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Yanping Xia
University of Illinois at Urbana-Champaign

Andrew W. Sommers
University of Illinois at Urbana-Champaign

Anthony M. Jacobi
University of Illinois at Urbana-Champaign

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A Numerical Model for Condensate Drops and Bridges Retained on the Air-Side Surface of Heat Exchangers

Yanping XIA[1], Andrew D. SOMMERS[2], Anthony M. JACOBI[3,]*

Department of Mechanical and Industrial Engineering, University of Illinois,
Urbana, IL, USA
[1]Fax: (217) 244-6534, Phone: (217) 333-2328, E-mail: xia@uiuc.edu
[2]Fax: (217) 244-6534, Phone: (217) 244-0778, E-mail: asommers@uiuc.edu
[3]Fax: (217) 244-6534, Phone: (217) 649-3162, E-mail: a-jacobi@uiuc.edu

*Indicate Corresponding Author

ABSTRACT

A model is developed to predict the shape of a drop or condensate bridge by numerically solving the Young-Laplace equation which governs the shape of the liquid-vapor interface, subject to a prescribed shape of the base contour and variation of the contact angle along the base contour. The model is successful in predicting the shapes and volumes of condensate elements on surfaces with widely varying hydrophilicity. In addition to its utility in predicting the volume (or mass) of condensate retained on fin surfaces as drops or bridges, this model can be used to study the effects of hydrophilicity on inter-fin or inter-louver condensate bridging.

1. INTRODUCTION

In a broad range of air-cooling applications, water retention on the air-side surface of heat exchangers is problematic, because it can reduce the air-side heat transfer coefficient, increase core pressure drop, and provide a site for biological activity. In refrigeration systems, the accumulation of frost on the heat exchanger requires periodic defrosting and attendant energy expenditure. When water is retained on these surfaces following the defrost cycle, ice is more readily formed in the subsequent cooling period, and such ice can lead to shorter operation times between defrost cycles. Understanding the shape and size of bridges and droplets is the key to understanding the mechanisms of droplet retention on a surface. The ability then to model these condensate elements accurately is imperative to designing a better heat transfer surface. Much work has already been done, both analytically and numerically, to model these structures on surfaces so only the most germane work will be presented here.

Dussan V and Chow (1983) studied static droplet shapes at critical conditions on an inclined surface for a drop contact line with straight-line segments on the sides. In this view, the droplet was assumed to be elongated and parallel-sided. This analysis was valid only in the limit of small contact angle, and Dussan V (1985) later extended this work to allow for larger contact angles. The model provided closed-form expressions for the maximum volume, speed, and wetted area of a droplet on a surface of inclination, α, but it required knowledge of the advancing and receding contact angles, θ_A and θ_R, as well as the slope of the contact angle with respect to the speed of the contact line, κ_R and κ_A. The most limiting restriction of this analysis was its assumption of small contact angle hysteresis. Dussan V (1987) later included the effects imposed by the motion of the surrounding fluid, but again the analysis was limited to a droplet with small contact angle hysteresis.

Briscoe and Galvin (1991) studied the critical volume of sessile and pendant droplets and found that the critical surface inclination angle, α_c, scaled with $V^{-2/3}$ for sessile droplets where V equals the volume of the droplet at incipient motion. They compared their data with the prediction of maximum volume given by Dussan V (1985) and reported reasonable agreement.

In a numerical study involving free energy minimization of fixed-volume droplets on a vertical surface, Milinazzo and Shinbrot (1988) sought to disprove the hypotheses that the wetted area of a droplet remains unchanged as the

Bond number increases from zero and that a bifurcation instability can occur suddenly. They found instead that the contact angle hysteresis increases with Bond number until either θ_A goes to π or θ_R goes to zero.

In a finite element solution of the Young-Laplace (capillarity) equation, Brown, *et al.* (1980) were able to solve for the shape of droplets on various surfaces of inclination. Their analysis did acknowledge the variation of the contact angle around the base contour, but it only considered the case of a circular base contour and a fixed contact line and predicted the horizontal contact angle, θ_H, to be intermediately located between the maximum and minimum contact angles of the drop, a behavior counter to experimental observations.

The two aforementioned numerical investigations along with an earlier one by Larkin (1967), who solved the capillarity equation using a finite difference technique, found the horizontal contact angle to lie somewhere between the advancing and receding contact angles. This finding, however, stands in contradiction to experimental work by MacDougall and Ockrent (1942) who reported the horizontal contact angle to be almost equal to the advancing contact angle.

Extrand and Kumagai (1995) studied contact angle hysteresis, droplet shape, and the retentive force for water and ethylene glycol droplets at the critical condition on polymer and silicon surfaces using a tiltable plane. They found that surfaces with large contact angle hysteresis produce more elongated drops. Similarly, the retentive force was found to increase with the elongation of the droplet.

In a numerical study of droplets at the critical condition, Dimitrakopoulos and Higdon (1999) solved for the droplet configuration that produced minimum contact angle hysteresis (i.e. θ_A -θ_R) for a specified advancing angle θ_A and Bond number. They equated the pressure contributions from gravity and surface tension at the liquid-vapor interface and set no requirements on base contour of the droplet. They found the droplet shape was elongated in the direction perpendicular to the gravitational force, a result inconsistent with experimentation.

In two recent reports by El Sherbini and Jacobi (2004), droplet shapes were studied experimentally. The droplet shape was approximated using a 'two-circle method' in which the droplet profile is fitted with two circles sharing a common tangent at the apex of the droplet. The volume was then calculated by integrating the profile around the circumference of the base. This method was found to accurately predict the volume of droplets, knowing only the contact angle and shape of the three-phase contact line. This method, however, was intended only for conventional surfaces of homogeneous roughness.

Fan and Wang (2003) performed a stability analysis of the liquid-bridging force between two surfaces. They found that solutions describing the equilibrium shape of bridges become physically unfeasible as the liquid volume approaches zero. Their analysis, performed by perturbing various equilibrium solutions and examining the change in system free energy, revealed that a liquid bridge may become unstable under certain circumstances. Fan and Wang attributed this phenomenon to a bridge breaking into smaller droplets as its volume decreases. Their analysis, however, was limited to the case of two-dimensional axial symmetric contact (*i.e.* liquid bridging between a solid sphere and flat surface) and two-dimensional plane strain contact (*i.e.* liquid bridging between a long cylinder and flat surface).

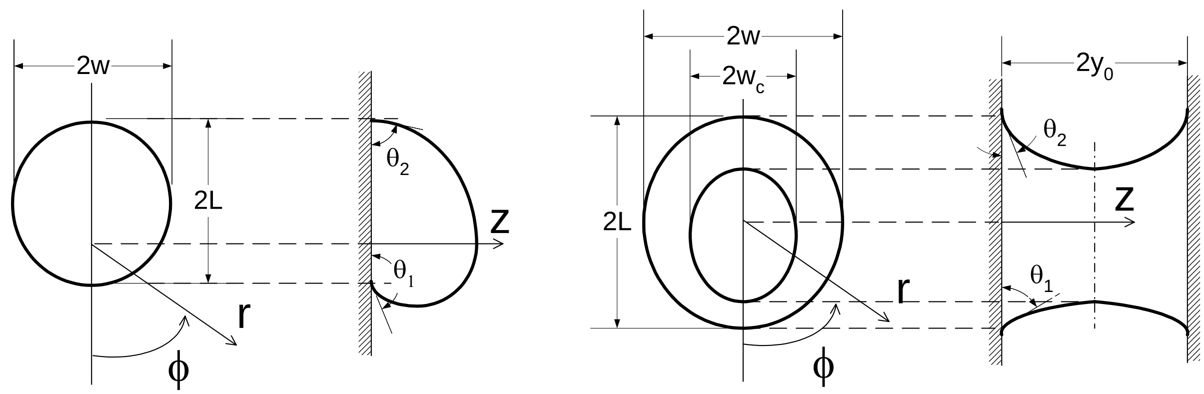
In a numerical paper on fluid bridges, Dimitrakopoulos and Higdon (2003) studied the specific conditions for displacement of three-dimensional bridges from solid boundaries in a pressure-driven Stokes flow. The bridges were assumed to be symmetric about their midplane, but fore-aft asymmetry was permitted due to deformation in the flow direction. The contact line of the bridge was then optimized to resist the largest flow rate while still adhering to the surfaces. The critical flow rate was found to be sensitive to the viscosity ratio of the bridge and was strongly affected by the plate spacing.

The technical literature is indeed replete with articles aimed at modeling droplets; however, only a very small subset of that research has attempted to model droplets and bridges simultaneously. Furthermore, most of these earlier works have had constraining limitations (*i.e.* small contact angle hysteresis, unrealistic base contour shapes, unusual azimuthal contact angle variations, etc.). The authors are aware of only a few papers that have modeled and compared the three-dimensional nature of condensate elements both numerically and experimentally.

In this paper, a numerical model to predict the shape of a drop or condensate bridge will be described. The predicted volume and shape will be compared to the experimental data and images condensate elements on surfaces with different hydrophilicities. A generally applicable model for predicting the volume (or mass) of condensate retained on fin surfaces as drops or bridges can be used to study the effects of hydrophilicity on inter-fin or inter-louver condensate bridging.

2. PROBLEM DESCRIPTION AND EXPERIMENTAL METHOD

The physical situations of interest, a droplet on a vertical surface and a bridge between two vertical surfaces, are shown in Figure 1(a) and 1(b). Cylindrical coordinate system is adopted: z-direction is vertical to the surface(s); in the case of a droplet, the origin is the projection on the vertical surface of the point on the droplet surface that has the largest z-coordinate (it is chosen in such a way to ensure for each pair of ϕ and r, there is only one z value on the droplet surface); in the case of a bridge, the origin is chosen to provide $r(\phi=0°)=r(\phi=180°)=L$; and the azimuthal angle ϕ equals zero when r-direction parallels the direction of gravity. When modeling a bridge, only half of the body is considered due to the symmetry according to the center plane.



(a) (b)

Figure 1: Schematics of a droplet on a vertical surface (a) and a bridge between two vertical surfaces (b).

The apparatus used to study drop shapes is described in details in ElSherbini and Jacobi (2004). The drop to be examined rested on the test surface, which was attached to a fixed plate. A camera and a light source were connected to an arm that rotated around the drop. The fixed plate and rotating arm were attached to a larger plate which can be tilted to different angles of inclination. Although the study in this work was conducted with vertical surface(s), it can be easily extended to surface(s) at different inclination angles. A digital camera with a 6:1 macro close-up lens was used to capture images of the droplets for future software processing. A xenon light was used at an angle of 180° from the camera, illuminating around the drop profile. The droplets studied in this work were water droplets, and were studied on a fixed plate inside a vaportight, transparent box that was saturated with water vapor to reduce the evaporation rate. Tests were conducted to verify that refraction through the box did not affect the recorded images.

A similar apparatus was developed to test bridge shapes. It consisted of two surfaces with controllable distance between them to simulate variable fin spacing. Both of the surfaces were inside a box to reduce the evaporation rate. Images of bridge profiles can be recorded at different azimuthal angles, similar to drops. The dimensions of bridge contours can be determined from locations of the edges of profiles taken at different azimuthal angles.

The droplet or bridge was injected onto the test surface(s) from a micro-syringe, which provided the volume measurement with an accuracy of ± 0.2 mm^3.

3. NUMERICAL VOLUME PREDICTION MODEL

The Young-Laplace equation may be written for any point on the liquid-vapor interface:

$$\Delta P = \gamma K \,, \tag{1}$$

where ΔP is the pressure difference across the interface, γ is the liquid surface tension, and K is the mean curvature. The pressure in equation (1) may be obtained from the hydrostatic pressure,

$$\Delta P = c_1 r \cos\phi + c_2 , \tag{2}$$

where $c_1 = \Delta\rho g$ ($\Delta\rho$ is the density difference across the interface and g is the acceleration of gravity), and $c_2 = \Delta P_0$, which is the pressure difference at the origin, and equals $\Delta\rho g r(\phi{=}180°, z{=}0)$. The surface of a droplet (or bridge) can be expressed as a function $r=r(\phi, z)$. Let the subscripts 'ϕ' and 'z' denote partial derivatives with respect to ϕ and z, respectively. The mean curvature at any point on the surface of the droplet (or bridge) can be expressed as:

$$K = \frac{-2r_\phi r_z r_{\phi z}/r^2 + r_{\phi\phi}/r^2\left(1+r_z^2\right) + r_{zz}\left(1+r_\phi^2/r^2\right) - r_\phi^2/r^3}{\left(1+r_\phi^2/r^2+r_z^2\right)^{3/2}} . \tag{3}$$

The basic idea of the volume prediction model is to numerically solve the Young-Laplace equation, that is, the partial differential equation for $r(\phi, z)$ resulted after substituting Eqs. (2) and (3) into (1). The PDE is solved under the boundary conditions including the shape of the base line and the contact angles along the base line. The boundary conditions are determined based on the observation of images captured using the digital camera at different azimuthal angles.

Note that at ϕ=0° and ϕ=180°, r_ϕ=0. And $r_{\phi\phi}$ equals to zero for the degenerate case. Thus Eq. (3) can be reduced to

$$K = \frac{r_{zz}}{\left(1+r_z^2\right)^{3/2}} . \tag{4}$$

Substitute Eqs. (4) and (2) (with ϕ=0° and ϕ=180°) into Eq. (1), we get a differential equation for the profile at ϕ=0° and ϕ=180°,

$$\frac{r_{zz}}{\left(1+r_z^2\right)^{3/2}} = \begin{cases} \left(c_1 r + c_2\right)/\gamma, & \phi = 0^\circ \\ \left(-c_1 r + c_2\right)/\gamma, & \phi = 180^\circ \end{cases} , \tag{5}$$

with boundary conditions

$$r_z\left(z=0\right) = \begin{cases} -\cot(\theta_1), & \phi = 0^\circ \\ \cot(\theta_2), & \phi = 180^\circ \end{cases} \quad \text{for droplets, and} \tag{6}$$

$$r_z\left(z=0\right) = \begin{cases} -\cot(\theta_1), & \phi = 0^\circ \\ \cot(\theta_2), & \phi = 180^\circ \end{cases} \quad \text{for bridges.} \tag{7}$$
$$r_z\left(z=y_0\right) = 0,\ \phi = 0^\circ,\ 180^\circ$$

Integrating Eq. (5) and rearranging, we have

$$r_z = -\frac{0.5c_1/\gamma z^2 + c_5 z + c_3}{\sqrt{1-\left(0.5c_1/\gamma z^2 + c_5 z + c_3\right)^2}} , \tag{8}$$

where c_3 and c_5 are constants and are determined using boundary conditions (6) and (7) for droplets and bridges respectively. Then Eq. (8) is solved numerically using finite-difference method, and the profiles for ϕ=0° and ϕ=180° are determined. For the profiles at other azimuthal angles, an initial guessed profile is obtained by assuming r_ϕ=0 and $r_{\phi\phi}$=0. Then starting from the contact line toward larger z-coordinate, the guessed profile is used to calculate the partial derivatives in Eq. (3) except for r_{zz}, and the r-coordinate at a higher z-coordinate is updated using r_{zz}. This process continues until convergence, and the coordinates of each point on the liquid-vapor interface are determined. Finally, the volume of the condensate element is calculate with

$$V = 2\sum_{\phi=0}^{\pi}\sum_{r=0}^{r(\phi)}\sum_{z=0}^{z(r,\phi)} r\Delta\phi\Delta r\Delta z\,. \tag{9}$$

4. RESULTS

The numerical results for droplets and bridges on three different surfaces will be shown. Table 1 lists the measurement of some critical geometries, contact angles and the volumes of these droplets and bridges. In calculating the numerical results, the boundary conditions were given such that the azimuthal variation of the contact angle around the contact line followed a third-order polynomial, and the base contour was assumed to be elliptical and continuous, as reported in ElSherbini and Jacobi (2004) for droplets. Figure 2 shows the variation of contact angle with azimuthal angle for a bridge on surface B. The contact angle can be fit by a third-degree polynomial of the azimuthal angle, with a coefficient of determination, R^2, of 0.99. Therefore, a third-order polynomial relation as proposed by ElSherbini and Jacobi was also used for bridges.

Table 1: Geometry, contact angle, and volume measurements

	Volume, V (mm^3)	**Contact angle, θ_1 (°)**	**Contact angle, θ_2 (°)**	**Contact line major axis, L (mm)**	**Contact line minor axis, w (mm)**	**Bridge center width, w_c (mm)**	**Half surface distance, y_0 (mm)**
Droplets on surface A	2.0	86	68	1.0	1.0	N/A	N/A
	4.0	86	59	1.5	1.3	N/A	N/A
	6.0	87	57	1.7	1.5	N/A	N/A
	8.0	97	56	1.7	1.6	N/A	N/A
Bridges on surface B	2.8	35	22	1.2	1.1	0.5	0.76
	7.0	40	23	1.4	1.5	0.8	1.1
	10.0	77	54	1.4	1.7	1.3	0.86
Bridges on surface C	5.0	89	77	1.1	1.3	1.4	0.59
	9.0	91	70	1.5	1.5	1.4	0.67
	11.0	85	59	1.7	1.4	1.3	0.83

The volume prediction obtained with the numerical model is compared to the volume measurement in Figure 3. The model prediction agrees with the experimental data within the accuracy of the volume measurement ($\pm$0.2 mm^3). The accuracy of the model mainly depends on how accurate the boundary conditions are provided. The results supported and extended the findings of ElSherbini and Jacobi (2004) on azimuthal variation of the contact angle and the base contour shape for droplets.

The numerical solution provides the coordinates of the drop or bridge surface, which are subsequently plotted for visualization, revealing the full, three dimensional shape of the retained condensate. Figure 4 and 5 compares the calculated shapes of a drop and two bridges to profiles captured using a digital camera at three different azimuthal angles. The profiles at several other azimuthal angles were also compared, but were not shown here. Good agreement was found in the comparisons, which further validates the numerical model. The dramatically different shapes of bridges shown in Figure 5 were caused by the different hydrophilicity of the surfaces they rested on. Surfaces with differing hydrophilicity exhibit different contact-angle variation along the base contour, which in turn significantly affects the shape of the condensate drop or bridge forming on the fin surfaces. Without any assumption on the shape of a droplet or a bridge (such as the 'two-circle method' proposed by ElSherbini and Jacobi, 2004), the model can be applied to any surface, given the azimuthal variation of the contact angle and the contact line shape are known. In addition to its utility in predicting the volume (or mass) of condensate retained on fin surfaces as drops or bridges, this model can be used to study the effects of hydrophilicity on inter-fin or inter-louver condensate bridging.

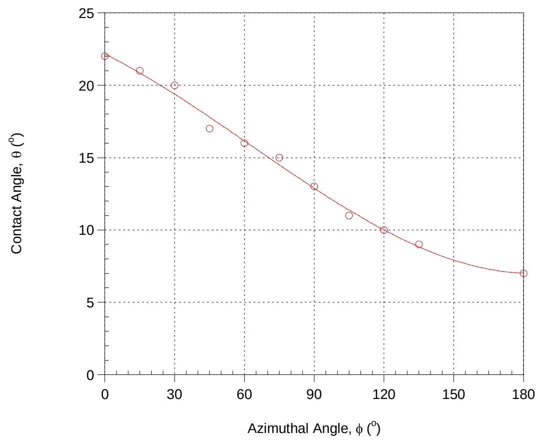


Figure 2: Variation of contact angle with azimuthal angle for a bridge on surface B. A third-degree polynomial fits the data with $R^2 = 0.99$.

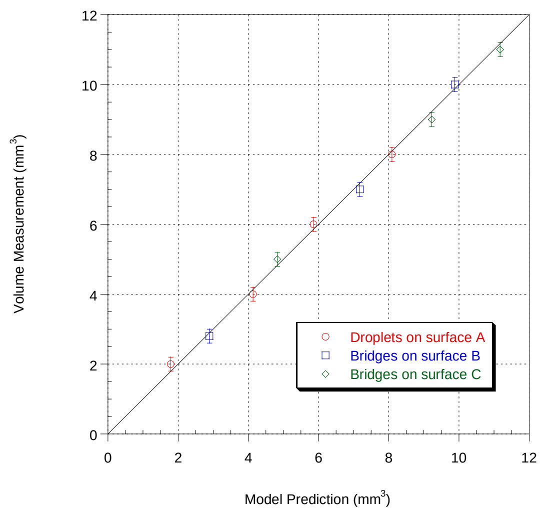
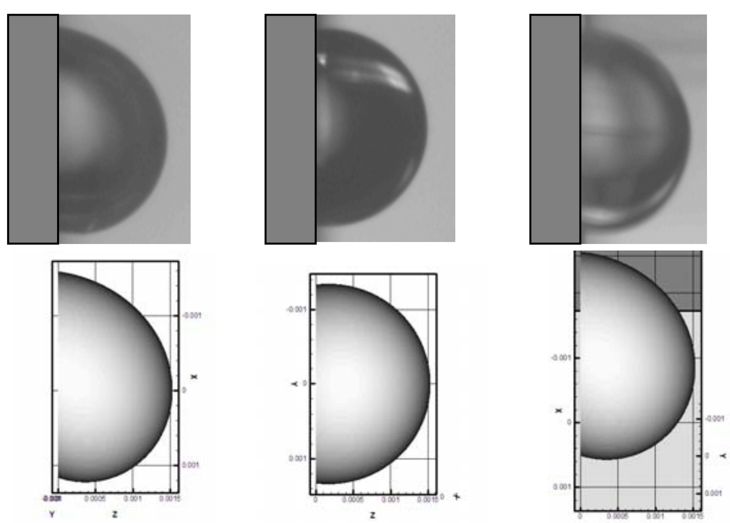
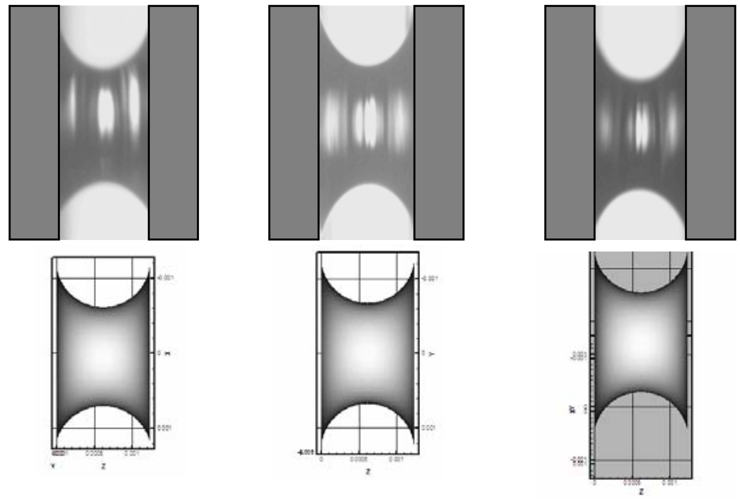


Figure 3: Predicted droplet and bridge volumes using the numerical model versus measured values.



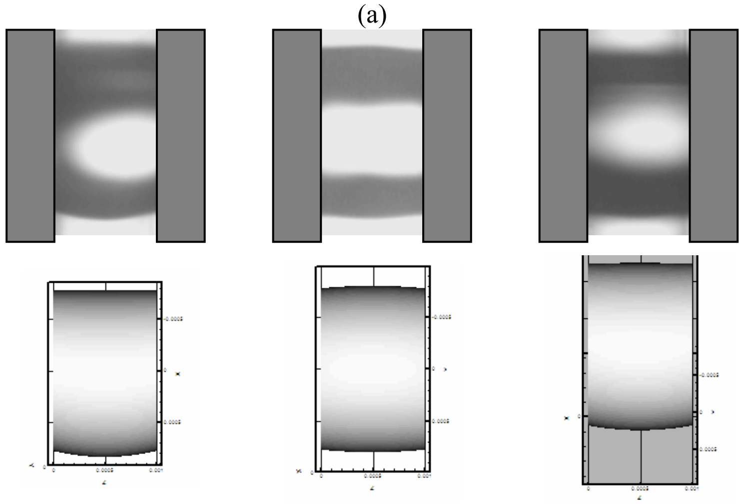
ϕ=0° ϕ=90° ϕ=150°

Figure 4: A comparison of the shape of a drop on surface A captured using a digital camera from three different azimuthal angles (ϕ, the picture shows a side view to the drop/bridge when ϕ=0°), to the predicted shape.



ϕ=0° ϕ=90° ϕ=150°

(a)



ϕ=0° ϕ=90° ϕ=135°

(b)

Figure 5: A comparison of the shapes of two bridges captured using a digital camera to the predicted shape: (a) on surface B; (b) on surface C..

5. CONCLUSIONS

A model is developed to predict the shape of a drop or condensate bridge by numerically solving the Young-Laplace equation which governs the shape of the liquid-vapor interface. The equation is solved under the boundary conditions of a prescribed shape of the base contour (contact line) and variation of the contact angle along the base contour. The predicted shapes are compared to those obtained experimentally. The model is successful in predicting the shapes and volumes of condensate elements on surfaces with widely varying hydrophilicity. In addition to its utility in predicting the volume (or mass) of condensate retained on fin surfaces as drops or bridges, this model can be used to study the effects of hydrophilicity on inter-fin or inter-louver condensate bridging.

NOMENCLATURE

g	acceleration of gravity	(m s^{-2})	**Greek Symbols:**		
K	mean curvature	(m^{-1})	ϕ	azimuthal angle	(°)
L	contact line major axis	(m)	γ	surface tension	(N m^{-1})

ΔP	pressure difference	(Pa)	θ	contact angle	(°)
V	droplet volume	(m^3)	θ_1	contact angle at ϕ=0°	(°)
w	contact line minor axis	(m)	θ_2	contact angle at ϕ=180°	(°)
w_c	bridge center width	(m)	ρ	density	(kg m^{-3})
y_0	half surface distance	(m)			

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