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Design of an asymmetric reluctance machine for a generator application

Bryan David Marquet
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By Bryan David Marquet

Entitled
DESIGN OF AN ASYMMETRIC RELUCTANCE MACHINE FOR A GENERATOR APPLICATION

For the degree of Master of Science in Electrical and Computer Engineering

Is approved by the final examining committee:

Scott Sudhoff 4/26/2016
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Approved by: V. Balakrishnan 4/26/2016
Head of the Departmental Graduate Program Date
DESIGN OF AN ASYMMETRIC RELUCTANCE MACHINE FOR A GENERATOR

APPLICATION

A Thesis
Submitted to the Faculty
of
Purdue University
by
Bryan David Marquet

In Partial Fulfillment of the
Requirements for the Degree
of
Master of Science in Electrical and Computer Engineering

May 2016
Purdue University
West Lafayette, Indiana
For my family.
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This work compares an Asymmetric Reluctance Machine (ARM) to a Uniform air Gap Reluctance Machine (UGRM) for a generator application. The ARM employs an asymmetric rotor pole to increase machine performance, specifically increasing torque density. A multi-objective optimization is employed to minimize mass and loss subject to design constraints. Two machines are compared in terms of their Pareto-optimal fronts.
CHAPTER 1. INTRODUCTION

1.1 Introduction

Permanent-magnet synchronous machines (PMSMs) are employed in a wide variety of applications due to their high torque density and low machine loss. However, magnet cost and fault performance have been issues in certain situations [1]. Induction machines (IMs) are also used in a wide range of machine applications. The issue of high loss due to the rotor windings, however, motivates researchers to explore alternate machines [2], [3].

Reluctance machines tend to have poor power factor, low efficiency, and low torque density compared to other classes of machines, but they do have a few desirable features. These features include low assembly cost and relatively benign fault conditions.

Improvements to the reluctance machine by altering rotor construction have been explored in the past. One suggestion was to remove the laminations of the rotors; however, this machine has low torque density and efficiency [4] [5]. In [6] and [7], a machine with axially aligned magnetic laminations on the rotor is investigated. The laminations are assembled to reduce reluctance in the axial direction, parallel to the laminations, and maximize reluctance in the direction normal to the lamination. In [1], it was claimed this
machine is competitive with an induction machine, but that the cost of construction is high because securing the laminations on the rotor is difficult.

An asymmetric rotor design is set forth in [8]. The goal of that machine’s asymmetry, however, is to reduce torque ripple, which is fundamentally different from the objective of this thesis. An asymmetric reluctance machine is introduced in [9]. In this study, the air gap above the pole face is developed as a function of the stator MMF at the maximum design torque. The result is an asymmetric rotor pole face and a machine that compares favorably to the Uniform air Gap Reluctance Machine (UGRM), specifically having a higher torque density.

This thesis adds to the work of [9] by considering a generator application of the ARM. The ARM sacrifices performance while rotating in one direction to increase the performance in the other direction. The ARM efficiency and torque density surpass those of a standard reluctance machine. This increased torque density is achieved by tapering the rotor pole face, introducing asymmetry into the machine.

The organization of this thesis is as follows. In chapter two, the ARM is described in detail, with a focus on the magnetic analysis of the machine and the taper on the pole face. Chapter three describes the design approach and fitness function used to construct the machine designs. Next, chapter four analyzes a case study for the machine. Finally, chapter five discusses the conclusions of the thesis, and proposes future work.
CHAPTER 2. ASYMMETRIC RELUCTANCE MACHINE

The asymmetric reluctance machine is a design for rotating machinery that aims to improve torque density compared to a UGRM. This chapter entails a detailed geometric description of the machine, focusing on the asymmetric rotor. Next, a magnetic field analysis will be set. Finally, a lumped parameter model is derived. This chapter sets the stage for a case study comparing the ARM to a UGRM.

2.1 Stator Geometry

The stator of the ARM is similar to that of other distributed winding machines. The stator consists of a backiron and teeth. The teeth are placed to allow space for winding the coils. As with most distributed winding machines, the stator is comprised of steel laminations, stacked in the axial direction.

Fig. 2.1 depicts the cross section of a uniform airgap reluctance machine. All definitions related to the stator are identical to those defined in [10]. The variables shown in Fig. 2.1 include the $as$, $bs$, and $cs$, axes, which are the three magnetic axes of the stator windings, and the $q$- and $d$- axes of the rotor. Also, the figure defines the mechanical rotor position, $\theta_{rm}$, the mechanical angular position relative to the rotor measured from the $q$-axis, $\phi_{rm}$, and the mechanical angular position relative to the stator measured from the $as$-axis, $\phi_{sm}$. These position variables are related by
\[
\phi_{sm} = \phi_{rm} + \Theta_{rm}
\]  
(2.1)

Mechanical positions are related to their electrical positions by

\[
\theta_r = \frac{P}{2} \Theta_{rm}
\]  
(2.2)

\[
\phi_s = \frac{P}{2} \phi_{sm}
\]  
(2.3)

\[
\phi_r = \frac{P}{2} \phi_{rm}
\]  
(2.4)

where \( P \) is the number of rotor poles.
2.2 Rotor Geometry

The geometry of the ARM rotor is similar to that of a UGRM. There is a shaft, an inert region, a rotor backiron and rotor teeth. In both the ARM and a UGRM, the teeth’s interaction with the stator MMF produces torque.

The unique feature of the ARM is the asymmetry involved with the rotor poles. Instead of the tooth being shaped to provide a constant air gap across the tooth face, the tooth will be tapered. Figure 2.2 displays the rotor of the ARM and describes pertinent variables.
The rotor tooth is composed of two parts: a base, and a tip. The span of the base and tip are denoted $\theta_{rpt}$ and $\theta_{rpb}$ respectively. The spanning angles are defined as

$$\theta_{rpt} = \frac{2\pi\alpha_d}{P}$$  (2.5)
where $\alpha_d$ is the fraction of the rotor pole tip to the electrical pole, and $\alpha_{rpb}$ is a fraction of the rotor pole base to the rotor pole tip.

The width of the rotor pole base and tip are denoted $w_{rpb}$ and $w_{rpt}$. They can be calculated, respectively, as

\begin{align}
    w_{rpb} &= 2r_{rpb} \sin\left(\frac{\theta_{rpb}}{2}\right) \\
    w_{rpt} &= 2r_{rg} \sin\left(\frac{\theta_{rpt}}{2}\right)
\end{align}

where $r_{rpb}$ is the radius to the end of the rotor tooth base.

In the same figure, $d_{rb}$ denotes the depth of the rotor backiron, $d_{rpb}$ denotes the maximum depth of the rotor tooth base, and $d_{rpt}$ denotes the depth of the rotor tooth tip, which is constant across the rotor pole tip. The depth of the rotor pole base is not constant. This is explored further in Section 2.2.3. The radius of the inert region of the rotor is $r_{i}$, the radius to the stator is defined as $r_{st}$, and the radius to the edge of the rotor backiron is called $r_{rb}$. Also, the airgap function $g(\phi_r)$ will be explored in greater detail later in Section 2.2.2.

### 2.2.1 Asymmetry

The motivation for an asymmetric rotor tooth is an increase in torque density. Figure 2.3 shows the developed diagram for a UGRM and the proposed ARM.
The upper trace in Fig. 2.3 shows the air gap of a UGRM, and the lower trace depicts the air gap of the ARM. In the figure above, $g_q$ and $g_d$ are the air gap lengths at the $q$- and $d$- axis, respectively. Also, $d_d$ denotes the depth of the rotor pole of the UGRM at the $d$-axis. The lower trace of Fig. 2.3 shows the air gap of the ARM. Here, $g_{\text{max}}$ and $g_{\text{min}}$ denote the maximum and minimum air gap length, respectively. The locations of the transition points on the rotor pole are denoted $\phi_{tpr1}$ and $\phi_{tpr2}$.

### 2.2.2 Path Length

The path length of the machine will now be discussed. While, the air gap is the radial distance from the stator steel to the rotor steel, the path length is different from the air gap in that the path length accounts for fringing flux, which is considered later in this section. This means that for all positions except for those corresponding to fringing flux the air gap and the path length are identical.
In [9], the path length was described as a function of the desired stator MMF at the maximum design torque; here, a different approach is utilized. Now, the unique feature of the machine, asymmetry, will be discussed.

A quadratic polynomial defines the path length, and the air gap, at all points above the rotor tooth. This polynomial, called $Q(\phi_r)$ helps to establish the path length $g(\phi_r)$, which spans the entire machine.

The path length above the tooth is described by

$$Q(\phi_r) = q_1 + q_2 \phi_r + q_3 \phi_r^2$$

(2.9)

where $q_1$, $q_2$, and $q_3$ are coefficients. The $Q(\phi_r)$ coefficients are found as follows

$$
\begin{bmatrix}
q_1 \\
q_2 \\
q_3
\end{bmatrix} =
\begin{bmatrix}
1 & \phi_1 & \phi_1^2 \\
1 & \phi_2 & \phi_2^2 \\
1 & \phi_3 & \phi_3^2
\end{bmatrix}^{-1}
\begin{bmatrix}
g_r(\phi_1) \\
g_r(\phi_2) \\
g_r(\phi_3)
\end{bmatrix}
$$

(2.10)

where $g_r(\phi_1)$, $g_r(\phi_2)$, $g_r(\phi_3)$ are the path lengths at the edges and center of the rotor tooth. Specifically, the positions at which these gaps occur are given as

$$
\begin{bmatrix}
\phi_1 \\
\phi_2 \\
\phi_3
\end{bmatrix} =
\begin{bmatrix}
\frac{\pi}{2} - \frac{\theta_{rpt,e}}{2} \\
\frac{\pi}{2} \\
\frac{\pi}{2} + \frac{\theta_{rpt,e}}{2}
\end{bmatrix}
$$

(2.11)

in which $g(\phi_r)$ is the path length at $\phi_r$ and $\theta_{rpt,e}$ is the angle spanned by the rotor tooth tip, electrically. This definition places the middle of the rotor pole on the d-axis, whereas in [9], the rotor pole is placed on the q-axis.
Figure 2.4 below shows a developed diagram of one electrical pole of the path length. In this model, the path length includes the air gap over the region in which a fringing effect of flux in the rotor pole occurs. This fringing effect causes flux to go into the rotor pole tip. A discussion on fringing flux is given in [10].

It is noted that the path length in the fringing regions is longer than the path length over the rotor pole face. In a UGRM, the fringing gaps are the same, but because of the asymmetry, the fringing gaps of the ARM are different. Specifically, the side of the pole with a larger path length at the edge of the pole will have a larger fringing path length. While the path lengths on either side are different lengths, the transition points at which fringing effects occur are the same. These transition points are defined as

\[
\phi_1 = \frac{\pi}{2} + \theta_{rpt,e} + \frac{p d_{rpt}}{2 r_{st}} \tag{2.12}
\]

\[
\phi_3 = \frac{\pi}{2} + \frac{\theta_{rpt,e}}{2} + \frac{p d_{rpt}}{2 r_{st}} \tag{2.13}
\]

where \(d_{rpt}\) is the depth of the rotor pole tip and \(r_{st}\) is the radius to the stator region. The value \(\frac{p d_{rpt}}{2 r_{st}}\) represents the electrical angle over which fringing effects occur. The derivation of the angle over which fringing flux occurs is based on a developed diagram. Thus, this angle is approximate.
While using this method to establish the path length vector it is still possible to create a symmetric, or uniform airgap, reluctance machine. In order to do this, the values of $g_r(\phi_r)$ must be identical. In that case the rotor pole face will be shaped in the same manner as the rotor pole face of a typical machine, which causes both the path length and the air gap to be symmetric.

The path length, $g(\phi_r)$, is more convenient to represent electrically for calculations. The air gap and path length repeat; the values defined below for positions relating from 0 to $\pi$ also can be applied from $\pi$ to $2\pi$. This repeating waveform is shown in Figure 2.3. The path length vector can be split into four discrete sections. First, the gap over the rotor backiron will be discussed. The value over the backiron is straightforward and defined as

$$g(\phi_r)=r_{st}-r_{rb}, \quad \left( \phi_r \leq \frac{\pi}{2} - \frac{\theta_{rpt,e}}{2} - \frac{P d_{rpt}}{2 r_{st}} \right) \cup \left( \frac{\pi}{2} + \frac{\theta_{rpt,e}}{2} + \frac{P d_{rpt}}{2 r_{st}} < \phi_r \leq \pi \right)$$

(2.14)
The next three sections of the path length relate in some way to the rotor teeth; two relate to fringing effects on either side of the tooth and the final section is the air gap above the rotor tooth. As a reminder, the air gap over the rotor tooth is equal to the path length over the rotor tooth. The path length over the fringing regions are as follows

\[ g(\phi_r) = Q\left(\frac{\pi}{2} - \frac{\theta_{rpt,e}}{2}\right) + \frac{\pi}{2} r_{st} \left(\frac{\pi}{2} - \frac{\theta_{rpt,e}}{2} - \phi_r\right), \quad \frac{\pi}{2} - \frac{\theta_{rpt,e}}{2} - \frac{P d_{rpt}}{2} r_{st} < \phi_r < \frac{\pi}{2} - \frac{\theta_{rpt,e}}{2} \]

(2.15)

\[ g(\phi_r) = Q\left(\frac{\pi}{2} + \frac{\theta_{rpt,e}}{2}\right) + \frac{\pi}{2} r_{st} \left(\phi_r - \left(\frac{\pi}{2} + \frac{\theta_{rpt,e}}{2}\right)\right), \quad \frac{\pi}{2} + \frac{\theta_{rpt,e}}{2} < \phi_r < \frac{\pi}{2} + \frac{\theta_{rpt,e}}{2} + \frac{P d_{rpt}}{2} r_{st} \]

(2.16)

where (2.15) describes the path length for the fringing effects at the beginning of the tooth, and (2.16) describes the path length for the fringing effects at the end of the tooth and \( Q(\phi_r) \) is the curve above the tooth defined in (2.9). Fringing is accounted for in the tooth tips, so the fringing spans the depth of the tooth tip. These equations demonstrate why the fringing paths are different from each other; the \( Q(\phi_r) \) value for each are not the same due to asymmetry.

The final section of the path length is above the rotor tooth face. The path length is defined here:

\[ g(\phi_r) = Q(\phi_r), \quad \frac{\pi}{2} - \frac{\theta_{rpt,e}}{2} < \phi_r < \frac{\pi}{2} + \frac{\theta_{rpt,e}}{2} \]

(2.17)
For the given definition above, the path length could become negative, which is undesirable. This issue is addressed in Chapter 3.

2.2.3 Area of Rotor Region

With some geometry established for the rotor, it is appropriate to discuss the area of the rotor of this machine in order to facilitate calculations of the mass. First, the area of the rotor pole base is derived. For this area, the rotor backiron may overlap with the rotor pole base. In addition, the asymmetry causes the calculation of the area of the rotor pole base to be non-trivial. The depth of the rotor pole base is not constant across the entire face.

The asymmetry may cause the rotor pole tip to overlap into the rotor pole base. These areas should not be counted twice. In order to find the true area of the rotor pole base, an integration is necessary. Figure 2.5 shows the area of the rotor pole base explicitly.

![Figure 2.5 Area of Rotor Pole Description](image)
The overlap with the backiron is discussed first. The machine model it defined such that the depth of the rotor pole base begins at the radius of the rotor backiron. This depth needs to be modified to take into account the area that goes below the radius of the rotor backiron. This is done with the variable \( \Delta_h \), shown in Figure 2.5. This height term is defined as

\[
\Delta_h = r_{rb} - h
\]  

(2.18)

where \( h \) is the height in Cartesian coordinates at which \( \Delta_h \) begins and is defined as

\[
h = \cos\left(\frac{\theta_{base}}{2}\right)r_{rb}
\]  

(2.19)

where \( \theta_{base} \) is the angle that rotor pole base spans at the adjusted height, shown in Figure 2.5.

The area of the rotor backiron that overlaps the rotor pole base is defined as

\[
a_{rb,over} = a_{arc} - a_{tri}
\]  

(2.20)

where \( a_{arc} \) is the area of the arc segment that \( \theta_{base} \) spans, defined as

\[
a_{arc} = \frac{1}{2} \theta_{base} r_{rb}^2
\]  

(2.21)

and \( a_{tri} \) is the area of the triangle that has to be subtracted, defined as

\[
a_{tri} = \frac{1}{2} w_{rp} h
\]  

(2.22)

Now, the overlap due to the rotor pole tip will be considered. The polar coordinates of each point across the rotor pole tip is known from Section 2.2.2. These polar coordinates, called the depth due to the rotor pole tip, \( d_t(\phi_r) \), can be defined as

\[
d_t(\phi_r) = r_r - g(\phi_r)
\]  

(2.23)
A straightforward integration to find the area is not yet possible. First, the depth due to
the rotor pole must be converted in Cartesian coordinates.

The Cartesian coordinates of point $i$ on the rotor polar face, $(x_i, d_i(x_i))$, are found
by employing a change of variables from the polar to the Cartesian system of $d_i (\phi_r)$. Now,
the depth due to the rotor pole tip must be adjusted by the height at which the rotor pole
base occurs. This results in

$$d_{rpb,adj}(x) = d_i(x) - d_{rpt} - r_b + \Delta_h$$  \hspace{1cm} (2.24)

Where $d_{rpb,adj}$ is the adjusted rotor pole base depth to be integrated.

Next, the area of integration, $a_{rect}$, is calculated as

$$a_{rect} = \int_{x_{b1}}^{x_{b2}} d_{rpb,adj}(x) dx$$  \hspace{1cm} (2.25)

where $x_{b1}$ and $x_{b2}$ are the $x$-coordinates corresponding with the width of the rotor pole
base. At last, the area of the rotor pole base can be defined as

$$a_{rpb} = a_{rect} - a_{rpb,over}$$  \hspace{1cm} (2.26)

The area of the rotor pole tip is straightforward and can be described as

$$a_{rpt} = w_d \cdot d_{rpt}$$  \hspace{1cm} (2.27)

The total area of a single rotor pole is given as

$$a_{rp} = a_{rpb} + a_{rpt}$$  \hspace{1cm} (2.28)
In addition, the area of the rotor backiron is defined as

$$a_{rb} = \pi (r_{rb}^2 - r_{ri}^2).$$  \hspace{1cm} (2.29)

Finally, the total area of the rotor is given as

$$a_r = a_{rb} + P(a_{rpb} + a_{rpt}).$$  \hspace{1cm} (2.30)

### 2.3 Field Analysis

In order to establish a design model it is helpful to derive an expression for the radial flux density. In the following analysis, the rotor is assumed to be at a position $\theta_r$. For this design model, the magnetomotive force (MMF) drops across steel are neglected, and the flux density in the steel will be constrained to ensure the validity of this assumption. Under these assumptions, and no rotor current, an application of Ampere’s law around the path given in Figure 2.2 leads to a conclusion that the air gap MMF drop is equal to the stator MMF. Thus,

$$F_g(\phi_{sm}) = F_s(\phi_{sm})$$  \hspace{1cm} (2.31)

where $F_g$ is the MMF drop in the air gap, and $F_s$ is the MMF in the stator.

Next, it is convenient to describe the stator MMF. The $a$-, $b$-, and $c$- phase currents into the machine are denoted as $i_{as}$, $i_{bs}$, and $i_{cs}$, or $\mathbf{i}_{abcs} = [i_{as} \ i_{bs} \ i_{cs}]^T$. The currents are wye connected, and can be expressed as

$$\mathbf{i}_{abcs} = \sqrt{2} I_s \begin{bmatrix} \cos(\theta_{esi}) \\ \cos(\theta_{esi} - \frac{2\pi}{3}) \\ \cos(\theta_{esi} + \frac{2\pi}{3}) \end{bmatrix}$$  \hspace{1cm} (2.32)
where \( I_s \) is the rms amplitude, and \( \theta_{esi} \) is the angular position of the stator current. This position can be related to the rotor position by

\[
\theta_{esi} = \theta_r + \phi_l
\]

(2.33)

where \( \phi_l \) is the current phase relative to rotor position.

The continuous description of the conductor density is given in [11] as

\[
\mathbf{n}_{abc} (\phi_{sm}) = N_p \begin{bmatrix} \sin \left( \frac{P\phi_{sm}}{2} \right) \\ \sin \left( \frac{P\phi_{sm} - \frac{2\pi}{3}}{2} \right) \\ \sin \left( \frac{P\phi_{sm} + \frac{2\pi}{3}}{2} \right) \end{bmatrix} - \alpha_3 N_p \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \frac{3P\phi_{sm}}{2} \]

(2.34)

where \( N_p \) is the maximum fundamental component of the conductor density expressed in conductors per radian and \( \alpha_3 \) is the relative amplitude of the third-harmonic component relative to the fundamental component. From (2.34) the \( a-, b-, \) and \( c-, \) phase winding functions of the machine can be expressed as

\[
\mathbf{w}_{abc} (\phi_{sm}) = \frac{2N_p}{P} \begin{bmatrix} \cos \left( \frac{P\phi_{sm}}{2} \right) \\ \cos \left( \frac{P\phi_{sm} - \frac{2\pi}{3}}{2} \right) \\ \cos \left( \frac{P\phi_{sm} + \frac{2\pi}{3}}{2} \right) \end{bmatrix} - \alpha_3 \frac{2N_p}{P} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \frac{3P\phi_{sm}}{2} \]

(2.35)

The stator MMF may be expressed as

\[
F_s (\phi_{sm}) = w_{as} (\phi_{sm}) i_a + w_{bs} (\phi_{sm}) i_b + w_{cs} (\phi_{sm}) i_c
\]

(2.36)

Simplifying (2.36) leads to
\[ F_s(\phi_{sm}) = 3\sqrt{2} N_p I_s \cos \left( \frac{P}{2} \phi_{sm} - \frac{P}{2} \theta_{sm} \right) \]  

(2.37)

With the stator MMF established, an expression for the MMF drop across the air gap can be established. For the rest of Section 2.3, \( \theta_r \) is taken as 0. By the definition of MMF drop,

\[ F_g(\phi_{sm}) = \int_{r_g - g_{eff}(\phi_{sm})}^{r_g} H(r, \phi_{sm}) \, dr \]  

(2.38)

where \( H(r, \phi_{sm}) \) denotes the radial component of the field intensity at radius \( r \) and position \( \phi_{sm} \), and \( g_{eff}^m(\phi_{sm}) \) is the effective path length as a function of position measured from the stator mechanically. The effective path length is related to the physical path length measured mechanically from the rotor, \( g^m(\phi_r) \), but incorporates the effects as stator slots into the analysis. Following the approach of [10], and remembering that \( \theta_r \) is set to 0, the mechanical effective air gap can be expressed as

\[ g_{eff}^m(\phi_{sm}) = c_s g^m(\phi_{sm}) \]  

(2.39)

where \( c_s \) is the stator Carter’s coefficient and the superscript ‘m’ denotes the mechanical representation of the path length vector. An in-depth derivation of Carter’s coefficient is given in [10].

The flux density \( B \) and field intensity \( H \) in the radial direction vary with radius. Considering a radial field, for radii \( r_g - g(\phi_{sm}) \leq r \leq r_s \), Gauss’ law yields

\[ B(r, \phi_{sm}) = \frac{r}{r} B_{rs}(\phi_{sm}) \]  

(2.40)

where \( B_{rs} \) is the flux density stator radius \( r_s \). In the air gap, the field intensity and flux density are related by
From (2.38) - (2.41), the MMF drop in the air gap may be expressed as

$$F_g(\phi_{sm}) = R_g(\phi_{sm})B_{rs}(\phi_{sm})$$  \hspace{1cm} (2.42)

where $R_g$ is a reluctance density given by

$$R_g(\phi_{sm}) = \frac{r_s}{\mu_0} \ln \left( 1 + \frac{g_m^{\text{eff}}(\phi_{sm})}{r_s - g_m^{\text{eff}}(\phi_{sm})} \right)$$ \hspace{1cm} (2.43)

Now, an expression for the flux density is found. Specifically,

$$B_{rs}(\phi_{sm}) = \frac{F_r(\phi_{sm})}{R_g(\phi_{sm})}$$ \hspace{1cm} (2.44)

The result from (2.44) establishes the flux density throughout the machine. Knowing the flux density allows core loss calculations and constraints on flux to prevent saturation.

The flux density in the rotor structure will now be considered. For convenience, the transition points are defined in electrical coordinates. It is noted that they are converted to mechanical coordinates for the flux calculations.

The rotor consists of backiron, tooth base, and tooth tip regions. Each component needs an expression for flux density to ensure the flux density constraints are not violated. For this study, fringing flux is considered for the rotor tooth tip, but nowhere else. In other words the rotor tooth base is not considered to have any fringing flux involved. Figure 2.6 shows a developed diagram for the flux density as a function of position measured relative to the rotor.
In Figure 2.6, the positions $\phi_1$-$\phi_5$ are defined in Chapter 2. Positions, $\phi_6$ and $\phi_7$, represent the transition points where the rotor pole base begins and ends, respectively.

They are defined as

$$\phi_6 = \frac{\pi}{2} - \frac{\theta_{rpb,e}}{2}$$

(2.45)

and

$$\phi_7 = \frac{\pi}{2} + \frac{\theta_{rpb,e}}{2}$$

(2.46)

There are multiple regions of flux across the rotor pole face that are considered. These regions are demonstrated in Figure 2.7.
First, there is $\Phi_{rpfr}$, the flux in the rotor pole tip on the right side, or the right fringing flux. Next, $\Phi_{rprr}$ is the flux in the rotor pole tip radial face on the right. The flux $\Phi_{rpri}$ is the sum of the flux on the right side. Similarly, on the left side, $\Phi_{rpfl}$ and $\Phi_{rpfl}$ are the left fringing flux and the left radial flux respectively. The flux $\Phi_{rp2}$ is the sum of the flux on
the left side. The variable $\Phi_{rpt3}$ is the flux through the rotor pole radial face directly above
the rotor pole base. These fluxes are defined as follows:

$$\Phi_{rptr} = \frac{\mathcal{L}_s}{\mathcal{P}_r} \int B_{rs} (\phi_{sm}) d\phi_{sm}$$  \hspace{1cm} (2.47)

$$\Phi_{rptr} = \frac{2}{\mathcal{P}_r} \int B_{rs} (\phi_{sm}) d\phi_{sm}$$  \hspace{1cm} (2.48)

$$\Phi_{rptlf} = \frac{2}{\mathcal{P}_r} \int B_{rs} (\phi_{sm}) d\phi_{sm}$$  \hspace{1cm} (2.49)

$$\Phi_{rplt} = \frac{2}{\mathcal{P}_r} \int B_{rs} (\phi_{sm}) d\phi_{sm}$$  \hspace{1cm} (2.50)

$$\Phi_{rptl3} = \frac{2}{\mathcal{P}_r} \int B_{rs} (\phi_{sm}) d\phi_{sm}$$  \hspace{1cm} (2.51)

$$\Phi_{rpt1} = \Phi_{rptr} + \Phi_{rptrf}$$  \hspace{1cm} (2.52)

$$\Phi_{rpt2} = \Phi_{rplt} + \Phi_{rptlf}$$  \hspace{1cm} (2.53)

and

$$\Phi_{rpb} = \Phi_{rpt1} + \Phi_{rpt2} + \Phi_{rpt3}$$  \hspace{1cm} (2.54)

It should be noted that, due to the taper, the distance for the fringing flux to travel on the
left side of the pole will not be the same as the distance for the fringing flux on the right.
Whether the magnitude of the distance is larger for the right or left side is determined by
the application of the machine. $P_{out}$

Now, the flux densities are given as
Finally, using symmetry and Gauss’ law, the flux in the rotor backiron may be expressed as

\[ B_{rb} = \frac{1}{2} \frac{w_{rpb}}{d_{rb}} B_{rpb} \quad (2.60) \]

This ends the discussion on flux in the rotor region. Equations for flux density were found in the rotor tooth tip, the rotor tooth, and the rotor backiron.

Next, the flux density in the stator region is considered. An in-depth radial field analysis of an identical stator shell along with derivations for the flux density in the stator backiron and teeth are unmodified from those derived in [10].

2.4 Lumped Parameter Model

This section establishes the lumped parameter model for the ARM. An expression for flux linkages, mutual inductance, and torque is presented. This section makes extensive use of the \( q \)- and \( d \)-axis variables.
Before inspecting either the stator or rotor geometry, it is helpful to establish the \( qd0 \) variables in the rotor reference frame. A change of variables transforms the 3-phase variables of stationary circuit elements to the rotor reference frame. This change of variables is called Park’s transformation and is expressed as

\[
f^r_{qd0s} = K^r T_{abcs}
\]

where \( f \) may be a voltage \( v \), flux linkage \( \lambda \), or a current \( i \). The quantity \( f_{abcs} \) is a vector of machine variables of the form

\[
f_{abcs} = [f_{as}, f_{bs}, f_{cs}]^T
\]

and \( f^r_{qd0s} \) is the corresponding vector of \( qd0 \) quantities which has the form

\[
f^r_{qd0s} = [f^r_{qs}, f^r_{ds}, f^r_{os}]^T
\]

where

\[
K^r_T(\theta_r) = \frac{3}{2} \begin{bmatrix}
\cos(\theta_r) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\
\sin(\theta_r) & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\]
In (2.64), $\theta_r$ is the electrical rotor position.

The $q$- and $d$- axis voltage equations in the rotor reference frame are given by [12] as

\[
v'_q = R_s i'_q + \omega_r \lambda'_{qs} + p \lambda'_{qs}
\]

\[
v'_d = R_s i'_d - \omega_c \lambda'_{qs} + p \lambda'_{ds}
\]

In (2.65) and (2.66), $R_s$ is the stator resistance, $\omega_r$ is the electrical rotor speed, and $p$ denotes $\frac{d}{dt}$. In addition, $\lambda'_{qs}$ and $\lambda'_{ds}$ denote the $q$- and $d$- axis flux linkages in the rotor reference frame, respectively.

Similar to [11], the flux linkage is made up of leakage and magnetizing components. Specifically,

\[
\lambda_{qd0} = \lambda_{qd0,m} + \lambda_{qd0,l}
\]

where $\lambda_{qd0,m}$ is the magnetizing flux linkage, and $\lambda_{qd0,l}$ is the leakage component. It will be found that the magnetizing component is the dominant term. The leakage flux linkage may be expressed as

\[
\lambda_{qd0,l} = L_{ls} i_{qs}.
\]

In (2.68), the leakage inductance, $L_{ls}$, is calculated as it is in [11].

In order to establish the magnetizing flux linkage, it is helpful to first describe the flux linkage in $a$-, $b$-, and $c$- phase variables. From [10], the magnetizing flux linkage may be expressed as

\[
\lambda_{a_b_c, m} = r_s \int_{\phi_m}^{2\pi} B_{rs} (\phi_m) w_{a_b_c} (\phi_m) d\phi_m
\]
Now, the flux density at the stator can be described as

\[ B_{es}(\phi_{sm}) = \frac{\mu_0 w_{abc}^T i_{abc}}{g_{eff}(\phi_{sm})} \]  

(2.70)

Next, equations (2.69) and (2.70) are transformed into \( q \)- and \( d \)-axis variables, yielding

\[ \lambda_{q0m} = \frac{2}{P} S \left[ \cos(\phi - \theta) \begin{array}{c} \cos(3\phi - \theta) \\ \sin(3\phi - \theta) \end{array} \right] \]  

(2.72)

Simplifying (2.71), yields

\[ \lambda_{q0m} = \frac{3}{2} r_i \int_{0}^{\frac{2\pi}{P}} \frac{w_{q0s} w_{q0s}^T}{g_{eff}(\phi_{sm})} d\phi_{sm} \]  

(2.73)

Thus,

\[ L_{q0m} = \frac{3}{2} r_i \int_{0}^{\frac{2\pi}{P}} \frac{w_{q0s} w_{q0s}^T}{g_{eff}(\phi_{sm})} d\phi_{sm} \]  

(2.74)

Finally, The ARM is transformed in the rotor reference frame; so, from [12], the torque can be described as

\[ T_r = \frac{3}{2} P \frac{1}{2} \left( \lambda_{qs}^r i_{qs}^r - \lambda_{qs}^r i_{qs}^r \right) \]  

(2.75)
CHAPTER 3. DESIGN APPROACH

This research employs a multi-objective optimization code to design the machine. For this study, the optimization is carried out using a genetic algorithm, and the objectives to minimize are mass and loss. Specifically, the optimization code used is GOSET, a MATLAB toolbox for single-objective and multi-objective genetic optimization [13]. To this end, first, design specifications are defined. Next, the design space is established, which defines the independent variables of a machine design. Finally, a fitness function which quantifies the performance of the design is set forth. This chapter details the design specifications, design space, and fitness function.

3.1 Design Specifications

The design specifications inform the algorithm of the desired machine capabilities. The specifications establish values such as the required output power, the speed of the machine and the maximum allowed current density. The specifications can be broken down into groups: operating points, mechanical limits, material limits, assumptions, semiconductor data and winding configurations. In this section, the optimization is posed in general; however, specific values are shown in order to simultaneously provide an example and document the upcoming case study in the next chapter. Table 3.1 gives the design specifications.
Table 3.1 Design Specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Description</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{out}$</td>
<td>Required Electrical Output Power</td>
<td>(W)</td>
<td>1e4</td>
</tr>
<tr>
<td>$\omega_{rm}$</td>
<td>Speed</td>
<td>(Rad/s)</td>
<td>60$\pi$</td>
</tr>
<tr>
<td>$T_{e rq}$</td>
<td>Required Torque</td>
<td>(Nm)</td>
<td>-1e4/(60$\pi$)</td>
</tr>
<tr>
<td>$v_{dc}$</td>
<td>DC Input Voltage</td>
<td>(V)</td>
<td>700</td>
</tr>
<tr>
<td>$r_{ss,lim}$</td>
<td>Outer Radius Limit</td>
<td>(m)</td>
<td>1</td>
</tr>
<tr>
<td>$l_{lim}$</td>
<td>Length Limit</td>
<td>(m)</td>
<td>1e10</td>
</tr>
<tr>
<td>$w_{clr}$</td>
<td>Winding Clearance to Top of Slot</td>
<td>(m)</td>
<td>0</td>
</tr>
<tr>
<td>$w_{sts}$</td>
<td>Width of Slot Liner for Spacing</td>
<td>(m)</td>
<td>.25e-3</td>
</tr>
<tr>
<td>$m_{lim}$</td>
<td>Mass Limit</td>
<td>(kg)</td>
<td>400</td>
</tr>
<tr>
<td>$P_{lt,lim}$</td>
<td>Loss Limit</td>
<td>(W)</td>
<td>1e6</td>
</tr>
<tr>
<td>$v_{tp, mx}$</td>
<td>Maximum Allowed Tip Speed</td>
<td>(m/s)</td>
<td>200</td>
</tr>
<tr>
<td>$a_{tar}$</td>
<td>Maximum Tooth Aspect Ratio</td>
<td>-</td>
<td>10</td>
</tr>
<tr>
<td>$a_{so}$</td>
<td>Slot Opening Factor</td>
<td>-</td>
<td>1.25</td>
</tr>
<tr>
<td>$g_{min}$</td>
<td>Minimum Air Gap</td>
<td>(m)</td>
<td>5e-4</td>
</tr>
<tr>
<td>$I_{allowed}$</td>
<td>Maximum Allowed Current Density</td>
<td>(A/m$^2$)</td>
<td>20e6</td>
</tr>
<tr>
<td>$f_f$</td>
<td>Safety Factor for Flux Level</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>$R$</td>
<td>Rotor Steel</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>$S$</td>
<td>Stator Steel</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>$T_{c, min}$</td>
<td>Minimum Curie Temperature</td>
<td>(C)</td>
<td>349</td>
</tr>
<tr>
<td>$C$</td>
<td>Conductor Type</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>$T$</td>
<td>Assumed Winding Temperature</td>
<td>(C)</td>
<td>25</td>
</tr>
<tr>
<td>$k_{pf}$</td>
<td>Packing Factor</td>
<td>-</td>
<td>.5</td>
</tr>
<tr>
<td>$l_{fs}$</td>
<td>Length of Front Shaft</td>
<td>(m)</td>
<td>0</td>
</tr>
<tr>
<td>$l_{bs}$</td>
<td>Length of Back Shaft</td>
<td>(m)</td>
<td>0</td>
</tr>
<tr>
<td>$t_{sl}$</td>
<td>Thickness of Slot Liner</td>
<td>(m)</td>
<td>2.5e-4</td>
</tr>
<tr>
<td>$r_{rs}$</td>
<td>Shaft Radius</td>
<td>(m)</td>
<td>.01</td>
</tr>
<tr>
<td>$l_{eo}$</td>
<td>End Winding Offset</td>
<td>(m)</td>
<td>.01</td>
</tr>
<tr>
<td>$\rho_{pt}$</td>
<td>Density of Potting Material</td>
<td>(kg/m$^3$)</td>
<td>0</td>
</tr>
<tr>
<td>$i_{rf}$</td>
<td>Inert Rotor Fraction</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>$f_{ratio}$</td>
<td>Ratio of Switching Frequency to Maximum Fundamental</td>
<td>-</td>
<td>20</td>
</tr>
<tr>
<td>$v_{dc, uf}$</td>
<td>DC Voltage Utilization Factor for Flux Weakening Control</td>
<td>(V)</td>
<td>.975</td>
</tr>
<tr>
<td>$v_{dc, tf}$</td>
<td>DC Voltage Test Factor</td>
<td>(V)</td>
<td>.99</td>
</tr>
<tr>
<td>$v_{f s}$</td>
<td>Forward Semiconductor Drop</td>
<td>(V)</td>
<td>1.5</td>
</tr>
<tr>
<td>$E_{sw o}$</td>
<td>Sum of Switching Loss Energies</td>
<td>(J)</td>
<td>150e-3</td>
</tr>
<tr>
<td>$v_{sw o}$</td>
<td>Nominal Voltage for Switching Loss Calculations</td>
<td>(V)</td>
<td>600</td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
<td>Unit</td>
<td>Value</td>
</tr>
<tr>
<td>----------</td>
<td>--------------------------------------------------</td>
<td>------</td>
<td>-------</td>
</tr>
<tr>
<td>$i_{sw0}$</td>
<td>Nominal Current for Switching Loss Calculations</td>
<td>(A)</td>
<td>600</td>
</tr>
<tr>
<td>$i_{pk,lim}$</td>
<td>Peak Current Limit</td>
<td>(A)</td>
<td>1e3</td>
</tr>
<tr>
<td>$n_{spp}$</td>
<td>Slots per Pole per Phase</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>$n_{spc}$</td>
<td>Number of Strands per Conductor</td>
<td>-</td>
<td>20</td>
</tr>
<tr>
<td>$f_{us}$</td>
<td>Forbid Upper Part of Slot</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>$PP_{mn}$</td>
<td>Minimum Number of Pole Pairs</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>$PP_{mx}$</td>
<td>Maximum Number of Pole Pairs</td>
<td>-</td>
<td>20</td>
</tr>
<tr>
<td>$J$</td>
<td>Number of Rotor Positions</td>
<td>-</td>
<td>20</td>
</tr>
<tr>
<td>$i_{test}$</td>
<td>Test Current</td>
<td>(A)</td>
<td>1</td>
</tr>
<tr>
<td>$s_d$</td>
<td>Number of Significant Digits</td>
<td>-</td>
<td>4</td>
</tr>
<tr>
<td>$r_{es}$</td>
<td>Length of Position Arrays (for reporting)</td>
<td>-</td>
<td>720</td>
</tr>
</tbody>
</table>

It will be convenient for the upcoming discussion on the fitness function to describe the design specifications with a structure, $D$, which is defined as a vector composed of each variable in Table 3.1.

As Table 3.1 shows, the design specification designates the specified material type. It is possible to allow the material type to be a part of the design space; however, for this study the material was chosen a-priori.

For this study, the rotor and stator material types, $R$, and $S$ are chosen to be ‘1’, which corresponds with M19 steel. Table 3.2 below gives the parameters on M19 steel.

<table>
<thead>
<tr>
<th>Steel Type</th>
<th>Mass Density</th>
<th>Flux Density to Avoid Saturation</th>
<th>$K_m$</th>
<th>$K_h$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>(kg/m³)</td>
<td>(T)</td>
<td>(J/m³)</td>
<td>(Js/m³)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M19</td>
<td>7402</td>
<td>1.3922</td>
<td>50.691</td>
<td>0.027513</td>
<td>1.3375</td>
<td>1.8167</td>
</tr>
</tbody>
</table>
The last four columns of Table 3.2 deal with the loss parameters of M19 steel. Specifically, they are parameters in the Modified Steinmetz Equation describing loss density as derived in [10]. The loss will be discussed in greater detail later.

The flux density limit is based on the point where the absolute relative permeability goes below 1000. This limit is to ensure that approximating the flux density versus field intensity relationship linearly is reasonable. Operating in the linear region simplifies calculations considerably, and is a fair assumption while enforcing the limit described above.

The design specification also chooses the conductor type. The conductor type value, $C$, of ‘1’ corresponds with copper. The parameters of copper are given in Table 3.3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$d$</th>
<th>$\sigma_0$</th>
<th>$J_{\text{lim}}$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>Conductor Type</td>
<td>Conductivity at $t_0$</td>
<td>Recommended Maximum Current Density</td>
<td>Mass Density</td>
</tr>
<tr>
<td>Unit</td>
<td>-</td>
<td>1/(Ohm-m)</td>
<td>(A/m$^2$)</td>
<td>(kg/m$^3$)</td>
</tr>
<tr>
<td>Value</td>
<td>Copper</td>
<td>5.59e7</td>
<td>7.6025e6</td>
<td>8890</td>
</tr>
</tbody>
</table>

### 3.2 Design Space

The design space consists of independent variables describing the machine, which are chosen from a range of values. Table 3.4 lists the design space.
Table 3.4 Design Space

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Unit</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Gene Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole Pairs</td>
<td>PP</td>
<td>-</td>
<td>1</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>Depth of Inert Region</td>
<td>$d_{ir}$</td>
<td>(cm)</td>
<td>.1</td>
<td>50</td>
<td>3</td>
</tr>
<tr>
<td>Depth of Rotor Backiron</td>
<td>$d_{rb}$</td>
<td>(cm)</td>
<td>.1</td>
<td>100</td>
<td>3</td>
</tr>
<tr>
<td>Depth of Rotor Pole Base</td>
<td>$d_{rpb}$</td>
<td>(cm)</td>
<td>.1</td>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>Depth of Rotor Pole Tip</td>
<td>$d_{rpt}$</td>
<td>(cm)</td>
<td>.1</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Depth of Tooth Base</td>
<td>$d_{tb}$</td>
<td>(cm)</td>
<td>.1</td>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>Stator Tooth Fraction</td>
<td>$\alpha_t$</td>
<td>-</td>
<td>.1</td>
<td>.9</td>
<td>2</td>
</tr>
<tr>
<td>Depth of Stator Backiron</td>
<td>$d_{sb}$</td>
<td>(cm)</td>
<td>.1</td>
<td>100</td>
<td>3</td>
</tr>
<tr>
<td>Active Length</td>
<td>$l$</td>
<td>(cm)</td>
<td>1</td>
<td>200</td>
<td>3</td>
</tr>
<tr>
<td>Fraction of rotor pole tip</td>
<td>$\alpha_d$</td>
<td>-</td>
<td>1e-3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Fraction of rotor pole base</td>
<td>$\alpha_{rpb}$</td>
<td>-</td>
<td>1e-3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Peak Phase Conductor Density</td>
<td>$N_{s1,star}$</td>
<td>(cond/ rad)</td>
<td>1</td>
<td>10,000</td>
<td>3</td>
</tr>
<tr>
<td>-(Q-axis Current)</td>
<td>$-\left(l_q\right)$</td>
<td>(A)</td>
<td>.01</td>
<td>500</td>
<td>3</td>
</tr>
<tr>
<td>D- axis Current</td>
<td>$l_d$</td>
<td>(A)</td>
<td>.01</td>
<td>500</td>
<td>3</td>
</tr>
<tr>
<td>Air Gap 1</td>
<td>$g_r(\phi_1)$</td>
<td>(mm)</td>
<td>.5</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Air Gap 2</td>
<td>$g_r(\phi_2)$</td>
<td>(mm)</td>
<td>.5</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Air Gap 3</td>
<td>$g_r(\phi_3)$</td>
<td>(mm)</td>
<td>.5</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

Using this design space, every other quantity of the machine can be calculated. It is noted that some independent variables are not included in the design space, because they are already defined in the design specifications, and are fixed prior to the initiation of the design process. The design space structure $\theta$, is defined as a vector comprising the variables described in Table 3.4.

The gene type is necessary because a genetic algorithm is used for this study. The three types of genes describe how the genes can vary; the types of genes are integer, linear, and exponential. In Table 3.4, gene type 1 corresponds to integer, type 2 corresponds to linear, and type 3 corresponds to exponential. An in-depth discussion of gene types is given in [10].
3.3 Fitness Function

After establishing the design space and design specifications, the creation of the fitness function must be specified. The fitness function evaluates the capability of the given machine design from the design space by evaluating two quantities: mass of the machine, and electric loss of the machine. In addition to finding the mass and loss, the fitness function evaluates the specific machine design against multiple constraints, which help to ensure the design feasibility.

Herein, the metrics of interest are mass and loss. Calculating the mass of the machine is fairly straightforward. The density of the material is known, and the volume is found within the fitness function. These can be related to the mass by

\[ m = \rho v \]  

(3.1)

The total mass is found as follows,

\[ M_t = m_{ss} + m_{rs} + m_{cd} + m_{ri} + m_{pt} + m_{rb} + Pm_{rp} \]  

(3.2)

In (3.2), \( m_{ss} \) is the mass of the stator shell, \( m_{rs} \) is the mass of the rotor shell, and \( m_{cd} \) is the total mass of the conductors. Also, \( m_{ri} \) is the mass of the inert region of the rotor, \( m_{pt} \) is the mass of the potting in the slots, \( m_{rb} \) is the mass of the rotor backiron, and \( Pm_{rp} \) is the total mass of the rotor poles, with \( m_{rp} \) representing the mass of a single rotor pole corresponding to the area described in Section 2.2.3.

The loss is calculated as follows

\[ P_t = P_e + P_{scd} + P_{sw} + P_c + P_p \]  

(3.3)
where $P_l$ is the loss of the machine, and the right hand side encompasses all of the components that make up the total loss. In particular, $P_r$ is the resistive loss, $P_{scd}$ is the semiconductor conduction loss, and $P_{ssw}$ is the semiconductor switching loss. In addition, $P_c$ is the core loss and $P_p$ is the proximity effect loss. All of the following equations for loss are derived in [10].

The resistive losses are defined as

$$P_r = 3R_sI_s^2$$

(3.4)

where $R_s$ is the stator resistance of the system.

In (3.3), $P_{scd}$ is given as

$$P_{scd} = \frac{6\sqrt{2}}{\pi} I_s N_{pw} V_{fs}$$

(3.5)

where $I_s$ is the stator current amplitude, $N_{pw}$ is the number of parallel windings, and $V_{fs}$ is the forward semiconductor drop. $P_{ssw}$ is defined as

$$P_{ssw} = \frac{6\sqrt{2}}{\pi} I_s V_{dc} N_{pw} E_{sw0} f_{sw0}$$

(3.6)

where $V_{dc}$ is the input dc voltage, $E_{sw0}$ is the sum of the switching loss energies, $f_{sw}$ is the switching frequency, $V_{sw0}$ is the nominal voltage of the switch, and $I_{sw0}$ is the nominal current of the switch.

The core losses are defined as

$$P_c = P_{ct} + P_{cb}$$

(3.7)
where $P_{ct}$ is the core loss in the teeth, and $P_{cb}$ is the core loss in the backiron. These losses are derived in [10], but they are found from an application of the Modified Steinmetz equation and the material loss parameters presented in Table 3.2.

The proximity effect loss is as follows

$$P_p = 3R_{prox}I_s^2$$

where $R_{prox}$ is the proximity effect resistance as set forth in [10].

Now, the design constraints are discussed. The constraints on the machine are incorporated into the final fitness value. For example, a constraint could be to ensure that the output power is equal to or above the desired value from the specifications, or that the flux entering a tooth is less than the flux limit for that type of material. Two comparator equations are used to enforce these constraints and are given below

$$
\text{lte}(x, x_{mx}) = \begin{cases} 
1, & x \leq x_{mx} \\
\frac{1}{1 + x - x_{mx}}, & x > x_{mx}
\end{cases}
$$

(3.9)

$$
\text{gte}(x, x_{mn}) = \begin{cases} 
1, & x \leq x_{mn} \\
\frac{1}{1 + x - x_{mn}}, & x > x_{mn}
\end{cases}
$$

(3.10)

In (3.9), above, $x_{mx}$ is the maximum allowed value, and $x$ is the input. Likewise, in (3.10), $x_{mn}$ is the minimum allowed value. Equation (3.9) returns a fitness value of ‘1’ if the input is less than or equal to the constraint. Equation (3.10) returns a fitness value of ‘1’ if the input is greater than or equal to the constraint. If the inputs do not meet the constraints, they are still assigned values. The constraint equations work so that if one input...
is closer to meeting a constraint than a second input, the first input’s value is closer to ‘1,’
giving it a higher fitness. This feature may help in achieving viable designs in the early
stages of the optimization [10].

There are many constraints imposed on the design to ensure geometric validity and
overall feasibility. First, the stator backiron depth is a dependent variable, so the depth must
be calculated and ensured to be greater than zero. This follows as

\[ c_i = \text{gte}(d_{sb}, 0). \]  

\[ (3.11) \]

Figure 3.1 Rotor Pole Base Constraint Motivation

The width of the rotor pole base cannot exceed \( \frac{\pi d_{rb}}{p} \) at the radius at the rotor pole
tip; however, because the pole drops down vertically instead of radially, it is possible for
the rotor pole base to be wider than the entire rotor backiron region. The width of the
backiron is not to be confused with the depth of the backiron; the width is shown in Figure
3.1 above and defined as
where $\theta_{\text{width}}$ is the maximum angle that could be spanned by a rotor pole tip if $\alpha_d$ is set to ‘1’, defined as

$$\theta_{\text{width}} = \frac{2\pi}{P}$$  \hspace{1cm} (3.13)

The optimization could trend to a rotor pole base with $\alpha_d$ and $\alpha_{rp_b}$ near ‘1’. This forces the rotor pole base to be wide.

This situation causes poles to intersect. This is undesirable because the fitness function does not incorporate this pole intersection in the field analysis. Thus,

$$c_2 = \text{lte} \left( w_{rp_b}, 2r_{rb} \sin \left( \frac{\theta_{\text{width}}}{2} \right) \right).$$  \hspace{1cm} (3.14)

In addition, a similar circumstance could arrive where the rotor pole tips have a large span, and to ensure the rotor pole tip fringing flux lines do not intersect, it is necessary to enforce

$$c_3 = \text{lte} \left( \frac{\theta_{rpt}}{2} + \frac{d_{rpt}}{r_{st}} \frac{\pi}{P} - \frac{\theta_{rpt}}{2} - \frac{d_{rpt}}{r_{st}} \right)$$  \hspace{1cm} (3.15)

where $\theta_{rpt}$ is the angle spanned by the entire rotor pole tip, $P$ is the total number of poles, and $\frac{d_{rpt}}{r_{st}}$ represents the angle added to the pole to include the fringing effect. It is noted that
the likelihood of these poles being this close to each other is small, but as a precaution the limit is included.

The winding constraints ensure the area of the conductor is greater than zero and the width of a conductor will fit inside the stator slot. They are computed as follows:

\[ c_4 = \text{lte}(a_c, 1e^{-9}) \]  (3.16)

\[ c_5 = \text{lte}(d_{ss}, \alpha_{so}, w_{so}). \]  (3.17)

In (3.16), \(a_c\) is the conductor area. In (3.17), \(d_{ss}\) is the diameter of a single strand, \(\alpha_{so}\) is the slot opening factor, which is set to 1.25, and \(w_{so}\) is the width of slot opening.

The stator tooth is constrained to be below a certain aspect ratio. The constraint required is

\[ c_6 = \text{lte}\left(\frac{d_{st}}{w_{tb}}, \alpha_{tar}\right) \]  (3.18)

where \(d_{st}\) is the depth of the stator tooth, \(w_{tb}\) is the width of the stator tooth base, and \(\alpha_{tar}\) is the tooth aspect ratio.

A few current constraints are also in place. First, the current density is limited to some value set in the design specifications. This constraint yields

\[ c_7 = \text{lte}\left(\max\left(\frac{I_s}{a_c}, J_{\text{allowed}}\right)\right) \]  (3.19)

where \(I_s\) is the stator current amplitude and \(J_{\text{allowed}}\) is the allowed current density. The stator current amplitude, \(I_s\), is found directly from \(I_q\) and \(I_d\), the \(q\)- and \(d\)-axis currents from the design space, as follows
\[ I_s = \frac{I_q^2 + I_d^2}{\sqrt{2}} \]  
\[ (3.20) \]

Similarly, there is a constraint on the peak current, as follows

\[ c_8 = \text{lte} \left( I_{pk}, I_{pk\text{lim}} \right) \] 
\[ (3.21) \]

where \( I_{pk} \) is the peak current in the machine and \( I_{pk\text{lim}} \) is the peak current limit set in the specifications. The peak current is defined as

\[ I_{pk} = \sqrt{2} \max(I_s) \] 
\[ (3.22) \]

The path length has to be constrained within a range to ensure a valid geometry. The constraints are as follows:

\[ c_9 = \text{lte} \left( g(\phi_r), r_{st} - r_{rb} \right) \] 
\[ (3.23) \]

\[ c_{10} = \text{gte} \left( g(\phi_r), g_{\min} \right) \] 
\[ (3.24) \]

These constraints force the path length to be greater than the minimum value without cutting into the rotor backiron. A design that has any point of \( g(\phi_r) \) in the restricted area is considered unviable.

The rotor tip speed is constrained to keep a machine mechanically stable. This constraint is as follows:

\[ c_{11} = \text{lte} \left( r_{st} \max(\omega_{rm}), v_{tpmx} \right) \] 
\[ (3.25) \]

where \( v_{tpmx} \) is the maximum allowed tip velocity, and \( \omega_{rm} \) is the mechanical rotor speed.

The mass and length are subject to constraints as well, which yields

\[ c_{12} = \text{lte} \left( m, m_{\text{lim}} \right) \] 
\[ (3.26) \]

and
where \( m \) is the mass of the machine, \( m_{\text{lim}} \) is the mass limit, \( l_t \) is the total length of the machine, and \( l_{\text{lim}} \) is the length limit. These limits are set in the design specifications.

The peak line-to-line voltage is expressed as

\[
v_{pk,tl} = \sqrt{3} \sqrt{(v_{qr})^2 + (v_{ds})^2}
\]

(3.28)

In order for the machine to perform properly, the peak line-to-line voltage must be less than the dc input voltage. So

\[
c_{14} = \text{lte}(v_{pk,tl}, v_{dc})
\]

(3.29)

The flux density in all parts of the machine is limited to the recommended flux density levels to avoid saturation as follows

\[
c_{15} = \text{lte}\left(\max\left(|B_t|, B_{s,\text{lim}}f_f\right)\right)
\]

(3.30)

\[
c_{16} = \text{lte}\left(\max\left(|B_b|, B_{s,\text{lim}}f_f\right)\right)
\]

(3.31)

\[
c_{17} = \text{lte}\left(\max\left(|B_{rpt}|, B_{r,\text{lim}}f_f\right)\right)
\]

(3.32)

\[
c_{18} = \text{lte}\left(\max\left(|B_{rpb}|, B_{r,\text{lim}}f_f\right)\right)
\]

(3.33)

\[
c_{19} = \text{lte}\left(\max\left(|B_{rb}|, B_{r,\text{lim}}f_f\right)\right)
\]

(3.34)

where \( B_t \) is the flux density in a stator tooth, \( B_b \) is the flux density in the stator backiron, \( B_{rpt} \) is the flux density in a rotor pole tooth, \( B_{rpb} \) is the flux density in a rotor pole base, \( B_{rb} \) is the flux density in the rotor backiron, and \( f_f \) is the factor of safety on the flux density.

Derivations for \( B_t \) and \( B_b \) are found in [10]. In addition, \( B_{s,\text{lim}} \) and \( B_{r,\text{lim}} \) are the flux density limits of the stator and rotor materials, respectively.
The last constraint applied is the power constraint, to ensure the machine is producing enough power. Thus

\[ c_{20} = \text{gte} \left( P_{\text{eout}}, P_{\text{out}} \right) \]  \hfill (3.35)

where \( P_{\text{eout}} \) is the electrical output power, and \( P_{\text{out}} \) is the required electrical output power.

The electrical output power is defined as

\[ P_{\text{eout}} = -T_e \omega_{\text{em}} - P_l \]  \hfill (3.36)

where \( T_e \) is the electromagnetic torque produced, and \( P_l \) is the power loss. As a reminder, the torque produced from this machine is negative, because it is acting as a generator.

At this point, the total constraint value can be defined as

\[ c_a = \frac{1}{N_c} \sum_{i=1}^{N_c} c_i \]  \hfill (3.37)

where \( N_c \) is the total number of constraints.

This optimization follows a similar process as in [14]. In order to minimize mass and loss, the objectives can be described as

\[ o_d = \begin{bmatrix} 1 & \frac{1}{M_t} & \frac{1}{P_l} \end{bmatrix}^T \]  \hfill (3.38)

Using (3.37) and (3.38), the fitness function is found as

\[ f = \begin{cases} \epsilon \left( c_a - 1 \right) \left[ 1 \right]^T & c_a < 1 \\ o_d & c_a = 1 \end{cases} \]  \hfill (3.39)
where $\epsilon$ is a small positive number. Using this approach, non-viable designs have a negative fitness which become less negative as they approach viability; on the other hand, viable designs have a fitness defined as the reciprocal of the mass and loss. The reciprocation is necessary because the optimization algorithm is set up to maximize objectives, and in this study minimization is desired.

Now, the operation of the fitness function will be considered. The pseudo-code in Figure 3.2 below describes the operation of the fitness function. In order to discuss the program flow, it is convenient to introduce structures that appropriately categorize the variables of the model. To aide this discussion, the subscripts $I$ and $D$ represent independent and dependent variables, respectively.

The design specification structure $D$, and the design space structure, $\Theta$ are discussed in Section 3.1 and Section 3.2 respectively. Next, the structure $G$ is considered, where $G$ denotes geometry. $G$ is defined as

$$G = \begin{bmatrix} G_I^T & G_D^T \end{bmatrix}^T$$

(3.40)

where the independent geometry vector is defined as

$$G_I = \begin{bmatrix} r_i & d_{rh} & d_{pb} & d_{ps} & d_{pb} & \alpha_i & l & \alpha_t & \alpha_{rpt} & P & \phi_{ss,1} \end{bmatrix}$$

(3.41)

In (3.41), two variables are new to the reader: $d_t$ is depth of a stator tooth base, and $\phi_{ss,1}$ is the center location of the first slot as discussed in [10]. It is also noted that many of the independent terms are defined in the design space structure; early in the fitness function these values are assigned to the geometry structure. The dependent geometry vector is defined as
\[ G_D = \begin{bmatrix} r_{rh} & r_{rg} & r_{rpb} & s_t & S_s & q_t & q_{st} & w_{sb} & r_{sb} & d_{st} & w_{so} & r_{ss} & a_{slt} & a_{s} & v_{st} & v_{sb} & \ldots \\ \theta_{rpb} & v_{st} & v_{rb} & r_{ri} & \theta_{rp} & \theta_{rpb} & w_{rpb} & w_{rpt} & v_{rpt} & v_{rpb} \end{bmatrix}^T \]

(3.42)

where the newly introduced variables are discussed in [10].

In addition, the structures \( S, R, W, \) and \( C \) are the stator material, rotor material, winding and coil material structures. These are identical to those described in [10]. Next, the current structure, \( I \), is also established [10]. The electrical structures \( E \) and \( E_2 \), are defined as

\[ E = \begin{bmatrix} R_s & L_{ls} & P_{stf} \end{bmatrix}^T \]

(3.43)

and

\[ E_2 = \begin{bmatrix} L_q & L_d \end{bmatrix}^T. \]

(3.44)

Finally, the fields structure, \( F \), is encapsulated as

\[ F = \begin{bmatrix} q_{re} & B_{rle} & pB_{rle} & B_{ble} & pB_{ble} & P_c \end{bmatrix}^T. \]

(3.45)

With these structures defined, the pseudo-code for the fitness function is set forth.

1. Initialization and material selection
   Assign field of material vectors (structures) \( S, R, C \), based on \( \theta \)
2. Calculate machine geometry
   Assign independent fields of \( G \) based on \( \theta \) and \( D \)
   Evaluate \( c_1 \) using (3.11)
   Calculate dependent fields of \( G \)
   Evaluate \( c_2, c_3 \) using (3.14), (3.15)
3. Winding calculations
   Assign independent fields of \( W \) based on \( \theta \) and \( D \)
   Calculate dependent fields of \( W \)
   Evaluate \( c_4 - c_6 \) using (3.16)-(3.17)
4. Current calculations
   Assign independent fields of \( I \) based on \( \theta \)
   Calculate dependent fields of \( I \)
   Determine constraint \( c_7, c_8 \) using, (3.19), (3.21)
5. Air Gap calculations
Assign air gap lengths to air gap vector, $g$, based on $\theta$
Calculate quadratic curve, $Q$, of best fit to air gap vector
Determine constraints $c_9$, $c_{10}$ using (3.23), (3.24)
Test constraints
Evaluate air gap $g$

6. Speed calculations
Assign $\omega_{rm}$ and $f_{sw}$ based on $D$
Evaluate constraint on tip speed, $c_{11}$, using (3.25)

7. Mass calculations
Compute total mass using 3.33
Calculate total length
Evaluate constraints $c_{12}, c_{13}$ using (3.26), (3.27)
Test constraints

8. Analyze electrical performance
Determine electrical parameter $E$ from $C$, $G$ and $W$
Determine electrical parameter $E_2$ from $\phi_r$, $C$, $G$, and $W$
Determine flux linkages
Determine line-to-line voltage using 3.21
Determine torque using 2.45
Evaluate constraint $c_{14}$ using (3.29)
Test constraint

9. Perform field analysis under operational conditions
Assign fields of $I$ as necessary
Determine $F$ from $G$, $W$, $I$, $\phi_r$, and $\omega_{rm}$
Compute $c_{15}, c_{16}$ using (3.30), (3.31)
Determine rotor flux density values
Compute $c_{21} - c_{23}$ using (3.32)-(3.34)
Test constraints

10. Compute losses
Compute semiconductor loss using 3.35 and 3.36
Compute resistive loss using 3.37
Compute core loss using 3.38
Compute proximity effect loss using 3.39
Compute total loss using 3.34
Compute constraint $c_{24}$ using (3.35)
Test constraint

11. Compute fitness using (3.39)
Return

Figure 3.2 Pseudo-code for Calculation of Fitness Function
CHAPTER 4. CASE STUDY

The design of the ARM is achieved using a multi-objective optimization where mass and electromagnetic loss are minimized subject to design constraints. In this chapter, a case study is considered based on an operating point with a 10-kW output power at 700 V and a speed of 1800 r/min. The Pareto-optimal front of the ARM is presented, as well as the parameter distribution of the study. Next, one design in particular is examined. In addition, one design ARM is examined using a 2D, linear FEA analysis. Finally, the ARM Pareto-optimal front is compared with the front of a UGRM.

4.1 Design Results

The Pareto-optimal front plots the non-dominated set of viable designs found in the optimization. Figure 4.1 depicts the Pareto-Optimal front of the ARM, minimizing mass and loss. The study is run over 2000 generations with a population size of 2000. ARM design #75, which is highlighted in the figure, will be analyzed in greater detail later in this section. Figure 4.1 displays a clear trade-off between mass and loss, which is expected.
4.1.1 Parameter Distribution

Before discussing design #75 specifically, the parameter distribution for the ARM is investigated. Figure 4.2 shows the parameter distribution for the entire ARM study. This parameter distribution plots each parameter from the designs in the Pareto-optimal front against decreasing mass; so, the highest mass is on the left of each subplot. The parameters are normalized to values between zero, which corresponds to the minimum allowed parameter value and one, corresponding to the maximum allowed parameter value. The plot is also tone mapped, so each color represents a distinct design.
The first parameter is the number of pole pairs. At first, the number of pole pairs decreases as mass decreases. Later, as the mass approaches the lower extreme, the number of poles increases, which is expected, it can also be seen that the length of those machines goes down as the poles increase. At first, as mass decreases the number of pole pairs decreases.

The second parameter, the depth of the inert region, does not truly converge, but trends toward a value for the lower mass designs. Next, parameter three, the depth of the rotor backiron, decreases as mass decreases, which is reasonable; a shorter backiron corresponds with less mass.

The fourth parameter, the depth of the rotor pole base, peaks when the number of poles is the lowest. Each individual rotor pole is responsible for carrying more flux. A machine with a longer rotor pole tends to have a wider spanning rotor pole tip, which
creates a path for more flux. Even if $\alpha_{rp_b}$, the rotor pole base fraction, between two designs is the same, a machine with a longer rotor pole has a larger radius to apply that fraction against, leading to a wider rotor pole tip.

The fifth parameter, the depth of the rotor pole tip, does not show strong convergence, but overall congregates near the lower limit. This is interesting; one consequence of a thin rotor pole tip is a small amount of fringing flux.

Parameter 6, the depth of the stator tooth base, trends down as mass decreases. Parameter 7, the stator tooth fraction does not truly converge, but stays within a reasonable range of values.

The stator backiron, parameter 8, decreases as mass decreases. A deep stator backiron lends directly to large mass, so this is result is expected. Parameter 9, the active length of the machine behaves similarly to the stator tooth fraction. The rotor tooth tip fraction, parameter 10, fluctuates for higher mass designs but seems to converge as mass decreases. Parameter 11, the rotor pole base fraction converges to one for the lower mass and higher pole designs. This means that the rotor pole tips are effectively the same width as the rotor pole base, and the rotor pole tip merely serves to aide in the construction of the airgap.

Now the windings and current will be considered. Parameter 12, the peak phase conductor density, seems to choose between two approximate values, with a significant gap in between. The current, defined by parameters 13 and 14 behaves in a similar manner; they are split between two values, with an upward trend apparent for the negative $q$-axis current. This is reasonable, as mass is decreasing, loss is increasing. Higher currents lead to more loss directly, as an application of Ohm’s Law.
The last three parameters describe the air gap. Overall, the taper performs as expected. The first two air gap values are smaller than the third, indicating a taper in the desired direction. It may be beneficial for the third air gap position maximum to be increased. This would allow those peak values to explore the space they desire.

4.1.2 Design # 75 Results

Now, design #75 of the solution set will be considered in detail. First, the parameters are given in Table 4.1. An analysis and discussion of the design follows.

<table>
<thead>
<tr>
<th>Gene</th>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole Pairs</td>
<td>PP</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>Depth of Inert Region</td>
<td>d_{ir}</td>
<td>0.192</td>
<td>(cm)</td>
</tr>
<tr>
<td>Depth of Rotor Backiron</td>
<td>d_{rb}</td>
<td>1.757</td>
<td>(cm)</td>
</tr>
<tr>
<td>Depth of Rotor Pole Base</td>
<td>d_{rp_{b}}</td>
<td>4.303</td>
<td>(cm)</td>
</tr>
<tr>
<td>Depth of Rotor Pole Tip</td>
<td>d_{rp_{t}}</td>
<td>0.2059</td>
<td>(cm)</td>
</tr>
<tr>
<td>Depth of Tooth Base</td>
<td>d_{tb}</td>
<td>2.985</td>
<td>(cm)</td>
</tr>
<tr>
<td>Stator Tooth Fraction</td>
<td>\alpha_t</td>
<td>0.6307</td>
<td>-</td>
</tr>
<tr>
<td>Depth of Stator Backiron</td>
<td>d_{sb}</td>
<td>1.444</td>
<td>(cm)</td>
</tr>
<tr>
<td>Active Length</td>
<td>l</td>
<td>23.52</td>
<td>(cm)</td>
</tr>
<tr>
<td>Fraction of rotor pole tip to one pole</td>
<td>\alpha_d</td>
<td>0.332</td>
<td>-</td>
</tr>
<tr>
<td>Fraction of rotor pole base to rotor pole tip</td>
<td>\alpha_{rp_{b}}</td>
<td>0.7913</td>
<td>-</td>
</tr>
<tr>
<td>Peak Phase Conductor Density</td>
<td>N_{s1_{star}}</td>
<td>77.7</td>
<td>(cond./rad)</td>
</tr>
<tr>
<td>Q-axis Current</td>
<td>I_{q}</td>
<td>-20.95</td>
<td>A</td>
</tr>
<tr>
<td>D- axis Current</td>
<td>I_{d}</td>
<td>11.39</td>
<td>A</td>
</tr>
<tr>
<td>Air Gap 1</td>
<td>g_{r}(\phi_1)</td>
<td>0.5000</td>
<td>(mm)</td>
</tr>
<tr>
<td>Air Gap 2</td>
<td>g_{r}(\phi_2)</td>
<td>0.9011</td>
<td>(mm)</td>
</tr>
<tr>
<td>Air Gap 3</td>
<td>g_{r}(\phi_3)</td>
<td>2.624</td>
<td>(mm)</td>
</tr>
</tbody>
</table>
From the parameters given in Table 4.1, the $q$- and $d$- axis flux linkages of the selected ARM are found to be $0.09429 \text{ Vsec}$ and $0.9684 \text{ Vsec}$, respectively. Recalling (2.55), this results in a torque of $-57.65 \text{ Nm}$, which is expected; a negative torque is desired for a generator application. The mass of this design is $57.7 \text{ kg}$, with nearly $70\%$ of that mass coming from the stator laminations and conductors. The total loss of the machine is $699.9 \text{ W}$. Based on the air gap parameters described above, the air gap across the pole face varies from $0.50 \text{ mm}$ to $2.4 \text{ mm}$. This is a modest variation compared to the overall size of the machine, but it is significant.

Figure 4.4 shows the radial cross section of the machine. All positions are defined as increasing in the counter-clockwise position. Likewise, a positive torque is defined by
rotor rotation in the counter-clockwise position. The tapering of the pole is evident, and the parameter values from Table 4.1 support this observation.

Figure 4.5 shows the path length as a function of position, displaying the taper of the machine in greater detail. In addition, the path length in the fringing region is noted as well. As the position moves away from the rotor pole face, the fringing distance increases linearly. In addition, the fringing path on the right side of the pole is not the same distance as the fringing path on the left side of the machine. This difference is due to the asymmetry of the pole face.

![Figure 4.4 Path Length vs. \( \theta_r \)](image)

The radial flux density waveform is given in Figure 4.6. It should be noted that the waveform has a different shape than that of a UGRM; moreover, the peak value is near the middle of the rotor pole tooth. In fact, the flux density increases well into the tooth region.
It is noted that the peak flux density found in both the rotor pole base and the stator teeth is .69 T, only 50% of the flux density allowed in the rotor pole base.

![Figure 4.5 Design #51 Radial Flux Density](image)

**4.2 Design Verification**

The ARM design described above is now studied with a 2D-linear FEA. First, the flux densities waveforms calculated by the two models are compared. Next, the flux linkages predicted by the design model are compared to that obtained by the FEA analysis. Finally, the average torque from the optimization is compared to that of the FEA analysis.

Figure 4.7 displays the stator tooth flux density found from the FEA analysis and the optimization code. These waveforms are measured at the final stator tooth and backiron segment corresponding to a mechanical pole.
As can be seen, the flux density found in the optimization has reasonable agreement with the FEA. The FEA validation in [9] achieved high fidelity, so this work seems to agree with past studies. Next, Figure 4.8 compares the stator backiron flux densities of each analysis.
Figure 4.7 Stator Backiron Flux Density

There is rough agreement between the two waveforms. The ripple in the backiron of the FEA analysis is due to the stator slots, which are not accounted for in the design model except through Carter’s coefficient. This design consideration eliminates torque ripple, so this discrepancy is expected between the two plots.

The flux linkage values of the designs are in Table 4.2.

<table>
<thead>
<tr>
<th>Value</th>
<th>Multi-Objective Optimization</th>
<th>FEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_q$, Vsec</td>
<td>-.09429</td>
<td>-.23911</td>
</tr>
<tr>
<td>$\lambda_d$, Vsec</td>
<td>.9684</td>
<td>.77581</td>
</tr>
</tbody>
</table>

Clearly, the studies do not agree on flux linkage. While the sign of the flux linkages agree, which leads both to having the desired negative torque, the magnitudes disagree by a significant amount. It seems that the flux linkage calculation in either the optimization or
the FEA analysis is incorrect. The difference in flux linkage values leads to torque values that do not correspond.

The torque computed by the design model is -57.65 Nm, and the torque computed from the FEA is -40.5891 Nm. This is an error percentage of 30%, and, again, suggests an issue in the flux linkages calculation in either the optimization or FEA model.

4.3 Machine Comparison

To test the metrics of the ARM against other systems, a similar multi-objective optimization study is run for a uniform airgap reluctance machine. The design specifications of the UGRM are the same as those of the ARM. Both machines use M19 steel for the laminations, and copper for the windings. Also, the design space and fitness function are nearly identical. Instead of allowing a quadratic curve to describe the air gap, the optimization code forces the air gap of the rotor pole face to be constant. Thus, the design space of the UGRM is the same as the design space of the ARM except for the omission of the final two parameters, \( g_r(\phi_2) \), and \( g_r(\phi_3) \). These parameters are no longer necessary. Figure 4.8 shows the Pareto-optimal fronts of each machine compared to each other.
It is observed that the ARM outperforms the UGRM over the entire front. It is noted that these results generally support those of [9]. In that study, a mass reduction of approximately 50% was found while comparing two machines, depending on the loss. The average mass reduction varies slightly across the front in [9], but the conclusion is a significant mass reduction across all designs. In this study, the mass reduction seems to be less, but still significant.

It is argued that allowing for more points to describe the air gap would have increased the performance of the ARM. By increasing the order of the curve, the function describing the air gap has the freedom to form as flexible of a curve as is beneficial to the machine. Using three points limits the airgap to a quadratic curve, which may not correspond with the best airgap distance for every point.

Figure 4.8 Comparison of ARM and UGRM Pareto-Optimal Fronts
Next, the specifications for ARM design #79 are compared to those of UGRM design #54. For this comparison, designs with comparable loss were chosen, specifically .7 kW. In this instance, the designs will have similar mass as well as loss. The design space variables are given in Table 4.3, and the radial cross section of UGRM design #54 is given in Figure 4.9.

<table>
<thead>
<tr>
<th>Table 4.3 UGRM Design #54 Design Space</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gene</strong></td>
</tr>
<tr>
<td>Pole Pairs</td>
</tr>
<tr>
<td>Depth of Inert Region</td>
</tr>
<tr>
<td>Depth of Rotor Backiron</td>
</tr>
<tr>
<td>Depth of Rotor Pole Base</td>
</tr>
<tr>
<td>Depth of Rotor Pole Tip</td>
</tr>
<tr>
<td>Depth of Stator Tooth Base</td>
</tr>
<tr>
<td>Stator Tooth Fraction</td>
</tr>
<tr>
<td>Depth of Stator Backiron</td>
</tr>
<tr>
<td>Active Length</td>
</tr>
<tr>
<td>Fraction of rotor pole tip</td>
</tr>
<tr>
<td>Fraction of rotor pole base</td>
</tr>
<tr>
<td>Peak Phase Conductor Density</td>
</tr>
<tr>
<td>Q-axis Current</td>
</tr>
<tr>
<td>D-axis Current</td>
</tr>
<tr>
<td>Air Gap 1</td>
</tr>
</tbody>
</table>
This design has $q$- and $d$- axis flux linkage values of -.1321 and .9363 Vsec respectively. The torque produced is -56.85 Nm. The mass of the machine is 77.14 kg, and the loss is .7 kW. As a reminder the mass and loss of the ARM being analyzed are 57.7 kg and .7 kW respectively.

Clearly, for a machine with nearly identical loss, the mass increases by a significant amount. This suggests that an increase in torque density was achieved; the machine consumed less mass for the same torque and power objectives.

While there are significant issues with the validation of the study, information can still be gathered from the results. First, the flux density calculations are the same for both the ARM and the UGRM, so, by extension, the consequences of any miscalculation are applied to both machines. This points to an imperfect design model that can still be trusted.
for the comparisons between these machines, if not for the actual values of torque they produce.

The air gap and the radial flux density for the UGRM are provided in Figure 4.10 and Figure 4.11, respectively.

![Figure 4.10 UGRM Design #54 Path Length vs. Position](image-url)
Figure 4.11 UGRM Design #54 Radial Flux Density vs. Position
CHAPTER 5. CONCLUSION

In this work, the ARM is presented. A design approach based on multi-objective optimization is set forth to study the ARM. A specific machine design has been examined against a linear, 2D-FEA analysis.

A comparison between two different electric machine topologies for a 10 kW, 1800-r/min generator application has been analyzed. The Pareto-optimal fronts trading off electromagnetic mass and electromagnetic loss suggest the ARM is favorable to the UGRM in a significant manner, though the results are questionable because the torque and flux linkage values are not validated by the FEA.

Future work is needed to fully explore an asymmetric machine. The first priority is to correct the design model so it can be validated by the FEA. After this is achieved, a 3D, non-linear FEA analysis might prove beneficial. In addition, a higher order function describing the air gap should be considered. When allowed more degrees of freedom, in this case one for the UGRM versus three for the ARM, the optimization clearly trended towards a tapered machine. In addition, studying this machine more calls for improving the design model. This could be done by employing a magnetic-equivalent circuit model.
REFERENCES
REFERENCES


