Analysis of Two-Phase Flow in Condensers and Evaporators Based on Two-Fluid Models: Numerical Model and Experimental Comparison

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ABSTRACT

A numerical study of the thermal and fluid-dynamic behaviour of the two-phase flow in conducts is presented. The numerical analysis is based on two-fluid models. The model allows consider that liquid and gas have different velocities and temperatures. The numerical simulation has been developed by means of the finite volume technique based on a one-dimensional transient integration of the continuity, momentum and energy equations. The gas distribution into the tube, pressure, velocities and temperatures of the liquid and gas phase are obtained to resolve the differential equations by means of the Newton-Raphson algorithm. The novelty in this work is how two-fluid model describes the distribution of the interface velocity and temperature differences along the two-phase region. The numerical comparative results show the verification of the numerical model developed, while the experimental comparative results validate the simulation. Experimental and analytical data obtained from literature are used in the validation task.

1. INTRODUCTION

The simultaneous flow of gas and liquid occur in different industrial applications. Evaporators and condensers are elements where always are presence a fluid in two states, liquid and gas, in function of the heat transfer and pressure.

This paper is focused on detailed one-dimensional numerical simulation of phase change phenomena within tubes. A non-equilibrium two-fluid model is used to working with two-phase flow problems. The basic field equation consists of two continuity equations, two momentum equations and two energy equations, one equation by each phase present. The application of two-fluid model that works well in numerical simulation of nuclear or chemistry equipments, where are used extensively, is now used in the numerical simulation of refrigeration systems.

Different empirical correlations are necessary to evaluate the mass, momentum and energy exchanged through the interface, geometric conditions, distribution of the gas and liquid phase into the tube and new terms that appear in the conservation equations. These correlations are known depending on flow pattern maps. A simplified flow pattern map is expressed in function of the gas volume fraction and the liquid and gas velocities.

The gas distribution into the tube, that is evaluated with the gas volume fraction, pressure, velocities and temperatures of liquid and gas are obtained to resolve the differential equations detailed above by means of the Newton-Raphson algorithm.

Different numerical aspects have been evaluated with the aim of verifying the quality of the numerical solution. Convergence errors, discretization errors, and numerical schemes have been analysed. Effects of the boundary conditions, initial value problem and mesh sizes over stability are also considered in the solution. The difference between the numerical simulation data obtained with quasi-homogeneous model and two-fluid model into the tube are illustrated.

2. MATHEMATICAL FORMULATION

Mathematical formulation is based on the application of the conservative equations on each phase, liquid and gas. Then, six-equations are obtained: continuity, momentum and energy for each phase. The assumed hypotheses are: one-dimensional flow, constant cross section, and negligible axial heat conduction in fluid, and heat radiation. The governing equations for the two phases are:

$$\frac{\partial(\rho_g \alpha_g)}{\partial t} + \frac{\partial(\rho_g \alpha_g v_g)}{\partial z} = \Gamma_g \qquad (1)$$

$$\frac{\partial(\rho_l \alpha_l)}{\partial t} + \frac{\partial(\rho_l \alpha_l v_l)}{\partial z} = \Gamma_l \qquad (2)$$

$$\frac{\partial(\rho_g \alpha_g v_g)}{\partial t} + \frac{1}{A}\frac{\partial(\rho_g \alpha_g v_g^{2} A)}{\partial z} = -\alpha_g \frac{\partial p}{\partial z} - (\rho_g \alpha_g g)\sin\theta - \frac{1}{A}\frac{\partial(\alpha_g \tau_{zzg})}{\partial z} + \Gamma_g (v_g - v_l) - C\alpha_g \alpha_l \rho_m \left[\frac{\partial(v_l - v_g)}{\partial t} + v_g \frac{\partial v_l}{\partial z} - v_l \frac{\partial v_g}{\partial z}\right] \qquad (3)$$

$$\frac{\partial(\rho_l \alpha_l v_l)}{\partial t} + \frac{1}{A}\frac{\partial(\rho_l \alpha_l v_l^{2} A)}{\partial z} = -\alpha_l \frac{\partial p}{\partial z} - (\rho_l \alpha_l g)\sin\theta - \frac{1}{A}\frac{\partial(\alpha_l \tau_{zzl})}{\partial z} - \Gamma_g (v_g - v_l) - C\alpha_g \alpha_l \rho_m \left[\frac{\partial(v_g - v_l)}{\partial t} - v_g \frac{\partial v_l}{\partial z} + v_l \frac{\partial v_g}{\partial z}\right] \qquad (4)$$

$$\frac{\partial(\rho_g \alpha_g h_g)}{\partial t} + \frac{1}{A}\frac{\partial(\rho_g \alpha_g h_g v_g)}{\partial z} = \frac{1}{A}\left(q_{wg} P_{wg}\right) + q_{ig} A_i + \Gamma_g h_g \qquad (5)$$

$$\frac{\partial(\rho_l \alpha_l h_l)}{\partial t} + \frac{1}{A}\frac{\partial(\rho_l \alpha_l h_l v_l)}{\partial z} = \frac{1}{A}\left(q_{wl} P_{wl}\right) + q_{il} A_i - \Gamma_g h_l \qquad (6)$$

In the last equations, $\boldsymbol{\alpha_g}$ is void fraction of gas, $\boldsymbol{\alpha_l}$ is void fraction of liquid (α_l =1.0-α_g), and $\boldsymbol{\Gamma_g}$ and $\boldsymbol{\Gamma_l}$ are the mass transfer rate per unit volume of the vapor and the liquid phase respectively. Mass transfer from liquid to gas is equal to mass transfer loss by liquid, this is $\boldsymbol{\Gamma_g = -\Gamma_l}$. In momentum equations $\boldsymbol{\tau_{zzg}}$ and $\boldsymbol{\tau_{zzl}}$ are shear stresses acting on the phase at the wall and at the interface, θ is the inclination angle of the pipe and C is the virtual mass coefficient. The virtual mass coefficient depends of regimen present in the flow. Finally, in the energy equations $\mathbf{q_{il}}$ and $\mathbf{q_{ig}}$ are interfacial heat transfer, $\mathbf{q_{wl}}$ and $\mathbf{q_{wg}}$ are heat transfer through wall and fluid, $\mathbf{P_{wl}}$ and $\mathbf{P_{wg}}$ are the part of the wall perimeter wetted by gas and liquid respectively and $\mathbf{A_i}$ is the interfacial area per unit of volume.

2.1 Discretization

The governing equation (1) to (6) can be integrated in terms of the local averaged fluid variables using the finite control volume technique. The concept of a staggered mesh has been used; therefore velocities are defined at the cell boundaries, while pressure, void fraction and enthalpy are located at the middle of volume.

The mass and momentum equation are used as a sum and a difference equations. This form allows degeneration of the model to homogeneous model when single phase occur. A semi-implicit method is used to obtain equations partially implicit in the time:

$$\left\{\alpha_g{}^n(\rho_g{}^{n+1}-\rho_g{}^n)+\alpha_l{}^n(\rho_l{}^{n+1}-\rho_l{}^n)+(\rho_g{}^n-\rho_l{}^n)(\alpha_g{}^{n+1}-\alpha_g{}^n)\right\}_j + \left\{\left[(\alpha_g\rho_g A)^n v_g{}^{n+1}-(\alpha_g\rho_g A)^n v_g{}^{n+1}\right]_{j+1/2}+\left[(\alpha_l\rho_l A)^n v_l{}^{n+1}-(\alpha_l\rho_l A)^n v_l{}^{n+1}\right]_{j-1/2}\right\}\frac{\Delta t}{V}=0 \quad (7)$$

$$\left\{\alpha_g{}^n(\rho_g{}^{n+1}-\rho_g{}^n)-\alpha_l{}^n(\rho_l{}^{n+1}-\rho_l{}^n)+(\rho_g{}^n+\rho_l{}^n)(\alpha_g{}^{n+1}-\alpha_g{}^n)\right\}_j + \left\{\left[(\alpha_g\rho_g A)^n v_g{}^{n+1}-(\alpha_g\rho_g A)^n v_g{}^{n+1}\right]_{j+1/2}-\left[(\alpha_l\rho_l A)^n v_l{}^{n+1}-(\alpha_l\rho_l A)^n v_l{}^{n+1}\right]_{j-1/2}\right\}\frac{\Delta t}{V}=2\Gamma_g\Delta t \quad (8)$$

$$\left((\alpha_g\rho_g)^n(v_g{}^{n+1}-v_g{}^n)+(\alpha_l\rho_l)^n(v_l{}^{n+1}-v_l{}^n)\right)_{j+1/2} + \left\{(\alpha_g\rho_g)^n{}_{j+1/2}\left[(2v_g)^n{}_{j+1}(v_g{}^{n+1}-v_g{}^n)_{j+1}+(v_g{}^n)^2{}_{j+1}-(2v_g)^n{}_j(v_g{}^{n+1}-v_g{}^n)_j+(v_g{}^n)^2{}_j\right]+ (\alpha_l\rho_l)^n{}_{j+1/2}\left[(2v_l)^n{}_{j+1}(v_l{}^{n+1}-v_l{}^n)_{j+1}+(v_l{}^n)^2{}_{j+1}-(2v_l)^n{}_j(v_l{}^{n+1}-v_l{}^n)_j+(v_l{}^n)^2{}_j\right]\right\}\frac{\Delta t}{2\Delta z}= \left\{(\rho^n{}_m g)_{j+1/2}+\Gamma_g{}^{n+1}(v_l{}^n-v_g{}^n)_{j+1/2}-F_{wg}(\alpha_g{}^n\rho_g{}^n v_g{}^{n+1})_{j+1/2}-F_{wl}(\alpha_l{}^n\rho_l{}^n v_l{}^{n+1})_{j+1/2}\right\}\Delta t -(p_{j+1}-p_j)^{n+1}\frac{\Delta t}{\Delta z} \quad (9)$$

$$\left(1+C\frac{\rho_m{}^2}{\rho_l\rho_g}\right)^n\left((v_g{}^{n+1}-v_g{}^n)-(v_l{}^{n+1}-v_l{}^n)\right)_{j+1/2}+\left\{(v_g{}^2{}_{j+1}-v_g{}^2{}_j)^{n+1}-(v_l{}^2{}_{j+1}-v_l{}^2{}_j)^{n+1}\right\}\frac{\Delta t}{2\Delta z}= -\left(\frac{\rho_l-\rho_g}{\rho_l\rho_g}\right)(p_{j+1}-p_j)^{n+1}\frac{\Delta t}{\Delta z}+\Gamma_g{}^{n+1}\left\{\frac{(\rho_m v_i)^n-(\alpha_g\rho_g)^n v_l{}^{n+1}+(\alpha_l\rho_l)^n v_g{}^{n+1}}{\alpha_g\rho_g\alpha_l\rho_l}\right\}_{j+1/2}\Delta t- \left\{F_{wg}(v_g{}^{n+1})_{j+1/2}-F_{wl}(v_l{}^{n+1})_{j+1/2}-(\rho_m{}^n F_i)(v_g-v_l)^{n+1}{}_{j+1/2}\right\}\Delta t \quad (10)$$

$$\left\{(\alpha_g\rho_g)^n(h_g{}^{n+1}-h_g{}^n)+(\alpha_g h_g)^n(\rho_g{}^{n+1}-\rho_g{}^n)+(\rho_g h_g)^n(\alpha_g{}^{n+1}-\alpha_g{}^n)- (\alpha_g)^n(p^{n+1}-p^n)\right\}_j+\left\{\left[(\alpha_g\rho_g h_g A)^n v_g{}^{n+1}\right]_{j+1/2}-\left[(\alpha_g\rho_g h_g A)^n v_g{}^{n+1}\right]_{j-1/2}\right\}\frac{\Delta t}{V}= (Q_{wg}+Q_{ig}+\Gamma_g h_g)\Delta t \quad (11)$$

$$\Big\{(\alpha_l\rho_l)^n(h_l^{\,n+1}-h_l^{\,n})+(\alpha_l h_l)^n(\rho_l^{\,n+1}-\rho_l^{\,n})+(\rho_l h_l)^n(\alpha_l^{\,n+1}-\alpha_l^{\,n})-$$
$$(\alpha_l)^n(p^{n+1}-p^n)\Big\}_j+\Big\{\Big[(\alpha_l\rho_l h_l A)^n v_l^{\,n+1}\Big]_{j+1/2}-\Big[(\alpha_l\rho_l h_l A)^n v_l^{\,n+1}\Big]_{j-1/2}\Big\}\frac{\Delta t}{V}= \qquad (12)$$
$$(Q_{wl}+Q_{il}-\Gamma_g h_l)\Delta t$$

2.2 Solver

Equations are resolved by means of a Newton-Raphson method. The six governing equations are written in a vector form as:

$$\vec{x}_{m+1}=\vec{F}(x)\cdot\vec{F}'(x)^{-1}+\vec{x}_m \qquad (13)$$

Where $\vec{F}(x)$ is a matrix that contain the six equations in function of variables, $\vec{F}'(x)$ is the Jacobian matrix, $\vec{x}_m$ is the value of the variables in the last iteration, and $\vec{x}_{m+1}$ is the value of the variable to be found. Based on this procedure, variables such as void fraction, pressure, velocities and enthalpies are iteratively evaluated for each control volume.

3. EMPIRICAL CORRELATION

Definition of the regimen is necessary to chose the correct empirical correlation. Factors such as frictional coefficients, heat transfer coefficients and interfacial area are obtained by means of the empirical correlations. Different regimen can be present into the inner tubes. Geometry, position, velocity and heat condition are some parameter that help to find the kind of regimen present. A simplified scheme is used to evaluate the regimen as function of the void fraction, velocity and position following Levy, S. (1999). Bubbly, slug, annular, mist, stratified flow and combination of them can occur in horizontal or vertical tubes.

Frictional forces between the tube wall and liquid phase F_{wl}, the tube wall and gas phase F_{wg} , and interfacial F_i, and heat transfer coefficient at the wall by liquid and gas, and at the interface have been evaluated by means of the empirical expression resumed by NUREG/CR-5535-V1 (1995).

Heat transfer from interface to gas phase Q_{ig} and to liquid phase Q_{il} are evaluated with heat transfer coefficients at interface. In the same form, heat transfer from wall to gas phase Q_{wg} and to liquid phase Q_{wl} are calculated with heat transfer coefficient assumed that only one phase is present in the flux.

$$Q_{ig}=H_{ig}(T_{sat}-T_g)A_i \quad \text{and} \quad Q_{il}=H_{il}(T_{sat}-T_l)A_i \qquad (14)$$

$$Q_{wg}=H_{wg}(T_w-T_g)A_{wg} \quad \text{and} \quad Q_{wl}=H_{wl}(T_w-T_l)A_{wl} \qquad (15)$$

Mass transfer rate per unit of volume Γ_g is the sum of the mass transfer at the interface Γ_{ig} by increment of energy of the one phase and the mass transfer from the wall to interface Γ_w by action of the external heat transfer. This value is obtained by means of the energy balances at the interface and at the wall.

$$\Gamma_{ig}=-\frac{(Q_{ig}+Q_{il})}{(h_g-h_l)_{sat}} \;,\quad \Gamma_w=-\frac{(Q_{wg}+Q_{wl})}{(h_g-h_l)_{sat}} \quad \text{and} \quad \Gamma_g=\Gamma_{ig}+\Gamma_w \qquad (16)$$

In these expression A_i is the interfacial area per unit of volume, A_{wg} and A_{wl} are the superficial area of the wall per unit of volume that are in contact with the gas phase and liquid phase respectively, and $(h_g - h_l)_{sat}$ is a difference of enthalpies at saturation condition. If an evaporation phenomenon is present Q_{wg} is considered zero and when a condensation phenomena is present Q_{wl} is zero. This affected only the evaluation of the mass rate transfer at the wall.

4. RESULTS

Three cases have been taken to test the code developed. The first case is the well-known water faucet problem. This case is widely reported as benchmark case in two fluid model works, Hewitt, G. F., Delhaye, J. M., Zuber N., (1987). It consists of a vertical pipe with 12 meters of length and 1 meter of diameter. Pipe is filled with water to next conditions: at the inlet void fraction 0.2, gas velocity 0.0 m/s, liquid velocity 10 m/s, and temperature 50ºC, and at the outlet pressure 1.0×10^5 Pa.

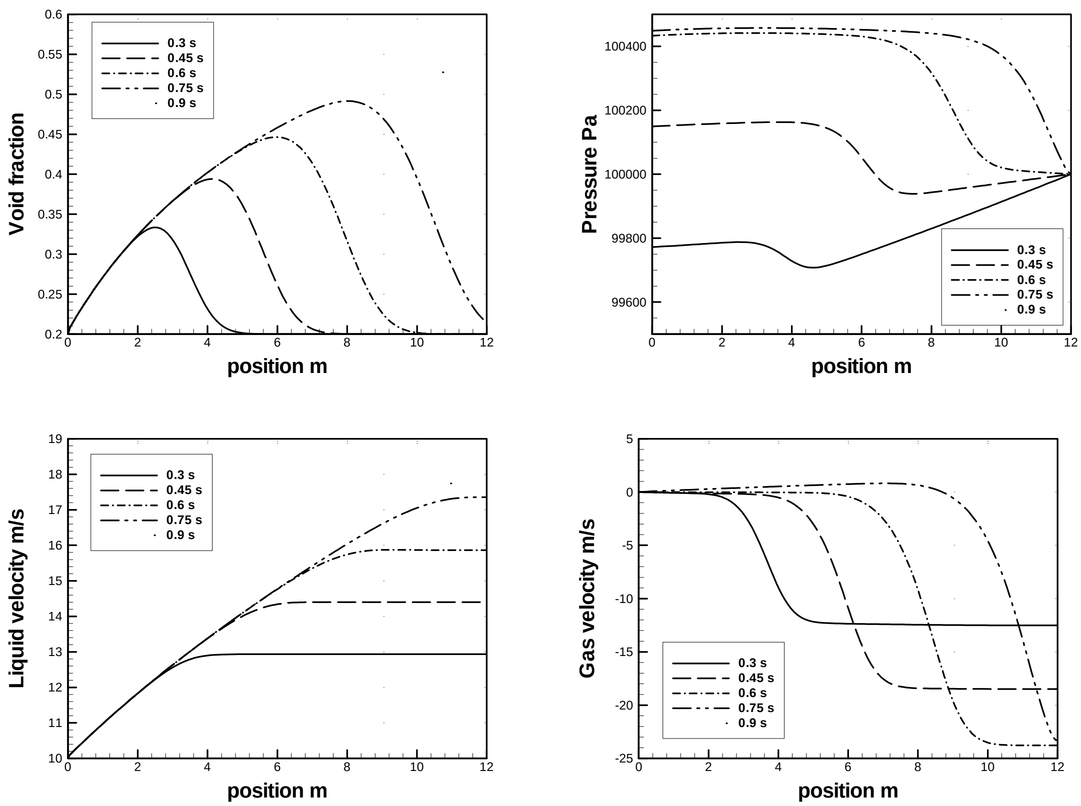

Figure 1. Water faucet results at the time.

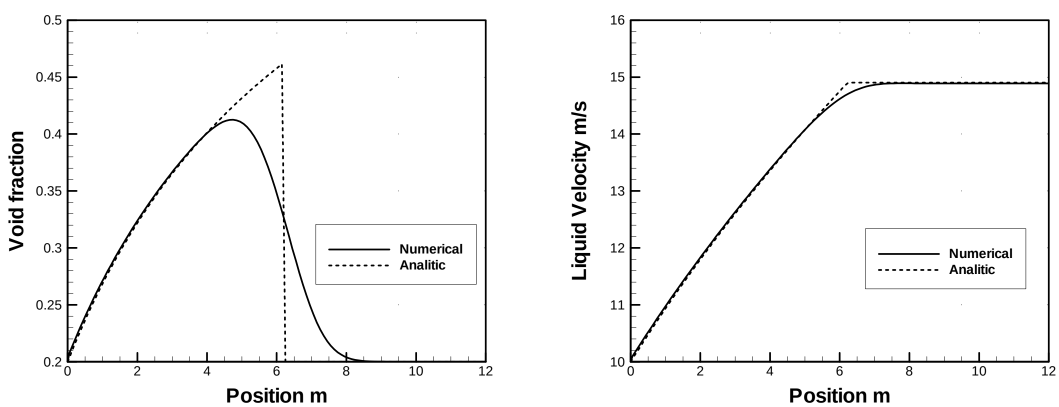

Figure 2. Water faucet solution at 0.5 s.

The water faucet assumes that frictional forces and thermal effects on interface are negligible, because they have a minor effect and is possible to obtain a simplification of the problem. It case help to see the gravity effect over the acceleration of the fluid. Illustrative results of this transient case are depicted in Figure 1, where evolution of the void fraction, pressure, liquid and gas velocities are showed at different times. Different mesh sizes and level of error were proof with this case. A mesh of 120-control volume, 1.0e-8 convergence criteria and a time rate of 0.0005 s. were used to solve this case. Our results are quite similar to results obtained by Cortes (2002). Furthermore, a comparison of the transient case with analytical solution at 0.5 s. to void fraction and liquid velocity are depicted in Figure 2.

The second case is a horizontal water steam evaporator. This case has been simulated with commercial code RELAP5, widely used in simulation of the thermal hydraulic systems in nuclear plants. A qualitative comparison between RELAP5 and our results have been done. The pipe has 6 meters length and 0.00815 meter of internal diameter. Boundary conditions at inlet are: gas velocity 1.1974 m/s, liquid velocity 0.0301 m/s, pressure 1.03361e5 Pa, temperature 100.5 ºC and void fraction 0.69. An external heat flux of 790 W/m^2 is applied.

This case presents a stratified flow regimen and empirical correlations have been chosen in function of this flow. Frictional forces and heat transfer coefficient at the wall and at the interface have been calculated and used in the numerical solution. The comparison is depicted in Figure 3. Behavior of the pressure, void fraction, mass fraction, temperatures and velocities are showed. Similar results have been obtained with our code. Differences between RELAP5 and our code are: pressure 0.1%, temperature 0.095%, and void fraction 4.3%. The possible differences are because we have not assumed the wall friction dissipation in the energy equations and neither the abrupt area change loss in the momentum equations. Furthermore, virtual mass term is negligible because the regime present is a stratified flow, therefore (C=0).

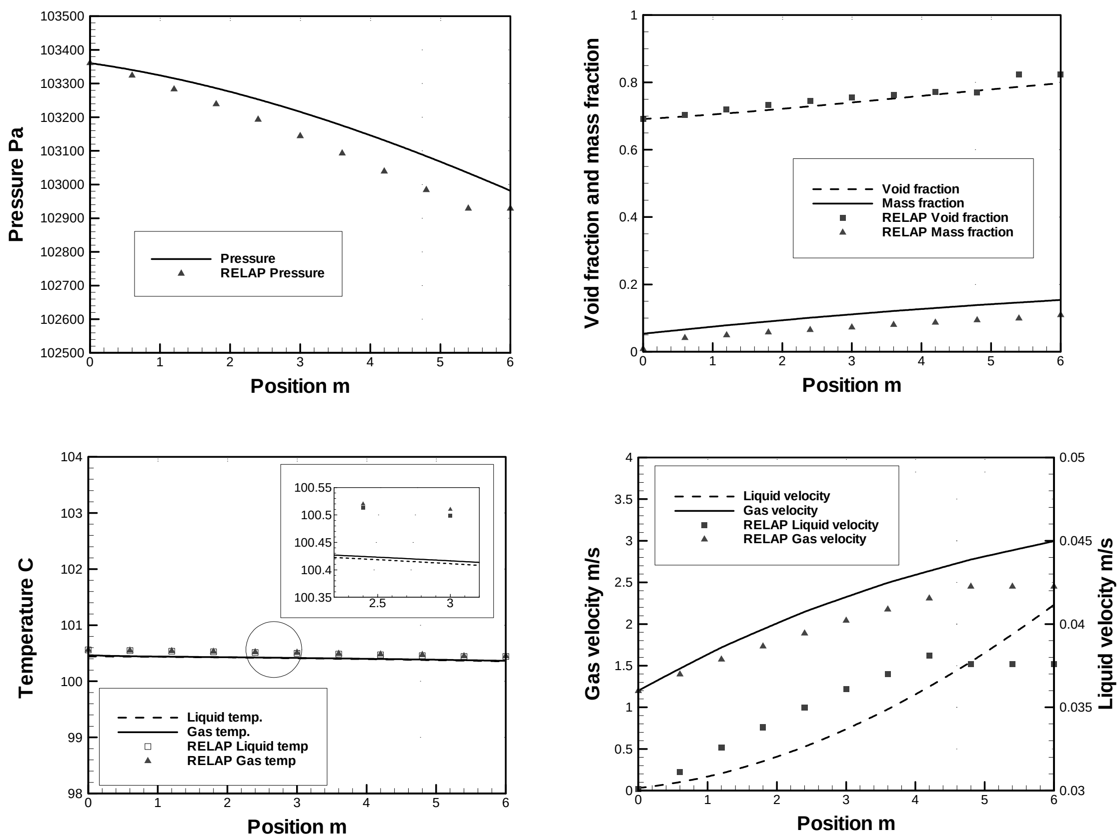

Figure 3. Water evaporator results.

The third case is a comparison between numerical results obtained with quasi-homogeneous model and two-fluid model of the evaporator that works with refrigerant R134a. A short section of the evaporator reported by Rigola, J., Morales, S., Raush, G., and Pérez-Segarra, C.D. (2004) has been used to contrast the results. In the inner of pipe with 0.00815 meter of diameter and 2 meter of length as test long.. The boundary conditions at the inlet of the pipe are: pressure 1.387×10^5 Pa, liquid velocity 0.076 m/s, gas velocity 1.310 m/s, liquid enthalpy 1.748×10^5 J/kg, gas enthalpy 3.88×10^5 J/kg and an external heat flux of 1320 W/m^2 is applied.

This case present stratified flow along of the test tube. Experimental results for pressure and temperature are included. Shorts differences have been obtained between numerical and experimental data such as is illustrated in Figure 4. Difference between experimental and two-fluid temperature data is of 0.93%, while experimental and quasi-homogeneous data is of 1.16% at 0.75 meter of tube. Similar differences have been obtained with the pressure, experimental and two-fluid difference is 0.18% while experimental and quasi-homogeneous difference is 0.683% at 2 meter of tube.

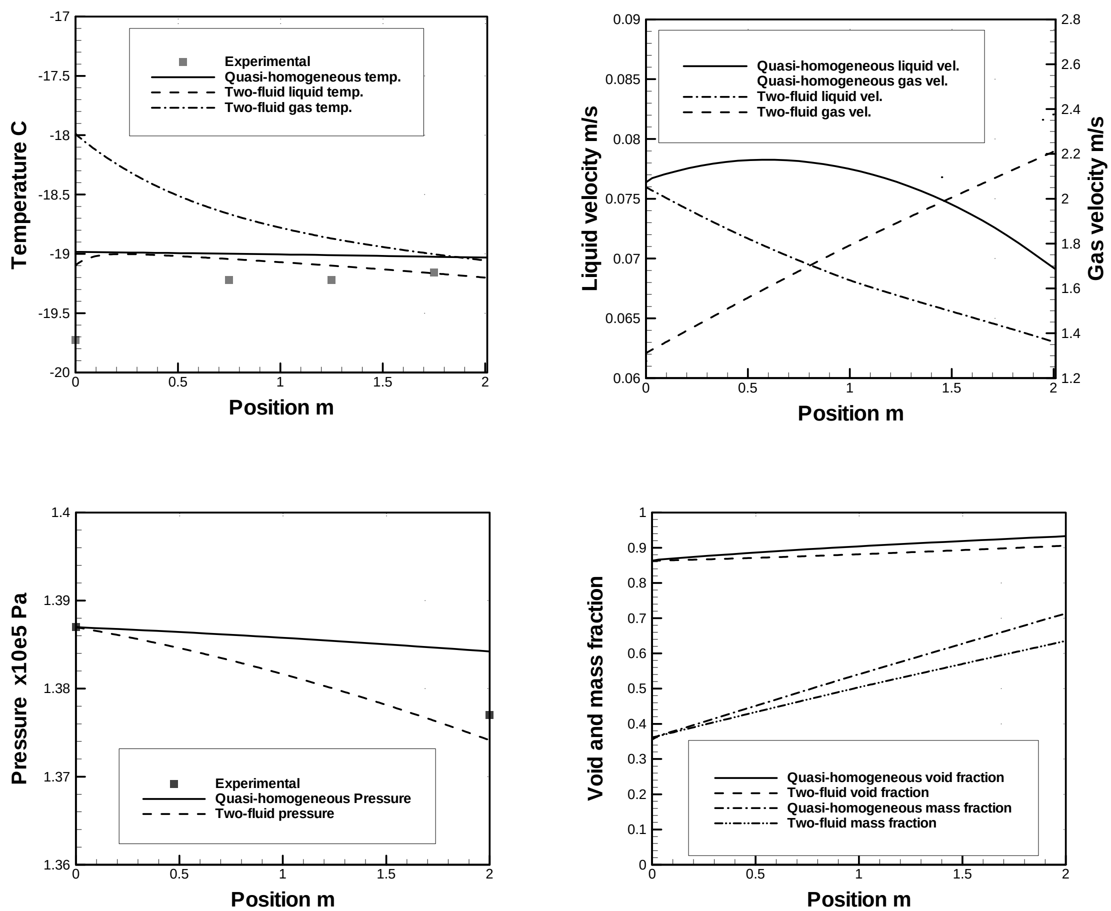

Figure 4. Evaporator with refrigerant R134a

5. CONCLUSIONS

This paper resumes our work with two-fluid model and its application on refrigeration systems. Our first task has been to study the regimen flow presented into horizontal and vertical pipes. A group of different empirical correlations have been collected and applied to develop a simulation code. Secondly, validation and verification of the code have been done with a benchmark case and comparison with a commercial code. A numerical simulation of the evaporator with R134a as refrigerant has also been performed. A comparison between quasi-homogeneous and two-fluid model has been done and short difference has been found. Two-fluid model gives more detail information about the different flow variables (temperature and velocities). The application of the two-fluid model to refrigerant systems allows a clearer understanding of the two-phase flow phenomena into evaporators and condensers.

NOMENCLATURE

α	void fraction	(--)
ρ	density	(kg/m^3)
Γ	mass rate transfer	($kg/s\ m^3$)
τ	shear stress	(N)
A	area	(m^2)
C	virtual mass coefficient	(--)
F	frictional factor	(--)
g	gravity	(m/s^2)
h	enthalpy	(J/kg)
P	perimeter	(m)
p	pressure	(Pa)
Q	heat flow per unit of volume	(W/m^3)
q	heat flow per unit of area	(W/m^2)
t	time	(s)
V	volume	(m^3)
v	velocity	(m/s)
z	length	(m)

Subscripts

g	gas
l	liquid
i	interface
m	mixture
wg	wall-gas
wl	wall-liquid
ig	interface-gas
il	interface-liquid

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