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Theoretical and Experimental Study on Valve Flutter

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ABSTRACT

A model for valve flutter is presented. Using classical theory of stability the onset of flutter could be predicted. Stability diagrams are presented, which enable the valve designer to take in at glance the flutter situation of a certain valve by a simple procedure. Flutter experiments with an enlarged model and with compressor valves are discussed, which confirm the flutter model. A similarity theory for non steady compressor behaviour is outlined.

Part I: THEORY OF VALVE FLUTTER

1. INTRODUCTION

The minimal demands on a system able to flutter are a chamber(volume V) with a spring loaded valve mounted on it, Fig.1a. A valve v allows to control the flow rate $\dot{V}$ passing the system. The spring loaded valve experiences a certain lift X depending on flow rate. Increasing flow rates cause increasing(stable) equilibrium positions X of the valve plate. Above a certain limit the equilibrium position becomes unstable and the plate starts an increasing oscillatory movement, until it is stopped by external influences, Fig.1b. Hence the flutter phenomenon may be defined and described as a problem of equilibrium stability which can be treated with classical theory of stability.

In compressors, valves are not subject to a constant flow rate but to a varying flow according to piston movement. In this case the motion of the valve plate may become unstable too. Problems of stability of motion are much more complicated than problems of stability of equilibrium and call e.g. for Ljapunow's theory of stability. Fortunately the motion of the plate in absence of flutter has a more or less quasisteady character so that classical theory of stability gives a good approach for flutter phenomena in compressors.

Often valve flutter is seen in conjunction with pressure oscillations in the piping. It should be noted however that these are two quite different phenomena though there may be interference between them. This paper is focussed on valve flutter and ignores completely pressure pulsations in the piping. Constant inflow and outflow to the system is assumed throughout this paper.

Fig.2a gives a sketch of the flutter model. Let us suppose that the valve plate -subject to flow- is in an equilibrium position X(pressure difference ΔP_{12}) and experiences a small initial disturbance x. As a consequence the flow across the valve increases, thus producing a decrease of mass content in volume V_1 and an increase in V_2. This in turn causes an isentropic pressure increase in V_2 and a decrease in V_1. This change in pressure difference Δp_{12} is not proportionate to x but is an integral effect. At the same time the plate motion acts like a piston between the two volumes and this also results in an isentropic change in pressure difference Δp_{12} (the socalled gas spring effect, [6]), this time proportionate to x. The signal flow is sketched in Fig.2b. The

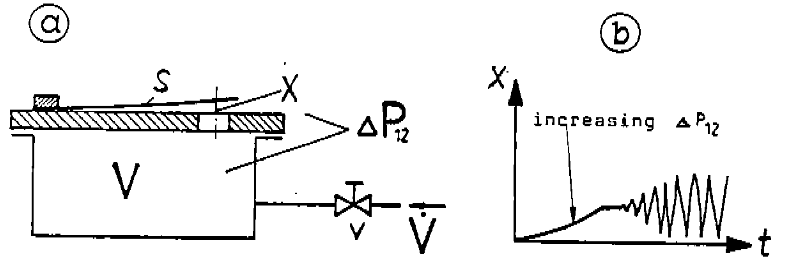


Fig.1 Valve flutter arrangement. a basic set up; s spring loaded valve(reed valve); v supply control valve. b onset of flutter.

changes in pressure difference cause -via non steady flow equation- a time delayed change in exit velocity W_2. This in turn changes the force on valve plate F_{pl} and -via valve platé motion equation- gives a feed-back to the initial disturbance x. According to the basic relations a certain phase shift φ results between x and F_{pl}. For $0° < \varphi < 180°$ the initial disturbance is amplified(→flutter); for $180° < \varphi < 360°$ damped out (→stable equilibrium position).

The traditional compressor process simulation model has a signal flow diagram(with resp. to flutter) as sketched in Fig.2c: the gas spring effect is ignored and quasisteady flow equations are used instead of non steady flow equations.Experiments back a flutter model according to Fig.2b.

From a point of view of energy flow, valve flutter occurs if sytem para-meters allow, that energy flows from gas flow to the valve plate. There

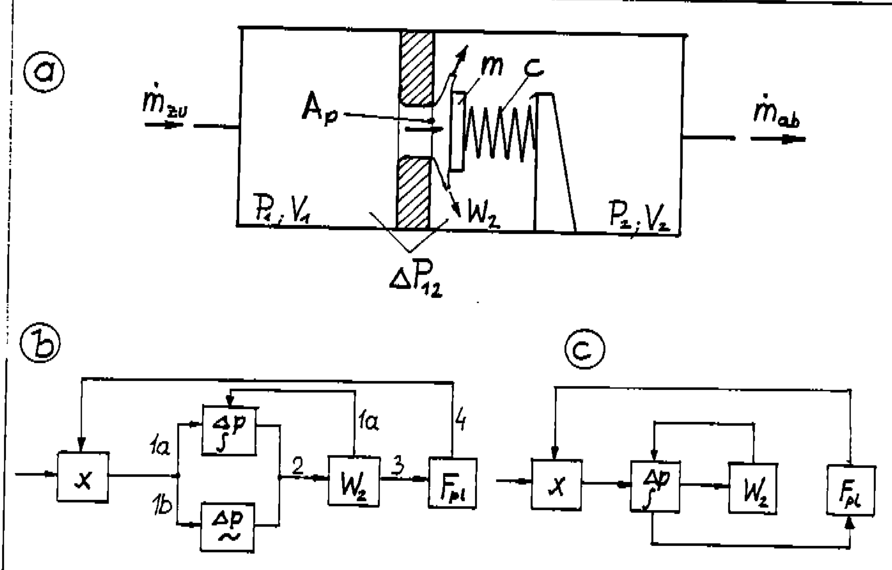


Fig.2 Flutter model. a basic arrangement and notation; b signal flow diagram for proposed flutter model; c signal flow diagram according to the traditional computer simulation of compressor process.

is a profound analogy to aircraft wing flutter: above a certain air speed energy flows from air flow to wing bending and torsional vibrations. This is expressed clearly by the parallelism in the basic equations for valve flutter and wing flutter.

2. BASIC EQUATIONS FOR VALVE FLUTTER

Non steady flow equations, equations for the gas spring effect and for the plate force have been derived elsewhere in these Proceedings [6]. These correspond to relations 1b,2,3 in Fig.2b. Missing are relations 1a and 4. Relation 1a concerns the change in pressure difference Δp_{12} due to transfer of a small mass m from V_1 to V_2 by a disturbance of the equilibrium position(x). For mathematical treatment we denote parameters at equilibrium position by capital letters and small oscillations about equilibrium position by small letters, according to

$X, W_2, \Delta P_{12}, F_{pl}$ equilibrium position

$x(t), w_2(t), \Delta p_{12}(t), f_{pl}(t)$ small oscillations

Refering to [6], Fig.4 we get

$$dP = kP_{m12} \cdot \frac{dm}{\rho}\left(\frac{1}{V_1} + \frac{1}{V_2}\right)$$

$$\frac{dP}{dt} = \Delta\dot{p}_{12} = kP_{m12} \cdot \frac{1}{\rho}\frac{dm}{dt}\left(\frac{1}{V_1} + \frac{1}{V_2}\right)$$

$$\frac{dm}{dt} = \dot{m}_{zu} - \rho LC_D(W_2+w_2)\cdot(X+x) = \rho LC_D XW_2 - \rho LC_D XW_2 - \rho LC_D(W_2x+Xw_2+xw_2)$$

Neglecting quantities small of second order(i.e. xw_2) results in

$$\Delta\dot{p}_{12} = -kP_{m12}LC_D(W_2x+Xw_2)\cdot\left(\frac{1}{V_1} + \frac{1}{V_2}\right)$$

The dynamic equation for the motion of the plate reads(see Fig.2a)

$$m\ddot{x} + d.\dot{x} + cx = f_{pl}$$

Here we have used a damping force $d.\dot{x}$ which is proportionate to plate velocity $\dot{x}$. Now we have collected the five relations according to the signal flow diagram of Fig.2b. Relations 1a and 1b simply could be added. So we may summarize the equations to

$$\Delta\dot{p}_{12} = -kP_{m12}\left[A_p r\dot{x}+LC_D(W_2x+Xw_2)\right]\cdot\left(\frac{1}{V_1} + \frac{1}{V_2}\right) \qquad (1)$$

$$\frac{\Delta p_{12}}{\rho} = \frac{W_2 w_2}{1+\beta} + J(X\dot{w}_2+W_2\dot{x}) + \frac{A_p W_2}{2LC_D}\dot{x} \qquad (2)$$

$$f_{pl} = \rho A_p W_2 w_2 \qquad (3)$$

$$m\ddot{x} + d.\dot{x} + cx = f_{pl} \qquad (4)$$

To get equ(2) and (3) from equ(8) and (9) in [6] one has to subtract steady state values which hold for equilibrium position and neglect quantities small of second order. The suppression of quantities small of second order corresponds to a linearization of the equations which is the first step in a classical stability investigation.

To get a better survey and to prepare similarity considerations we change to non dimensional variables according to

$$\overline{x} = x/X \qquad \overline{f}_{pl} = f_{pl}/F_{pl}$$

$$\overline{w}_2 = w_2/W_2 \qquad \overline{t} = \omega_0 t \quad \text{with } \omega_0 = \sqrt{c/m} \tag{5}$$

$$\overline{\Delta p}_{12} = \Delta p_{12}/\Delta P_{12}$$

To simplify further we use, beginning from now, the following abbreviations

$$v = \frac{1}{V_1} + \frac{1}{V_2} \qquad \text{volume function}$$

$$\dot{()} = \frac{d()}{d\overline{t}} = \omega_0 \frac{d()}{dt} \qquad \text{derivative with resp. to } \overline{t} \tag{6}$$

Introducing equ(5) and (6) into (1),(2),(3),(4) we get -after some mathematical procedure- the following non dimensional and linearized basic equations for the flutter problem with 6 nondimensional constants:

$\dot{\overline{\Delta p}}_{12} = -C_1(\overline{w}_2+\overline{x}) - C_2\dot{\overline{x}}$	(7)	in Fig.2b relation	1
$\Delta p_{12} = 2\overline{w}_2+C_5\dot{\overline{w}}_2+C_6\dot{\overline{x}}$	(8)		2
$\overline{f}_{pl} = 2\overline{w}_2$	(9)		3
$C_3(\ddot{\overline{x}}+\overline{x}) + C_4\dot{\overline{x}} = \overline{f}_{pl}$	(10)		4

$$C_1 = \frac{kP_{m12}LC_DXW_2v}{\omega_0 \cdot \Delta P_{12}} \qquad \text{mass transfer parameter}$$

$$C_2 = \frac{kP_{m12}A_pXrv}{\Delta P_{12}} = C_3 \cdot \frac{\Delta c \cdot c_p}{c} \qquad \text{gas spring parameter}$$

$$C_3 = \frac{cX}{c_pA_p\Delta P_{12}} \qquad \text{spring characteristic parameter} \tag{11}$$

$$C_4 = \frac{d\omega_0 X}{c_pA_p\Delta P_{12}} = C_3 \cdot \frac{d\,\omega_0}{c} \qquad \text{damping parameter}$$

$$C_5 = \frac{2JX\omega_0}{W_2 \cdot (1+\beta)} = C_{6'} \qquad \text{gas inertia parameter}$$

$$C_6 = C_{6'}+C_{6''} = \frac{2JX\omega_0}{W_2 \cdot (1+\beta)} + \frac{A_p \cdot \omega_0}{LC_DW_2(1+\beta)} \qquad \text{non steady flow parameter}$$

c designates the spring constant. If the characteristic of the spring is nonlinear, i.e. c=c(X), the local spring constant at the lift position X, whose stability is investigated, is applicable, Fig.3. The spring characteristic parameter C_3 may be interpreted easily according to Fig.3. For all springs with linear characteristic having no prestress for X=0 we have

$$C_3 = 1 \qquad \text{for linear spring without prestress} \tag{12}$$

This is true for nearly all reed valves. The 6 non dimensional constants $C_1 \ldots C_6$ are formed from 16 constants having different dimensions ($k, P_{m12}, L, C_D, X, W_2, v, \omega_0, \Delta P_{12}, A_p, r, c, d, J, \beta, c_p$), but flutter is controlled by the 6 non dimensional constants only.

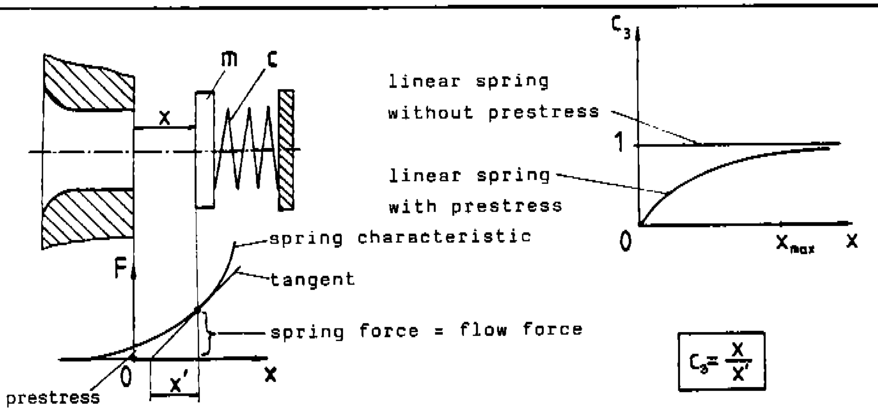


Fig.3 Relations concerning spring characteristic parameter

Equ(7),(8) and (9),10) may be combined to 2 equations as follows

$$\ddot{\overline{w}}_2 + \frac{2}{C_5}\dot{\overline{w}}_2 + \frac{C_1}{C_5}\overline{w}_2 = -\frac{C_6}{C_5}\ddot{\overline{x}} - \frac{C_2}{C_5}\dot{\overline{x}} - \frac{C_1}{C_5}\overline{x} \qquad (13)$$

$$\ddot{\overline{x}} + \frac{C_4}{C_3}\dot{\overline{x}} + \overline{x} = \frac{2}{C_3}\overline{w}_2 \qquad (14)$$

If we put $w_2=0$(≙ no flow changes) the left side of equ(14) describes the damped oscillations of the valve plate. The natural frequency becomes

$\overline{\omega}_o = 1$ (with resp. to $\overline{t}$)

$\omega_o = \sqrt{c/m}$ (with resp. to t) natural frequency $f_o = \frac{1}{2\pi}\sqrt{\frac{c}{m}}$

If we put $x=0$(≙ valve plate fixed) the left side of equ(13) describes the flow and pressure oscillations of an acoustical system consisting of V_1, V_2 and the acoustical mass in the port area. This system is of the Helmholtz-resonator type. The acoustical natural frequency becomes

$$\overline{\omega}_{g,o} = \sqrt{C_1/C_5} \qquad \omega_{g,o} = \omega_o \cdot \sqrt{C_1/C_5} \quad \text{nat.frequ. } f_{g,o} = f_o \sqrt{C_1/C_5} \qquad (15)$$

The ratio of the both natural frequencies becomes

$$\boxed{f_{g,o}/f_o = \sqrt{C_1/C_5}}$$

This quantity is of great importance for valve flutter if gas inertia is appreciable. Equ(15) gives the basis for the experimental determination of the inertia parameter J according to equ(13) in [6].

The right sides of equ(13),(14) are the coupling terms of the two vibration systems. Equ(13),(14) are very similiar to the equations of aircraft wing flutter(coupled bending and torsional vibrations with energy input from air flow),[7].

First of all we are interested to predict if under certain conditions (given constants $C_1 \ldots C_6$) flutter will occur or not. Following the standard procedure prescribed by classical theory of stability(see e.g. [8]) one has to start with the basic (linearized) equations (7)....(10) and has to put the solutions:

$$\bar{x} = Ae^{\alpha\bar{t}} \qquad \text{with } \alpha = a+ib \quad i=\sqrt{-1}$$
$$\bar{w}_2 = Be^{\alpha\bar{t}} \qquad B = B_1+iB_2$$
$$\overline{\Delta p}_{12} = De^{\alpha\bar{t}} \qquad D = D_1+iD_2 \tag{16}$$
$$A, B_1, B_2, D_1, D_2 \text{ real numbers}$$

hence $\bar{x} = Ae^{a\omega_0 t}\left[\cos(b\omega_0 t) + i.\sin(b\omega_0 t)\right]$ etc.

Introducing (16) into (7)...(10) results in the socalled <u>characteristic equation</u> of the problem. This equation and their solutions are the key for estimating stability. The characteristic equation for our problem is of the fourth order and reads

$$a_0\alpha^4 + a_1\alpha^3 + a_2\alpha^2 + a_3\alpha^1 + a_4 = 0 \tag{17}$$

$$\text{with } a_0 = \tfrac{1}{2}C_5 \qquad a_3 = 1+C_2/C_3+\tfrac{1}{2}C_1C_4/C_3$$
$$a_1 = 1+\tfrac{1}{2}C_4C_5/C_3 \qquad a_4 = \tfrac{1}{2}C_1+C_1/C_3 \tag{18}$$
$$a_2 = \tfrac{1}{2}C_1+C_4/C_3+\tfrac{1}{2}C_5+C_6/C_3$$

There are 4 solutions of this characteristic equation which migth be real or complex. Usually there is at least one pair of conjugate complex solutions $\alpha_{1,2}$ (corresponding to the oscillatory solution of the spring mass system)

$$\alpha_{1,2} = a \pm ib \qquad \text{or} \qquad \alpha_{1,2} = a \pm ib$$
$$\alpha_{3,4} = \bar{a} \pm i\bar{b} \qquad \alpha_3 = \bar{a}$$
$$\alpha_4 = \bar{\bar{a}}$$

A computer has to be used to solve equ(17). For stability($\hat{=}$no flutter) it is necessary that all 4 solutions have negativ real parts($a<0, \bar{a}<0, \bar{\bar{a}}<0$).The value "b" is responsible for the frequency, which in nearly all cases exceeds the natural frequency of the valve plate($b>1$)

$$f = f_0 . b \tag{19}$$

Amoung the 4 solutions $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ the one with the biggest value a is of interest for us, for simplicity denoted by α . The others correspond to motions which decline more rapid to zero. They are of interst only if certain initial conditions have to be fulfilled. If there are two pairs of conjugate complex solutions, usually b is near to 1 and $\bar{b}$ near to $f_{g,o}/f_o$. So the first pair $\alpha_{1,2}$ may be attributed to the spring-mass system, and the second pair $\alpha_{3,4}$ to the acoustic system. Often for the latter the solutions become real numbers($\bar{a}, \bar{\bar{a}}$) and hence an aperiodic solution for $\bar{x}$ results. Usually the spring-mass system is dominating, but under special conditions the acoustical system may become dominating (high pressure compressors).

It is not possible to discuss here the many aspects of the solutions of equ(17). The reader is refered to [1].Some results of general interest which in many cases allow a quick survey of the flutter situation are presented in the next section.

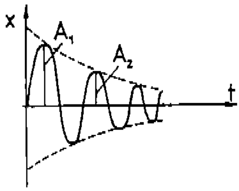
The procedure for solving a flutter problem may be sumed up as follows:

- Find the 6 constants $C_1 \ldots C_6$ from the parameters of the problem equ(11)
- Find the 5 coefficients $a_0 \ldots a_4$ of the characteristic equation equ(18)

- Solve the characteristic equation(17), which is of fourth order, with a computer. If gas inertia is negligible($C_5=0$), this equation is of third order and may be solved even with a pocket calculator. Usually C_1, C_2 are of primary importance only($C_3=1$, $C_4 \approx 0$).

- Estimate the 4 solutions: if all real parts are negative($a<0, \bar{a}<0...$) the sytem is stable and flutter will not occur. If one or more real parts are positive, the system is unstable(flutter).

- The grade of stability (or flutter) may be estimated by the ratio of the amplitudes of 2 successive oscillations, Fig.4:



$$A_2 : A_1 = e^{2a\pi/b} \qquad (20)$$

Fig.4 Definition of a quantity to measure the grad of stability or flutter

3.STABILITY DIAGRAMS

Based on computer solutions of equ(17) for various parameters according to equ(11) stability diagrams may be constructed. Stability diagrams presented are restricted to $C_3=1$(i.e. for valves with linear spring and without prestress, e.g. reed valves) and to $C_5=0$(gas inertia neglected). Instead of using C_1 in diagrams we better use

$$C_o=C_1/C_2 = \frac{LC_D}{\omega_* A_p r} W_2 \sim W_2 \sim \sqrt{\Delta P_{12}}; \quad \text{velocity parameter} \qquad (21)$$

The effect of non steady work exchange is included in the diagrams. Fig.5 shows a C_o, C_2-stability diagram for 3 different parameters $r(1+B)$ ($r>1$ being the correction coefficient for the gas spring effect, see Fig.5 in[6], in most cases $B \approx 0$).

For a certain valve, working between volumes V_1, V_2 under a given pressure P_{m12}, C_2 is a fixed number. For increasing ΔP_{12}(i.e. W_2) C_o rises parallel to the C_o-axis until the stability limit is reached. A further rise causes flutter.

Fig.6 shows a C_o, C_2-stability diagram which allows also for various damping parameters C_4. Damping forces act stabilizing. For $C_4>0,75$ flutter is excluded under all conditions.

Fig.7 gives a C_o, C_2-stability diagram for $C_4=0$ with parameters $A_2:A_1$ according to equ(20) indicating the "grade" of stability or flutter. For a valve subject to steady flow, the stability limit is of great importance and states if flutter will occur or not. For a valve working in a compressor it makes not so much difference if we have $A_2:A_1=0,97$ or 1,03 because we are interested in the behaviour of the valve in a time of about 5 oscillating periods following the initial disturbance caused by the opening delay. Hence for a compressor valve we may demand

$$A_2 : A_1 < 0,7$$

This means that after 2 oscillations the initial amplitude reduces to about 50% and to 25% after 4 oscillations.

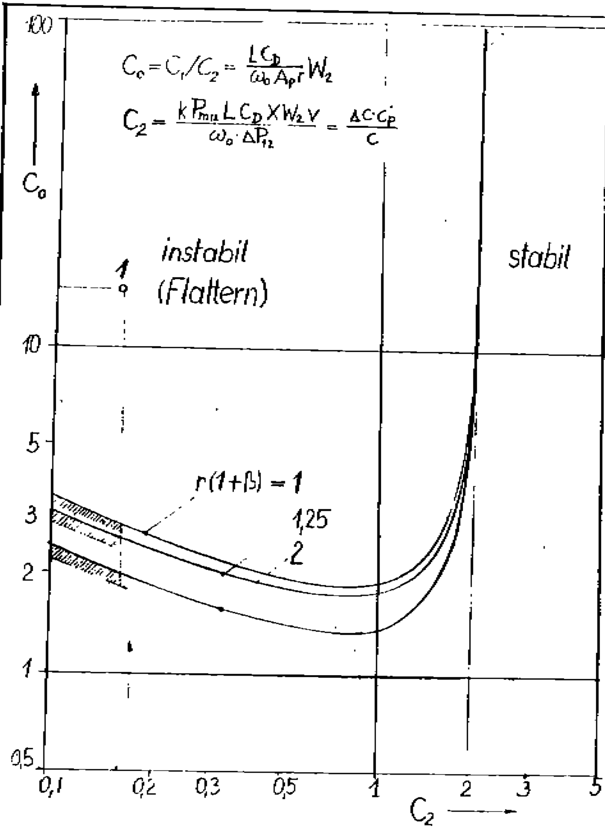


Fig.5 C_o,C_2-Stability diagram for 3 parameters r(1+β)
C_3=1; C_4=0; C_5=C_6,=0 i.e.damping and gas inertia neglected

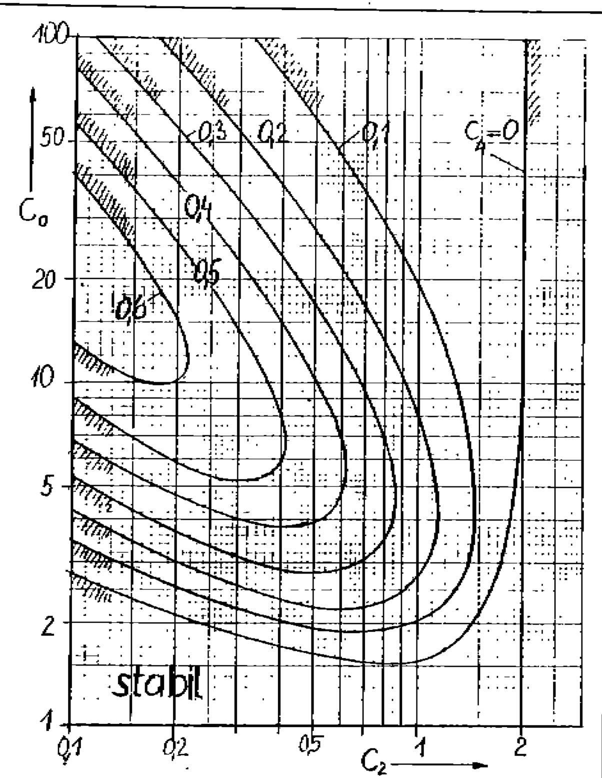


Fig.6 C_o,C_2-Stability diagram for r(1+β)=1.5;
C_3=1; C_5=C_6,=0(gas inertia neglected) and
various damping parameters C_4.

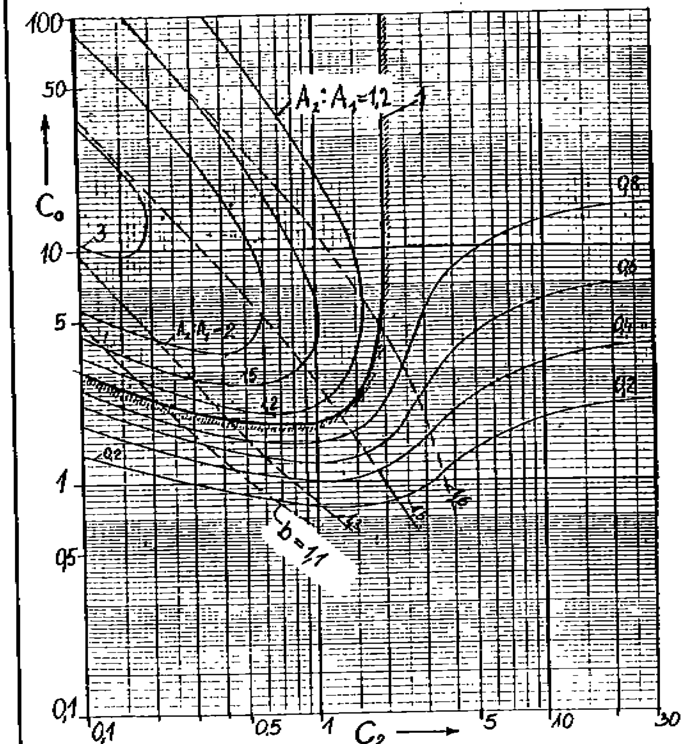


Fig.7 C_o,C_2-Stability diagram for $r(1+B)=1.25$;$C_3=1$ $C_4=0$; $C_5=C_{6'}=0$(damping and gas inertia neglected) with parameterlines for amplitude- and frequency ratio

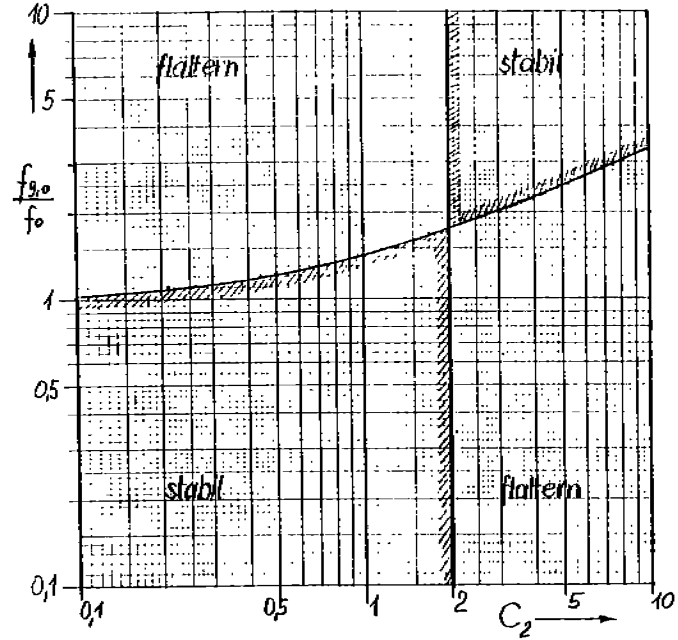


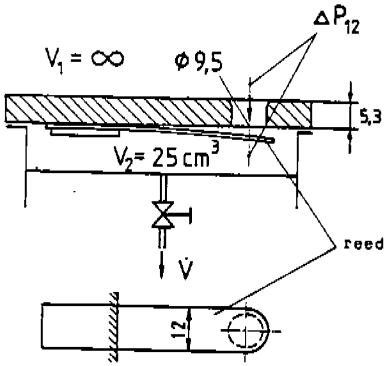
Fig.8 $f_{g,o}/f_o$-C_2-Stability diagram for systems with appreciable gas inertia effect.$C_3=1$; $C_4=0$;$C_{6'}=0$, (damping and non steady work exchange neglected).

Fig.7 indicates that for $C_2>2$ in all cases stability is achieved. Such cases often occur in compressor valves when working near top dead center or with high pressures. For a valve working in a compressor, the successive points of state may be plotted in Fig.7 thus estimating the flutter situation. The diagram in Fig.7 neglects gas inertia. If gas inertia is appreciable similiar diagrams may be constructed but such a diagram is valid for a specific valve only.

Fig.8 gives a stability diagram for systems with appreciable gas inertia effect. To enable a general diagram of this type the non steady work exchange effect has to be neglected. Instead of C_0 the ratio of the natural frequencies $f_{g,o}/f_o$ is used as a variable.

4.EXAMPLE

The application of the theory is demonstrated by the following example, Fig.9. A suction valve works with air at conditions given in Fig.9. $\dot{V}$ corresponds to the momentanous piston displacemen rate.



$c=230N/m$	$P_{m12}=10$ bar
$f_o=125Hz$	$\rho=11.6\ kg/m^3$
$C_D=0,6$	$\dot{V}=2.53.10^{-4}m^3/s$
$c_p=1$	$d=0.00184\ Ns/m$
$\beta=0$	$J=4.96$ equ(4) in [6]

This results in an equilibrium position:

$X=0.71mm$ $\Delta P_{12}=2300$ Pa

$W_2=19.9m/s$

Fig.9 Data for the example

Constants $C_1...C_6$ and coefficients for the characteristic equation become

$C_1=7.852$ $C_2=1.660$($r=1.357$ with $D=12$, Fig.5 in [6]) $C_3=1$

$C_4=0.0063$ $C_5=0.259$ $C_6=0.414$

$a_o=0.130$ $a_1=1$ $a_2=4.476$ $a_3=2.685$ $a_4=11.78$

Solutions of characteristic equation: $\alpha_{1,2}=0.020\pm i1.658$ $a>0$ ⟶ flutter

$\alpha_{3,4}=-3.885\pm i4.177$

Ratio of 2 successive amplitudes:

$$A_2:A_1=e^{2a\pi/b}=e^{2.0.020.\pi/1.685}=1.08$$

Frequency of flutter:

$$f=b.f_o=1.658.125=211Hz$$

For a first estimation we may also use Fig.7. Differing to our example are: $r(1+\beta)=1.25$(our example: 1.357); $C_5=0$(our example $C_5=0.259$); $C_6=C_{6'}=0.155$(our example $C_6=0.414$). To use the diagram we have to form $C_o=C_1/C_2=4.73$; for $C_2=1.660$ one gets from the diagram Fig.7:

$$A_2:A_1=1.1 \qquad b=1.58$$

This gives a good approach to the calculated results, which means that the influence of gas inertia in this special example is still small.