Valve Throttling, Its Influence on Compressor Efficiency and Gas Temperatures, Part I

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VALVE THROTTLING, ITS INFLUENCE ON COMPRESSOR EFFICIENCY AND GAS TEMPERATURES

Part I. Full load operation

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Abstract

The paper derives simple formulas for temperature rises in the compressed gas due to leakage and work losses in valves, to estimate volumetric and isentropic efficiencies of reciprocating compressors at full load, by proposing simple analytical approaches to every single one of the factors of influence considered.

1 INTRODUCTION

Due to a number of physical effects unavoidable in real reciprocating compressors, the volume of gas transported per compression cycle from the suction line to the discharge line is smaller than the volume displaced by the piston, and the gas inside this already reduced volume is at a higher temperature and hence thinner than the gas at nominal suction conditions p_1 and T_1. Both effects reduce compressor capacity. Furthermore, the work W_{real} required by a real compressor will be higher than the work W_{is} necessary for isentropic compression of the same mass of gas from p_1, T_1 to p_2 and $T_{2,is}$, and it is the task of the engineer to minimize these adverse effects. Many approaches have been proposed so far to forecast volumetric and energetic efficiencies of reciprocating compressors, we shall mention here only those that have recently been presented here at Purdue, in particular

- *H.J. Kleinert* and *H. Najork* (1986 [1]) use a number of different efficiencies (indicated, isentropic, of utilization, volumetric, of denseness etc.), some of them being defined by analytical formulas, others are determined empiricially, to forecast compressor performance.
- *T. Morel* and *R. Keribar* (1988 [2] and [3]) use elaborate Finite Element Methods to completely model the compressor and forecast gas flows and heat transfer as well as gas and component temperatures.
- *J.A. McGovern* (1990 [4]) presents a simple and clear physical approach to account for some of the effects to be considered hereafter.
- *E.H.Machu* (1990 [5]) uses a system of differential equations to account not only for valve throttling, but also for heat transfer and valve leakage to find indicated power, gas temperatures and efficiencies of delivery.

If the papers presented in [1] or [4] cover only part of the problem, the methods presented in [2] and [3], or [5] require highly specialized software and time consuming computations, they are therefore not available to everybody and, above all, they are unable to allow a rapid quantification of the different factors of influence. In the present paper, pulsations are neglected, i.e. pressures in suction and discharge plenums are assumed constant, all formulas assume ideal gas behaviour with constant specific heats C_p and C_v.

2 EFFICIENCIES OF A RECIPROCATING COMPRESSOR

To compare different designs, the following efficiencies are useful:

1. *The volumetric efficiency* η_v is defined geometrically as the ratio of effective intake volume to stroke volume. The resulting η_v is the base length of the isentropic part of the indicator diagram.

$$\eta_v = \frac{\textit{length of the effective suction stroke}}{\textit{length of piston stroke}} = 1 - \Delta z_{ex} - \Delta z_s \tag{2.1}$$

where Δz_{ex} is the well known expansion loss

$$\Delta z_{ex} = s\left[\frac{Z_1}{Z_2}\left(\frac{p_2}{p_1}\right)^{\frac{1}{\kappa}} - 1\right] \tag{2.2}$$

The other volume loss Δz_s is due to heavy suction valve throttling, it occurs only for $q > 0.16$ approximately, when cylinder pressure in the outer dead center is still below suction plenum pressure.

$$\Delta z_s \approx \max\left(0, \frac{q - 0.16}{5}\right) \qquad \text{where} \qquad q = \left(\frac{V_{st}}{\Phi}\right)^2 \cdot \frac{\omega^2}{8} \cdot \frac{\varrho_1}{p_1} \approx \frac{\Delta p_{max}}{p_1} \tag{2.3}$$

This definition of Δz_s seems more correct than using the ratio of cylinder pressure in the outer piston dead center to nominal suction pressure p_1, since, as long as cylinder pressure is smaller than p_1, the suction valve will be open, gas will still flow into the cylinder even after the piston has passed its outer dead center position, and $\frac{p_{cyl}}{p_1} \neq \frac{1+s-\Delta z_s}{1+s}$.

2. With M as the mass of gas delivered per compression cycle, nominal suction gas density ϱ_1 at p_1, T_1, and $T_{1,s}$ as the temperature inside the working chamber at p_1, after the intake stroke and before compression, the *efficiency of delivery* λ is defined as

$$\lambda = \frac{M}{V_{st}\,\varrho_1} = (\eta_v - \Delta\lambda_l)\,\frac{T_1}{T_{1,s}} = \lambda^*\,\frac{T_1}{T_{1,s}} \tag{2.4}$$

such that M is given by

$$M = \varrho_1\,\lambda\,V_{st} = \frac{p_1}{RT_1Z_1}\,\lambda\,V_{st} = \frac{p_1}{RT_{1,s}Z_1}\,\lambda^*\,V_{st} \tag{2.5}$$

3. Leaking valves cause a volume loss $\Delta\lambda_l^*$ as well as a mass loss $\Delta\lambda$. Since valve leakage distorts the indicator diagram, compression and expansion lines are no more isentropes, and it is preferrable not to account for it in η_v. Indeed, if a valve is not broken but only worn normally, leakage is not excessive and its influence cannot be taken with sufficient precision from a measured indicator diagram, except perhaps when using sophisticated diagnostics software ([7], [8]) or other rather complex computer programs ([2], [5]). An approximate formula for estimating $\Delta\lambda_l$ analytically, together with the associated temperature rise, is given below, equations 4.4 and 4.7.

4. The *isentropic efficiency* η_{is} is defined as the ratio between W_{is}, the work required for lossfree isentropic compression, to W_{real}, which is the work required by the real cycle to compress a mass of gas M, i.e.

$$\eta_{is} = \frac{W_{is}}{W_{real}} \approx \frac{h_{2,is} - h_1}{h_{dpl} - h_1} = \frac{T_{2,is} - T_1}{T_{dpl} - T_1} \tag{2.6}$$

The right hand sides of equation 2.6 after the approximate sign assume $\Delta Q = 0$, see equation 6.1. W_{is} is given by

$$W_{is} = p_1\,V_{st}\,\eta_v\,\frac{T_1}{T_{1,s}} \cdot \frac{Z_1 + Z_2}{2Z_1} \cdot \frac{\kappa}{\kappa - 1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}} - 1\right] \tag{2.7}$$

Note that W_{is} (equation 2.7) is the work necessary for compressing a mass of gas M under temperature T_1, and not $T_{1,s}$, the gas temperature inside the working chamber at $p_{cyl} = p_1$, after the suction event, before compression. W_{is} corresponds therefore to the part of the indicator diagram between p_1 and p_2, with base length $\eta_v \frac{T_1}{T_{1,s}}$ which is smaller than η_v.

3 SURVEY OF THE TEMPERATURE CHANGES OF THE GAS

During its path through the cylinder (in the wider sense of the word, i.e. including suction and discharge plenums) of a reciprocating compressor operating at full load, the gas will take the following temperatures, starting from nominal suction temperature T_1 in the suction pipe:

T_{spl} is the suction plenum temperature at pressure p_1. T_{spl} can differ from T_1 due to late closure of the suction valve allowing hotter gas to flow back, due to heat exchange and due to valve leakage, the only effect to be considered here, for full load operation.

$T_{1,s}$ is the temperature in the working chamber at the end of the suction stroke, at $p_{cyl} = p_1$. $\Delta T_{1,s} = T_{1,s} - T_{spl}$ can be found from equation 6.8 when ΔQ_s and ΔW_s are known. It should be noted that temperature $T_{1,s}$ is the one having the biggest impact on compressor capacity and isentropic efficiency: In fact, the higher $T_{1,s}$, the smaller will be the mass of gas contained in the working chamber at the beginning of the compression stroke, while the work required for isentropic compression remains unchanged. For this reason, ample suction valve flow areas are more important for good efficiencies than ample discharge flow areas.

$T_{2,is} = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\kappa-1}{\kappa}}$ is the temperature at p_2 on the isentrope starting at p_1, T_1.

$T_{2,c}$ is the temperature in the working chamber at the end of the compression stroke at $p_{cyl} = p_2$, on the isentrope (i.e. neglecting any heat exchange during compression, and considering discharge valves to be tight) starting from p_1, $T_{1,s}$, i.e. $T_{2,c} = T_{1,s}\frac{T_{2,is}}{T_1}$.

$T_{2,d}$ is the final discharge temperature of the real compression cycle, in the discharge plenum, at $p_{cyl} = p_2$, to be found from 6.15.

T_{dpl} is the temperature in the discharge pipe, as the result of mixing of two gas streams, one from the head end and one from crank end working chamber, both leaving their discharge valves with different temperatures $T_{2,d}$, and having possibly undergone heat exchange in the discharge plenum:

$$T_{dpl} = \frac{(M\,T_{2,d})_{HE} + (M\,T_{2,d})_{CE}}{M_{HE} + M_{CE}} \tag{3.1}$$

To be exact, each of these states has to be associated with its own compressibility factor Z, however, for simplicity, we shall assume that, on the suction side

$$Z_1 \approx Z_{spl} \approx Z_{1,s} \quad \text{and on the discharge side} \quad Z_2 \approx Z_{2,is} \approx Z_{2,c} \approx Z_{2,d} \approx Z_{dpl} \tag{3.2}$$

4 INFLUENCE OF VALVE LEAKAGE

Even when closed, a leaking valve offers a small flow area A_l considered to be proportional to total length of seat lands, also called valve opening periphery, of that valve, where leakage lift h_l is the factor or proportionality. Typically, we can assume $h_l = 0.25 \times 10^{-3}[mm] = 10^{-5}[in]$. With nominal lift h corresponding to a nominal valve port area A_{gap}, we have

$$A_l = A_{gap}\,\frac{h_l}{h} \tag{4.1}$$

4.1 Volume loss due to valve leakage

Figure 1 shows in the upper part a typical p-θ-chart of a reciprocating compressor operating at full load, and in its lower part the resulting leakage gas flow rates $\frac{dM}{d\theta}$. The areas under these curves represent $\oint \frac{dM}{d\theta} d\theta = \Delta M_l$, the total mass of gas leaking back. With small leakages, neither the indicator diagram nor the gas temperatures will differ significantly from those when operating with tight valves. With this simplification we can make the following statements as to the masses of gas leaking back and their impact on compressor capacity:

1. Leakage through the suction valve:

   (a) During the expansion event, the gas leaking from the working chamber to the suction plenum does not influence compressor capacity, since all the gas leaking back will be taken in immediately afterwards, in addition to the gas normally taken in from the suction plenum.

   (b) During the intake stroke, the suction valve is open and can by definition not be leaking.

   (c) During compression and discharge, any gas leaking back towards the suction plenum reduces the mass of gas in the cylinder, and hence compressor capacity.

2. Leakage through the discharge valve:

   (a) Any particle of gas leaking back through the closed discharge valve during expansion and intake is missing in the discharge plenum, and is in surplus inside the working chamber, since it takes the place of another particle of gas that otherwise would have come in from the suction plenum. Consequently, leakage through the discharge valve during the expansion and intake reduces compressor capacity.

   (b) During compression, any particle of gas leaking back through the discharge valve will immediately be recompressed and discharged shortly afterwards, consequently it does not reduce compressor capacity.

The above statements can be summarized as follows:

- During the first half of a crank shaft revolution, it is the leakage through the discharge valve that reduces compressor capacity, it is given by the area hatched vertically in Figure 1.
- During the second half of a crank shaft revolution, it is the leakage through the suction valve that reduces compressor capacity, this reduction is given by the area hatched horizontally in Figure 1.

From Figure 1, it can be seen that the sum of the areas hatched horizontally and vertically (representing the total leakage volume loss per compression cycle through suction and discharge valves with identical leakage areas A_l) is only slightly smaller than the area of the rectangle a–b–c–d which equals the mass of gas leaking back per crankshaft revolution through the discharge valve during zero load operation, with the suction side being unloaded, discharge plenum pressure and temperature being those of full load operation. This mass of gas can therefore easily be calculated. With reference to Figure 1, we define a correction factor φ (in most cases $0.6 \leq \varphi \leq 1.0$)

$$\varphi = \frac{\textit{sum of the hatched areas}}{\textit{area of the rectangle a–b–c–d}} \tag{4.2}$$

Let p_2, T_{dpl} be the gas state function in the discharge plenum during full load operation, ϱ_{dpl} and Z_{dpl} the corresponding gas density and real gas compressibility factor, further let u_m be the mean gas velocity in the suction valve gap area A_{gap}. With $\varepsilon = \max(0.5, \frac{p_{downstream}}{p_{upstream}})$ as the expansion ratio, and $\Psi \approx \sqrt{(1-\varepsilon)\varepsilon^{\frac{1.526}{\kappa}}}$ as the expansion function (see [5]), the rate of gas flow $\dot{M}_l$ through the closed but leaking discharge valve, as well as the mass of gas $\dot{M}_{100\%}$ delivered during full load operation per compression cycle with tight valves, are given by

$$\dot{M}_l = A_l\, \varrho_{dpl}\, \varphi\, \Psi \sqrt{2RT_{dpl}Z_{dpl}} \qquad \text{and} \qquad \dot{M}_{100\%} = 0.5\, \varrho_1\, A_{gap}\, u_m\, \lambda^* \qquad \left[\frac{kg}{s}\right] \tag{4.3}$$

With the ratio of gas densities $\frac{\varrho_{dpl}}{\varrho_1} \approx \left(\frac{p_2}{p_1}\right)^{\frac{1}{\kappa}}$, the reduction in efficiency of delivery $\Delta\lambda_l^*$ (the suffix * signifying *not yet accounting for temperature increase in the suction plenum*) is given by the ratio of $\dot{M}_l$ over $\dot{M}_{100\%}$

$$\Delta\lambda_l^* = \frac{\dot{M}_l}{\dot{M}_{100\%}} \approx \varphi\, \frac{0.5 \times 10^{-3}}{h\, u_m\, \lambda^*}\, \Psi \sqrt{2\,RT_{dpl}}\, \frac{\varrho_{dpl}}{\varrho_1} \tag{4.4}$$

4.2 Rise in suction plenum temperature due to valve leakage

Any gas leaking back from the discharge plenum to the working chamber, or from the working chamber to the suction plenum will increase the enthalpy and hence the temperature downstream. This effect will take place for *all gas particles leaking back*, independantly of whether they reduce efficiency of delivery (and correspond to the hatched areas in Figure 1) or whether they do not, i.e. enthalpy is carried backwards from the discharge side to the suction plenum by a mass of gas $> M_l$.

On the other hand, only the gas leaking back from the discharge plenum (the place with the highest cycle temperature T_{dpl}) will carry highest specific enthalpy h_{dpl}, those leaking through the suction valve are generally cooler. To simplify matters, we shall not consider a bigger leakage mass (the one that reduces efficiency of delivery, and the one that does not) carrying a smaller specific enthalpy, but calculate total enthalpy input into the suction plenum due to gas leakage to be equal to $M\,\Delta\lambda_l^*\,h_{dpl}$. Mixing in the suction plenum of a mass of gas $M\,(\lambda^* - \Delta\lambda_l^*)$ from the suction line with T_1, with another mass $M\,\Delta\lambda_l^*$ leaking back through closed suction valves and having a temperature T_{dpl}^* will result in a mean temperature T_{spl} in the suction plenum

$$M\,T_{spl}\,\lambda^* = M\,T_1\,(\lambda^* - \Delta\lambda_l^*) + M\,T_{dpl}^*\,\Delta\lambda_l^* \quad \text{or} \quad \frac{T_{spl}}{T_1} = 1 + \frac{\Delta\lambda_l^*}{\lambda^*}\left(\frac{T_{dpl}^*}{T_1} - 1\right) \tag{4.5}$$

Assuming that the increase in final temperature in the discharge plenum T_{dpl} is approximately proportional to the increase in suction plenum temperature T_{spl}, we have

$$\frac{T_{dpl}}{T_1} = \frac{T_{dpl}^*}{T_1}\cdot\frac{T_{spl}}{T_1} \tag{4.6}$$

This increased T_{dpl} in the discharge plenum will in turn send hotter leakage gases to the suction plenum and increase there T_{spl}. Substituting for $\frac{T_{dpl}}{T_1}$ from equation 4.6 in 4.5 we get the new relationship

$$\frac{T_{spl}}{T_1} \approx \frac{\lambda^* - \Delta\lambda_l^*}{\lambda^* - \Delta\lambda_l^*\frac{T_{dpl}^*}{T_1}} \qquad \text{and} \qquad \Delta\lambda = \Delta\lambda_l^*\,\frac{T_{spl}}{T_1} \tag{4.7}$$

$\Delta\lambda_l$ is an estimate for the real loss in efficiency of delivery due to valve leakage, for small $\Delta\lambda_l^* < 5\%$.

4.3 Conclusion

From equation 4.4 it can be seen that the leakage loss $\Delta\lambda_l$ is the higher,

1. the smaller the product of valve lift h multiplied by mean gas velocity u_m in the valve gap,
2. the smaller the efficiency of delivery λ,
3. the smaller the molecular weight $\bar{M}$ of the compressed gas, since the gas constant $R = \frac{8314.5}{\bar{M}}$ is inversely proportional to $\bar{M}$.

Other leakages may occur along piston rings, they are not considered here. Leakages along piston rod packings leave the system and have no influence on temperatures.

5 HEAT TRANSFER BETWEEN THE DUCTS OF THE SUCTION VALVE AND THE GAS FLOWING THEREIN

The following is a very simplified approach, just enough to estimate the order of magnitude of this influence. If it appeared to be of any significance, some research work would have to be done, in particular on heat transfer constants α. Neglecting the heat which eventually is transferred by conduction between the suction valve and the surrounding valve pocket, the amount of heat given to the flowing gas during intake must equal the amount of heat exchanged with the valve during the remainder of the cycle. Since heat transfer coefficients increase with flow velocity, it is sufficient to look at the heat transferred when there is no flow in the valve to get an estimate for ΔQ. Considering the generated surface of the guard (or of the seat in the case of a discharge valve) perpendicular to the valve's sealing surface as a rectangle, whose base length equals the valve opening perimeter $\frac{A_{gap}}{h}$ and whose height to the guard's or seat's thickness t, the surface of the suction valve A_{SV} (3 × the basis surface: valve plate, upper and lower face of the guard) or of the discharge valve A_{DV} (only one basis surface) exposed to the working chamber can roughly be estimated as

$$A_{SV} \approx 3D_{SV}^2\frac{\pi}{4} + A_{gap}\frac{t_{guard}}{h} \qquad \text{and} \qquad A_{DV} \approx D_{DV}^2\frac{\pi}{4} + A_{gap}\frac{t_{seat}}{h} \tag{5.1}$$

where D_{SV} or D_{DV} are the valve's outer diameters. Simplifying by assuming further that gas temperature in the working chamber equals T_1 during one half of a revolution of the crankshaft, and equals $T_{2,is}$ during the other half, the suction valve's surface temperature T_{SV} will be the arithmetic average between $T_{2,is}$ and T_1. With these simplifications, we get for the heat ΔQ_s given to the suction valve's surface during half a crankshaft revolution, and the corresponding temperature rise $\Delta T_{Q,s}$

$$\Delta Q_s = A_{SV}\,\alpha\frac{T_{2,is} - T_1}{2}\cdot\frac{30}{n} = MC_p\,\Delta T_{Q,s} \tag{5.2}$$

The heat transfer constant α is, in a first approximation, proportional to Reynolds number to the power 0.8, see equation 7.4. Reynolds number in turn depends on gas velocity (whose variations inside the working chamber are neglected, since we consider heat transfer between gas in the working chamber and the suction valve during compression and discharge events) and gas density. With molar mass $\tilde{M} = 22.711\varrho_n$ we get

$$\alpha \approx \alpha_n\left(\frac{\varrho_1}{\varrho_n}\right)^{0.8} \qquad C_p = R\frac{\kappa}{\kappa - 1} \qquad R = \frac{8314.5}{22.7\varrho_n} = \frac{366}{\varrho_n} \tag{5.3}$$

$\alpha_n = 35\,[W/m^2 K]$ at standard atmospheric conditions is a good assumption. From equations 5.2, 5.3 and 2.5 and rearranging, we get in SI-units (366 is not dimensionless!)

$$\Delta T_{Q,s} = T_1\frac{0.04}{n}\cdot\frac{\kappa - 1}{\kappa}\cdot\frac{A_{SV}\,\alpha_n}{V_{st}\,\lambda}\left(\frac{p_n}{p_1}\cdot\frac{T_1 Z_1}{T_n Z_n}\right)^{0.2}\left(\frac{T_{2,is}}{T_1} - 1\right) \tag{5.4}$$

The same algorithm can of course be used for the heat exchange during flow through the discharge valve. The heat given up there reduces the temperature of the gas arriving in the discharge plenum, and increases the gas temperature in the working chamber during the expansion event, which in turn may reduce η_v.

6 RISE IN GAS TEMPERATURE DUE TO WORK LOSSES IN VALVES

Let us draw a control surface around the working chamber of a compressor which is pumping gas from p_1, T_1 to p_2, T_2. When conditions are stationary, the sum of all streams of energy entering or leaving the control surface (mechanical work ΔW supplied by the piston, heat transfer ΔQ from the walls to the gas, change in internal energy $M(u_2 - u_1)$ and change in flow work $M(p_2 v_2 - p_1 v_1)$) must equate to zero. Remembering that specific enthalpy h is defined as $h = u + pv$, putting $\Delta T = T_2 - T_1$ and $\Delta h = h_2 - h_1 = C_p\,\Delta T$, we get the required relationship

$$\Delta W + \Delta Q = \Delta H = M\Delta h = M\,C_p\,\Delta T = M\,R\frac{\kappa}{\kappa - 1}\,\Delta T \tag{6.1}$$

According to equation 6.1 and with constant C_p or κ, the two temperature rises, ΔT_Q due to ΔQ and ΔT_W due to ΔW can be found separately and then added.

6.1 Temperature rise due to the suction work loss

Let $\Delta p_{i,SV}$ be the mean effective pressure of the suction work loss in the indicator diagram referring to the full length of the indicator diagram (i.e. $\Delta p_{i,SV} = \eta_v \Delta p_{m,SV}$), with gas density ϱ_1 in the suction plenum and constant volume flow rate $\dot{V}_1 = \frac{n}{60} \cdot \frac{\lambda M}{\varrho_1}$

$$\Delta p_{i,SV} = 3.75 \frac{\eta_v}{\lambda^2} \varrho_1 \left(\frac{\dot{V}_1}{\Phi_{SV}} \right)^2 \tag{6.2}$$

(see [11] or [12], chapter 2) then the suction work loss, with suction plenum temperature T_{spl} instead of T_1, is given by

$$\Delta W_s = V_{st} \, \Delta p_{i,SV} \frac{T_1}{T_{spl}} \tag{6.3}$$

Applying equation 6.1 to the suction stroke, the temperature rise from T_{spl} to $T_{W,s}$ is obtained by substituting from equations 2.5 and 6.3 in 6.1. We obtain

$$V_{st} \, \Delta p_{i,SV} \frac{T_1}{T_{spl}} = \frac{V_{st} p_1}{R \, T_{W,s} \, Z_1} \lambda^{*} R \frac{\kappa}{\kappa - 1} (T_{W,s} - T_{spl}) \tag{6.4}$$

or

$$\frac{\Delta p_{i,SV}}{p_1} \cdot \frac{T_1}{T_{spl}} = \frac{\lambda^{*}}{Z_1} \cdot \frac{\kappa}{\kappa - 1} \left(1 - \frac{T_{spl}}{T_{W,s}} \right) \tag{6.5}$$

For abbreviation, we introduce a constant C_s

$$C_s = \frac{\Delta p_{i,SV}}{p_1} \cdot \frac{T_1}{T_{spl}} \cdot \frac{Z_1}{\lambda^{*}} \cdot \frac{\kappa - 1}{\kappa} \tag{6.6}$$

such that equation 6.5 can be written in the form $C_s = 1 - \frac{T_{spl}}{T_{W,s}}$, and we finally get

$$\frac{T_{W,s}}{T_{spl}} = \frac{1}{1 - C_s} \quad \text{or} \quad \Delta T_{W,s} = T_{spl} \frac{C_s}{1 - C_s} \tag{6.7}$$

The total temperature increase during intake is obtained by adding those from equations 5.4 and 6.7:

$$\Delta T_{1,s} = \Delta T_{W,s} + \Delta T_{Q,s} \tag{6.8}$$

6.2 Temperature rise due to the discharge work loss

Neglecting heat exchange inside the working chamber and assuming tight suction and discharge valves, compression will be isentropic, and the mass of gas M given by equation 2.5 and entering through the suction valve will be the same as the one leaving through the discharge valve. With these assumptions, the discharge stroke will start from inside the working chamber from p_2 and $T_{2,c}$ according to equation 6.9.

$$T_{2,c} = T_{1,s} \left(\frac{p_2}{p_1} \right)^{\frac{\kappa - 1}{\kappa}} \tag{6.9}$$

The discharged gas will arrive in the discharge plenum with temperature $T_{2,d}$, the final discharge temperature of the real compression cycle. Let ν be the length of the discharge stroke, then we have from continuity

$$\nu = \eta_v \frac{\varrho_{1,s}}{\varrho_{2,c}} = \eta_v \left(\frac{p_1}{p_2} \right)^{\frac{1}{\kappa}} \frac{Z_2}{Z_1} \tag{6.10}$$

It follows from 6.10 that equation 6.2 can be used to find the discharge loss $\Delta p_{i,DV}$ as well, using Φ_{DV} instead of Φ_{SV}. The work required to overcome the pressure drops in the discharge valves is then given by

$$\Delta W_d = V_{st}\,\Delta p_{i,DV}\,\frac{T_{2,is}}{T_{2,c}} = V_{st}\,\Delta p_{i,DV}\,\frac{T_1}{T_{1,s}} \tag{6.11}$$

Substituting in equation 6.1, considering that the work input ΔW_d causes the gas temperature to increase by $\Delta T_{W,d}$

$$\frac{T_1}{T_{1,s}}\,V_{st}\,\Delta p_{i,DV} = \frac{V_{st}\,p_1}{R\,T_{1,s}\,Z_1}\,\lambda^*\,R\,\frac{\kappa}{\kappa-1}\,\Delta T_{W,d} \tag{6.12}$$

Simplifying, using equations 6.9 and 6.10 and rearranging, we get

$$\frac{T_1}{T_{1,s}}\cdot\frac{\Delta p_{i,DV}}{p_2} = \frac{\nu}{Z_2}\cdot\frac{\kappa}{\kappa-1}\cdot\frac{1}{T_{2,c}}\,\Delta T_{W,d} \tag{6.13}$$

Again, if for abbreviation we introduce a constant C_d

$$C_d = \frac{\Delta p_{i,DV}}{p_2}\cdot\frac{Z_2}{\nu}\cdot\frac{\kappa-1}{\kappa}\cdot\frac{T_1}{T_{1,s}} \tag{6.14}$$

we obtain, with $\Delta T_{Q,d}$ found in analogy to equation 5.4

$$\Delta T_{W,d} = T_{2,c}\,C_d \qquad \text{and} \qquad \Delta T_{2,d} = \Delta T_{W,d} + \Delta T_{Q,d} \tag{6.15}$$

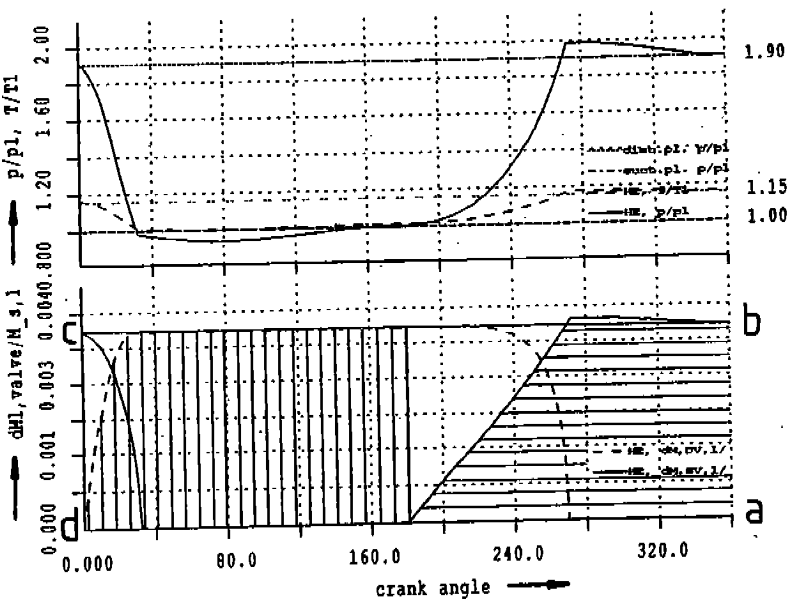

Figure 1: Pressure and temperature in the working chamber,
leakage gas flows in suction and discharge valves, drawn over crank angle