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L. Rosa
University of Padua; Italy

R. Tosato
University of Padua; Italy

ONE DIMENSIONAL NON-STEADY FLOW IN COMPRESSOR PIPES SIMULATED BY A MODIFIED INVERSE MARCHING METHOD OF CHARACTERISTICS.

L.ROSA - R.TOSATO

Department of Mechanical Engineering

University of Padua (Italy)

SUMMARY

Here the problem of a correct prediction of transient pressures and pulsations in reciprocating compressors is considered, with a particular approach to connecting pipes and their specific boundary conditions (valves, closed and open ends, cylinder, etc.).

The possibility of employing the method of characteristics for gas flow is well known. We are using a numerical solution method which decreases the mathematical errors of smearing because the number of linearizations is much less than with mesh methods.

We suggest to divide pipes in three sections:

- the sections at the right and left ends have to be very short (one cell) and mesh methods are used to apply the boundary conditions;
- the intermediate section is the longest part (more cells) and uses a modified inverse method which makes linear interpolations to determine $\Delta\lambda$, $\Delta\beta$ and ΔAA along characteristic lines, but not to determine λ, β and AA at initial points of characteristic lines.

NOTATION

a	sonic velocity
aref	sonic velocity at reference conditions
aA	sonic velocity at reference pressure
A = a/aref	non-dimensional sonic velocity
AA = aA/aref	entropy level of non-homentropic flow
C_p, C_v	specific heats of gas
$k=C_p/C_v$	ratio of specific heats (ideal gas is considered)
K1 = (k+1)/(k-1)/2	constant
K2 = (3-k)/(k-1)/2	constant
D	diameter
F	area of cross section
f	friction factor
p	gas pressure
pref	pressure at reference conditions
q	heat transfer rate per unit mass
T	time
Z=aref*T/xref	non-dimentional time
u	gas velocity
U=u/aref	non-dimensional gas velocity
x	distance from the left end of the pipe
X=x/xref	non dimensional distance
xref	reference length
β=A-U*(k-1)/2	Riemann variable of characteristic
λ=A+U*(k-1)/2	Riemann variable of characteristic

INTRODUCTION

The non-steady flow in a pipe is assumed to be one-dimensional with a good correspondance to the reality and two kinds of flows are usually considered:

a) the homentropic flow of a perfect gas;
b) the non-homentropic flow with area changes, friction at walls, heat transfer and entropy gradients.

Many publications have fully described non-steady compressible flow basic equations and their solutions /1,2,3,4,5/.

By the method of characteristics continuity, momentum and energy conservation partial differential equations are trasformed into three total differential equations (compatibility equations) along three characteristic directions:

- the path line characteristic U,
- the λ characteristic U+A,
- the β characteristic U-A.

The Riemann variables λ and β and the entropy level AA are used to define all physical parameters and the fluid velocity u in the pipe.

For the non-homoentropic flow:

I) along a λ characteristic line

a) direction condition

$$dX/dZ = U+A = K1*\lambda - K2*\beta, \tag{1}$$

b) compatibility condition

$$\begin{aligned} dA+(k-1)/2*dU = d\lambda = &-(k-1)/2*A*U/F*dF/dX*dZ+A*dAA/AA- \\ &-(k-1)/2*2*f*xref/D*U**3/|U|*(1-(k-1)*U/A)*dZ+ \\ &+(k-1)**2/2*q*xref/aref**3/A*dZ \end{aligned} \tag{2}$$

II) along a β characteristic line

a) direction condition

$$dX/dZ = U-A = K2*\lambda - K1*\beta, \tag{3}$$

b) compatibility condition

$$\begin{aligned} dA-(k-1)/2*dU= d\beta = &-(k-1)/2*A*U/F*dF/dX*dZ+A*dAA/AA+ \\ &+(k-1)/2*2*f*xref/D*U**3/|U|*(1+(k-1)*U/A)*dZ+ \\ &+(k-1)**2/2*q*xref/aref**3/A*dZ \end{aligned} \tag{4}$$

III) along a path line U

a) direction condition

$$dX/dZ = U = (\lambda-\beta)/(k-1), \tag{5}$$

b) compatibility condition

$$dAA=(k-1)/2*AA/A**2*(q*xref/aref**3+2*f*xref/D*|U**3|)*dZ \tag{6}$$

Pressure p and velocity u at every point are evaluated from the values of λ, β and AA at this point, by the relationships between pressures, velocity and speeds of sound:

$$(p/pref)**((k-1)/(2*k)) = a/aA = A/AA = (\lambda+\beta)/AA/2 \qquad (7)$$

$$u/aref = U = (\lambda-\beta)/(k-1) \qquad (8)$$

For the homentropic flow, only two characteristic lines are used (λ and β) and equations (2) and (4) are simpler and reduced to $d\lambda = d\beta = 0$ when the cross area F is constant.

The graphical solution of the characteristic equations is precise, but it is only simple for homentropic flow and very complex for the non-homentropic one. Numerical solution is always preferred. It is based on the direct transformation of total differential equations (1) ... (6) into finite difference equations and on their integration advancing the solution for a series of time steps ΔZ and space steps ΔX, starting from initial conditions at time Z=0.

NUMERICAL SOLUTION OF EQUATIONS

For non-homentropic flow, according to Zucrow and Hoffman /3/, three methods are usually employed to construct a finite difference grid and for marching through a flow field (X,Z) with a numerical algorithm:

a) the <u>direct marching methods</u> locate the solution point at the intersection of two characteristics (U+C, U-C or U) extending from two known initial data points; the values of flow properties at the third initial data point are evaluated by interpolation;

b) the <u>inverse marching methods</u> specify a priori the locations of the solution points (on lines of constant Z+ΔZ time and constant X coordinate for mesh method) and three characteristics are extended from the solution point to three initial points; three interpolations are used to determine flow properties at these initial data points;

c) the <u>modified inverse marching methods</u> only locate a priori the Z+ΔZ coordinate of the solution point and determine its X coordinate along a characteristic passing through one known initial point (non mesh method); two interpolations need to calculate properties at a further two initial points.

Linear interpolations and other simplified assumptions, introduced to achieve an acceptable solution with a reasonable computing time, reduce the accuracy of the solution, in comparison with the graphical solution proposed by Jenny /1/ and experimental results. The solution is smeared and this mathematical damping prevents the model from predicting higher frequency pressure pulsations usually experimentally observed in reciprocating compressors. These pulsations have to be accounted for to simulate the dynamics of the valves.

The mesh method introduces the greatest smearing of the solution because of the amount of interpolations and because the interpolations are based on flow properties at points outside the domain of dependence.

Usual simplified assumptions are:

- the values of variables appearing in the equations are those existing at time Z;
- the characteristics connecting times Z and Z+ΔZ are straight and the slope is equal to the value at the beginning of the step;
- the pipe length is divided into a low number of elements.

- no iteration is carried out to account for the changes of variables during the step;
- a low number of characteristics is used and the total amount of old and new ones introduced at complex boundary conditions is controlled to avoid it becoming too large.

For non-homentropic flow, Pajri, Corberan and Boada /7/, discovered that the linear interpolations of λ and β along the grid interval may introduce errors and sharp pressure gradients. To improve the accuracy of numerical results, they used a modified non-mesh method along pathlines and a mesh method for λ and β calculations with linearizations made on the pressure values.

Also, the numerical solution mesh method for homentropic flow uses linear interpolations in such a way that smearing errors are still present and Benson /4/ plotted the difference of results comparing them with graphical solution results. Numerical procedure can apply more complex algorithms, as shown by Woollatt /6/ to avoid many linear interpolations.

We already developed an algorithm that allows to avoid a lot of linearizations for non homentropic flow and it was applied to transient flow of air and water vapour /8/ in pipes having simple boundary conditions, with a close coincidence between numerical and experimental results.

The proposed method is now explained and it can be obviously adopted to simulate non-steady flow in reciprocating compressors and their piping systems. To show the accuracy of results, two examples of homentropic flow will be simulated and compared with graphical solution. The data is exactly the same as the examples solved and illustrated in Benson's book /4/.

CALCULATION METHOD

Authors experimented that non-mesh inverse methods and the method proposed by Woollatt are only relatively simple for pipes having open and closed ends. In different cases, for pipes with the complex boundary conditions characteristic of compressors (the cylinder, valves, nozzles, junctions of pipes, sudden enlargements and contractions), the number of characteristics λ and β increases rapidly, those of the same family intersect. Algorithms become more complex and computing time longer, especially because of boundary conditions.

Internal of pipe

To reduce this problem we prefer to adopt:

- a <u>non-mesh method</u>, over the entire length of the pipe, to calculate AA along the U characteristics,
- a <u>mesh method</u>, in the cells near the boundaries, to calculate λ and β in the fixed nodes;
- a <u>non-mesh modified method</u>, in the interior part of the pipe, to calculate λ e β along the corresponding characteristics;

By this method λ e β lines pass from the interior part of the pipe to the cells near the boundaries, and vice-versa, without any reflections and then without any number rise. All linearizations are avoided in the internal part of the pipe and only one characteristic, at the end of each time step, arrives at each boundary end of the pipe.

This method is also applicable in the homentropic case, without any integration of the equations along the pathlines U. Fig. 1 considers this kind of flow.

Therefore, referring to fig. 1, we consider a pipe whose length L has been divided into a number N of equal elements. Initial conditions (λ, β and AA), at the time Z=0 and at the fixed points 1, 2, ... N+1, are known. The U characteristics start from these points, the λ characteristics from 1, 2, ... N points and the β ones from 2, 3, ... N+1 points. The same number marks a characteristic line and its intersection points with lines of Z and Z+ΔZ times.

We divided the pipe into:

I section between 1 and 2 mesh points,
II section between 2 and N mesh points,
III section between N and N+1 mesh points.

At the generic instant Z, the values of variables β, λ ed AA are known at corresponding moving points as results of previous steps. To determine flow conditions at Z+ΔZ time it is necessary to limit the value of ΔZ by the Courant-Friedrich-Lewy condition.

For all three sections we determine, at Z+ΔZ time, the positions of pathlines U (new points 1', 2', ... N+1') and the values of the variable AA (eq.6) by a non-mesh method, in these moving points, using values at old points.

Inside the I section, a mesh method is used to calculate λ at the fixed point 2, at time Z+ΔZ, using variables at points 1 and 2 at time Z.

Inside the section II we first locate the positions of new points 2", 3", ... N-1" and then, by linear interpolation, the values here of AA and of the variable ΔAA of eq. 2. The values of λ at these new moving points are calculated by eq. 2, using the variables of initial points 2", 3", ... N-1".

Inside the III section the values of λ at mesh points N and N+1 are evaluated by a mesh method using the variables at initial points N-1, N and N+1.

The calculation of β values at mesh points 1, 2 and N, inside I and III section, and at new points 3"', 4"', ... N"' inside the II section, is obviously made in the same manner and therefore we haven't described it.

The λ value in the left boundary (point 1) and β value at the right end (point N+1) are easily obtained by the well known boundary condition equations.

To increase the accuracy, an iterative procedure can be introduced using values still obtained at solution moving points at Z+ΔZ time.

Values of λ, β and AA at mesh points, at time Z+ΔZ, are evaluated by linear interpolation.

The total number of U characteristics is maintained equal to N+1 introducing a new U line when a U line intersects the boundary ends and eliminating a U line when another U line enters from a pipe connected at the boundary end.

Also the number of λ e β lines is maintained equal to N-1 inside the II section of the pipe. When two lines of the same family intersect, they are substituted by only one or by two close and parallel characteristics.

For the first time step, the method is the same, the difference being that at the initial time all moving points are coincident with fixed mesh points.

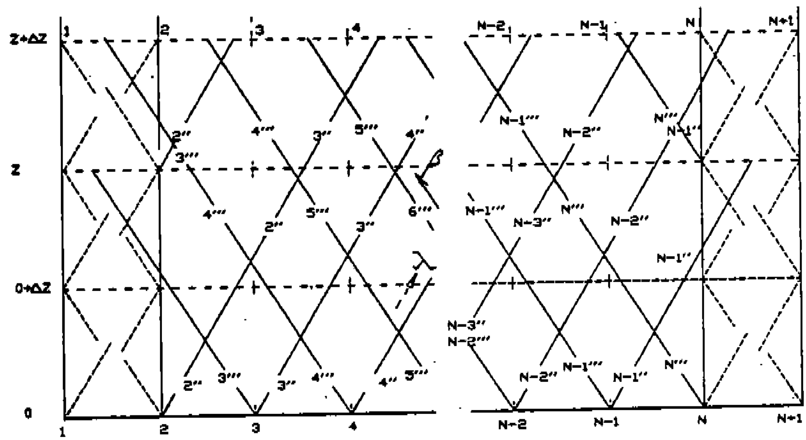

fig. 1. Characteristics λ *and moving points 2", 3", N-1", characteristics* β *and moving points 3"', 4"', ..., N"', fixed points in the position diagram, for a homentropic flow, in a pipe divided into N intervals.*

Boundary Conditions

Only one characteristic arrives at the pipe-end at the end of the time step and this is the simplest way to solve boundary condition calculations. The use of specific equations is done at the end of the time step.

TEST OF NUMERICAL METHOD FOR HOMENTROPIC FLOW

In order to test the numerical method, a Fortran program was written to simulate homentropic flow. Two subroutines allow to evaluate the positions of Riemann characteristics and the values of λ é β at moving and mesh points. Other subroutines simulate the normal boundary conditions of compressor pipes. The program considers the total number N of elements of three sections as a variable and it is possible to vary, from 2 to N, the number of cells of external sectors I and II in order to evaluate the influence of linearizations. No iteration procedure is used.

Example 1

In the first example a pipe connects an infinite reservoir, at the pressure of 1.5 bar and temperature of 300 K, to the atmosphere. The air (k=1.4) flows through the pipe, 1 m long, when the atmospheric end of the pipe is suddenly opened. Waves which start from the atmospheric end propagate as rarefaction to the reservoir side and reflect as compression waves which travel to the atmospheric end.

Pressures P1 at the open end, P3 at pipe entry and P2 at mid-point, when N=40, 30, 20 (continuous lines) and N=10 (dotted line) and only for one mesh in section I and III, are plotted against the time T, in fig. 2.

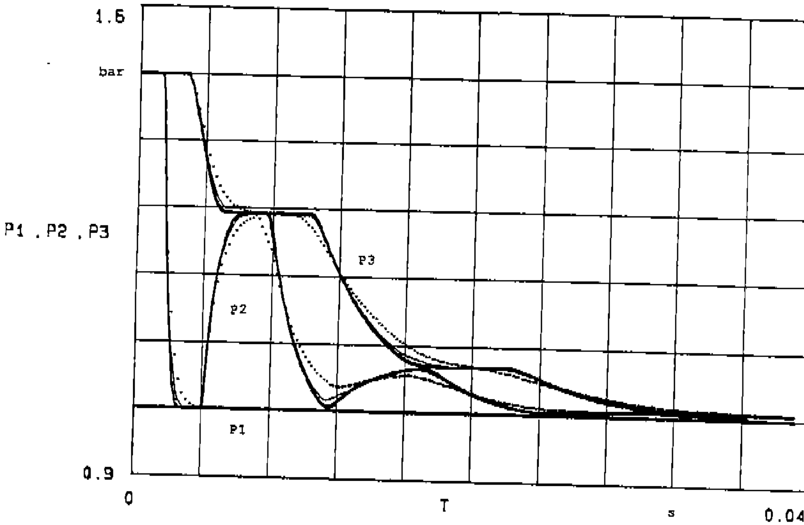

fig. 2. Pressures at the open end P1, at pipe entry P3 and at mid-point P2, as a function of the time T. Effect of the number N of meshes on the accuracy.

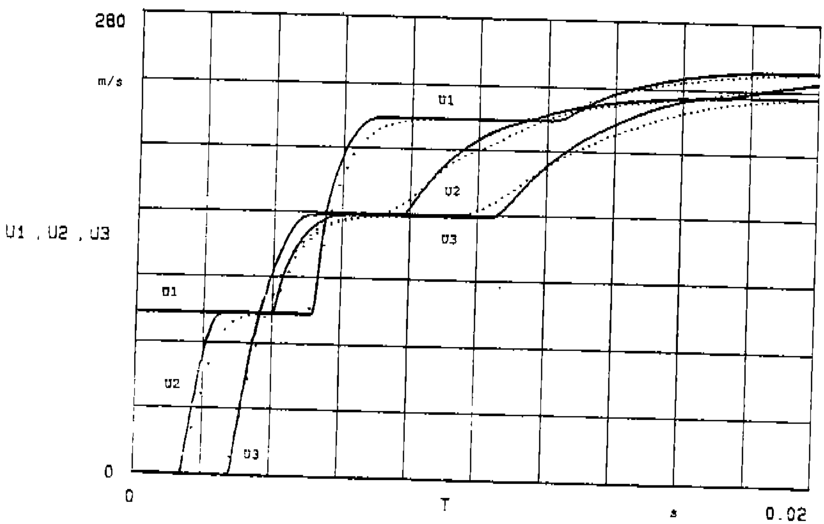

fig. 3. Velocity U1, U2 and U3 curves.

Velocity diagrams of the same points are plotted in fig. 3.

For N=40, 30 and 20 these diagrams are very close to results obtained by graphical method. The shapes of the curves are not smoothed out and for this reason the method is able to simulate high frequency pressure pulsations.

Pressure diagrams calculated by the proposed method (continuous line) and by mesh-method (dotted line) are plotted in Fig. 4, with N=30.

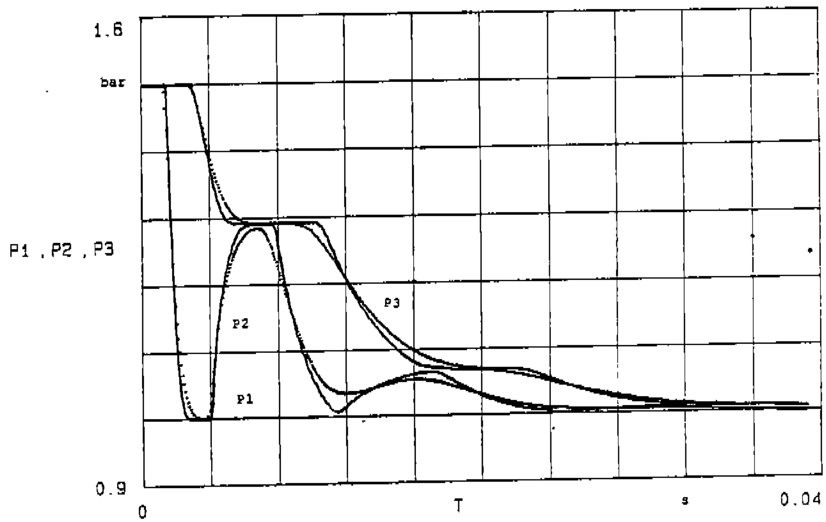

fig. 4. Comparison between mesh method (dotted lines) and the proposed method (continuous line) for N=30. Effect of linearizations.

Example 2

In the second example a pipe, L=1.219 m long, filled with air (k=1.4), is initially at a pressure of 3.041 bar and a temperature of 135 C. Gas is suddenly discharged into the atmosphere (1 bar) through a nozzle having an area ratio of 0.25. Pressure waves travel up and down the pipe to finally bring down the pressure to the atmospheric value.

Variations of pressures P1, P2 and velocities U1, U2 , at two points at a distance of L/12 from the closed end (point 1) and from the nozzle (point 2), obtained by the proposed method, are plotted in fig. 5 and fig. 6.

Different curves refer to N=48, 36, 24 (continuous lines) and N=12 (dotted lines).

Numerical results are very close to graphical ones for continuous curves. For N=12, the two external sections of the pipe, where the mesh method is used, corrispond to 2/12 of the entire length L and linearizations, as in the previous example, become an important source of errors.

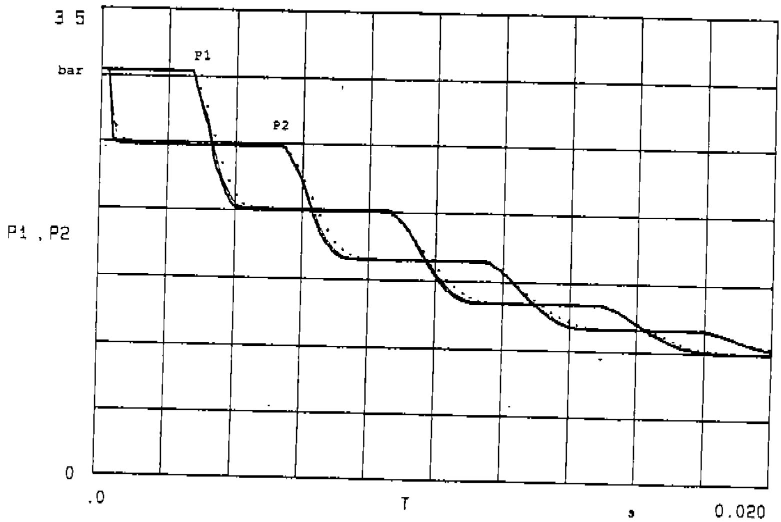

fig. 5. Pressures P1 at 1/12 L and P2 at 11/12 L from the closed end of a pipe with a partially open end.

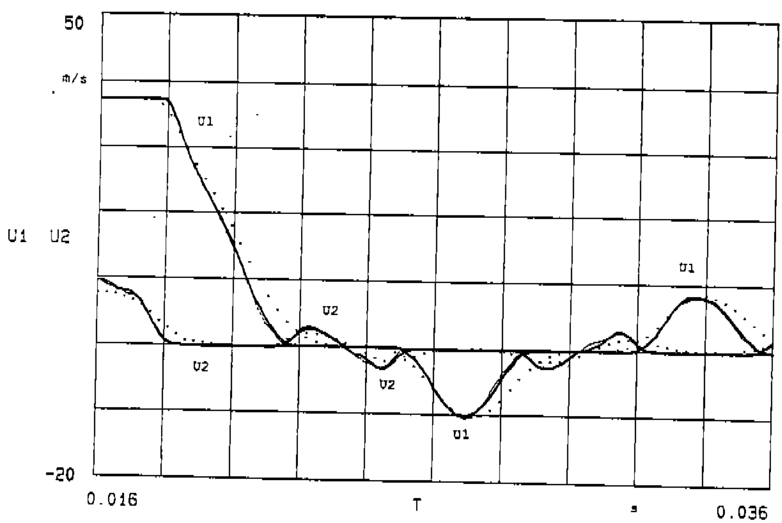

fig. 6. Residual velocities U1 at 1/12 L and U2 at 11/12 L from the closed end of a pipe with a partially open end.

CONCLUSIONS

Theoretical analysis has been carried on for homentropic flow in a pipe and compared with results obtained by graphical solution. Information on the possibility for the model to represent the flow with a high degree of accuracy is positive. The influence of residual linearizations still present in the proposed method is negligible, if the pipe is divided into a number of finite elements higher than 20. The method here described for homentropic and non-homentropic flow achieves, in this condition, the same accuracy as the graphical method of characteristics.

ACKNOWLEDGMENT

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