Fall 2014

Evaluation of methods for estimating prestress losses in high-strength structural concrete

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By Mayren Yelitza Mata Carrillo

Entitled EVALUATION OF METHODS FOR ESTIMATING PRESTRESS LOSSES IN HIGH-STRENGTH STRUCTURAL CONCRETE

For the degree of Master of Science in Civil Engineering

Is approved by the final examining committee:

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Julio A. Ramirez

Approved by Major Professor(s): ________________________________

Approved by: Dulcy M. Abraham 12/04/2014

Head of the Department Graduate Program Date
EVALUATION OF METHODS FOR ESTIMATING PRESTRESS LOSSES
IN HIGH-STRENGTH STRUCTURAL CONCRETE

A Thesis
Submitted to the Faculty
of
Purdue University
by
Mayren Y. Mata Carrillo

In Partial Fulfillment of the
Requirements for the Degree
of
Master of Science in Civil Engineering

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Purdue University
West Lafayette, Indiana
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ABSTRACT


The purpose of this study was to evaluate the applicability of several current approaches used to estimate losses in prestressed concrete members with compressive strengths greater than 15ksi. The scope of the study focused on time-dependent losses for normal weight concrete bonded applications. The approaches evaluated were the PCI Design Handbook (2010) method, the AASHTO Specifications (2012) refined method, the PCI Bridge Design Manual (2003) time-dependent analysis using both the AASHTO (2012) and the PCI-BDM (2003) creep and shrinkage models, and a time-step method developed by Swartz (2010). The methods were compared to existing data on prestress losses from twenty-two specimens with compressive strengths from 11ksi to 18ksi. However, a paucity of data existed for strengths higher than 15ksi, with only seven specimens available. Therefore, insufficient data was available to justify a change to the current limit. Based on the comparison of all approaches evaluated, the PCI-BDM (2003) time-dependent method using the PCI-BDM (2003) creep and shrinkage models was shown to give conservative estimates close to the measured losses from the available specimen data. If a simpler analysis is desired, the AASHTO (2012) refined method could be applied. Although, caution is recommended when using this method, since the analysis conducted in this study showed that it could result in an underestimation of the losses within the range of existing data. Considering the scatter in the available
data, it is recommended that more tests be carried out in order to properly evaluate extension of current approaches to design concrete strengths greater than the current limit of 15ksi. Guidance is given in this thesis on the key design parameters that should be considered in such experimental evaluation.

**Keywords:** prestress losses, high-strength concrete, creep, shrinkage, AASHTO, PCI.
CHAPTER 1. INTRODUCTION

1.1. Need for Study

The use of high strength concrete (HSC) and the strength defining it have increased over the years. In the 1950s, 5ksi was considered high strength, while currently 8ksi is defined to be the lower limit for a HSC classification. There are specific examples of bridges with strengths around 13ksi to 14ksi, such as the North Concho river overpass and the Louetta road overpass, and it is common to experience higher strengths than specified. The use of concrete with higher compressive strengths has been increasing throughout the years as often can lead to a reduction in the overall cost of the project and improved life-cycle performance. Using a higher concrete compressive strength can result in smaller members, larger spacing between sections, longer beams, reduced dead loads resulting in smaller foundations, and also lower frequency of maintenance due to higher resistance. Another important benefit is that a larger compressive strength in the concrete allows the use of larger prestress force which helps with the control of cracking in long flexural members, resulting in improved performance.

Prestress loss estimation plays an important role in the design and analysis of prestressed concrete members. The designer needs to accurately estimate the prestress losses. Underestimation of losses can result in cracking of the concrete during service loads and transfer. Over-predicting them can lead to a design that is too conservative and may cause a larger camber from a higher specified initial prestressing force.

To date, the study of time-dependent prestress losses has mostly focused on concrete with normal compressive strength or in the lower range of high strength. In addition,
the current methods available for the calculation of losses are limited to strengths of 12ksi in the Precast/Prestressed Concrete Institute Bridge Design Manual (PCI-BDM, 2003) approach and 15ksi for the American Association of State Highway Officials Specifications (AASHTO, 2012), restricting the designer’s options. Efforts to study the loss calculation methods for strengths higher than 15ksi have been carried out during the last decade with the most recent one almost seven years ago. Extrapolating the available methods to higher strengths is questionable since the behavior of high-strength concrete (HSC) is different than normal strength concrete and impacts estimation of prestress losses. HSC usually experiences lower shrinkage and creep due to the lower water-to-cement ratio and stronger aggregates used in the mix impacting time-dependent losses.

1.2. Objectives, Approach, and Scope of Work

The goal of this study was to evaluate the current approaches to estimate losses in normal-weight prestressed concrete members with bonded strands and with design compressive strengths higher than 15ksi. The performance of the methods was assessed by comparing them to available data from previous studies.

An extensive literature review was conducted in order to understand prestress losses and the impact of high-strength concrete in their estimation. Next, the emphasis was on evaluating the current methods available from design codes, manuals, and proposed in previous studies. The methods were used to calculate the losses on sections with strengths greater than 13ksi evaluated in previous research, and the results were compared to the available measured data. From this, conclusions and recommendations were drawn on whether the methods could give a reasonable estimate of the losses and if the specified limits should be modified.
The scope of work of this study focused on the methods currently most frequently used by designers: the PCI Design Handbook (2010) method, the AASTHO (2012) refined method, and the PCI-BDM (2003) time-dependent analysis; along with a time-step method developed by Swartz (2010) using the creep, relaxation, and shrinkage models defined by AASHTO (2012). This study focused on the long-term losses for simple span, normal weight concrete bonded applications.

1.3. Summary of Content

The necessary background information is given in Chapter 2. High-strength concrete and prestress losses are discussed, and each method studied is explained. Chapter 3 contains a discussion of the methods and an explanation of the findings from comparing the estimates obtained using each method to the measured values from available data. Chapter 4 presents the findings and conclusions of the study together with recommendations for future work.
CHAPTER 2. REVIEW OF PRESTRESS LOSSES AND ESTIMATION APPROACHES

2.1. Prestress Losses

Several factors can reduce the initial jacking force introduced in the prestressing strands. Determining this reduction in the prestressing force is important since under-predicting it can cause cracking of the concrete under service loads. While over-predicting the prestress losses can give a design that is too conservative, and inefficiencies such as girder span length limitations and requiring larger initial prestressing forces that end up causing more camber than desired (Waldron, 2004). Therefore, it is important to understand the sources of losses and the methods that are available to estimate them.

The loss of prestressing force in the tendons is divided into two major categories: instantaneous and time-dependent. The first ones are those that occur immediately during stressing and include friction, anchorage seating, and elastic shortening of the concrete. Time-dependent losses are due to concrete creep and shrinkage, and steel relaxation. The focus of this research was on effects of using higher concrete strengths on prestress losses related to time-dependent effects.

2.1.1. Instantaneous Losses

In posttensioning applications, frictional losses occur from the rubbing between the duct and the tendons at the moment of jacking. There are two components: curvature and wobble. The first one is due to the changes of angles in the tendon profile, while the
latter is from unintentional angle changes and depend highly on the type of cover and tendons used. Anchorage losses are due to the tendons loosening caused by the strands retracting at release pulling the anchors until the wedges lock. This loss only affects a part of the member and is called the anchor setting length, which is dependent on the friction losses. The type of wedges used and the strand stress greatly affect the amount of slippage. Similarly in pretensioned applications, friction losses can occur during stressing at strand drape points and during anchorage seating; however, typically these potential losses are compensated by overstressing of the strands during pretensioning operation.

Elastic shortening is the shortening of the concrete member as the prestress is being transferred, which causes a reduction in the length of the prestressing steel. In effect, reducing the strand length leads to a lower prestressing force. This change in stress due to elastic shortening is calculated whenever a load is applied, and can result in either a loss at transfer or possibly a gain when superimposed loads are placed. The calculations will depend on whether it is a pretension or a post-tension application. In post-tension, the loading sequence of the tendons is a factor since losses occur at the previously tensioned strands as each of the subsequent tendons are stressed. If there is only one tendon, then there is no elastic shortening loss because it would happen before it is anchored. In the case of unbonded applications, the change in stress in the steel is not the same as the stress change in the concrete at the same level because of the ability of the steel to slide, so the average change in stress of the concrete at the level of prestress is considered in the calculations instead. This is also applicable in the case of creep calculations.

2.1.2. **Time-Dependent Losses**

The time dependent losses are those that change over the life of the structure. These losses are the effects of steel relaxation as it is under sustained stress, concrete creep,
and concrete shrinkage. Although they are caused by different sources, they are all dependent of each other. The estimation of the time-dependent losses is essential for the calculation of deflections in the member, which are a performance factor needed to control to assure an acceptable final design. In the case of time-dependent losses, increasing the initial stress will not necessarily reduce them, as it is usually done to compensate for instantaneous losses. A higher initial stress can actually lead to an increase in losses due to creep and relaxation.

2.1.2.1. Creep

Creep is the ability of the concrete to continue to experience strain increases under sustained stresses. There are two components: basic and drying. Basic creep occurs when the element does not exchange water with its surroundings, meaning the specimen is in hydro-equilibrium with the environment. Drying creep is the additional deformation to basic creep, when the specimen exchanges moisture with the ambient. Creep ultimately shortens the member, causing a reduction in stress in the tendons. Some of the factors affecting creep besides time include the member size and shape, concrete age at loading, loads applied, stress level, relative humidity, curing conditions, and even aggregate type.

According to Report 496 written for the National Cooperative Highway Research Program (NCHRP) by Tadros, Al-Omaishi, Seguirant, and Gallt (2003), the member size and geometry greatly influence the amount of creep experienced and the rate at which it develops, with the ultimate creep being much smaller for bigger sections. The compressive strength of the element also affects creep, given that higher strengths can resist larger deformations (Tadros et al., 2003).

The curing method also influences creep. The NCHRP report 595 explained that if two specimens of the same strength are compared, the 1-day heat cured member will experience less creep than the 7-day moist cured one (Rizkalla, Mirmiran, Zia, Russell, &
Mast, 2007). Members that are air-cured will show higher creep than those that are moist-cured; also accelerating the cement hydration can lower creep by 30% to 50% for specimens that are steam cured (Waldron, 2004). Relative humidity also influences creep behavior, with a higher relative humidity resulting in a reduction in creep deformations due to the exchange of water for the drying creep component.

2.1.2.2. Shrinkage

Shrinkage is the change in volume of the member, independent of the loads applied. Drying shrinkage occurs when the specimen loses the free water present in the concrete to the environment. This water exchange occurs when the humidity in the member is greater than in the environment. Autogenous shrinkage is defined as the use of the water in the concrete to serve in the hydration of the cement; where the volume of hydrated cement is smaller than the combination in volume of dry cement and water. The reduction in volume over time caused by shrinkage leads to a lower stress in the strands. Factors that affect the amount of shrinkage and the rate at which it is experienced include the member size and geometry, the relative humidity, and curing conditions, among others.

The strength of the concrete used affects shrinkage in that mixes with lower water-cement ratios are used in order to achieve larger compressive strengths. The lower the amount of free water available in the element, the less it will be lost to the environment. The size of the specimen affects the volume change expected. It is easier to lose water for smaller members because it takes less time and effort for the water in the inside of the member to reach the environment. Also, the relative humidity affects the shrinkage loss in that the larger the humidity differential between the ambient and the specimen, the greater the water loss. Finally, specimens that are heat-cured experience less shrinkage than moist-cured experiments, and higher strength concretes also show a
lower volume change (Rizkalla et al., 2007). Waldron (2004) explained that when higher temperatures are used in accelerated curing, the shrinkage experienced is lowered.

2.1.2.3. Relaxation

A decrease in stress in the steel strands while keeping a constant strain is called relaxation. It is related to the other sources of losses because relaxation at a certain time depends on the stress level at that same moment, which varies depending on creep and shrinkage. Since the stresses are constantly reducing over time due to losses, the relaxation rate decreases. Greater initial stresses and temperatures increase the steel relaxation losses; however, low relaxation strands are commonly used now and the relaxation is significantly lower than stress-relieved strands (Tadros et al, 2003).

2.2. High Strength Concrete

2.2.1. Definition

High strength concrete (HSC) was previously defined by the American Concrete Institute (ACI) Committee 363 (1992) as having a design compressive strength of 6ksi or higher. Myers (2008) explained that the lower limit was revised to 8ksi in 2002 after discussions of setting an even higher level. This definition does not include concrete that uses “exotic materials or techniques,” which encompasses artificial aggregates, polymer impregnated, and epoxy concrete (ACI 363, 1992).

The committee’s report clarified that the lower limit specified does not mean that a sudden change in behavior occurs there, or that the production and testing methods change. The mechanical properties actually progress gradually from the lower strength to the higher strength concretes. This lower limit was selected as a way of defining when special care in the production process is needed (ACI 363, 1992).
Higher compressive strengths in the concrete are achieved by modifying the typical mix design used for normal strength concrete (NSC). Several factors are to be considered, such as achieving a high release strength faster to minimize the time spent in the casting bed, getting a higher strength without affecting workability from the lower water-cement ratios used, and material availability in the area, among other variables. The desired high strengths are achieved by changing the typical mix design by adding chemical admixtures, using special aggregates, etc. The ACI 363 (1992) report stated that “specially selected pozzolanic and chemical admixtures are employed, and the attainment of a low water-cementitious ratio is considered essential” (p. 8).

2.2.2. History of Applications

The production and use of HSC concrete has increased throughout the years. 5ksi was considered high in the 1950s, then strengths between 6ksi and 7.5ksi were produced in the 1960s, and in the 1970s strengths of 9ksi were available (ACI 363, 1992). Recently, the codes have been modified to extrapolate their current equations to strengths up to 15ksi; however, much higher strengths can be reached. An extreme example is ultra-high performance concrete which is defined as those that can reach compressive strength levels higher than 20ksi. This type of concrete is not in the scope of this study.

The continuing use of HSC and the constant research being carried out to get a better understanding of its behavior show that the use of HSC could keep increasing along with the availability of more adequate equations for property and behavior predictions.

There are several examples of HSC uses. In the case of buildings, the use of HSC minimizes member size which yields to more rentable floor space. It is mostly used in columns and shear walls, since the higher strengths allow the members to resist higher compressive forces. Some examples of HSC buildings include the Two Union Square building in Seattle, where design strength of 19ksi was specified to obtain a high modulus of elasticity. Other examples are the Trump Tower in Chicago with 16ksi, the
225 West Wacker Drive building in Chicago with 14ksi, and the Key Bank Tower in Cleveland with 12ksi.

The use of prestressed HSC is more common in bridge applications. In this case, the use of prestressed HSC can be found both in long-span and short-to-medium span bridges. Although HSC in prestressed applications is commonly used, strengths higher than 12ksi are not often seen. Examples include the Louetta road overpass with 13.1ksi, and the North Concho river overpass with 14ksi (Myers, 2008).

2.2.3. Advantages and Disadvantages

The decision to use HSC in any given project ultimately depends on the advantages outweighing the disadvantages. A drawback is that higher strengths mean the member needs to sit in the casting bed for longer to reach the desired initial compressive strength; if these higher initial strengths are reached too fast, then cracking is a concern. Longer spans can cause member transportation issues, where the cost can increase due to special permits requirements and route feasibility, among others. Longer spans can also mean instability at the time of transportation and handling, but it depends on the member type used, i.e. tubs and boxes are more stable than bulb tees. Material availability is a common disadvantage. Not every region counts with the aggregates needed for reaching certain strengths and materials have to be imported. Quality control is also a concern, since it is more elaborate than for NSC and some places do not have available people with the necessary expertise. Another problem is that HSC can lead to the use of smaller members, making it difficult to accommodate the tendons. This limitation can be fixed by using larger tendon diameters and by using external post-tensioning. Finally, modifying the casting beds to include the larger amounts of tendons can become expensive.

HSC can sometimes be a more economically feasible option. Benefits include that HSC allows the use of longer spans and larger spacing between members. These are
convenient in reducing the number of piers or members needed. Shallower members are also possible, which can help with height clearances. Smaller members can reduce dead loads which leads to smaller foundations. If maintenance is a concern, HSC can reduce the frequency in which it is needed. Generally, the resistance to abrasion is related to the compressive strength of the concrete, hence the required maintenance is minimized with higher strength, allowing the bridge to stay open for traffic. The cost per volume of HSC is higher, but it is still used because the mentioned benefits offset the additional cost.

2.2.4. Properties

The behavior of the material properties of the concrete also changes as higher strengths are achieved. These properties include the stress-strain relationship, modulus of elasticity, strength gain, creep, shrinkage, and modulus of rupture, among others. Time dependent properties relevant to the calculations of losses in the stress of the tendons are briefly explained in this section. Creep and shrinkage are directly related to prestress loss calculations and are discussed in Sections 2.2.4.1 and 2.2.4.2.

2.2.4.1. Creep

As mentioned in Section 2.1.2.1, the creep experienced by the concrete member impacts the stress losses in the tendons, and the compressive strength of the element affects the creep behavior by experiencing lower losses at higher strengths.

Rizkalla et al. (2007) described that creep behavior for HSC is similar to NSC in that the creep rate decreases with time. The maturity of the concrete at the time the loads are applied can greatly change the ability to resist creep, with a more mature concrete having better resistance. Khan, Cook, and Mitchell (1997) explained that “the creep of the high-strength concrete is much more sensitive to the age of loading than the normal
and medium strength concretes, with very early-age loading resulting in significantly higher creep” (p. 4). In addition to this, the applied stress also influences creep behavior. The AASHTO Specifications (2012) limit the ratio of applied stress to the concrete’s compressive stress to 60%. Waldron (2004) explained that some studies have determined that for HSC, creep and applied stresses are proportional up to possibly 65%, which means that the AASHTO limitation is still in that range of proportionality.

HSC normally experiences lower creep than NSC if both are loaded to comparable stress levels, due to the differences in mix design. Waldron (2004) discussed that HSC has a lower water-cement ratio, which is a characteristic that helps reduce the deformations due to the reduction of free water in the concrete. In the case of prestressed concrete, Type III cements (rapid hardening) are usually used. Since the creep deformations occur in the cement paste and not in the aggregates, the early high stiffness gained by the cement in prestressed applications reduces the creep the member experiences. As the cement paste creeps, the load is transferred to the aggregates. The rougher the aggregate the better the load is transferred, and the stiffer the aggregate the better it can resist deformations (Waldron, 2004).

### 2.2.4.2. Shrinkage

The free water available is low for high-strength concrete because of its low water-cement ratio; hence shrinkage is less for HSC than for NSC. HSC in prestressed applications is used for larger members, and smaller free water losses are expected in bigger sections due to longer amount of travel needed from the inside of the member to the environment. Despite of the smaller shrinkage expected on HSC due to this behavior, the NCHRP report 595 by Rizkalla et al. (2007) explained that there is little change in the shrinkage experienced by members with strengths between 10 and 18ksi.
2.2.4.3.  Modulus of Elasticity

The modulus of elasticity is a material property that varies with time and depends on several components of the mix design. Tadros et al. (2003) explained that the weight and compressive strength of the concrete are indirect variables that are commonly considered in the calculation of the modulus to represent the fundamental factors. It is also a way of maintaining the design equations simple for the user to understand and to reduce the amount of calculation work needed. In addition, there is a lot of information that is unknown to the engineer in the design stage of a project, while the design strength and whether it will be lightweight or normal weight concrete can be simply specified. Therefore, the modulus of elasticity is an important factor in the estimation of losses since the compressive strength of a member affects the losses experienced and because it constantly varies over time.

There are many parameters that affect the stiffness of the material such as moisture content, porosity, cement paste stiffness, aggregates used, and any other components used in the mix. The curing conditions play an important role in the strength gain behavior of the member. Rizkalla et al. (2007) explained tests done to determine the relationship between curing conditions and compressive strength. It was found that specimens with 1-day heat curing showed a much faster strength gain at the beginning but then plateau into a lower ultimate strength. Specimens that were cured by continuous moist and by 7-day moist both showed a close to same strength gain in the first 10 days but then the latter obtained a larger 28-day compressive strength (Rizkalla et al., 2007). The report concluded that in the cases of HSC, moist curing beyond the 7-day period is not needed because there is not a considerable strength gain due to the low permeability of the mix. The aggregate type used is another important parameter. The NCHRP report 496 explained that the aggregates used are big contributors to the stiffness of the concrete (Tadros et al., 2003).

Accurate prediction of the modulus of elasticity is a topic frequently discussed in research projects related to prestress losses. A more accurate estimate of the modulus
of elasticity results in a better approximation of the prestress losses. Several prediction formulas have been derived and are empirical relations between the modulus of elasticity, the unit weight, and the compressive strength. Therefore, there are some limitations to their accuracy and applicability due to the scatter of the data and to tests being done up to a certain strength. The shown equations are for normal weight concrete cases.

Equation (2-1) is defined in the AASHTO Specifications (2012). AASHTO (2012) specifies that it should be used when there is a lack of measured data. It is applicable for unit weights between 0.090kcf and 0.155kcf and up to a 15ksi strength. The $K_1$ factor is for correcting the aggregate source. This factor adjusts the prediction to be more applicable to the local materials used since the stiffness of the aggregate varies by region. $K_1$ is usually taken as 1.0 unless tests prove otherwise. This same equation is also defined by the American Concrete Institute (ACI 318, 2010) and by the Precast/Prestressed Concrete Institute Bridge Design Manual (PCI-BDM, 2003).

\[
E_c = 33,000K_1w_c^{1.5}\sqrt{f_c'} \tag{2-1}
\]

Where:

- $E_c$ is the modulus of elasticity of the concrete;
- $w_c$ is the unit weight of the concrete in kcf;
- $f_c'$ is the specified compressive strength in ksi;
- $K_1$ is a factor for correcting the aggregate source.

The ACI Committee 363 report (1992) stated that the AASHTO (2012), ACI 318 (2010) and PCI-BDM (2003) equation usually overestimates the modulus of elasticity. ACI 363 (1992) adopted Equation (2-2). Its limitations are that it does not have a factor to account for the aggregate type and it is only applicable for strengths ranging from 3ksi to 12ksi.

\[
E_c = \left(\frac{w_c}{0.145}\right)^{1.5} \left(1000 + 1265\sqrt{f_c'}\right) \tag{2-2}
\]
The Comité Euro-International du Beton-Fédération Internationale de la Précontrainte (CEB-FIP) Model Code (1990) also has a prediction formula, Equation (2-3), with a coefficient that accounts for the aggregate type used. The $\alpha_E$ coefficient is defined as 0.7 for sandstone, 0.9 for limestone, 1.0 for quartz, and 1.2 for basalt. The unit weight of the concrete is not part of the formula.

$$E_c = 3100\alpha_E\left(\frac{f_{cm}}{1.44}\right)^{1/3}$$

(2-3)

Where:

- $E_c$ is the elastic modulus for a 28-day concrete age and zero stress;
- $f_{cm}$ is the mean compressive strength of the concrete;
- $\alpha_E$ modifies the modulus for the strength of the aggregate being used.

Tadros et al. (2003) claimed that the formula by AASHTO (2012) provides a better prediction to the test values observed than the formula by ACI 363 (1992). This report proposed a new formula (Equation (2-5)) that only considers the compressive strength, accounts for the aggregate, and provides an upper and lower limit. Rather than directly having the unit weight in the formula, the report defined it as in Equation (2-4). The unit weight limit on the formula is 0.155kcf. The $K_1$ factor in this formula is a way of adjusting the modulus of elasticity from the national average to that of the local average. This factor is equal to one when both averages are the same. $K_2$ represents the upper and lower bounds. A lower bound is preferred for prestress loss and deflection calculations, while the upper bound is better for crack control since it is conservative.

Table 2.1 shows sample values for $K_1$ and $K_2$. A limitation with this method is that local data is required to take advantage of the K factors, while the K in Equation (2-1) is only dependent on testing the specific case being used.

$$w_c(kcf) = 0.140 + \frac{f_c'}{1000} < 0.155kcf$$

(2-4)
\[ E_c = 33,000 K_1 K_2 \left( 0.140 + \frac{f'_c}{1000} \right)^{1.5} \sqrt{f'_c} \]  
\hspace{1cm} (2-5)

Table 2.1 – K Factors for the NCHRP 496 Equation. Tadros et al. (2003)

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</tbody>
</table>

Another NCHRP study done by Rizkalla et al. (2007) resulted in report 595: Application of the LRFD Bridge Design Specifications to High-Strength Structural Concrete. Data for strengths higher than 15ksi is rare to find, but in this study concrete strengths up to 18ksi were tested. From it, a formula for calculating the modulus of elasticity was proposed (Equation (2-6)). It is mentioned that it slightly overestimates the modulus of elasticity measured for the database used. This \( K_1 \) value is to account for the aggregate force and shall be one unless physical tests prove otherwise and as approved by an authority of the jurisdiction. This is the same definition for \( K \) given in Equation (2-1).

\[ E_c = 310,000 K_1 w_c^{2.5} (f'_c)^{1/3} \]  
\hspace{1cm} (2-6)

Figure 2.1 graphs the equations explained and experimental data used by Tadros et al. (2003) and Rizkalla et al. (2007). For cases where the unit weight is considered, both 0.140kcf and 0.150kcf were plotted to show the range of results and because the unit weight varied throughout the available data. For the CEB-FIP (1990) formula, the constants for sandstone and basalt were used to show the range of the equation. In other cases with constants for the aggregate stiffness, a value of one was used.
This graph confirms the trends explained in the literature. Although there is a lot of scatter in the data, it can be seen that the equation by ACI 363 (1992) usually underpredicts the modulus of elasticity. The AASHTO Specifications (2012) and NCHRP report 595 (Rizkalla et al., 2007) predictions are very similar and seem to be a better representation of the data. The CEB-FIP (1990) model curves encompass most of the data, but it is hard to establish how accurate it is without knowing the aggregate type actually used. The NCHRP report 496 (Tadros et al., 2003) prediction overestimates after 14ksi and shows a steeper slope of the curve when compared to all the other equations. There are very few data points after 14ksi and most of the equations have 15ksi as their upper limit of applicability. It can be seen that most make similar predictions up to 14ksi, and then the curves start spreading apart. Therefore, it is hard to conclude which formula is a better prediction for higher strengths.

Figure 2.1 – Concrete Compressive Strength vs. Modulus of Elasticity (Adapted from Tadros et al., 2003; Rizkalla et al., 2007)
2.3. Prestress Losses Estimation Methods

2.3.1. PCI Design Handbook (2010)

The Precast/Prestressed Concrete Institute Design Handbook (2010) provides a method for approximating the prestress losses. This is a refined method in that the stress changes due to each loss source are calculated separately. The total losses (Equation (2-7)) are estimated by adding the effects from elastic shortening, creep, shrinkage, and relaxation. The anchorage seating losses and the friction losses, when applicable, also need to be considered. Creep losses are calculated following Equation (2-8).

\[ TL = ES + CR + SH + RE \] (2-7)

\[ CR = K_{cr} \frac{E_{ps}}{E_{c}} (f_{cir} - f_{cds}) \] (2-8)

\[ f_{cir} = K_{cir} \left( \frac{p_1}{A_g} + \frac{p_1 e^2}{I_g} \right) - \frac{M_g e}{I_g} \] (2-9)

\[ f_{cds} = \frac{M_{sd} e}{I_g} \] (2-10)

Where:

- TL is the total prestress loss;
- ES is the loss due to elastic shortening;
- CR is the loss due to creep;
- SH is the loss due to shrinkage;
- RE is the loss due to relaxation;
- \( K_{cr} \) is 2.0 for normal-weight concrete, and 1.6 for sand-lightweight concrete;
- \( E_{ps} \) is the modulus of elasticity of the prestressing steel;
- \( E_c \) is the modulus of elasticity of the concrete at 28-days;
- \( f_{cir} \) is the stress in the concrete at the center of gravity of the steel right after transfer;
- $f_{cds}$ is the stress in the concrete at the center of gravity of the steel due to the superimposed permanent dead loads;
- $K_{cr}$ is 0.9 for pretensioned strands;
- $p_i$ is the initial prestressing force after losses due to anchorage and friction;
- $A_g$ is the gross area of the concrete member;
- $I_g$ is the gross moment of inertia of the concrete member;
- $e$ is the eccentricity of the center of gravity of the prestress with respect to the center of gravity of the section;
- $M_g$ is the moment due to the self-weight of the member, along with any other permanent dead loads present at the moment of prestressing;
- $M_{sd}$ is the moment due to superimposed permanent dead loads that were applied after stressing the strands.

In the case of unbonded post-tension, the average compressive stress in the concrete through the length of the member at the level of the prestress centroid is used to calculate the stress in the concrete due to the prestressing force. This is due to strain compatibility no longer being applicable since the tendons can move within the duct, experiencing different stresses than the concrete at a given section. Next, the shrinkage and relaxation losses are calculated using Equations (2-11) and (2-12) below.

$$SH = (8.2 \times 10^{-6})K_{sh}E_{ps}(1 - 0.06V/S)(100 - RH) \quad (2-11)$$

Table 2.2 - $K_{sh}$ values for post-tensioned applications

<table>
<thead>
<tr>
<th>Days</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{sh}$</td>
<td>0.92</td>
<td>0.85</td>
<td>0.80</td>
<td>0.77</td>
<td>0.73</td>
<td>0.64</td>
<td>0.58</td>
<td>0.45</td>
</tr>
</tbody>
</table>

$$RE = [K_{re} - J(SH + CR + ES)]C \quad (2-12)$$
Table 2.3 - \( K_r e \) and J values

<table>
<thead>
<tr>
<th>Tendon Type</th>
<th>( K_r e )</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 270, stress-relieved</td>
<td>20,000</td>
<td>0.150</td>
</tr>
<tr>
<td>Grade 250, stress-relieved</td>
<td>18,500</td>
<td>0.140</td>
</tr>
<tr>
<td>Grade 240 or 235, stress-relieved</td>
<td>17,600</td>
<td>0.130</td>
</tr>
<tr>
<td>Grade 270, low-relaxation</td>
<td>5,000</td>
<td>0.040</td>
</tr>
<tr>
<td>Grade 250, low-relaxation</td>
<td>4,630</td>
<td>0.037</td>
</tr>
<tr>
<td>Grade 240 or 235, low-relaxation</td>
<td>4,400</td>
<td>0.035</td>
</tr>
<tr>
<td>Grade 145 or 160, stress-relieved</td>
<td>6,000</td>
<td>0.050</td>
</tr>
</tbody>
</table>

Where:

- \( K_{sh} \) accounts for the time between the end of moist-curing and the application of the prestressing force. It is 1.0 for pretensioned members, and for post-tensioned members Table 2.2 applies.
- V/S is the volume-to-surface ratio;
- RH is the relative humidity;
- \( K_r e \) and J values are defined in Table 2.3;
- \( f_{pu} \) is the ultimate strength of the prestress.

For stress-relieved strands, Equation (2-13) is used when \( f_{pi}/f_{pu} \) is between 0.75 and 0.70, Equation (2-14) is used when this ratio is between 0.70 and 0.51, and Equation (2-15) when the ratio is less than 0.51.

\[
C = 1 + 9 \left( \frac{f_{pi}}{f_{pu}} - 0.7 \right) \tag{2-13}
\]

\[
C = \frac{f_{pi}}{f_{pu} \times 0.19} \left( \frac{f_{pi}}{0.85} - 0.55 \right) \tag{2-14}
\]
\[ C = \frac{f_{pi}}{f_{pu}} \]  
\[ C = \frac{f_{pi}}{3.83} \]  
\[ (2-15) \]

For low-relaxation strands, Equation (2-16) is used when \( f_{pi}/f_{pu} \) is greater than 0.51, while Equation (2-17) is used when \( f_{pi}/f_{pu} \) is less than or equal to 0.54.

\[ C = \frac{f_{pi}}{f_{pu}} \left( \frac{f_{pi}}{f_{pu}} - 0.55 \right) \]  
\[ (2-16) \]

\[ C = \frac{f_{pi}}{f_{pu}} \]  
\[ (2-17) \]

\[ f_{pi} = \frac{P_i}{A_{ps}} \]  
\[ (2-18) \]

### 2.3.2. AASHTO Specification (2012)

The AASHTO LRFD Bridge Design Specifications (2012) present a refined method that splits the loss calculation into two stages: before and after casting the deck. The losses are calculated individually and then added together, as shown in Equation (2-19), where the elastic shortening, friction and anchorage losses have to be previously determined when applicable. Equation (2-20) shows just the change in the prestressing steel stress caused by the time-dependent losses.

\[ \Delta f_{pT} = \Delta f_{pES} + \Delta f_{pF} + \Delta f_{pA} + \Delta f_{pLT} \]  
\[ (2-19) \]

\[ \Delta f_{pLT} = \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR} \]  
\[ (2-20) \]

Where:

- \( \Delta f_{pT} \) is the total loss of prestress in the steel;
- \( \Delta f_{pES} \) is the loss due to elastic shortening;
- $\Delta f_{pF}$ is the loss due to friction applicable for post-tension applications;
- $\Delta f_{pA}$ is the loss due to anchorage set;
- $\Delta f_{pLT}$ is the total loss of prestress in the steel due to long-term losses: creep, shrinkage and relaxation;
- $\Delta f_{pSR}$ is the loss due to shrinkage;
- $\Delta f_{pCR}$ is the loss due to creep;
- $\Delta f_{pR}$ is the loss due to relaxation after transfer.

AASHTO (2012) divides the time-dependent loss calculation equations into two intervals: between transfer and deck placement, and between deck placement and final time. Creep losses in the initial interval can be calculated according to Equation (2-21) below.

$$\Delta f_{pCR} = \frac{E_p}{E_{ci}} f_{cgp} \Psi(t, t_i) K_{id}$$  \hspace{1cm} (2-21)

Where:
- $E_p$ is the modulus of elasticity in the prestressing steel;
- $E_{ci}$ is the modulus of elasticity of the concrete at transfer;
- $f_{cgp}$ is the concrete stress at the center of gravity of the prestressing steel due to the prestressing force at transfer and the self-weight of the member at the maximum moment section;
- $\Psi(t, t_i)$ is the girder creep coefficient, defined later;
- $K_{id}$ is the transformed-section coefficient, which considers the time-dependent interaction of the concrete and the bonded steel, defined as:

$$K_{id} = \frac{1}{1 + \frac{E_p}{E_{ci}} \frac{A_{ps}}{A_g} \left(1 + \frac{A_g e_{pg}^2}{I_g}\right) \left[1 + 0.7 \Psi(t, t_i)\right]}$$  \hspace{1cm} (2-22)

Where:
- $A_{ps}$ is the area of the prestressing strands;
- $A_g$ is the gross cross-sectional area of the member;
- $e_{pg}$ is the eccentricity of the prestressing force with the centroid of the member;
- $I_g$ is the moment of inertia of the gross section.

$$\psi(t, t_i) = 1.9 k_s k_{hc} k_f k_{td} t_i^{-0.118}$$  \hspace{1cm} (2-23)

$$k_s = 1.45 - 0.13 \frac{V}{S} \geq 1.0$$  \hspace{1cm} (2-24)

$$k_{hc} = 1.56 - 0.008H$$  \hspace{1cm} (2-25)

$$k_f = \frac{5}{1 + f'_{cl}}$$  \hspace{1cm} (2-26)

$$k_{td} = \frac{t}{61 - 4f'_{cl} + t}$$  \hspace{1cm} (2-27)

Where:

- $t$ is the maturity of the concrete (in days). For creep, $t$ is the age of the concrete between the time of loading and the time being considered;
- $t_i$ is the age of the concrete at time of loading (in days);
- $k_s$ is a factor for the volume-to-surface ratio effect;
- $k_{hc}$ is the humidity factor;
- $k_f$ is a factor for concrete strength effects;
- $k_{td}$ is a time development factor;
- $V/S$ is the volume-to-surface ratio (in inches)
- $H$ is the relative humidity (in percent).

The V/S factor, $k_s$, is an approximation from Equations (2-28) and (2-29) for creep and shrinkage calculations, respectively. These were developed empirically for V/S ratios no greater than 6.0 inches. AASHTO (2012) recommends that the more detailed equations should be used in specialized projects, in which the deformations at any time are important because the intermediate values are more sensitive to changes at different ages of the concrete than at ultimate. The simplified version was used in this study.
\[
k_c = \left[ \frac{26e^{0.36(V/S)} + t}{t + 45 + t} \right] \left[ \frac{1.80 + 1.77e^{-0.54(V/S)}}{2.587} \right]
\]

\[
k_s = \left[ \frac{26e^{0.36(V/S)} + t}{t + 45 + t} \right] \left[ \frac{1064 + 94(V/S)}{923} \right]
\]

Shrinkage losses between transfer and the moment the deck is placed is calculated using Equation (2-30).

\[
\Delta f_{pSR} = \varepsilon_{bid} E_p K_{id}
\]

\[
\varepsilon_{bid} = k_s k_h s k_f k_{ld} (0.48 \times 10^{-3})
\]

\[
k_{hs} = 2 - 0.014H
\]

Where:

- \( \varepsilon_{bid} \) is the strain in the girder due to concrete shrinkage between transfer and deck placement;
- \( k_{hs} \) is the humidity factor.

There are several ways of approximating the relaxation losses between transfer and deck placement. First and most commonly used is to assume this loss to be 1.2ksi in the case of low-relaxation strands. Second, Equation (2-33) gives an appropriate estimation. Or third, Equation (2-34) can be used for a more accurate prediction. This last method accounts for the effects that the shrinkage and creep have in changing the overall size of the concrete member therefore changing the tension originally defined in the prestressing steel. Given the relatively small contribution relaxation has in the total losses, the 1.2ksi approximation was used in this study.

\[
\Delta f_{pR} = \frac{f_{pt}}{K_L} \left( \frac{f_{pt}}{f_{py}} - 0.55 \right)
\]
\[ \Delta f_{PR} = \left[ f_{pt} \log(24t_i) \left( \frac{f_{pt}}{f_{py}} - 0.55 \right) \right] \left[ 1 - \frac{3(\Delta f_{pSR} + \Delta f_{pCR})}{f_{pt}} \right] K_{id} \]  \hspace{1cm} (2-34)

Where:

- \( f_{pt} \) is the stress right after transfer in the prestressing strands. It must be more than 0.55\( f_{py} \);
- \( K_L \) accounts for the kind of steel used. Unless accurate data is available, it will be taken as 30 for low-relaxation strands, and 7 for any other prestressing steel;
- \( f_{py} \) is the prestressing steel yield strength;
- \( K'_L \), similar to \( K_L \), will be taken as 45 for low-lax and 10 for stress relieved steel.

The second time interval that AASHTO (2012) considers is any time after deck placement. The equations and procedures remain similar to the first interval, with only changes related to the presence of the deck such as using the composite section properties. The creep and shrinkage experienced by the concrete girder can be determined by Equations (2-35) and (2-36) respectively. Concrete deck shrinkage is found using Equation (2-38). The prestressing steel relaxation can be found in the same manner as for the time before placing the deck. AASHTO (2012) explains that research has found that approximately half of the relaxation loss happens before the deck placement, hence this loss can be set equal to what was previously used: 1.2ksi.

\[ \Delta f_{pCD} = \frac{E_p}{E_{ci}} f_{gpt} \left[ \Psi_b(t_f, t_i) - \Psi_b(t_d, t_i) \right] K_{df} + \frac{E_p}{E_c} \Delta f_{cd} \Psi_b(t_f, t_d) K_{df} \]  \hspace{1cm} (2-35)

\[ \Delta f_{pSD} = \varepsilon_{bdf} E_p K_{df} \]  \hspace{1cm} (2-36)

\[ K_{df} = \frac{1}{1 + \frac{E_p A_{ps}}{E_{ci} A_c} \left( 1 + \frac{A_c e_{pc}^2}{I_c} \right) \left[ 1 + 0.7 \Psi_b(t, t_i) \right]} \]  \hspace{1cm} (2-37)

\[ \Delta f_{pSS} = \frac{E_p}{E_c} \Delta f_{cdf} K_{df} \left[ 1 + 0.7 \Psi_b(t_f, t_d) \right] \]  \hspace{1cm} (2-38)
\[ \Delta f_{cad} = \frac{\varepsilon_{ddf} A_d E_{cd}}{1 + 0.7 \Psi_d(t_f, t_d)} \left( \frac{1}{A_c} - \frac{e_{pc} e_d}{I_c} \right) \]  

(2-39)

Where:

- \( \Delta f_{pCD} \) is the loss in prestress due to creep, between deck placement and final time;
- \( \Psi_b(t_f, t_i) \) is the creep coefficient of the girder at the final time due to the load at transfer;
- \( \Psi_b(t_d, t_i) \) is the creep coefficient of the girder at the time of the deck placement due to the load at transfer;
- \( K_{df} \) is the transformed-section coefficient, which considers the time-dependent interaction of the concrete and the bonded steel, for the interval between deck placement and final;
- \( \Delta f_{cd} \) is the concrete stress change at the strand centroid by the long-term losses in the time between transfer and deck placement along with the superimposed loads and the weight of the deck.
- \( \Psi_b(t_f, t_d) \) is the creep coefficient of the girder at the final time due to the deck placement load;
- \( \Delta f_{pSD} \) is the loss in prestress due to shrinkage, between deck placement and final time;
- \( \varepsilon_{bdf} \) is the strain in the girder due to concrete shrinkage between deck placement and final. It is determined using the same equation than for \( \varepsilon_{bid} \) while using different time variables.
- \( A_c \) is the gross composite section area;
- \( e_{pc} \) is the eccentricity between the prestressing strands and the centroid for the composite section;
- \( I_c \) is the gross composite section moment of inertia;
- \( \Delta f_{pSS} \) is the loss in prestress due to shrinkage of the deck;
- $\Delta f_{\text{cdf}}$ is the concrete stress change at the strand centroid due to the concrete deck shrinkage;
- $\varepsilon_{\text{ddf}}$ is the strain in the concrete deck due to shrinkage;
- $A_d$ is the area of the concrete deck;
- $E_{\text{cd}}$ is the modulus of elasticity for the concrete deck.
- $\Psi_d(t_f, t_d)$ is the creep coefficient of the deck at the final time due to the deck placement load;
- $e_d$ is the eccentricity between the gross composite centroid and the deck.

2.3.3. PCI Bridge Design Manual (2003)

The Precast/Prestressed Concrete Institute Bridge Design Manual (PCI-BDM, 2003) has provisions for calculating the prestress losses. The latest edition changed its previous method (2003) to match the AASHTO Specifications explained in Section 2.3.2. In addition to this refined method, the PCI-BDM (2003) also explains a time-dependent analysis which can be used to calculate losses. This analysis remains the same in the current edition. The time-step method can estimate the stress in the prestressing steel at any time desired. It can also use any creep, shrinkage and relaxation models. Following, is a description of these models from the PCI-BDM (2003) along with an explanation of the time-step method.

Since the elastic modulus increases with time at a decreasing rate, an age-adjusted elastic modulus is used in the calculations. The adjustment is done by considering a creep coefficient $C$, and a factor $\chi$ that accounts for the age of the concrete.

$$E_{\text{c}}^*(t, t_0) = \frac{E_c(t_0)}{1 + \chi(t, t_0)C(t, t_0)}$$ \hspace{1cm} (2-40)

Where:

- $E_{\text{c}}^*(t, t_0)$ is the age-adjusted modulus of elasticity;
- $E_c(t_0)$ is the modulus of elasticity of the concrete at transfer;
- $\chi(t, t_0)$ is the aging coefficient. There are methods for calculating it, but a good approximation is to use 0.7 if the concrete is young at the beginning of the interval, and 0.8 in all other cases. For special cases in which the load is instantaneously applied, a value of 1 should be used;
- $t$ and $t_0$ are the time desired and the time at the end of curing, respectively, in days;
- $C(t, t_0)$ is the creep coefficient, defined by Equations (2-41) and (2-42) for high strength concrete.

$$C(t, t_0) = C_uk_{st}\frac{(t - t_0)^{0.6}}{(12 - 0.5f'_c) + (t - t_0)^{0.6}}$$

(2-41)

$$C_u = 1.88k_{la}k_hk_s$$

(2-42)

$$k_{st} = 1.18 - 0.045f'_c$$

(2-43)

$$k_{la} = 1.13(t_{la})^{-0.094}$$

(2-44)

$$k_{la} = 1.25(t_{la})^{-0.118}$$

(2-45)

$$k_R = 1.586 - 0.0084H \quad \text{for } 40 \leq H \leq 100$$

(2-46)

$$k_s = \frac{2}{3}(1 + 1.13e^{-0.54V/s})$$

(2-47)

Where:

- $C_u$ is the ultimate concrete creep coefficient as defined in Equation (2-42);
- $k_{st}$ is a correction factor for high strength concrete;
- $f'_c$ is the concrete compressive strength at 28 days, between 4ksi and 12ksi;
- $k_{la}$ is a correction factor for the loading age. Equation (2-44) is for steam cured and Equation (2-45) applies for moist cured;
- $k_h$ is a correction factor for relative humidity;
- $k_s$ is a correction factor for the size of the member;
- $t_{la}$ is the loading age.
Shrinkage strains are calculated according to Equations (2-48) and (2-49). Concrete shrinkage strain calculations depend on the way the member was cured. Equation (2-48) applies for steam cured for 1 to 3 days, while Equation (2-49) applies for concrete moist-cured for seven days. The following equations are for concrete strengths between 4ksi and 12ksi.

\[
S(t, t_0) = (545 \times 10^{-6})k_{cp}k_hk_sk_{st}\frac{t - t_0}{(65 - 2.5f'_c) + (t - t_0)}
\]

(2-48)

\[
S(t, t_0) = (545 \times 10^{-6})k_{cp}k_hk_sk_{st}\frac{t - 7}{(45 - 2.5f'_c) + (t - 7)}
\]

(2-49)

\[
k_h = 2.00 - 0.0143H \quad \text{for} \ 40 \leq H \leq 80
\]

(2-50)

\[
k_h = 4.286 - 0.0429H \quad \text{for} \ 80 < H \leq 100
\]

(2-51)

\[
k_s = 1.2e^{-0.12V/s}
\]

(2-52)

\[
k_{st} = 1.2 - 0.05f'_c
\]

(2-53)

Where:

- \( S(t, t_0) \) is the shrinkage strain experienced in the time interval;
- \( k_{cp} \) is the factor for moist curing period used other than 7 days, see Table 2.4.

<table>
<thead>
<tr>
<th>Moist Curing Period (days)</th>
<th>Shrinkage Factor ( k_{cp} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.20</td>
</tr>
<tr>
<td>3</td>
<td>1.10</td>
</tr>
<tr>
<td>7</td>
<td>1.00</td>
</tr>
<tr>
<td>14</td>
<td>0.93</td>
</tr>
<tr>
<td>28</td>
<td>0.86</td>
</tr>
<tr>
<td>60</td>
<td>0.79</td>
</tr>
<tr>
<td>90</td>
<td>0.75</td>
</tr>
</tbody>
</table>
The intrinsic relaxation of the strands is defined in Equation (2-54). For grade 270 low-relaxation strands, \( K_r \) and \( f_y \) are 45 and 243 ksi respectively. In the case of stress-relieved, \( K_r \) is 10 and \( f_y \) is 229.5 ksi. This equation is only applicable for \( \frac{f_{pl}}{f_y} \geq 0.55 \).

\[
f_r(t, t_0) = \frac{f(t_0)}{K_r} \left[ \frac{f(t_0)}{f_y} - 0.55 \right] \log_{10} \left( \frac{24t + 1}{24t_0 + 1} \right)
\]

Where:
- \( f_r(t, t_0) \) is the relaxation of the strands in the time step;
- \( f(t_0) \) is the tensile stress at the beginning of the interval;
- \( f_y \) is the yield strength of the strand;
- \( K_r \) is a material constant, which is 45 for low-relaxation strands, and 10 for stress-relieved strands;
- \( t_0 \) is the age of the concrete at the beginning of the interval, in days;
- \( t \) is the age of the concrete at the desired time after loading, in days.

The PCI-BDM (2003) describes a detailed method for calculating the time-dependent effects in the member. It is a time-step analysis used in cases when a more rigorous assessment is desired. Calculating the age-adjusted modulus of elasticity defined in Equation (2-40) is the first step in the analysis. The second step is calculating the modular ratio for each element. The modular ratio \( n_k \) is the elastic modulus of element \( k \) divided by the adjusted elastic modulus of the concrete. Next, as the third step, the transformed section properties (area, centroid, and moment of inertia) are calculated. Fourth, the initial stains in the given time interval for each element are calculated. These are the shrinkage, creep and relaxation strains and curvature resulting from the stresses previously applied. The non-prestressing steel normally does not have initial stresses. Shrinkage strains are calculated from Equations (2-48) and (2-49), as previously discussed. Creep calculations for high-strength applications are determined by multiplying the creep coefficient, Equation (2-41), with the strain of the concrete found
in step seven from the previous interval. Strains due to steel relaxation are based on Equation (2-54).

The fifth step is calculating the theoretical restraint forces for each element, $N_{0k}$ and $M_{0k}$, which are then summed up to find the restraint forces of the section, $N_0$ and $M_0$, as in Equations (2-55) through (2-58).

\[
N_{0k} = -E^* \varepsilon_{0k} A_k \tag{2-55}
\]
\[
N_0 = \sum N_{0k} \tag{2-56}
\]
\[
M_{0k} = -E^*_{ck} I_k \phi_{0k} \tag{2-57}
\]
\[
M_0 = \sum \left[ M_{0k} - N_{0k} (y_k - y) \right] \tag{2-58}
\]

Where:

- $N_{0k}$ is the theoretical normal force in element $k$;
- $E^*$ is the age-adjusted modulus for element $k$;
- $\varepsilon_{0k}$ is the initial strain in the element, which are creep and shrinkage in the case of concrete and relaxation for steel;
- $A_k$ is the area of element $k$;
- $N_0$ is the total theoretical normal force in the section;
- $M_{0k}$ is the theoretical restrain moment in element $k$;
- $E^*_{ck}$ is the age-adjusted modulus of elasticity for element $k$;
- $I_k$ is the moment of inertia of element $k$;
- $\phi_{0k}$ is the initial curvature in the element;
- $y_k$ is the distance from an edge to the centroid of element $k$;
- $y$ is the distance from an edge to the centroid of the section.

Next, the restraint forces are subtracted from the applied forces to calculate the total strains, shown in Equations (2-59) and (2-60). Seventh is calculating the strains for each element using Equations (2-61) and (2-62).
\[ \varepsilon = \frac{N - N_0}{E\varepsilon A} \]  \hspace{1cm} (2-59)

\[ \phi = \frac{M - M_0}{E\varepsilon I} \]  \hspace{1cm} (2-60)

\[ \varepsilon_k = \varepsilon - \phi(y_k - y) \]  \hspace{1cm} (2-61)

\[ \phi_k = \phi \]  \hspace{1cm} (2-62)

Where:

- \( \varepsilon \) is the total strain in the section;
- \( N \) is the applied force;
- \( A \) is the area of the section;
- \( \phi \) is the curvature of the section;
- \( M \) is the applied moment on the section;
- \( \varepsilon_k \) is the strain in element \( k \);
- \( \phi_k \) is the curvature of element \( k \).

Finally, the eighth step is to calculate the element forces and elastic strains.

\[ N_k = E_{ck}^{*}\varepsilon_{0k}A_k + N_{0k} \]  \hspace{1cm} (2-63)

\[ \varepsilon_{fk} = \frac{N_k}{E_{ck}^{*} A_k} \]  \hspace{1cm} (2-64)

\[ M_k = E_{ck}^{*} I_k \phi_0 k + M_{0k} \]  \hspace{1cm} (2-65)

\[ \phi = \frac{M_k}{E_{ck}^{*} I_k} \]  \hspace{1cm} (2-66)

Where:

- \( N_k \) is the force in element \( k \);
- \( \varepsilon_{fk} \) is the elastic strain in element \( k \);
- \( M_k \) is the moment in element \( k \);
- \( \phi_{fk} \) is the elastic curvature in element \( k \).
Steps four and five are omitted when the intervals are of zero length, which are those where a discrete event occurred, such as the transfer of prestress, deck placement, and load application. The smaller the time step, the more refined the analysis will be. These steps are to be performed on an individual cross-section for each time step. Although the method can calculate the losses at any time, the PCI-BDM (2003) recommends the following significant events with the corresponding typical duration of the interval:

- Stand relaxation before transfer – 12 to 24 hours.
- Transfer of prestress – 0 days.
- Creep, shrinkage and relaxation of beam after transfer – 30 days to 1 year.
- Placement of cast-in-place deck – 0 days.
- Creep, shrinkage and relaxation of composite deck and beam – 7 days to 6 months.
- Application of superimposed dead load on composite deck and beam – 0 days.
- Creep, shrinkage and relaxation of composite deck and beam – 25 years or more.

2.3.4. **Swartz (2010)**

Swartz (2010) presented two methods for calculating prestress losses. One is a time-step method, and the other one is called the “Direct Method,” which was developed as a simplification of the AASHTO (2012) refined method. The Direct Method was not considered in this thesis. Swartz (2010) explained that the main idea behind this time-step method is to discretize the element into several horizontal layers, and that a prestress loss approximation can be found by knowing the strain distribution at each step. At every interval, the strains are calculated by accounting for creep, shrinkage and relaxation, and the elastic response due to the loads applied.

The assumptions used for this method include constant stresses per time step, uniform shrinkage through the section, and plane sections remain plane. This method is only
applicable for bonded prestressing steel since one assumption calls for a perfect bond between the steel and the concrete so that strain compatibility could be applied. Another assumption was creep superposition, for which the author allows the use of a creep recovery factor for when there is not a full creep recovery in cases of stress reversal.

The first step in this method is to calculate the stress at each level of the discretized member. This stress will be zero in the first time step, while in the following steps it will be the stress found at the end of the preceding interval. The second step is to calculate the creep strains at each element, Equation (2-69), by subtracting the elastic strain, Equation (2-68), from the total strain, Equation (2-67). Third, the shrinkage strain is calculated and added to the creep strain, giving the total inelastic strain. The creep coefficient and the shrinkage strain are obtained using the equations from AASHTO (2012) previously explained.

\[
\varepsilon_{T_k}(t_i) = \sum_{j=1}^{i-1} \Delta\sigma(t_j) \left[ \frac{1}{E_c(t_j)} + \frac{\Psi(t_i,t_j)}{E_c(t_j)} \right]
\]  

(2-67)

\[
\varepsilon_{k\text{elastic}} = \frac{\sigma_{k\text{elastic}}}{E_c}
\]  

(2-68)

\[
\varepsilon_{cr} = \varepsilon_{T_k} - \varepsilon_e - \varepsilon_{sh}
\]  

(2-69)

\[
\varepsilon_I = \varepsilon_{sh} + \varepsilon_e
\]  

(2-70)

Where:

- \(k\) is the layer being considered;
- \(\varepsilon_{T_k}\) is the total stress-related strain;
- \(\Delta\sigma\) is the stress increment in the previous step, to be defined later;
- \(E_c\) is the modulus of elasticity of the concrete;
- \(\Psi\) is the creep coefficient, using Equation (2-23) previously defined in the AASHTO (2012) method;
- \(\varepsilon_{k\text{elastic}}\) is the elastic strain;
- \( \sigma_{elastic} \) is the elastic stress found in the previous step, to be defined later;
- \( \varepsilon_c \) is the creep strain;
- \( \varepsilon_{sh} \) is the shrinkage strain, using Equation (2-31) for \( \varepsilon_{bid} \) previously defined in the AASHTO (2012) method;
- \( \varepsilon_l \) is the inelastic strain.

The fourth step is to get the constants that will be used to solve a system of equations to be defined later, along with the axial forces and effective moments experienced due to internal stresses.

\[
A = \sum_{k=1}^{m} E_c(t_i)A_k + E_pA_p
\]

\( (2-71) \)

\[
B = \sum_{k=1}^{m} E_c(t_i)A_ky_k + E_pA_p d_p
\]

\( (2-72) \)

\[
C = \sum_{k=1}^{m} E_c(t_i)A_ky_k^2 + E_pA_p d_p^2 + E_s A_s d_s^2
\]

\( (2-73) \)

\[
N_l = \sum_{k=1}^{m} E_c(t_i)A_k \varepsilon_{IK}(t_i)
\]

\( (2-74) \)

\[
N_p = E_p A_p \varepsilon' J
\]

\( (2-75) \)

\[
N_d = \sum_{k=1}^{m} E_c(t_i)A_k (\varepsilon_{0d} - y_k \Phi_d)
\]

\( (2-76) \)

\[
M_l = \sum_{k=1}^{m} E_c(t_i)A_k \varepsilon_{IK}(t_i)y_k
\]

\( (2-77) \)

\[
M_p = E_p A_p \varepsilon' J d_p
\]

\( (2-78) \)

\[
M_d = \sum_{k=1}^{m} E_c(t_i)A_k (\varepsilon_{0d} - y_k \Phi_d) y_k
\]

\( (2-79) \)
\[ \varepsilon_R = \frac{\sigma_R}{E_p} \]  \hspace{1cm} (2-80)

\[ \varepsilon_J = \frac{f_{pi}}{E_p} \]  \hspace{1cm} (2-81)

\[ \varepsilon'_J = \varepsilon_J - \varepsilon_R \]  \hspace{1cm} (2-82)

Where:

- \( m \) is the total number of layers;
- \( A_k \) is the area of the element being considered;
- \( E_p \) is the modulus of elasticity of the prestressing steel;
- \( A_p \) is the area of the prestressing steel;
- \( y_k \) is the distance from the centroid of the element considered to the top of the deck;
- \( d_p \) is the distance from the centroid of the prestressing steel to the top of the deck;
- \( E_s \) is the modulus of elasticity of the mild steel;
- \( A_s \) is the area of the mild steel;
- \( d_s \) is the distance from the centroid of the mild steel to the top of the deck;
- \( N_I \) is the axial force due to creep internal stresses;
- \( N_P \) is the axial force due to the initial strand tension internal stresses;
- \( \varepsilon'_J \) is the effective strain due to the jacking force on the prestressing steel considering relaxation losses;
- \( N_d \) is the axial force due to the internal stresses for layers in the deck;
- \( \varepsilon_0 \) is the reference strain in the strain profile, found in previous time-step and explained later. For \( N_d \) and \( M_d, \varepsilon_{0d} \) is the strain in the deck by using the strain profile and setting \( y_k \) as the distance to the layer in the deck;
- $\phi$ is the reference curvature in the strain profile, found in previous time-step and explained later. For $N_d$ and $M_d$, $\phi_d$ is the strain in the deck by using the strain profile and setting $y_k$ as the distance to the layer in the deck;
- $M_I$ is the effective moment due to creep internal stresses;
- $M_P$ is the effective moment due to the initial strand tension internal stresses;
- $M_d$ is the effective moment due the internal stresses for layers in the deck;
- $\varepsilon_R$ is the strain due to relaxation;
- $\sigma_R$ is the stress due to the relaxation strain calculated using AASHTO (2012) methods defined before;
- $f_{pi}$ is the initial jacking stress, usually $0.75f_{pu}$;
- $f_{pu}$ is the tensile strength of the prestressing steel.

After all these constants and parameters have been found, the system of equations below can be solved simultaneously by simplifying it with the matrix solutions defined by Equations (2-87) through (2-89).

$$N = N_I + N_P + N_d$$  \hspace{1cm} (2-83)

$$M = M_I + M_P + M_d + M_{applied}$$  \hspace{1cm} (2-84)

$$A\varepsilon_0(t_i) - B\phi(t_i) = N$$ \hspace{1cm} (2-85)

$$B\varepsilon_0(t_i) - C\phi(t_i) = M$$ \hspace{1cm} (2-86)

$$K = \begin{bmatrix} A & -B \\ B & -C \end{bmatrix}$$ \hspace{1cm} (2-87)

$$f = \begin{bmatrix} N \\ M \end{bmatrix}$$ \hspace{1cm} (2-88)

$$\begin{bmatrix} \varepsilon_0 \\ \phi \end{bmatrix} = K^{-1}f$$ \hspace{1cm} (2-89)

Where:

- $N$ is the total axial force;
- $M$ is the total moment;
- $M_{\text{applied}}$ is the moment due to the applied loads.

Finally, the total and elastic strains along with the elastic stress and stress increase at each level are calculated. From the strain and curvature found from the system of equations, the strain at the level of the prestressing steel is found, then transformed to stress in the steel.

$$
\varepsilon_{\text{Ktotal}} = \varepsilon_0 - y_k\phi
$$

$$
\varepsilon_{\text{Kelastic}} = \varepsilon_{\text{Ktotal}} - \varepsilon_{\text{sh}} - \varepsilon_{\text{cr}} - \varepsilon_0d
$$

$$
\sigma_{\text{Kelastic}} = \varepsilon_{\text{Kelastic}}E_c
$$

$$
\Delta\sigma = \sigma_{\text{Kelastic}}(t_i) - \sigma_{\text{Kelastic}}(t_{i-1})
$$

$$
\varepsilon_s = \varepsilon_0 - y_s\phi - \varepsilon'_f
$$

$$
\sigma_s = \varepsilon_sE_p
$$

Where:

- $\varepsilon_{\text{Ktotal}}$ is the total strain;
- $\Delta\sigma$ is the stress increment between the current time step and the previous step;
- $\varepsilon_s$ is the prestress strain;
- $\sigma_s$ is the prestress stress.

All the explained steps are to be repeated for the amount of intervals the designer decides to use. The total losses can be found by getting the difference between the initial stress at jacking and the stress in the prestressing steel at the final time step.
CHAPTER 3. EVALUATION OF METHODS TO ESTIMATE LOSSES

This section discusses the methods explained in Section 2.3, followed by findings from comparing available data and estimated values using these procedures.

3.1. Discussion of Methods


The method for calculating losses explained in the Precast/Prestressed Concrete Institute Design Handbook (2010) is a simple process that, unlike the lump-sum methods, still separates the contributions from each source of loss. The PCI Design Handbook (2010) mentions that a different method should be used if the case is not a normal design scenario. However, it does not indicate what a normal case entails and fails to mention any important limitation to the procedure that other design methods usually specify, such as the maximum compressive strength to which it can be applied. This is the only method in this study that does not use an age-adjusted modulus of elasticity approach. This is because this method only provides an estimate of the losses for the end of the service life, and it does not separate the process into stages, such as before and after deck placement as the AASHTO (2012) refined method does. Principally this is owed to the application often to building type pretensioned members.
3.1.2. **AASHTO Specifications (2012)**

The AASHTO Specifications (2012) mention two methods for calculating the time-dependent losses. One is an approximate method used to estimate the total losses as part of the preliminary design. This method was not discussed in Chapter 2 because it is a lump-sum method that does not separate the different loss sources and does not consider the time dependent behavior. A more detailed analysis is desired if strengths higher than 15ksi are being used. The second method explained is the refined method, which was discussed in Section 2.3.2. This method allows understanding the behavior before and after casting the deck, and resulted from the research explained in the NCHRP report 496 (Tadros et al., 2003). A draft of the “Guide to Estimating Losses” by the ACI-ASCE Committee 423 (2014) explained that the method can be used in calculating losses for any pretensioned member, not just bridge girders.

The limit on compressive strength is 15ksi. The creep and shrinkage models developed by Tadros et al. (2003) were based on lab specimens with concrete strengths varying from 2ksi to 17ksi. The prestress loss provisions were derived and compared to measured data obtained by instrumenting seven girders in bridges across the county with compressive strengths between 9ksi and 11ksi (Tadros et al., 2003). The method was also compared to available data from 31 girders with specified compressive strengths varying from 5.3ksi up to 14.0ksi, with almost 30% of them being higher than 12.0ksi. Therefore, the method was extrapolated to strengths up to 15ksi.

The method is only for use with normal weight concrete. AASHTO (2012) states that a time-step analysis is needed for lightweight concrete. The scope of this thesis is normal weight concrete but it is worth noting that a study showed that the AASHTO (2012) refined method can be used on high-performance lightweight concrete but it will yield conservative results while the lump-sum method will underestimate the losses (Khan & Lopez, 2005). On geometry, the volume-to-surface (V/S) ratio was developed empirically considering a maximum ratio of 6.0in. This restraint is not a concern since typical cross-
sections used do not exceed it, such as AASHTO beams which have a V/S of around 3in and typical decks with a V/S of around 4in.

Rizkalla et al. (2007) proposed changing the time development factor defined in Equation (2-27) to Equation (3-1). The report mentioned that the results from Equation (2-27) are negative once the initial strength is greater than 15ksi in the first few days (Rizkalla et al., 2007). The authors acknowledged that an initial strength that high is not normally used, but the factor needs to be all inclusive considering that post-tension applications could show a compressive strength higher than 15ksi at the moment it is loaded. Another comment made is that the time-development factor shows a sudden slope change in the creep for strengths higher than 12ksi also for the first few days after loading (Rizkalla et al., 2007). Figure 3.1 to Figure 3.4 below show these behaviors along with the result from the proposed equation by Rizkalla et al. (2007).

\[
k_{td} = \frac{t}{12 \left( \frac{100 - 4f'_{ci}}{f'_{ci} + 20} \right) + t}
\]  

(3-1)

Figure 3.1 - Time-development factor for 8ksi initial strength
Figure 3.2 - Time-development factor for 12ksi initial strength

Figure 3.3 - Time-development factor for 16ksi initial strength
Figure 3.4 - Time-development factor for 18ksi initial strength

Swartz (2010) explained that compared to the other adjustment factors used to calculate the shrinkage strains, the “time-development factor is of least importance for prestress loss estimates because the shrinkage at final time is of primary importance. The rate of shrinkage strain becomes secondary” (p. 16). Changing the time-development factor equation to the one proposed by Rizkalla et al. (2007) will not result in significant changes in the overall final loses for the usual release strength cases, but it will yield to better estimates for the cases in which the release strength is much higher.

Another issue regarding the AASHTO (2012) refined method is its complexity. Compared to its previous version from the 2004 AASHTO Specifications, the current refined method requires a large amount of steps and calculations (Garber, 2014). The specifications do not provide an easy to understand step-by-step explanation or examples to follow along. The method is divided into two stages considering three main events: release, deck casting, and ultimate life. Research done by Swartz (2010) and Garber (2014) arrived to the same conclusion that dividing the calculations into before and after the deck calculations is not necessary and complicates the process. They
mention that the final losses do not vary much when the separation in time is omitted. Both authors developed their own methods with only one stage of analysis. Both claim that they are much simpler than the AASHTO (2012) refined method while giving comparable results. These proposed simplified methods were not considered in this study. Although it is true that using this method in the estimation of losses involves a lot of steps, the process becomes easier to understand and apply once an example is seen. In addition, it might be desired in some cases to know the losses before and after the deck is cast, including the shrinkage gain due to the deck placement.

Finally, if a more detailed analysis is needed, a time-step method should be used. AASHTO (2012) does not explain a specific time-step procedure; instead, it cites papers for the user to reference. From these sources, the time dependent analysis from the PCI-BDM (2003) was chosen, explained in Section 2.3.3, and discussed below.


The Precast/Prestressed Concrete Institute Bridge Design Manual (2003) time-dependent analysis allows the designer to choose the desired amount of time intervals to use. The smaller the intervals, the more refined the analysis will be. The major loading events in the life of the member have to be steps in the analysis. These events are the tensioning of the strands, the transfer of the prestress (release), the placement of the deck, and the application of the superimposed dead loads. Figure 3.5 shows the stress behavior through the analysis from the NCHRP report 496 (Tadros et al., 2003), and Table 3.1 shows the intervals explained in the PCI-BDM (2003). Steps in time can be defined in between these major intervals if a more refined estimate is desired.
Table 3.1 - Beam Lifetime Intervals (PCI-BDM, 2003)

<table>
<thead>
<tr>
<th>Interval</th>
<th>Event</th>
<th>Typical Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Strand relaxation before transfer</td>
<td>12 to 24 hours</td>
</tr>
<tr>
<td>2</td>
<td>Transfer of prestress</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Creep, shrinkage, and relaxation of beam after transfer</td>
<td>30 days to 1 year</td>
</tr>
<tr>
<td>4</td>
<td>Placement of cast-in-place deck</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Creep, shrinkage, and relaxation of composite deck and beam</td>
<td>7 days to 6 months</td>
</tr>
<tr>
<td>6</td>
<td>Application of superimposed dead load on the composite deck and beam</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>Creep, shrinkage, and relaxation of composite deck and beam</td>
<td>25 years or more</td>
</tr>
</tbody>
</table>

Figure 3.5 - Stress versus time in the strands in a pretensioned concrete girder (Tadros et al., 2003)

Time-step methods are recommended when a more detailed analysis is needed. This applies to cases such as segmental construction, where there is the need of knowing the stresses at different times throughout the construction process. This time-dependent analysis has the ability of using any creep and shrinkage model. In this study, the models
explained in the AASHTO Specifications (2012) and the PCI-BDM (2003) were used, which have compressive strength limits of 15ksi and 12ksi, respectively.

This time-step method uses an age-adjusted modulus of elasticity. It calculates the change in stresses for each interval and then adds them at the end to obtain the final stress desired. It is recommended that this analysis is not done manually. The section properties and stresses are continuously changing, while the general steps taken in each interval are repetitive. Therefore, it is not difficult to code and it will save time if different results need to be evaluated by varying several design inputs.

The PCI-BDM (2003) has examples that help the user understand the time-step procedure better. However, these examples are done until right after transfer, making it difficult for the designer to clearly understand how to apply it once the deck is cast. Also, although the creep and shrinkage strains are calculated at the beginning of each step, the method does not separate the losses caused by each source, as the AASHTO (2012) refined method and the PCI Design Handbook (2010) method do.

3.1.4. **Swartz (2010)**

Different creep and shrinkage models could be implemented in the time-step method by Swartz (2010). This study used the AASHTO (2012) models. There are some differences between this method and the PCI-BDM (2003) time-dependent analysis. First, instead of calculating the change in stresses experienced in each interval to then add them at the end of the analysis, the steel stress found at the end of each interval is with respect to the initial time. To find the stress change experienced in a given step, the steel stress estimated in the previous interval needs to be subtracted from the stress found in the current time-step. When compared to the PCI-BDM (2003) time-dependent analysis, it is important the designer understands that the time datum being used is different since it changes the calculation of the creep and shrinkage strains, and the steel relaxation
stress. Also, as mentioned in Section 2.3.4, the derivation of this method was based in the assumption that a perfect bond between the concrete and the strands exist.

When a time-step is defined before the design concrete strength is achieved, the method expects the designer to use the modulus of elasticity from the strength at that specific time. This information is not easily available to the engineer at the moment of design and assumptions would need to be made. The method uses an age-adjusted modulus of elasticity when calculating the total strain at each interval. However, it does not use the transformed or composite section properties accounting for this varying modulus of elasticity. Also, the method separates the section into several horizontal layers. This process may be tedious because one must decide how many layers to use, and calculate each of their areas and distances from the top of the deck. Furthermore, this method can be harder to follow along and understand since it uses constants and solves simultaneous equations using matrices. As mentioned in the PCI-BDM (2003) discussion in Section 3.1.3, it is also recommended that this method is not used by hand calculations and it should be automated. Finally, Swartz has a complete example for this method, which helps understand how to apply it once the deck is cast, unlike the PCI-BDM (2003) example. However, this example has a few errors, so it needs to be followed with caution.

3.2. Findings

3.2.1. Available Data
The methods explained were used to estimate losses for beams studied in previous research. The focus was sections with compressive strengths higher than 13ksi, and the measured material properties were used when available. Table 3.2 gives a summary of the sections used. Finding available prestress loss data for concrete with compressive
strength higher than 15ksi was difficult. From the twenty-two girders considered, only seven were higher than 15ksi.

Table 3.2 - Summary of Girders Studied

<table>
<thead>
<tr>
<th>Study</th>
<th>Specimen</th>
<th>Section</th>
<th>Measured $f'_c$ (ksi)</th>
<th>Age at Loss Measurement (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choi (2006); Choi et al. (2008); Rizkalla et al. (2007)</td>
<td>10PS-N</td>
<td>AASHTO Type II</td>
<td>11.81</td>
<td>222</td>
</tr>
<tr>
<td></td>
<td>14PS-N</td>
<td></td>
<td>15.66</td>
<td>228</td>
</tr>
<tr>
<td></td>
<td>18PS-N</td>
<td></td>
<td>18.11</td>
<td>232</td>
</tr>
<tr>
<td></td>
<td>10PS-1S</td>
<td></td>
<td>13.19</td>
<td>189</td>
</tr>
<tr>
<td></td>
<td>14PS-1S</td>
<td></td>
<td>15.53</td>
<td>184</td>
</tr>
<tr>
<td></td>
<td>18PS-1S</td>
<td></td>
<td>14.49</td>
<td>199</td>
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<td>10PS-5S</td>
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<td>11.49</td>
<td>120</td>
</tr>
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<td></td>
<td>14PS-5S</td>
<td></td>
<td>16.16</td>
<td>143</td>
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<td></td>
<td>18PS-5S</td>
<td></td>
<td>18.06</td>
<td>175</td>
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<tr>
<td>Gross and Burns (2000)</td>
<td>E13</td>
<td>AASHTO Type IV</td>
<td>13.70</td>
<td>422</td>
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<tr>
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<td>N32</td>
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<td></td>
<td>BT-56</td>
<td>AASHTO BT-56</td>
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</tbody>
</table>

The smallest section analyzed was an AASHTO Type II. The specimens are discussed in detail in papers by Choi (2006), Choi et al. (2008), and Rizkalla et al. (2007), which originated from the same research project. Nine beams were tested in the lab. The first number in the specimen name is the design compressive strength of the concrete. The specified strengths were 10ksi, 14ksi, and 18ksi but the actual values obtained varied from 11.49ksi to 18.11ksi. From the nine girders tested, only one did not reach its specified strength: 18PS-1S. The last number in the specimen name indicates the case
for the deck. An “N” shows that no deck was cast, “1S” is a 1ft wide deck, and “5S” is a
5ft wide deck. The thickness of the deck was 8in and all the members used 0.5in
diameter strands. Although these were the smaller sections in the considered data, they
contained the largest compressive strengths found: 18.11ksi and 18.06ksi. All three
10ksi cases were considered because one of them had strength higher than 13ksi.
Measured compressive strengths at release were not found and the design values were
not noted by the author either. Assumptions on the initial strengths were made by
matching the loss estimations using the AASHTO (2012) refined method to the
predictions reported in the paper using the design values. Compressive strengths at
transfer were chosen as 7.5ksi, 8ksi and 9ksi for design compressive strengths of 10ksi,
14ksi and 18ksi cases, respectively. Finally, the measured values tabulated in Table 3.3
below are losses calculated using the measured modulus of rupture and cracking
moment.

The next research study considered was done by Gross and Burns (2000), where two
different bridges in Texas were instrumented during construction to monitor their
behavior. One is the North Concho River Overpass with AASHTO Type IV sections and
measured compressive strengths ranging from 13.70ksi to 14.83ksi. The other bridge is
the Louetta Road Overpass, also located in Texas. It uses Texas U-beam sections and
shows a range in strength from 13.29ksi to 14.32ksi. The compressive strength was not
the only changing variable between girders within the same bridge. The width of the
deck, the number of strands, and the time in which the losses were measured also
changed. The deck thicknesses were between 7.25in and 7.5in, the widths varied from
6ft to 16ft, and all members used 0.6in diameter strands.

The final set of data resulted from research done by Canfiled (2005). Two different
girders were tested in the lab, an AASHTO Type IV and an AASHTO BT-56. They both had
compressive strengths above 15ksi, a 5ft wide and 8in thick deck, and used 0.6in
diameter strands. One issue encountered with this data set was that several specimens
were tested for compressive strength and modulus of elasticity using different curing
methods, making unclear which would be the best representation of the actual beam. Also, from the measured loss values reported, the contribution from the steel relaxation was not a measurement but rather an estimate based on information from the strand producer.

3.2.2. Analysis of Data

The losses for each of the girders were estimated with five different procedures: (1) PCI Design Handbook (2010) method; (2) AASHTO (2012) refined method; (3) PCI-BDM (2003) time-step method using the AASHTO (2012) creep and shrinkage models; (4) PCI-BDM (2003) time-step method using the PCI-BDM (2003) creep and shrinkage models; and (5) time-step method explained by Swartz (2010) using the AASHTO (2012) creep and shrinkage models. Table 3.3 through Table 3.6 below show the measured data and estimated values for each study. Figure 3.6 through Figure 3.9 following them illustrate the ratio of estimated to measured data for each girder and method.

The first set of data corresponds to the AASHTO Type II girders by Rizkalla et al. (2007). Table 3.3 shows the losses estimated remained fairly consistent through the methods, varying from 26.4ksi to 48.6ksi. All results were conservative, with the losses being overestimated in many cases by over 50% regardless of the strength used, as seen in Figure 3.6. The PCI Design Handbook (2010) method gave the most conservative predictions, with an estimate of more than double the measured losses for five out of the nine specimens. For cases with strengths higher than 15ksi, the estimated values for different strengths within the same deck case did not show a significant change; Table 3.3 shows only a 1ksi difference, except for the Swartz (2010) and the PCI Design Handbook (2010) methods. This is not consistent with the measured data, where the losses for the 18ksi specimens were 4ksi to 6ksi larger. Some methods estimated a bigger loss for the 18ksi cases than the 14ksi cases, while others showed a smaller loss. These observations demonstrate discrepancies among the methods.
Table 3.3 - Measured vs. Calculated Values for Data by Rizkalla et al. (2007)

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Figure 3.6 - Estimated-to-Measured Ratios for the Data by Rizkalla et al. (2007)
The next data set belongs to the North Concho Overpass Bridge from Gross and Burns (2000). The strengths for these beams varied between 13ksi and 14ksi. Again, the PCI Design Handbook (2010) method showed the largest overestimation of the losses, mostly from 20ksi to 36ksi more, as shown in Table 3.4. For the other methods, the losses were mostly underestimated by up to 15ksi. There is no clear pattern observed regarding compressive strength, but the PCI-BDM (2003) time-step method using the PCI-BDM (2003) creep and shrinkage models generated the closest predictions. Table 3.4 shows the estimates using the AASHTO (2012) refined method and the Swartz (2010) time-step method were within 2ksi of each other. The predictions using the PCI-BDM (2003) method using the different creep and shrinkage models were within 5.2ksi from each other. However, the difference between the AASHTO (2012) refined method and the Swartz (2010) time-step method loss predictions from the PCI-BDM (2003) time step method was much higher, reaching up to a 14ksi difference.

<table>
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<th>Specimen</th>
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<th>Measured Loss (ksi)</th>
<th>Measured Loss (ksi)</th>
<th>Measured Loss (ksi)</th>
<th>Measured Loss (ksi)</th>
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</thead>
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<td>AASHTO Refined</td>
<td>PCI-BDM Time-Step</td>
<td>PCI-BDM Time-Step</td>
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<td>45.9</td>
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<td>68.2</td>
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</table>
Table 3.5 shows that the PCI Design Handbook (2010) method also overestimated the losses the most for the U-beams. But in this case, the losses predicted were much closer to the measured values and to the estimates from the other methods. The AASHTO (2012) refined method and the Swartz (2010) time-step method predicted smaller losses than measured in all beams. The Swartz (2010) method and the PCI-BDM (2003) time-step method using the AASHTO (2012) models gave the closest estimates, showing an average difference to the measured values of about 2.5ksi. The main difference is that the PCI-BDM (2003) estimated conservative results in most specimens while Swartz (2010) was unconservative for all, as seen in Table 3.5 and Figure 3.8. The rest of the estimates were also acceptable showing, on average, a difference from the measured values of 7.4ksi (20%) for the PCI Design Handbook (2010), 5.3ksi (13%) for the AASHTO (2012) refined method, and 4ksi (11%) for the PCI-BDM (2003) using the PCI-BDM (2003) models. The PCI-BDM (2003) method using the different models each gave estimates
within 3ksi from each other. The results from Swartz (2010) method were up to 7.5ksi different from the PCI-BDM (2003) estimates, and 3.6ksi from the AASHTO (2012) refined method. The PCI-BDM (2003) estimates were up to 11ksi different from the AASHTO (2012) refined method. This shows that in this set of data, the methods were more consistent against each other and the measured data than in the previous set.

Table 3.5 - Measured vs. Calculated Values for Data by Gross and Burns (2000) for the Louetta Road Overpass

<table>
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<th>Specimen</th>
<th>Measured Loss (ksi)</th>
<th>Calculated Loss (ksi)</th>
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</thead>
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<td>S16</td>
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<tr>
<td>S25</td>
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<td>41.3</td>
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Figure 3.8 - Estimated-to-Measured Ratios for the Data by Gross and Burns (2000) for the Louetta Road Overpass
The two beams tested by Canfield (2005) had compressive strengths of over 15ksi. The PCI Design Handbook (2010) overestimated their losses by almost twice the measured values, as seen in Figure 3.9. Table 3.6 shows the rest of the methods predicted the losses within 5ksi of the measured values. Only one case under estimated them by less than 1ksi. The time-dependent analysis by the PCI-BDM (2003) using the AASHTO (2012) models showed the best estimates, followed by Swartz (2010). All methods, excluding the PCI Design Handbook (2010), were within 4ksi of each other.

Table 3.6 - Measured vs. Calculated Values for Data by Canfield (2005)

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<td>BT-56</td>
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<td>42.8</td>
<td>39.5</td>
<td>42.7</td>
<td>41.1</td>
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</table>

Figure 3.9 - Estimated-to-Measured Ratios for the Data by Canfield (2005)
It is important to note that the PCI Design Handbook (2010) estimates the losses at the final time of the life of the member while the other methods calculated them for the age at the moment of testing. Another factor that influences the results for the time-step methods is the number of intervals used. The steps used were the major events with an additional step in between. More refined results are expected with a larger amount of time-steps. However, this was tried for a few specimens using the Swartz (2010) method and the results did not show a considerable change, deeming the numbers of steps used in this study as adequate. Finally, some input values affect the results more than others. The compressive strength of the concrete at release is an important variable, and measured values were known for most of the specimens. However, the strength at transfer was not known for the beams from the research by Rizkalla et al. (2007). This could be a reason why this set of data had the largest differences between measured and calculated losses.

### 3.3. Summary of Analysis and Findings

Section 3.1 presented the different methods described in Section 2.3. The PCI Design Handbook (2010) method is a simple analysis that separates the contributions from each loss source and does not use an age-adjusted modulus of elasticity. The AASHTO (2012) refined method is a time-dependent procedure that uses an age-adjusted modulus of elasticity and separates the analysis in two stages: before and after casting the deck. This method also calculates the contribution from each loss source separately. The PCI-BDM (2003) and the Swartz (2010) time-step methods are more complex and detailed analyses of the losses. The life of the member is divided into several time intervals. The smaller the intervals, the more refined the analysis. These methods also use an age-adjusted elastic modulus approach and allow the user to find the stresses at any point of the life of the member. However, they find the total stress remaining in the tendons without providing a breakdown of each source of loss.
Section 3.2.2 compared existing data on losses against estimates using the methods explained. The sample used focused on strengths greater than 13 ksi. There were twenty-two specimens with seven having a measured compressive strength greater than 15 ksi. Table 3.7 below gives a summary of the information shown in Figures 3.6 through 3.9. From this table, it can be seen that the PCI Design Handbook (2010) method overestimated the losses the most. Both the AASHTO (2012) refined method and the time-step method developed by Swartz (2010) underestimated the losses for half of the specimens. The PCI-BDM (2003) time-step method using the AASHTO (2012) creep and shrinkage models gave estimates closer to the test data than the PCI-BDM (2003) creep and shrinkage models; however, the AASHTO (2012) models underestimated the losses more often.

Table 3.7 - Summary of Findings

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CHAPTER 4. FINDINGS, CONCLUSIONS, AND RECOMMENDATIONS

4.1. Findings and Conclusions

Following, are findings and conclusions drawn from the review of relevant works in the literature from Chapter 2 and the analysis of test data conducted in Chapter 3. The analysis in Section 3.2.2 was based on available data from twenty-two specimens with compressive strengths varying from 11ksi to 18ksi. From these, thirteen members were between 13ksi and 14ksi, and seven had compressive strengths higher than 15ksi, as shown in Table 3.2. The conclusions drawn from the analysis of those data are regarding the performance of the methods reviewed in Section 3.1 and their applicability to strengths higher than the specified limit of 15ksi.

4.1.1. Findings and conclusions from literature review

The modulus of elasticity of concrete is a property that is constantly changing over time and affects loss calculations. Section 2.2.4.3 discussed the modulus of elasticity and different formulas available to estimate it. When several equations are compared, the large amount of scatter in the data makes it difficult to determine the best fit, as depicted in Figure 2.1. The data used was obtained from the NCHRP reports 496 and 595 by Tadros et al. (2003) and Rizkalla et al. (2007), respectively, with compressive strengths ranging from 1ksi to 18ksi. From Figure 2.1, the equation by ACI 363 (1992) usually underestimated the data. The equation presented by Tadros et al. (2003) in the NCHRP report 496 overestimated most of the measured values with compressive
strengths larger than 12ksi. The CEB-FIP (1990) predictions using two extreme constants to account for aggregate source encompassed most of the data; however, the aggregate type used in the tested members is unknown and specific data points cannot be compared to a given prediction line from this method in Figure 2.1. The estimates by the proposed equation in NCHRP report 595 by Rizkalla et al. (2007) using a unit weight of 140psf underestimated most values. Figure 2.1 shows that the equation defined in the AASHTO Specifications (2012), the PCI Bridge Design Manual (PCI-BDM, 2003), and the ACI 318 (2010) building code gave a better representation by overestimating about half of the data and underestimating the other half. Considering the scatter, the equation for predicting the modulus of elasticity defined by the AASHTO Specifications (2012) is an adequate approximation.

4.1.2. Findings and conclusions from analysis conducted in Chapter 3
A benefit of the PCI Design Handbook (2010) method is that it does not require a large amount of input variables and calculations, and the explanation of the method is straightforward. Figures 3.6 through 3.9 in Section 3.2.2 and Table 3.7 in Section 3.3 show a large discrepancy between measured and calculated losses with predicted-to-measured ratios ranging from 68% to 310%, with an average of 163%. These figures and Tables 3.3 through 3.7 show that the losses were overestimated by more than 40% in most cases, with some being more than twice the measured values. They also show that only two out of the twenty-two specimens were underestimated by 2% and 32%. The cases with strengths higher than 15ksi showed some of the highest overestimations, with predicted-to-measured ratios ranging from 165% to 310%, and an average of 209%. Therefore, the simplicity of the PCI Design Handbook (2010) method leads to estimates significantly higher than the measured values.

The AASHTO Specifications (2012) refined method is a simpler form of a time-step analysis that separates the loss calculation into only two stages: before and after deck
placement. Figures 3.6 through 3.9 and Table 3.7 show that the ratios between the estimated losses and the measured data are 110% on average, with a minimum of 76% and a maximum of 213%. It can be seen in Section 3.2.2 that three of the seven cases above 15ksi were overestimated within 11% of the measured values, while the remaining four specimens showed predictions over 30% and up to 113% of the test data. On average, the estimates for the cases with strengths above 15ksi were overestimated by 38%. Hence, the AASHTO (2012) refined method gives, on average, reasonable results for all the strengths considered. However, it can result in underestimation for specimens within the compressive strength applicability limit or a significant overestimation of the losses, especially for strengths above 15ksi.

The time-step method developed by Swartz (2010) divides the section into layers. Calculating the area and distance to the centroid of each element makes the method cumbersome and time consuming. From observing Figures 3.6 through 3.9 in Section 3.2.2 and Table 3.7 in Section 3.3, the method by Swartz (2010) has estimated-to-measured ratios ranging from 0.74 to 2.39, with an average of 1.14. Tables 3.3 through 3.7 show that all the seven cases with compressive strengths higher than 15ksi had a larger predicted loss than measured, with an average of 1.46. Three of these seven specimens were overestimated within 15% of the test data, while the rest were overestimated by more than 40%. These findings show that the time-step method by Swartz (2010) gives conservative predictions for compressive strengths higher than 15ksi but can considerably underestimate the losses for strengths below 15ksi.

The time-dependent method explained in the PCI Bridge Design Manual (PCI-BDM, 2003) was applied using the creep and shrinkage models from the AASHTO Specifications (2012) and from the PCI-BDM (2003). The results using the models by AASHTO (2012) showed a minimum estimated-to-measured ratio of 0.87 and a maximum of 1.84, with an average of 1.16, as seen in Figures 3.6 through 3.9 in Section 3.2.2. Tables 3.3 through 3.7 show that the prestress losses for eight out of the twenty-two specimens were underestimated, with only one of those having a compressive strength greater
than 15ksi. The figures in Section 3.2.2 demonstrate that cases with strengths higher than 15ksi had an average ratio of 1.35, a minimum of 0.98 and a maximum of 1.84. Hence, using the AASHTO (2012) creep and shrinkage models in the PCI-BDM (2003) time-step method yields to estimated losses close to measured values, with mostly conservative results for strengths above 15ksi.

As shown in Figures 3.6 through 3.9 and Table 3.7, using the PCI-BDM (2003) creep and shrinkage models in the PCI-BDM (2003) time-step method gave ratios between predicted and measured values from 0.96 to 2.07, with an average of 1.28. It can be seen in Tables 3.3 through 3.7 that from all twenty-two specimens, only two were underestimated by less than 4%. All test data with compressive strengths higher than 15ksi had conservative results. The figures in Section 3.2.2 illustrate that the estimated-to-predicted ratios for these seven members were between 1.06 and 2.97, with a 1.49 average. Therefore, the PCI-BDM (2003) time-step method and creep and shrinkage models give conservative predictions.

When the methods are compared against each other, the AASHTO Specifications (2012) refined method is more calculation intensive than the PCI Design Handbook (2010) method, but it is simpler than the time-dependent methods. Section 3.2.2 and the conclusions above show that the AASHTO (2012) refined method and the time-step method by Swartz (2010) had the predicted-to-measured ratios closest to unity, with a 1.10 and 1.14 respectively. However, both methods also underestimated the results for half the data, with a predicted loss 26% lower than measured in the worst case, as seen in Figures 3.7 and 3.8. Hence, loss estimates using the Swartz (2010) method were comparable to those by the AASHTO (2012) refined method, but with a considerable additional amount of work needed.

By comparing Sections 2.3.2 and 2.3.3, both the AASHTO (2012) and the PCI-BDM (2003) creep and shrinkage models applied in the PCI-BDM (2003) time-step analysis use factors to account for the same conditions: time of loading, member size, compressive
strength, relative humidity, and time development. The main difference is that the volume-to-surface area and the strength factors are divided into different equations for creep and shrinkage calculations in the PCI-BDM (2003) model, while the AASHTO (2012) model simplifies them to only one. As shown in Section 3.2.2 and the findings above, using the AASHTO (2012) models gave closer results to the measured values, with a 1.16 average ratio, than using the PCI-BDM (2003) model which showed a 1.28 average. However, the latter model gave more conservative predictions. Therefore, using the creep and shrinkage models and the time-step method by PCI-BDM (2003) was found to have the best combination of conservatism and predicted-to-measured ratios.

The AASHTO Refined method and both time-step methods using the AASHTO material models underestimated the losses for most specimens with strengths between 13.0ksi and 14.9ksi. Possible reasons include that these members were field monitored instead of lab tested, as the rest. Also, the contribution to the total losses from the relaxation in the steel was estimated instead of measured. Finally, these were larger sections with a much higher level of prestress. The impact of this observed underestimation should be considered depending on the specific application such as camber calculations, cracking prevention, etc.

As described in Sections 3.2.1, 3.3, and 4.1, only seven out of the twenty-two specimens used had compressive strengths greater than 15ksi: four with 15ksi, one with 16ksi, and two with 18ksi. Tables 3.3 and 3.6 show that the estimate of prestress losses for all but one of these members were conservative with only one instance where losses were underestimated by 2%. Although Rizkalla et al. (2007) concluded that the AASHTO (2012) refined method was applicable for strengths up to 18ksi, the calculated losses are overly conservative, overestimation of the measured losses from as low as 10% to as high as 113% for that specific data set was noted as shown in Figure 3.6 and Table 3.3. In addition, the ratios of estimated losses to measured values for the members with compressive strengths below 15ksi ranged from 0.68 to 2.46. This shows that there is significant scatter, including for strengths within the methods’ applicability limits. The
scatter warns that only seven specimens is not an adequate sample size. Hence, there is not enough data to determine if the methods are applicable to strengths higher than 15ksi.

4.2. Recommendations and Needed Research

4.2.1. Recommendations from this Study
- The PCI-BDM (2003) time-step method using the PCI-BDM (2003) creep and shrinkage models is recommended for analysis, as proved in Section 4.1.2.
- The current 15ksi strength limit needs to be kept. Increasing the limit cannot be justified based on the observed variation in the estimates for the small data sample available.

4.2.2. Needed Research
Testing members with measured compressive strengths above 15ksi is needed. Before testing and during the design phase of the testing program, there is value in carrying out a sensitivity study to understand better which parameters affect the losses most. The input values need to be varied accordingly, such as using an appropriate release compressive strength, levels of prestress, and section sizes for a given design compressive strength. This preliminary analysis would help determine what kind of testing would be more beneficial. Comparing the results from this sensitivity analysis using several methods is ideal and it could further ratify the results from this study.

This thesis focused on bonded applications in normal-weight concrete. Further work is needed if prestress losses for unbonded tendons are being evaluated. The same general procedure is recommended: doing an extensive literature review, evaluating the performance of the existing methods along with a sensitivity study, and testing of specimens if deemed necessary.
REFERENCES
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APPENDIX
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A = area of section.

$A_c =$ gross composite section area.

$A_d =$ area of concrete deck.

$A_g =$ gross cross-sectional area of member.

$A_k =$ area of element $k$.

$A_p =$ area of prestressing strands.

$A_{ps} =$ area of prestressing strands.

$A_s =$ area of the mild steel.

$C (t,t_0) =$ creep coefficient.

$CR =$ loss due to creep.

$C_u =$ ultimate concrete creep coefficient.

$d_p =$ distance from centroid of prestressing steel to top of the deck.

$d_s =$ distance from centroid of mild steel to top of deck.

$e =$ eccentricity of center of gravity of prestress with respect to center of gravity of section.

$e_d =$ eccentricity between gross composite centroid and deck.
\( e_{pc} \) = eccentricity between prestressing strands and centroid of composite section.

\( e_{pg} \) = eccentricity of prestressing force with centroid of member.

\( E^* \) = age-adjusted modulus for element \( k \).

\( E^*_c(t, t_0) \) = age-adjusted modulus of elasticity.

\( E^*_{ck} \) = age-adjusted modulus of element \( k \).

\( E_c \) = modulus of elasticity of concrete.

\( E_c(t_0) \) = modulus of elasticity of concrete at transfer.

\( E_{ci} \) = modulus of elasticity of concrete at transfer.

\( E_{cd} \) = modulus of elasticity for concrete deck.

\( E_p \) = modulus of elasticity of prestressing steel.

\( E_{ps} \) = modulus of elasticity of prestressing steel.

\( E_s \) = modulus of elasticity of mild steel.

\( E_S \) = loss due to elastic shortening.

\( f'_c \) = specified compressive strength.

\( f(t_0) \) = tensile stress at beginning of interval.

\( f_{cds} \) = stress in concrete at center of gravity of steel due to superimposed permanent dead loads.

\( f_{cgp} \) = concrete stress at center of gravity of prestressing steel due to prestressing force at transfer and self-weight of member at maximum moment section.

\( f_{cir} \) = stress in concrete at center of gravity of steel right after transfer.
\( f_{cm} = \) mean compressive strength of concrete.

\( f_{pi} = \) initial jacking stress.

\( f_{pt} = \) stress in prestressing strands right after transfer.

\( f_{pu} = \) ultimate strength of prestress.

\( f_{py} = \) prestressing steel yield strength.

\( f_y = \) yield strength of strand.

\( H = \) relative humidity.

\( I_c = \) gross composite section moment of inertia.

\( I_g = \) moment of inertia of gross section.

\( I_k = \) moment of inertia of element \( k \).

\( J = \) values defined in Table 2.3

\( k_{cp} = \) factor for moist curing period used other than 7 days

\( k_f = \) factor for concrete strength effects.

\( k_h = \) factor for relative humidity.

\( k_{hc} = \) humidity factor for creep.

\( k_{hs} = \) humidity factor for shrinkage.

\( k_{la} = \) factor for loading age.

\( k_s = \) factor for size of the member.

\( k_{st} = \) factor for high strength concrete.

\( k_{td} = \) time development factor.
$K_1 = $ factor for aggregate source.

$K_{cir} = 0.9$ for pretensioned strands.

$K_{cr} = 2.0$ for normal-weight concrete. $1.6$ for sand-lightweight concrete.

$K_{df} = $ transformed-section coefficient between deck casting and final time.

$K_{id} = $ transformed-section coefficient between transfer and deck casting.

$K_L = $ factor for kind of steel used. $30$ for low-relaxation strands. $7$ for any other prestressing steel.

$K'_L = $ material constant. $45$ for low-lax. $10$ for stress relieved steel.

$K_r = $ material constant. $45$ for low-lax. $10$ for stress relieved steel.

$K_{re} = $ values defined in Table 2.3.

$K_{sh} = $ factor for time between the end of moist-curing and application of prestress force.

$m = $ total number of layers.

$M = $ total moment.

$M = $ applied moment on section.

$M_{ok} = $ theoretical restrain moment in element $k$.

$M_{applied} = $ moment due to applied loads.

$M_d = $ effective moment due internal stresses for layers in deck.

$M_g = $ moment due to self-weight of member, along with permanent dead loads present at moment of prestressing.

$M_l = $ effective moment due to creep internal stresses.
\( M_k \) = moment in element \( k \).

\( M_P \) = effective moment due to initial strand tension internal stresses.

\( M_{sd} \) = moment due to superimposed permanent dead loads applied after stressing strands.

\( N \) = total axial force.

\( N \) = applied force.

\( N_0 \) = total theoretical normal force in section.

\( N_{0k} \) = theoretical normal force in element \( k \).

\( N_d \) = axial force due to internal stresses for layers in the deck.

\( N_I \) = axial force due to creep internal stresses.

\( N_k \) = force in element \( k \).

\( N_P \) = axial force due to initial strand tension internal stresses.

\( P_i \) = initial prestress force after instantaneous losses.

\( \text{RE} \) = loss due to relaxation

\( \text{RH} \) = relative humidity.

\( S(t, t_0) \) = shrinkage strain experienced in time interval.

\( \text{SH} \) = loss due to shrinkage.

\( t \) = age of concrete at desired time after loading.

\( t_i \) = age of the concrete at time of loading.

\( t_{la} \) = loading age.
\( t_0 \) = age of concrete at beginning of interval.

\( w_c \) = unit weight of concrete.

\( TL \) = total prestress loss.

\( V/S \) = volume-to-surface ratio.

\( y \) = distance to centroid of section.

\( y_k \) = distance to centroid of element \( k \).

\( \alpha_E \) = factor for strength of aggregate used.

\( \Delta f_{ES} \) = loss due to elastic shortening.

\( \Delta f_{cd} \) = concrete stress change at strand centroid by long-term losses between transfer and deck placement along with the superimposed loads and weight of deck.

\( \Delta f_{cdf} \) = concrete stress change at strand centroid due to concrete deck shrinkage.

\( \Delta f_{PA} \) = loss due to anchorage setting.

\( \Delta f_{pCD} \) = loss in prestress due to creep, between deck placement and final time.

\( \Delta f_{PCR} \) = loss due to creep.

\( \Delta f_{PF} \) = loss due to friction in post-tension applications.

\( \Delta f_{pLT} \) = total loss of prestress in steel due to long-term losses.

\( \Delta f_{PR} \) = loss due to relaxation after transfer.

\( \Delta f_{psD} \) = loss in prestress due to shrinkage between deck placement and final time.

\( \Delta f_{psR} \) = loss due to shrinkage.

\( \Delta f_{psS} \) = loss in prestress due to shrinkage of deck.
\( \Delta f_{pT} = \text{total loss of prestress in steel.} \)

\( \Delta \sigma = \text{stress increment between current time step and previous step.} \)

\( \varepsilon = \text{total strain in section.} \)

\( \varepsilon_0 = \text{reference strain in strain profile.} \)

\( \varepsilon_{0k} = \text{initial strain in element.} \)

\( \varepsilon_{bdf} = \text{strain in girder due to concrete shrinkage between deck placement and final.} \)

\( \varepsilon_{bid} = \text{strain in girder due to concrete shrinkage between transfer and deck placement.} \)

\( \varepsilon_{cr} = \text{creep strain.} \)

\( \varepsilon_{ddf} = \text{strain in the concrete deck due to shrinkage.} \)

\( \varepsilon_k = \text{elastic strain in element } k. \)

\( \varepsilon'_{j} = \text{effective strain due to jacking force on prestressing steel considering relaxation losses.} \)

\( \varepsilon_{l} = \text{inelastic strain.} \)

\( \varepsilon_k = \text{strain in element } k. \)

\( \varepsilon_{\text{ke}lastic} = \text{elastic strain.} \)

\( \varepsilon_{K\text{total}} = \text{total strain} \)

\( \varepsilon_R = \text{strain due to relaxation.} \)

\( \varepsilon_s = \text{prestress strain.} \)

\( \varepsilon_{sh} = \text{shrinkage strain.} \)

\( \varepsilon_{Tk} = \text{total stress-related strain.} \)
\( \sigma_{\text{elastic}} \) = elastic stress found in the previous step.

\( \sigma_R \) = stress due to relaxation strain.

\( \sigma_s \) = prestress stress.

\( \phi \) = reference curvature in strain profile

\( \phi_{0k} \) = initial curvature in element

\( \phi_{ik} \) = elastic curvature in element k.

\( \phi_k \) = curvature of element k.

\( \chi(t,t_0) \) = aging coefficient.

\( \Psi \) = creep coefficient.

\( \Psi(t,t_i) \) = girder creep coefficient.

\( \Psi_b(t_d, t_i) \) = creep coefficient of girder at the time of deck placement due to load at transfer.

\( \Psi_b(t_f, t_d) \) = creep coefficient of girder at final time due to deck placement load.

\( \Psi_b(t_f, t_i) \) = creep coefficient of the girder at final time due to load at transfer.

\( \Psi_d(t_f, t_d) \) = creep coefficient of deck at final time due to deck placement load.