method that uses the GA to find an initial guess in the neighborhood of the
global optimum and a non-global solver to zero in on the solution.

The solutions determined by our hybrid method match very closely with the
solution found by the ADS optimization toolbox, validating both the matrix
representation of the circuit and the optimization methodology. Moreover, the presented
matrix method is much more convenient for optimizing different sized networks quickly.
In ADS, a new simulation must be built for each network. The optimization of the circuit
simulator is also much more expensive, taking minutes to yield the result in table 5 for
even this simple network. This validates our tool for the optimization of WPT networks.

3.4 Demonstration of Power Splitting in Circuit Simulation

Next, I will show circuit simulations confirming the algorithm’s ability to perform
power splitting via impedance matching control. While enforcing such constraints might
not always result in optimal PTE for the entire system, it demonstrates the algorithm’s
potential utility in applications such as dynamic impedance matching where the designer
would like to enforce a particular power distribution across the network. For this
example, we will optimize power splitting for the same four-port five-resonator system
described earlier (see figure 13 and table 3). Now, we choose to enforce equality
constraints such that the power delivered to port two is twice that of ports three and four.
This creates equality constraints exactly of the form described in equations 3.7-3.9.
Table 6: Results of enforcing power splitting equality constraints in optimization

<table>
<thead>
<tr>
<th></th>
<th>Port 1</th>
<th>Port 2</th>
<th>Port 3</th>
<th>Port 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{ijopt}$</td>
<td>0.7828</td>
<td>0.5861</td>
<td>0.8208</td>
<td>0.8205</td>
</tr>
<tr>
<td>$K_{ijopt}$ (Ohms)</td>
<td>31.0244</td>
<td>4.6458</td>
<td>6.5061</td>
<td>6.5036</td>
</tr>
<tr>
<td>$C_p$ (pF)</td>
<td>402.30</td>
<td>3371.03</td>
<td>2214.52</td>
<td>2215.69</td>
</tr>
<tr>
<td>$C_s$ (pF)</td>
<td>2246.02</td>
<td>7483.70</td>
<td>13181.29</td>
<td>13173.93</td>
</tr>
<tr>
<td>$</td>
<td>S_{11}</td>
<td>^2 \times 100$</td>
<td>n/a</td>
<td>41.82%</td>
</tr>
</tbody>
</table>

Note that for this optimization, only the GA component was used as the SQP optimization would sometimes become unstable when solving for the equality constraints directly. In order to achieve greater precision, I assigned 16 bits per variable, instead of the 10 used in previous runs. This is the only case in this work where this much accuracy was needed; all other uses of the algorithm will use the following parameters: 0 for the lower bounds, 10 for the upper bounds, and 10 bits for every variable. The GA also required many more generations to achieve convergence, showing that power splitting is a computationally expensive process for this algorithm.

The results from table 6 show the algorithm was able to enforce power splitting to a certain margin of error. It makes sense that enforcing such a constraint using indirect methods would be difficult, as it requires the algorithm to find a balance between power splitting and power optimization. The results in table are replicated in ADS as shown in figure 17.
The peak PTEs show excellent agreement with the custom algorithm, demonstrating that the careful use of equality constraints can allow the designer to create IM networks which enforce power splitting.
CHAPTER 4. APPLICATION AND VALIDATION OF THE GENERALIZED COUPLING MATRIX AS AN OPTIMIZATION TOOL

4.1 Validation of Complex Load Matching

One of the novelties of this work is our ability to impedance match for complex loads. The theory discussed in chapter 2 allows the matching network to account for complex loads and achieve the same performance as the real load counterparts, assuming the same coupling and Q factor. This is important because most applications of WPT involve delivering the energy to a rectifier for AC-DC conversion, which typically takes on some complex input impedance. As an example, measured the input impedance of a particular rectifier (see figure 18) with a network analyzer. At an input power of 15 dBm and operating frequency of 13.56 MHz, the input impedance was measured as 43.2 – j*111.3 Ohms.
Figure 17: (a) An example of AC-DC conversion circuitry often seen as the input impedance for WPT. (b) Circuit equivalent to transform a 50-ohm load into the complex impedance given by the AC-DC circuitry.

We reproduced this input impedance using a capacitor network, allowing us to take network analyzer measurements using a relevant complex load. To show that our method can achieve complex load matching, we find optimal IM values for the same coil system for a real (50 ohm) and complex load shown in figure 18. The coil parameters (coupling coefficient, Q factors, and inductances) are measured using a network analyzer. The test setup is shown in figure 19.

Figure 18: Experimental setup of measuring peak PTE when coils are optimized for real (50-ohm) and complex (43.2-j111.2)ohm load.
The smaller, Rx coil has the capacitor network shown in figure 18 to make the load complex. The transmit coil was 16 cm in diameter and the receive coil was 7.5 cm in diameter. Both have two turns, with an inter-winding spacing of 1 cm and are constructed with 10 AWG wire. The measured parameters of the coils are summarized in table 7. Again, the parameters in table 7 are the only input needed for the algorithm to optimize the PTE.

Table 7: Summary of component values for the system pictured in figure 19

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{12opt}$</td>
<td>0.01</td>
</tr>
<tr>
<td>$Q_1$</td>
<td>430</td>
</tr>
<tr>
<td>$Q_2$</td>
<td>300</td>
</tr>
<tr>
<td>$L_1$</td>
<td>1259 nH</td>
</tr>
<tr>
<td>$L_2$</td>
<td>482 nH</td>
</tr>
<tr>
<td>$Z_S$</td>
<td>50 Ohms</td>
</tr>
<tr>
<td>$Z_L$</td>
<td>43.2-j111.3 Ohms</td>
</tr>
<tr>
<td>$f_0$</td>
<td>13.56 MHz</td>
</tr>
</tbody>
</table>

In this case we will optimize the coils as series resonators and use K-inverters. The resulting capacitors needed are summarized in table 8.
Table 8: Optimal capacitors to generate the K-inverters used to optimize the system summarized in table 7.

<table>
<thead>
<tr>
<th>Tx Coil</th>
<th>Rx Coil</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_{S1opt} = 0.0978$</td>
<td>$E_{2opt} = 0.3030$</td>
</tr>
<tr>
<td>$K_{S1opt} = 7.1588 \text{ Ohms}$</td>
<td>$K_{2opt} = 12.7635 \text{ Ohms}$</td>
</tr>
<tr>
<td>$C_{p1} (\text{pF})$</td>
<td>$C_{s1} (\text{pF})$</td>
</tr>
<tr>
<td>1707.23</td>
<td>116.77</td>
</tr>
</tbody>
</table>

Circuit simulations in ADS were compared with the constructed coils shown in figure 19. The Smith chart in figure 20 shows the small discrepancy between the simulated and measured results for the optimized coil system. As can be seen, the differences in the frequency response are very small, certainly within tolerances in our measurement system and in our capacitors. This strongly suggests the validity of my model for analyzing complex loads.

Figure 19: Smith chart comparing simulated optimal IM for complex load with measured results.
4.2 Application to Cavity Resonator System

One of the great utilities of the generalized coupling matrix is the variety of modalities one can apply it to. While it is intuitive from the circuit model that one can model WPT from a network of coils, other structures can be modeled as resonators as well, allowing optimal impedance matching. As an example, I will demonstrate how cavity resonator enabled WPT can be modeled with the generalized coupling matrix [44].

4.2.1 Background of Cavity Resonator WPT

A microwave cavity consists of mostly closed conductive structure that confines electromagnetic fields. Based on the geometry of the structure, it can give rise to many modes of resonance at particular frequencies, in a manner similar to how an organ pipe serves as an acoustical resonator. At particular modes of resonance, a pattern of standing electromagnetic waves occurs inside the cavity [45].

The idea behind cavity resonator WPT is to place a small receive structure inside the cavity resonator, such as a loop coil, that can harvest the standing magnetic waves and deliver energy to a load. This was first demonstrated using coupled mode theory [46] [47]. However, these systems do not achieve optimal PTE due to a lack of impedance matching. A later work creates a circuit model to represent the system and successfully uses network analysis to optimize PTE [44]. Because of this optimization, the cavity resonator system was successfully used to power implanted devices in rats, allowing a large cavity resonator to serve as a wireless powering environment for long term animal studies.
In this section, I will show how the circuit model can be combined with the existing generalized coupling matrix, thus showing the versatility of the matrix as a synthesis tool.

4.2.2 The Cavity Resonator as a Circuit Model

For narrowband approximations, cavity resonators can be modeled as parallel RLC circuits [48]. Because of this, I will use the shunt case of the general coupling matrix as discussed in chapter 2. All couplings will be modeled as J inverters instead of K inverters, as was done for the coil to coil case where it was more intuitive to model the system as series RLC resonators.

Cavity resonators must also be excited by some source to generate the standing fields; this is typically performed by electrical or magnetic coupling via a probe, loop, or iris from another EM source [45]. It can be shown that the excitation via a probe is an electrical coupling, which was discussed in chapter 2 [49]. The probe itself also introduces capacitance to the source load. Figure 22 shows the translation from the described circuit model to the coupling matrix compatible model.
Figure 21: (a) Circuit representation of the cavity resonator, demonstrating electric coupling from probe to cavity, magnetic coupling from cavity to coil, and external coupling from cavity to load. (b) Coupling replaced with pi network circuit equivalents. (c) BPF model of cavity resonator WPT.

Note I model the magnetic coupling as a J-inverter, as discussed in chapter 2.

Tuning on the source side must occur by adjusting the probe length, which will change the mutual capacitance between the cavity and the source. Tuning on the load side is achieved with capacitors that realize the J inverter described in chapter 2. The actual values of the lumped elements that make up the cavity can be determined through $S_{11}$ measurements, as described in Dr. Kajfez’s work [48].

### 4.2.3 Application of the Generalized Coupling Matrix

A picture of the resonant cavity used is shown in figure 23 and the dimensions are shown in table 9. The entire cavity is constructed on 1100 series aluminum for high conductivity.
We excite our cavity at the lowest mode of resonance, the TM_{110} mode. Using our VNA to take S_{11} measurements with a characteristic impedance of 50 ohms, we can determine the relevant circuit parameters of the cavity system, namely C_{probe}, C_{m}, C_{1}, R_{1}, and L_{1}. The initial probe length was 84 mm. For the receiver, we constructed a 7 mm
diameter, two tightly wound turn receive coil made of 22 AWG Cu magnet wire. A VNA was also used to measure the lumped values of the receive coil (L₂ and R₂). The results of the measurements are summarized in table 10.

Table 10: Summary of measured resonator parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Aluminum Cavity Resonator</th>
<th>Receive Resonator</th>
</tr>
</thead>
<tbody>
<tr>
<td>f₁₁₀</td>
<td>346.6 MHz</td>
<td>346.6 MHz</td>
</tr>
<tr>
<td>Lₙ</td>
<td>0.19455 nH</td>
<td>68.0 nH</td>
</tr>
<tr>
<td>Q₀n</td>
<td>1027</td>
<td>67.48</td>
</tr>
<tr>
<td>Cₙ</td>
<td>1083.8 pF</td>
<td>3.10 pF</td>
</tr>
<tr>
<td>lₚrobe</td>
<td>84 mm</td>
<td>n/a</td>
</tr>
<tr>
<td>Cₘ</td>
<td>3.55 pF</td>
<td>n/a</td>
</tr>
<tr>
<td>Cₚrobe</td>
<td>2.16 pF</td>
<td>n/a</td>
</tr>
<tr>
<td>k₁₂</td>
<td>0.002</td>
<td></td>
</tr>
</tbody>
</table>

n/a = not applicable

Knowing we have source and load impedance of 50 ohms (where the source is also loaded by the small probe capacitance, we now have the resonator Q factor and couplings, allowing us to compute our coupling matrix. Although the optimization algorithm is certainly overkill for a two resonator two port system, it demonstrates the versatility of the tool for WPT optimization. The results of the algorithm tell us the optimal values of the Jₛ₁ and J₂l, which we can translate into values of Cₘ, Cₚ, and Cₛ using the equations described in chapter 2. A relationship between probe length and the mutual and probe capacitance is obtained empirically and shown in figure 24.
Figure 23: Empirical relationship between the length of the probe and the mutual and probe capacitance.

Table 11 shows the calculated and implemented values obtained for optimization of the cavity system. We implement slightly higher values to account for the high frequency variation associated with surface mount 0201 capacitors soldered on the printed circuit board. Setting the length of the probe to 84 mm makes $C_m$ close to the calculated optimum.

Table 11: Optimal capacitors to generate the $J$-inverters used to optimize the system summarized in table 10 ($k_{12opt} = 0.002$).

<table>
<thead>
<tr>
<th>Optimal IM</th>
<th>$E_{S1opt} = 0.0340$</th>
<th>$E_{S2opt} = 0.1294$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$J_{S1opt} = 0.0073 \text{ S}$</td>
<td>$J_{2Lopt} = 0.0015 \text{ S}$</td>
</tr>
<tr>
<td></td>
<td>$C_m \text{ (pF)}$</td>
<td>$C_{p2} \text{ (pF)}$</td>
</tr>
<tr>
<td>Calculated</td>
<td>3.34</td>
<td>2.41</td>
</tr>
<tr>
<td>Implemented</td>
<td>3.55*</td>
<td>2.7</td>
</tr>
</tbody>
</table>

As a comparison, figure 25 shows $S_{21}$ as predicted by our coupling matrix model, as predicted by ADS circuit simulation of the circuit shown in figure 22, and our empirical results.
The model shows agreement between our matrix analysis, our circuit model, and our empirical results. This validates the coupling matrix formulation for the shunt resonator case and demonstrates the versatility of the tool for multiple WPT applications.

4.3 Concluding Remarks

By deriving the generalized coupling matrix with a focus towards WPT applications, I argue this work presents the most versatile and useful tool for WPT impedance matching synthesis to date. The \( n+m \) generalized coupling matrix can account for finite Q, mixed couplings, complex loads, and networks of arbitrary size. While many different tools could be used to optimize the matrix, I have shown that the custom hybrid method developed in this work can find optimal IM values for large networks, power splitting networks, complex loads, and systems with mixed coupling such as the cavity resonator system. Table 12 shows a comparison of the performance of the model made in this work with previous modes of analysis.
Table 12: Comparison of this work to other methods for WPT optimization

<table>
<thead>
<tr>
<th>Model can (account for):</th>
<th>Sample et al. [50]</th>
<th>Ha et al. [23]</th>
<th>Ean et al. [24]</th>
<th>Kiani et al. [29]</th>
<th>Nguyen et al. [51]</th>
<th>This Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Achieve Optimal IM</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Finite Q</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Mixed couplings</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Complex Loads</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Arbitrary Sized Networks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Power Splitting</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>

4.3.1 Future Work

Moving forward, I would like to demonstrate further evidence of the utility of the generalized coupling matrix by optimizing IM for mid-field powering. Hopefully, this can be done by accounting for media interactions as dielectric resonators and characterizing their couplings and Q factors, allowing us to include tissue into the circuit model. I believe the optimization of mid-field powering will prove extremely useful for biomedical applications. Moreover, it would further show the utility of the $n+m$ general coupling matrix to experimentally demonstrate optimization of mid-field powering methods, as this work has dealt with exclusively near-field methods.

Although the $n+m$ generalized coupling matrix presented here is quite useful, it still makes some assumptions, namely in the narrowband approximation. In future works, I would like to extend the coupling matrix to account for multi-modal resonators, making the model more accurate over a large bandwidth, not just near the resonant frequency.
If the two aforementioned forays are successful, I would like to experiment with modeling systems as discrete resonators. If possible, it would be interesting to see if electromagnetic problems could be simulated as discrete resonators with parameters based on material properties. This would allow the user to simulate for the flow of energy directly, without solving for the electric or magnetic field distributions, which could prove very useful telemetry path loss models, or RF ablation applications where energy must be concentrated in a particular region of tissue.

Finally, I would like to spend more time formalizing the optimization problem discussed in the beginning of chapter 3. I believe the \( n+m \) matrix could be useful for control algorithms. For the resonator cavity application, the small animal moving around the cavity disrupts the field distribution, resulting in small changes to the circuit model. This means the optimal probe length and resonant frequency shift slightly. I believe a more computationally inexpensive version of the optimization shown in chapter 3 would serve as a useful control algorithm for a dynamically tuned resonant cavity to achieve optimal PTE.
LIST OF REFERENCES
LIST OF REFERENCES


VITA

Kyle Thackston is most recently from Tucson, Arizona, but has also lived in Illinois, Hawaii, Colorado, Kansas, Virginia, Maryland, Indiana, and New Jersey. He received his Bachelor of Science in Biomedical Engineering from Purdue in 2015 and received the Haselby Outstanding Biomedical Engineering Senior Student Award. After being accepted into the 5th year MS program, Kyle was awarded the Charles C. Chappelle Fellowship, providing funding for his 5th year of graduate study. He has worked in Center for Implantable devices led by Professor Irazoqui for two years, specializing in wireless power transfer for medical devices. After graduation, Kyle intends to continue graduate studies and obtain a PhD. Kyle plans to spend his career furthering the physiological integration of electronics in healthcare.