International Compressor Engineering Conference School of Mechanical Engineering

1992

Inertia Effect of the Fluid Particles on the Lubricant Flow in a Dynamic Thrust Bearings

S. K. Padhy
West Virginia University

S. N. Dwivedi
West Virginia University

INERTIA EFFECT OF THE FLUID PARTICLES ON THE LUBRICANT FLOW IN A DYNAMIC THRUST BEARINGS

Sisir K. Padhy & S. N. Dwivedi
Department of Mechanical & Aerospace Engineering
West Virginia University, Morgantown, WV 26506.

ABSTRACT

Closed form equations are derived from the fundamental principles for the evaluation of volume flow rate of lubricant in between the pads of a thrust bearing accounting for the centrifugal inertia effect. Two types of configuration i.e. parallel disk and nonparallel disk configurations are considered. The theoretical analysis is presented.

1. INTRODUCTION

Thrust bearings are one of the most commonly used bearing configurations in the industrial applications utilizing parallel disk configuration. Although the mechanism of the hydrodynamic pressure development in this bearing is still not fully understood, it is clear that there has to be adequate supply of lubricant to prevent metal to metal contact. The lubricant flows in between the disks by virtue of pressure gradient that exists between the outer radius and inner radius. For static conditions the expression for the volume flow of lubricant is well published in the literature. However very few works have been reported on the effect of the centrifugal effect of the fluid particles on the volume flow, when one disk is in motion. Again it seems to have no work reported considering the misalignment of the shaft or the thrust pads which can induce nonparallelism. In the present paper closed form solutions are derived for the calculation of the volume flow rate of the lubricant in between the pads of a dynamic thrust bearings. The effect of taper that may be caused due to the misalignment of the shaft or production error, is also described. The equations are derived from the fundamental principles.

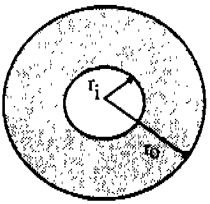
2. PREVIOUS WORK

Wilcock and Booser have presented an empirical formula [1] for calculation of fluid flow in a thrust bearing. They concluded that no method is available to compute the flow rate accounting for the rotation of the thrust face (one disk). Fuller [2] describes an analytical expression for the fluid flow in between two parallel disks in static condition. Shivamoggi [3] describes centrifugal flow due to a rotating disk. His discussions are limited to the flow of fluid in absence of an imposed radial pressure gradient and the case is appropriate for a disc rotating in fluid rather than the flow in between two discs in dynamic conditions. Again no closed formulation is presented for the volume of fluid flow. Neglecting convective inertia Osterle & Hughes [4] presented the radial pressure gradient when one disk is rotating. Szeri & Adams[5] analyzed the case of laminar through flow between closely spaced rotating disks. Their work is concerned with radial pressure profile. They solved the equations of continuity and motion with thin film approximation using numerical (modified predictor-corrector) methods to obtain the pressure profiles and the results compared with the experimental data very well. Laminar radial incompressible flow between two parallel plates was discussed by Savage [7] for air as the fluid. A solution for the pressure distribution was obtained by perturbing the creeping flow solution. Inertia effect of the fluid was considered. Their solution agreed well with the experimental results. However the study was primarily concerned with velocities in r and z directions and rotational effect i.e. velocity in θ direction was not considered. In another study Raal [8] discussed the radial source flow between parallel disks. Finite difference method was used to solve for the pressure and velocity distributions. Wark and Foss [9] described the forces caused by the radial outflow between the parallel disks. Their work was primarily experimental and acceptable agreement was found with theoretical analysis. Radial outflow between a smooth stationary disk and a grooved rotating disk was examined analytically as well as experimentally by Missimer and Johnson [10]. Their work emphasized on the flowrate and drag moment. A finite difference scheme was employed to solve the problem. In a very interesting paper Gans, Johnson and Malanoski [11] studied the separation and the pressure variations in between two stationary plates as a function of load and mass flowrate. They conclude that a slightly concavity ($1.6X10^{-3}$ of mean face slope) in one plate can significantly reduce the pressure requirement for an air support system. However no rotational effects were considered.

3. PRESENT WORK

The present work deals with analytical derivations for the fluid flow between two disks from fundamental principles when they are in parallel & tapered configurations. While one disk is stationary, the other disk is rotational. The fluid flows outwards in between the disks. In deriving the expression, centrifugal effect of the fluid particles are considered. Following assumptions are made in this derivation:

- The fluid flow is laminar.
- Flow is fully developed.
- Velocity profile due to centrifugal effect is linear along the z-direction.
- Roughness of the surfaces have no effect on the flowrate of the fluid.
- The pressure variation in θ and z directions are negligible compared to that of in r-direction.
- The fluid is incompressible.



[Figure 1. Disk geometry. Fluid flows from inside to outside]

The flow is axisymmetric and is therefore most convenient to use cylindrical coordinates. A differential control volume [Figure 2] is used in the form of a differential annular ring. The differential dimensions are dr, dz and $r\,d\theta$. For a fully developed steady flow the component of the momentum equation along the r-direction reduces to

$$F_r=0 \quad (1)$$

or

$$\Sigma(\text{Pressure Forces} + \text{Shear Forces} + \text{Inertia Forces})=0$$

Considering the pressure at the center of the annular control volume is p, then the force at inner face is

$$\left(p-\frac{\partial p}{\partial r}\frac{dr}{2}\right)\left(r-\frac{dr}{2}\right)d\theta dz$$

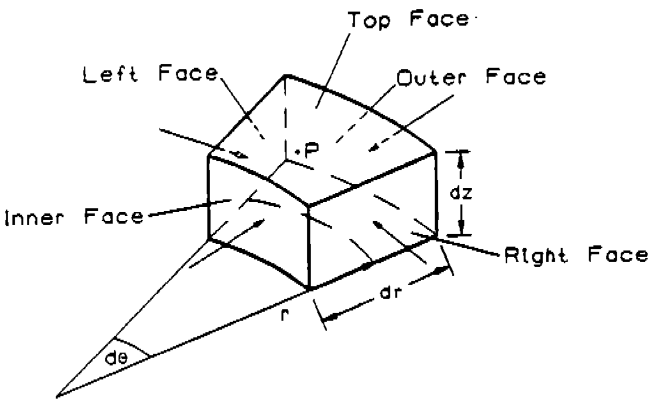
Pressure force at the outer face is

$$\left(p+\frac{\partial p}{\partial r}\frac{dr}{2}\right)\left(r+\frac{dr}{2}\right)d\theta dz$$

Now the component if pressure force the left and right side faces along the r-direction is

$$2(pdrdz)\sin\left(\frac{d\theta}{2}\right)\equiv 2(pdrdz)\left(\frac{d\theta}{2}\right)=(pdrdz)d\theta$$

as angle θ associated with the control volume is very small.



[Figure 2. Control volume and its dimensions]

The shear forces act on the top and bottom face of the control volume. If the shear stress at the center of the element is τ, then shear force at the bottom surface of the control volume is

$$-\left(\tau-\frac{\partial\tau}{\partial z}\frac{dz}{2}\right)rd\theta dz$$

and the shear force on the top face is

$$\left(\tau+\frac{\partial\tau}{\partial z}\frac{dz}{2}\right)rd\theta dz$$

Accounting fluid inertia, the body force of the control volume element becomes

$$\frac{m\forall^2}{r}$$

where the $\forall$ is the tangential velocity of the fluid element, m is the mass of the fluid element and r is the radius of the center of the mass. As the velocity v (not the velocity profile of the fluid moving through the two disks which is in z-r plane) of the fluid is assumed to be linear in the z-θ plane, at any height z the velocity term becomes

$$\forall = \left(\frac{vz}{h}\right)^2$$

Substituting this in the above equation and writing the mass term in terms of density ρ and volume, the body force becomes

$$\frac{\rho r d\theta dr dz (vz/h)^2}{r}$$

Substituting all the forces in to equation (1), we have

$$\left(p-\frac{\partial p}{\partial r}\frac{dr}{2}\right)\left(r-\frac{dr}{2}\right)d\theta dz + \left(p-\frac{\partial p}{\partial r}\frac{dr}{2}\right)\left(r-\frac{dr}{2}\right)d\theta dz + (pdrdz)d\theta - \left(\tau-\frac{\partial \tau}{\partial z}\frac{dz}{2}\right)rd\theta dz$$
$$+\left(\tau+\frac{\partial \tau}{\partial z}\frac{dz}{2}\right)rd\theta dz + \frac{\rho r d\theta dr dz (vz/h)^2}{r} = 0$$

Neglecting higher order terms this equation simplifies to

$$\frac{\partial \tau}{dz} = \frac{\partial p}{\partial r} - \frac{\rho}{r}\left(\frac{vz}{h}\right)^2 \tag{2}$$

Assuming a Newtonian fluid the shear stress term is given by

$$\tau = \mu \frac{du}{dz}$$

Recognizing that τ is a function of z only, $\partial\tau/\partial z$ becomes

$$\frac{\partial \tau}{dz} = \mu \frac{d^2u}{dz^2} \tag{3}$$

With substitution for $d\tau/dz$ in the equation (2), equation (2) becomes

$$\frac{d^2u}{dz^2} = \frac{1}{\mu}\frac{\partial p}{\partial r} - \frac{\rho}{\mu r}\left(\frac{vz}{h}\right)^2 \tag{4}$$

Integration of equation (4) once yields

$$\frac{du}{dz} = \frac{1}{\mu}\frac{\partial p}{\partial r}z - \frac{\rho v^2 z^3}{3\mu h^2 r} + c_1 \tag{5}$$

Integrating again the velocity profile can be expressed as

$$u = \frac{1}{\mu}\frac{\partial p}{\partial r}\frac{z^2}{2} - \frac{\rho v^2 z^4}{12\mu h^2 r} + c_1 z + c_2 \tag{6}$$

The constants c_1 and c_2 are evaluated using the boundary conditions. At $z=0$, $u=0$. Consequently $c_2=0$ and at $z=h$, $u=0$. Hence

$$0 = \frac{1}{\mu}\frac{\partial p}{\partial r}\frac{h^2}{2} - \frac{\rho v^2 h^4}{12\mu h^2 r} + c_1 h$$

This gives

$$c_1 = \frac{\rho v^2 h^3}{12\mu h^2 r} - \frac{1}{\mu}\frac{\partial p}{\partial r}\frac{h}{2}$$

and equation (6) becomes

$$u=\frac{1}{2\mu}\frac{\partial p}{\partial r}\left(z^2-hz\right)+\frac{\rho v^2}{12\mu h^2 r}\left(h^3 z-z^4\right) \tag{7}$$

The volumetric flowrate is given by

$$Q=\int_0^h u.2\pi r.dz \tag{8}$$

Substituting equation (7) for u in equation (8) yields

$$Q=\int_0^h\left(\frac{1}{2\mu}\frac{\partial p}{\partial r}\left(z^2-hz\right)+\frac{\rho v^2}{12\mu h^2 r}\left(h^3 z-z^4\right)\right).2\pi r.dz \tag{9}$$

Integration of equation (9) yields an expression for the fluid flowrate in terms of pressure gradient, i.e.

$$Q=-\frac{\pi r h^3}{6\mu}\frac{\partial p}{\partial r}+\frac{\pi\rho h^3 v^2}{20\mu}$$

or

$$-\frac{\partial p}{\partial r}=\frac{6\mu Q}{\pi r h^3}-\frac{3\rho v^2}{10r} \tag{10}$$

Writing v in terms of angular velocity of the upper disk and radius of rotation we have

$$-\frac{\partial p}{\partial r}=\frac{6\mu Q}{\pi r h^3}-\frac{3\rho\omega^2 r}{10} \tag{11}$$

Assuming the pressure variation in θ-direction as well as z-direction is negligible as compared to that of r-direction, the equation (11) becomes

$$-\frac{dp}{dr}=\frac{6\mu Q}{\pi r h^3}-\frac{3\rho\omega^2 r}{10} \tag{12}$$

It should be noted that equation (12) is identical to the radial pressure gradient equation of Osterle & Hughes [3].

Case I. Flow between parallel disks

Integration of equation (12) yields

$$-\int_{p_i}^{p_o}dp=\int_{r_i}^{r_o}\frac{6\mu Q}{\pi h^3}\frac{dr}{r}-\int_{r_i}^{r_o}\frac{3\rho\omega^2}{10}rdr$$

or

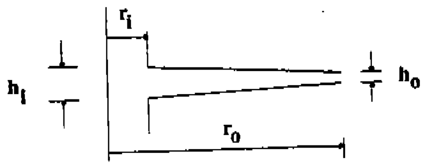
$$p_i-p_o=\frac{6\mu Q}{\pi h^3}\ln\left(\frac{r_o}{r_i}\right)-\frac{3\rho\omega^2}{20}\left(r_o^2-r_i^2\right) \tag{13}$$

Equation (13) gives the expression for Q as

$$Q=\frac{\pi h^3\left(p_i-p_o\right)}{6\mu\ln\left(r_o/r_i\right)}+\frac{\pi\rho\omega^2 h^3\left(r_o^2-r_i^2\right)}{40\mu\ln\left(r_o/r_i\right)} \tag{14}$$

Case II. Flow between nonparallel disks

Taking the clearance between the disks at the outer radius is h_o, slope of the upper disk from the outside to inside as α and that of the lower disk as β [Figure 2] the total clearance at the inner radius is



[Figure 2. Tapered thrust bearing pads]

$h_i = h_o + (r_o - r_i)(\sin\alpha + \sin\beta)$

and at any radius r the clearance becomes

$h = h_o + (r_o - r_i)(\sin\alpha + \sin\beta)\,(r_o - r)/(r_o - r_i)$

or

$h = h_o + [d(r_o - r)/(r_o - r_i)] = h_o + [d(1-x)/(1-s)] = h_o[1 + d(1-x)/(1-s)\,h_o]$

or (15)

$h = h_o[1 + b(1-x)]$

Where $d = (r_o - r_i)(\sin\alpha + \sin\beta)$, $x = r/r_o$, $s = r_i/r_o$, $b = d/h_o(1-s)$

Now writing equation (12) again we have

$$-\frac{dp}{dr} = \frac{6\mu Q}{\pi r h^3} - \frac{3\rho\omega^2 r}{10}$$

Now changing variable of integration from r to x and substituting h from equation (15) we have

$$-\frac{dp}{dx} = \frac{6\mu Q}{\pi h_o^3 x(1+b-bx)^3} - \frac{3\rho\omega^2 r_o^2 x}{10} \qquad (16)$$

The first term on the right hand side of equation (16) has to be integrated using partial fraction. Expanding this term using partial fractions we get

$$\frac{6\mu Q}{\pi h_o^3}\left[\frac{A}{x} + \frac{C}{(1+b-bx)} + \frac{D}{(1+b-bx)^2} + \frac{E}{(1+b-bx)^3}\right]$$

This term has two poles at x=0 and x=(1+b)/b and the coefficients are determined to be

$$A = \frac{1}{(1+b)^3};\ C = \frac{b^3}{(1+b)^3};\ D = \frac{-b^2}{(1+b)^2};\ E = \frac{b}{(1+b)}$$

Substituting these coefficients appropriately and noting that the integration limits are changed to s to 1 form that of r_i to r_o, equation (16) upon integration yields the following result

$$p_i - p_o = \frac{6\mu Q}{\pi h_o^3}\left[\frac{\ln\{(1+b-bs)^{b^2}/s\}}{(b+1)^3} - \frac{b^2(1-s)}{(b+1)^2(1+b-bs)} + \frac{b(2+b-bs)(1-s)}{2(1+b)(1+b-bs)^2}\right] - \frac{3\rho\omega^2(r_o^2 - r_i^2)}{20} \qquad (17)$$

By algebraic manipulation the lubricant volume flow rate is determined to be

$$Q = \frac{\left[\frac{\pi h_o^3(p_i - p_o)}{6\mu} + \frac{\pi h_o^3\rho\omega^2(r_o^2 - r_i^2)}{40\mu}\right]}{\left[\frac{\ln\{(1+b-bs)^{b^2}/s\}}{(b+1)^3} - \frac{b^2(1-s)}{(b+1)^2(1+b-bs)} + \frac{b(2+b-bs)(1-s)}{2(1+b)(1+b-bs)^2}\right]} \qquad (18)$$

When b is zero, i.e. there is no taper on the disks, equation (18) reduces to equation (14)

4. DISCUSSIONS

Equation for the fluid flow for the parallel pad configuration, reveals that the volume flow rate increases with the centrifugal effect. At higher speed and higher ratio of outer diameter to inner diameter of the thrust pad, the inertia effect (centrifugal effect) can be significant. For low speed operations and a smaller ratio of outer to inner diameter, the inertia effect may be neglected. In absence of an pressure fed lubrication i.e. Δp is zero, the centrifugal term will be the only driving factor for the fluid to flow from inside to outside.

Analyzing the equation for the tapered case, it is found that with positive value of taper the flow is higher than that of parallel case and with negative taper the flow is less than that of the parallel case. However with negative taper the denominator can be zero depending on the s and the ratio of the taper to the outer clearance. It becomes indeterminate when the taper equals to negative value of the outer clearance irrespective of the s value, i.e. when

$$d = -h_o$$

the logarithmic term becomes undefined.

5. CONCLUSION

A theoretical analysis is presented for the calculation of volume of lubricant flow in between parallel as well as nonparallel pads when one is rotating. Closed form equations are derived from the fundamental principles assuming a laminar & incompressible flow. It is found that a positive taper is preferred over a negative taper from lubricant flow stand point.

REFERENCES

1. Wilcock, D. F. & Booser, E. R., Bearing Design & Application, McGraw-Hill, New York, 1957.
2. Fuller, D. D., Theory & Practice of Lubrication for Engineers, John Wiley & Sons, New York, 1984.
3. Shivamoggi, B. K., Theoretical Fluid Dynamics, Martinus Nijhoff Publishers, Boston, 1985.
4. Osterle, J. F. & Hughes, W. F., Inertia Induced Cavitation in Hydrodynamic Thrust Bearings, *Wear*, Vol. 4, pp. 228-233, 1957/8.
5. Szeri, A. & Adams, M. L., Laminar Throughflow Between Closely Spaced Rotating Disks, *J. of Fluid Mechanics*, Vol. 86, pp. 1-14, 1978.
6. Dowson, D., Inertia Effects in Hydrostatic Trust Bearing, *Trans. of A.S.M.E., J. of Basic Engineering*, pp. 227-234, June, 1961.
7. Savage, S. B., Laminar Radial Flow Between Parallel Plates, *Trans. of A.S.M.E., J. of Applied Mechanics*, Vol. 31, No. 4, pp. 594-596, 1964.
8. Raal, J. D., Radial Flow Between Parallel Disks, *J. of Fluid Mechanics*, Vol. 85, pp. 401-416, 1978.
9. Wark, C. E. & Foss, J. F., Forces Caused by the Radial Outflow Between Parallel Disks, *Trans. of A.S.M.E., J. of Fluids Engg.*, Vol. 106, No. 3, pp. 292-297, Sept. 1984.
10. Missimer, J. R., & Johnson, W. S., Flow Between a Smooth Stationary Disk and Grooved Rotating Disk, *Trans. of A.S.M.E., J. of Lubrication Technology*, Vol. 104, No. 2, pp. 248-254, April, 1982.
11. Gans, R. F., Johnson, A. G. & Malanoski, Jr., S. B., Radial Flow Between Axisymmetric Nonparallel Plates of Small Slope, *Trans. of A.S.M.E., J. of Tribology*, Vol. 110, No. 2, pp. 342-347, April, 1988.