1992

Computer Aided Design of a Twin Rotor Screw Refrigerant Compressor

Z. Zhou
Shanghai College of Architectural and Municipal Engineering; P. R. China

Zhou, Z., "Computer Aided Design of a Twin Rotor Screw Refrigerant Compressor" (1992). *International Compressor Engineering Conference.* Paper 836.
https://docs.lib.purdue.edu/icec/836

COMPUTER AIDED DESIGN OF A TWIN-ROTOR SCREW REFRIGERANT COMPRESSOR

Zhou Zicheng

Shanghai College of Architectural and Municipal Engineering
71,Chi-Feng Road, Shanghai, 200092, The People's Republic of China

ABSTRACT

A computer aided design method for a twin-rotor screw refrigerant compressor has been developed which includes the profile generation, the meshed line and contact line plotting, the pressure distribution diagram calculation and plotting, the blow hole area calculation, the milling cutter profile calculation and plotting etc. Some current computer routines and numerical methods have been introduced to this program to solve the complex mathematical analysis. The method is different from the conventional analytical method.

1. INTRODUCTION

The profiles of refrigerant screw compressor rotors are composed of several segments. The male rotor's profile goes into mesh with the profile of female rotor, their mathematical relation is rather complex. To design a screw refrigerant compressor by mathematical analysis is an arduous and time-consuming task, while using computer numerical method and computer plotting, the design procedure will be easier and the results will be more accurate. The method presented in this paper introduces some current computer plotting routines and numerical methods into the screw compressor design, and can be used for new compressor design.

2. THE PROFILES OF MALE AND FEMALE ROTORS, THE MESHED LINE AND THE CONTACT LINE

The profile shape of a twin-screw refrigerant compressor rotors is fundamental to compressor construction and performance. There are several famous profiles used in industry such as the SRM-D profile, GHH profile, X profile and the new HITACHI profile etc. A good profile will cause shorter contact line length, smaller blow hole area, therefore, the compressor performance will be better.

The profile is made up of several segments. Each segment has its geomentrical shap and can be expressed by a mathematical equation. The parameters in the mathematical equation are chosen by comparison the contact line length and blow hole area.

The coordinate points of profile, the meshed line and contact line can be calculated according to their mathematical equations. Fig.1 shows the end profile shapes of male and female rotors. The meshed line and the contact line are shown on Fig.2 and Fig.3.

3. THE PRESSURE DISTRIBUTION DIAGRAM

In the procedure of screw refrigerant compressor design, the pressure distribution diagram is used for determining the discharge port shape and the applied forces to the rotors. At the first period of compression, the lobes are partial into the interlobes. The pressure and volume in the enclosed interlobe can be expressed as follows:

$$p_i = p_s \left(\frac{V_0}{V_0 - V_r} \right)^m \qquad (1)$$

where, p_i - the pressure in the enclosed interlobe at the rotational angle φ of male rotor.

p_s - the pressure in the enclosed interlobe at the beginning of compression.

V_0 - the total enclosed interlobe volume of male and female rotors.

V_r - the decreasing enclosed interlobe volume at angle φ when the lobes invade and occupy the enclosed interlobe volume.

m - the compression exponent.

The volume V_r can be calculated by integrating the invade and occupy area on the perpendicular plane multiplied by the distance Δz in the direction of rotor axle when the male rotor turns an angle $\Delta\varphi$. Therefore, the pressure p_i in equation (1) can be solved. Fig. 4 shows the invade and occupy area and Fig.5 shows the pressure distribution diagram.

4. THE BLOW HOLE AREA

The blow hole area is a small triangular-shaped area formed by the housing cusp and male and female rotor tips. Most of screw refrigerant compressors have a blow hole area. It causes the refrigerant gas leakage from the higher pressure cavity to the lower pressure cavity, and decreases the compressor performance.

To calculate the blow hole area, data of rotor's end profile and meshed line have to be given.

On the stationary coordinates $OX_1Y_1Z_1$ and moving coordinates $ox_1y_1z_1$ of female rotor, the blow hole area is on the plane formed by hausing cusp line SS and tip T of meshed line. Coordinates of hausing bore cusp can be found by the following formula

$$X1s = \frac{1}{2A}\left(RO_1^2 - RO_2^2 + A^2\right) \tag{2}$$

$$Y1s = -\sqrt{RO_1^2 - X1s^2} \tag{3}$$

where, RO_2 - outside radius of male rotor.

RO_1 - outside radius of female rotor.

A - the least distance between axis of the rotors.

The following characteristic points can be found by calling in Search Subroutine.

a) Position of male rotor's vertex H,

b) Position of meshed line tip T,

c) Point G on male rotor and B on female rotor that will come in contact at meshed line tip T.

Designing profile of rotor, vertex of male rotor is placed on the moving axis ox_2. If ox_2 of moving coordinates coincides with OX_2 of stationary coordinates, the rotor is said to be in its neutral position (Fig.1).

Applying the relation $\tan\alpha = y/x$, the following angles can be found:

a) The angle φ_{HS}, male rotor turns from its neutral position, so that vertex H reaches hausing cusp S. This is the angle at which the gas begins to leak. See Fig. 6a.

b) The angle φ_{GT}, male rotor turns from its neutral position, so that point G of profile reaches meshed line tip T. This is the angle at which leakage of gas ends. See Fig. 6d.

In this interval (i.e. φ_{HS} to φ_{GT}) we find out the blow hole area. The length of segment on line ST that is not covered by male or female rotor is calculated first. Integrating this length along OZ axis, we obtain the blow hole area. See Fig. 6c.

The equation of straight line ST in slope-intercept from $y=C + mx$ is to be found first. Then in the interval of φ_{HS} to φ_{GT}, taking step length $\Delta\varphi$, transform points on curve HG from moving coordinates ox_2y_2

to stationary coordinates OX_2Y_2. Formula for transformation is as follows:

$$X_2 = x_2 \mathrm{COS}\varphi - y_2 \mathrm{SIN}\varphi \tag{4}$$
$$Y_2 = x_2 \mathrm{SIN}\varphi + y_2 \mathrm{COS}\varphi \tag{5}$$

To find out the intersecting point N of straight line ST to curve HG at different angle of rotation, we call in Least Squre Curve Fitting Subroutine to approximate curve HG with the form of quadratic curve ($y = a_1 + a_2x + a_3x^2$) and call in Secant Subroutine to find the root of the following equation.

$$(a_1 - c) + (a_2 - m)x + a_3x^2 = 0 \tag{6}$$

This root is the x-coordinates of point N. Its y coordinate is $y = C + mx$. NT is the length of segment on line ST covered by male rotor.

The way to find out the length of segment SR on line ST covered by female rotor at corresponding angle of rotation is similar.(Fig. 6c) When female rotor turns a certain angle, point U on the vertex touches line ST and line ST begins to be covered by the profile of female rotor (Fig. 6b) When female rotor rotates $i\varphi_{GT}$ from its neutral position, point V on profile reaches meshed line tip T. i is the ratio of number of lobes on male rotor to that on female rotor. In similar way, we got the length on line ST covered by female rotor when curve UV rotates from $i\varphi_{HS}$ to $i\varphi_{GT}$.

$$h = ST - SR - TN \tag{7}$$

here h is the height of blow hole at different angle of rotation.

When male rotor turns an angle $\Delta\varphi$, any point on profile of male rotor moves a distance Δz in the direction of OZ.

$$\Delta z = r_{2i} \cdot \tan\beta \cdot \Delta\varphi \tag{8}$$

Where, r_{2i} - pitch circle radius of male rotor.
$\beta = H_2 / 2\pi r_{2i}$ - nelical angle
H_2 - lead

Once h and Δz have been found for different angle of rotation, the blow hole area can be calculated by calling subroutine of Simpson's Method of Integration.

5. THE DISC MILLING CUTTER PROFILE

Fig. 7 shows the scheme of screw rotor and disc milling cutter. In the system of coordinates, OXYZ is the stationary coordinates for rotor. oxyz is the moving coordinates for rotor. Ou Xu Yu Zu is the stationary coordinates for cutter. All of them belong to right-handed system. OX coincides with OuXu. The angle ψ between OZ and OuZu is called install angle. The least distance Ac between OuZu and OZ is called center distance between rotor and cutter.

The functional relation for data of end profile can be expressed as follows:

$$XO = XO\ (I) \tag{9}$$

$$YO = YO\ (I) \tag{10}$$

Data of end profile are represented in moving coordinates of rotor. Sequential number I of nodal point is the parameter of profile fuction too.

During the cutting time, the cutter profile forms a rotational surface which is tangential to screw rotor lobe surface. The contact equation gives the condition which had to fullfilled for cutter to be in contact with lobe surface. It is derived from the principle that rotational surface of milling cutter and lobe surface have tangential planes in common. The following equation is the contact equation :

$$\begin{aligned} &AO\cdot COS(\tau(I))+BO\cdot SIN(\tau(I))+CO\cdot\tau(I)\cdot SIN(\tau(I))+\\ &+DO\cdot\tau(I)\cdot COS(\tau(I))+EO = 0 \end{aligned} \tag{11}$$

where,

$$AO = 0.5\cdot XO(I)\cdot COS(\pi/2-\psi)\cdot AZ(I)+AX(I)\cdot P\cdot Ac\cdot SIN(\pi/2-\psi) \tag{12}$$

$$BO = \mp 0.5\cdot YO(I)\cdot COS(\pi/2-\psi)\cdot AZ(I)\mp AY(I)\cdot P\cdot Ac\cdot SIN(\pi/2-\psi) \tag{13}$$

$$CO = -P^2\cdot AX(I)\cdot COS(\pi/2-\psi) \tag{14}$$

$$DO = \mp P^2\cdot AY(I)\cdot COS(\pi/2-\psi) \tag{15}$$

$$EO = -0.5\cdot P\cdot AZ(I)\cdot SIN(\pi/2-\psi)-0.5\cdot Ac\cdot AZ(I)\cdot COS(\pi/2-\psi) \tag{16}$$

P = H/2 screw parameter

H - lead of rotor

AX(I), AY(I) and AZ(I) are the first derivatives of XO(I), YO(I) and $ZO(I) = XO(I)^2 + YO(I)^2$ to I respectively. AX(I), AY(I) and AZ(I) are proportional to the components of helical surface normal.

If there are plus and minus sign befor any term, the upper sign is true for right hand rotor, while the lower sign is ture for left hand rotor.

Solving contact equation (11), we obtain relation between warp angle of rotor and the corresponding parameter of profile when cutter and lobe surface are in contact.

The lobe surface equation is the expression of helical lobe surface in stationary coordinates of rotor.

$$\left.\begin{aligned} X1(I) &= XO(I)\cdot COS(\tau(I)) \mp YO(I)\cdot SIN(\tau(I)) \\ Y1(I) &= \pm XO(I)\cdot SIN(\tau(I)) + YO(I)\cdot COS(\tau(I)) \\ Z1(I) &= P\cdot\tau(I) \end{aligned}\right\} \quad (17)$$

Where, the upper sign is true for right hand rotor, while the lower sign is true for left hand rotor.

The expression of the locus of contact points (i.e. contact line) of cutter and rotor in rotor's stationary coordinates is obtained when we substitute the corresponding values of I and $\tau(I)$, obtained by solving contact equation (11) in to lobe surface equation (17).

The contact line equation in cutter's coordinates is as follows:

$$\left.\begin{aligned} Xu(I) &= X1(I) - Ac \\ Yu(I) &= Y1(I)\cdot COS\psi \pm Z1(I)\cdot SIN\psi \\ Zu(I) &= \mp Y1(I)\cdot SIN\psi + Z1(I)\cdot COS\psi \end{aligned}\right\} \quad (18)$$

Substituting X1(I), Y1(I) and Z1(I) into the above equation (18), we obtain the expression of contact line in cutter's ststionary coordinates.

Equation of milling cutter profile is the expression of milling cutter profile in cylindrical coordinates Ou Ru Zu.

$$\left.\begin{aligned} Zu &= Zu(I) \\ Ru &= \sqrt{Xu^2 + Yu^2} \end{aligned}\right\} \quad (19)$$

where Ru is the radius of milling cutter at Zu.

The included angle between surface normal vector and cutter's axis vector Ou Zu can be expressed as follows:

$$\varepsilon = ARCCOS\left\{\mp\frac{P\cdot AX(I)\cdot SIN\psi + AZ(I)\cdot COS\psi}{\sqrt{P^2[AX(I)^2 + AY(I)^2] + [\frac{1}{2}AZ(I)]^2}}\right\} \quad (20)$$

According to the principle of equidistance between profile surfaces, we stipulate the clearance along lobe surface normal. ε is the angle required in calculating modified profile of milling cutter.

In equation (20), the upper sign is true for right hand rotor, while the lower sign is true for left hand rotor.

The equation for modified profile of female rotor milling cutter is as follows:

$$\left.\begin{aligned} Zu1(I) &= Zu(I) + \delta\cdot COS(\varepsilon) \\ Ru1(I) &= Ru(I) + \delta\cdot SIN(\varepsilon) \end{aligned}\right\} \quad (21)$$

where,

δ - clearance along lobe surface normal, designated.

To obtain normal clearance we modify lobe surface of female rotor.

The profile of disc milling cutter from data of profile can be calculated with numerical method. It is of vital importance to find out first derivatives AX(I), AY(I) and AZ(I) accurately. We call in subroutine of cubic spline.

Spline is a fuction for interpolating . To calculate value of function at interpolating point with cubic spline, we have to calculate the second and first derivatives of the function at every node point. Therefore we may call in the first derivatives that already exist in the spline subroutine.

But when we call in subroutine of spline, the first derivatives at the terminals of profile must be given first.

For some of the rotors,it is not difficult to find out these two first derivatives analytically, particulerly when the terminal profiles are circular arcs. If it is difficult to find them analytically, three-point formula of forward or backward finite difference may be considered to apply.

To find out the root of contact equation, we may call in the subroutine of secant method for root determination. The interval for root searching may be taken to be from $-\pi/2$ to $\pi/2$. In general, number of roots in this interval will not exceed two. The rule for determining which one of the roots is the better is as follows. The necessary condition is that the profile of cutter must be continues. The sufficient condition is that the one giving smaller radius of milling cutter circle is to be selected.

Fig. 8 shows the profile of milling cutter for female rotor, which is obtained by numerical calculation based upon data of rotor's profile. The rotors are shown in Fig.1 .

6. CONCLUSIONS

1. A computer aided design for twin-rotor screw refrigerant compre-

ssor has been developed. Some complex calculations in screw refrigerant compressor design can be solved and plotted. The method is different from convetional method.

2. The feature of this method is that, generating the rotor's profile first, then the other design items can be solved by using those data with numerical method.

3. This method can be extended to the applied forces analysis and plotting.

7. REFERENCES

1. Amocob. П. E., ВИНТОВЫЕ КОМПРЕССОРНЫЕ СПРАВОЧНИК. МАШГИЗ,1977.

2. Rinder Laurenz, SCHRAUBEN VERDICHTER. Springer Verlag,1979.

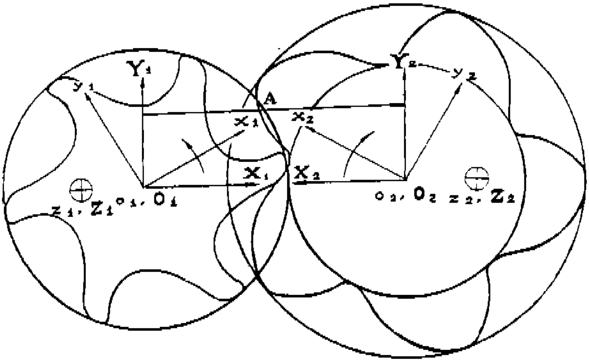

Fig. 1 The End Profile Shapes of Male And Female Rotors

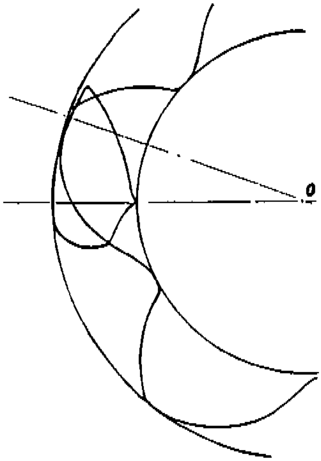

Fig.2 The Meshed Line

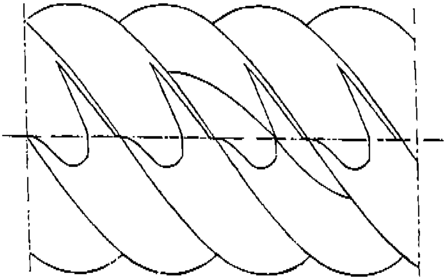

Fig.3 The Contact Line

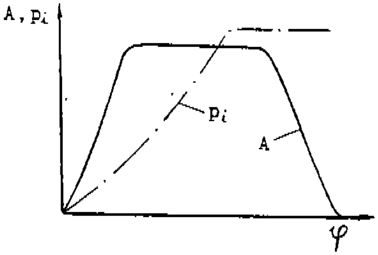

Fig.5 The Pressure Distribution Diagram

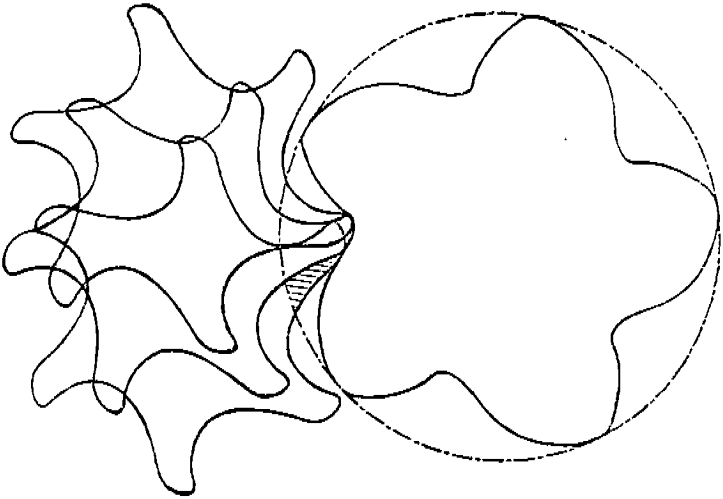

Fig.4 The Invade and Occupy Area

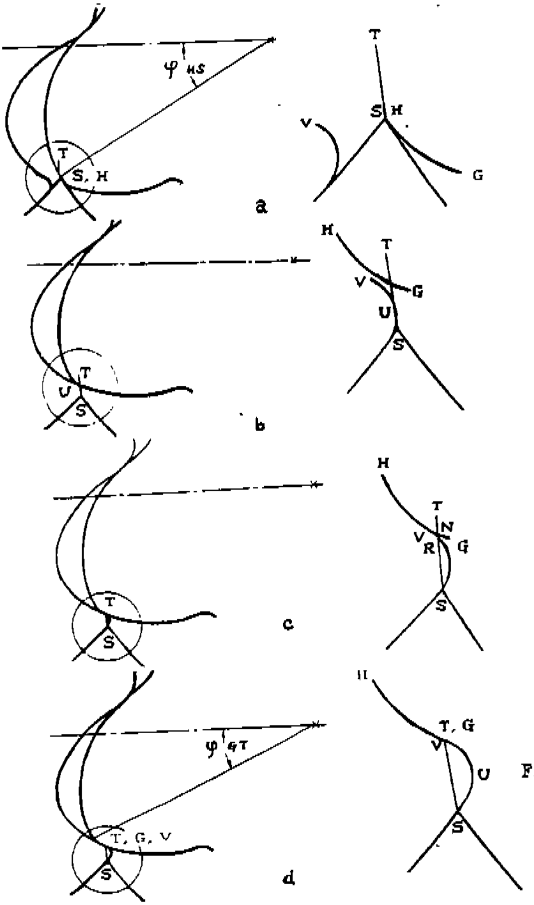

Fig.6 The Blow Hole Area

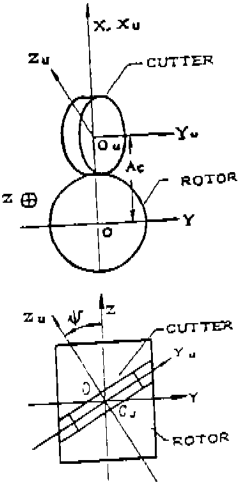

Fig.7 System of Coordinates
For Rotor And Cutter

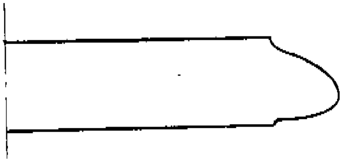

Fig.8 Profile of Milling Cutter For
Female Rotor