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Tariq A. Al-Azab
Al-Balqa Applied University

Rebhi A. Damseh
Al-Balqa Applied University

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Tariq A. Al-Azab[1], **Rebhi A. Damseh**[2]

[1] Mechanical Engineering Department, Al-Al-Salt Community College
Al-Balqa Applied University, P.O.Box 7003 (19117), Salt-Jordan
E mail: tazab@bau.edu.jo
Tel: +962-5-3532519
Fax: +962-5-3555735
[2] Mechanical Engineering Department, Al-Huson University College
Al-Balqa Applied University, P.O.Box 50, Irbid-Jordan
E mail: Rdamseh@yahoo.com

* Corresponding Author Rdamseh@yahoo.com

ABSTRACT

Closed form solution for a mathematical model describing the transient behavior of a micro channel tube cooler is presented. The model predicts the temperature distribution in the micro channel cooler for variable inlet fluid temperature. In terms of the presented model, the effect of different design parameters on performance is discussed.

1. INTRODUCTION

Micro channel heat exchangers have received much attention due to their ultimate potential for cooling a flowing fluid in different devices. The main advantage of micro channel coolers is their extremely high heat transfer area per unit volume.

Recent studies on the application of carbon dioxide refrigeration technology, a micro channel evaporators and gas coolers have to be used in order to sustain the high working pressure and for application in automobile air conditioning, the small size of micro channel heat exchanger is one important feature for the designers (Pettersen *et* al, 2000a) and (Pettersen *et* al, 2000b) .
By using liquid as a coolant, higher heat transfer rates can be obtained due to the higher thermal capacity of the fluid. Peng *et* al. (1996) conducted experiments for water flow with different micro channels to determine the influence of fluid flow, thermal conditions and micro channel size on the flow and forced convective heat transfer characteristics. The flow resistance in the micro channel also was investigated experimentally, and empirical correlations were proposed for its estimation. New experimental measurements for pressure drop and heat transfer coefficient were made by (Rahman, 2000). Test was performed on channels of different depths and using water as the working fluid. These measurements were used to calculate local and average Nusselt number and coefficient of friction in the device for different flow rates, channel size and configuration.

The developments of compact air-cooled micro channel were discussed by Wu and Little (1983). They measured the flow friction and heat transfer characteristics of gases flowing through micro channels, and observed that the convective heat transfer characteristic were different from conventionally sized channels. Also the frictional pressure drop for the laminar flow was found to be higher than the classical predictions. Kleiner *et* al (1995) investigated theoretically and experimentally forced air cooling micro channel parallel plate fin heat sink and tubes. Tube size was observed to have a significant impact on optimum heat sink design.
In this study, the dimensionless groups affecting the micro channel tube design are defined. The thermal behavior of a micro channel tube cooler is studied using theoretical model and the effect of each dimensionless group on thermal design is investigated.

2. MATHEMATICAL FORMULATION AND SOLUTION

The mathematical model describing the dynamic thermal behavior of the micro channel condenser is based on the following assumptions:

1. There are two separate phases, fluid and solid, which are not in thermal equilibrium. This is the case for low thermal conductivity fluid.
2. The fluid is one dimensional plug flow.
3. The fluid is laminar and Newtonian.
4. The thermal properties of the fluid and solid domains are constant and uniform.
5. Thermal and mass diffusion are assumed to be negligible.

Now, referring to Fig. 1, the governing energy equations for the solid and fluid domains are given respectively as:

$$\rho_s A_s C_c \frac{\partial T_s}{\partial t} = U_l A_l (T_f - T_s) - U_u A_u (T_s - T_\infty) \qquad (1)$$

$$\rho_f A_f C_f \frac{\partial T_f}{\partial t} + \dot{m}_f C_f \frac{\partial T_f}{\partial x} = -U_l A_l (T_f - T_s) \qquad (2)$$

Equations (1) and (2) are subjected to the following initial and boundary conditions:

$$T_f (0, x) = T_s (0, x) = T_\infty \qquad (3)$$

$$T_f (t, 0) = T_o (t) \qquad (4)$$

By using the dimensionless parameters defined in the nomenclature, equations (1 to 4) become:

$$\frac{\partial \theta_s}{\partial \tau} = R_2 (\theta_f - \theta_s) - R_1 \theta_s \qquad (5)$$

$$\frac{\partial \theta_f}{\partial \tau} + \frac{\partial \theta_f}{\partial X} = -R_3 (\theta_f - \theta_s) \qquad (6)$$

$$\theta_f (0, X) = \theta_s (0, X) = 0 \qquad (7)$$

$$\theta_f (\tau, 0) = \theta_o (\tau) \qquad (8)$$

Where

$$R_1 = \frac{U_u A_u \rho_f A_f L}{\rho_s A_s C_s \dot{m}_f}, \qquad R_2 = \frac{U_l A_l \rho_f A_f L}{\rho_s A_s C_s \dot{m}_f}, \qquad R_3 = \frac{U_l A_l L}{C_f \dot{m}_f}$$

In order to solve the above equations for θ_f and θ_s, the Laplace transform technique will be used. Noting that $L\{\theta(\tau, X)\} = w(s, x)$, The Laplace transform of equations (5) and (6) yields:

$$SW_s = R_2 \left(W_f - W_s\right) - R_1 W_s \qquad (9)$$

$$SW_f + \frac{\partial W_f}{\partial X} = -R_3 \left(W_f - W_s\right) \qquad (10)$$

And the boundary condition given in equation (8) is reduced to:

$$W_f (S, 0) = W_o (S) \qquad (11)$$

Solving for W_s from equation (9), substituting the result into equation (10) and rearranging yields:

$$W_s = \frac{R_2}{S + R_{12}} W_f \qquad (12)$$

$$\frac{\partial W_f}{\partial X} + \left(S + R_3 - \frac{R_5}{S + R_{12}} \right) W_f = 0 \qquad (13)$$

Where

$R_{12} = R_1 + R_2\,, \qquad R_5 = R_2 R_3$

Now, solving equation (13) for W_f , applying equation (11) and rearranging yields:

$$W_f = W_o(S)\exp\left[-\left(S + R_3 - \frac{R_5}{S + R_{12}}\right)X\right] \tag{14}$$

Equation (12) can be rewritten as:

$$W_s = \frac{R_2}{S + R_{12}}\left\{W_o(S)\exp\left[-\left(S + R_3 - \frac{R_5}{S + R_{12}}\right)X\right]\right\} \tag{15}$$

The solid and the fluid domain temperatures can be found by considering the following inversion formulas (Roberts *et* al 1996).

$$L^{-1}\{\exp(-ns)h(s)\} = H(\tau - n)R(\tau - n) = \begin{cases} F(\tau - a) & \text{if} \quad \tau > a \\ 0 & \text{if} \quad \tau < a \end{cases}$$

$$L^{-1}\left\{\frac{\exp(a/(s^{\nu} + b))}{s^{\nu} + b}\right\} = \exp(-b\tau)(\tau / a)^{\frac{\nu-1}{2}} I_{\nu-1}(2\sqrt{a\tau})$$

$$L^{-1}\{f(s)g(s)\} = \int_0^{\tau} F(\tau - u)G(u)du$$

Where $R(\tau - n)$ is the unit step function, . u is a dummy variable. Now define $\tau^* = (\tau - X),\ a = R_5 X$, $b = R_{12}\,,\ \nu = 1\,,\ n = X$. It's clear that the exact form of temperature distribution depends on the inlet fluid temperature. The inlet fluid temperature to the cooler for constant operating conditions of carbon dioxide refrigerating cycle is constant (Yin *et* al 2000).

$\theta_o(\tau) = \theta_o$ and as a result $W_o(S) = \dfrac{\theta_o}{S}$

In terms of the above consideration, equation (14) and (15) are inverted to yield:

$$\begin{aligned} &\theta_f(\tau^*, X) = \theta_o \exp(-R_3 X)\{\exp(-R_{12}\tau^*)BF + R_{12}\psi\} && \text{for} \quad \tau^* \geq 0 \\ &\theta_f(\tau^*, X) = 0 && \text{for} \quad \tau^* < 0 \end{aligned} \tag{16}$$

$$\begin{aligned} &\theta_s(\tau^*, X) = \theta_o R_2 \exp(-R_3 X)\psi && \text{for} \quad \tau^* \geq 0 \\ &\theta_s(\tau^*, X) = 0 && \text{for} \quad \tau^* < 0 \end{aligned} \tag{17}$$

Where

$$BF = I_0\left(2\sqrt{R_5 X \tau^*}\right)$$

$$\psi = \int_0^{\tau^*} \exp(-R_{12}u) I_0\left(2\sqrt{R_5 X u}\right) du$$

To simplify the computational procedure, the above integral can be rewritten as:

$$\psi = \frac{1}{a_2}\sum_{k=0}^{\infty}\frac{(R_5 X / R_{12})}{(k!)^2}\left[k! - \exp(-R_{12}\tau^*)\sum_{n=0}^{k}\frac{k!}{(k-n)!}\left(R_{12}\tau^*\right)^{k-n}\right]$$

3. RESULTS AND DISCUSSION

Figure 2 shows the temperature variation of the solid at the end of the cooler with time, at the same time the temperature difference between inlet and outlet fluid temperature is presented. After such time, the steady state is reached where solid and fluid temperatures will not be affected any more. At that time one can judge the cooler performance.

Figure 3 shows the solid temperature distribution with axial location of the micro channel tube heat exchanger. These results are obtained from the theoretical solution. As clear from this figure, the temperature profile attains its linear shape when the system reaches its steady state performance. This is predicted since the thermal losses to the ambient are uniform with the axial location.

At this stage, it's of great importance to notice the dimensionless groups, R_1 R_2 and R_3, and their effect on temperature distribution of both fluid and solid domains. Figure 4 presents the temperature distribution of solid and fluid domains through the cooler length. The heat dissipated through the cooler is the recognized thermal parameter which defined the cooler performance. The heat dissipated is a function of the fluid temperature difference across the cooler length. Since the inlet fluid temperature is constant, the fluid temperature at the cooler exit as shown from Figure 4 is varied with the recommended dimensionless groups. The extend at which those parameters affecting the fluid temperature difference is shown in Figure 5. In the literature, researchers do not judge the performance of such system based on its defined efficiency during its dynamic response. In fact such a judgment will give misleading conclusions. As a result, concern is focused on the dissipated heat at steady state conditions. It is clear from figure 5 that increasing R_1 improves the cooler performance. This is a predicted result since R_1 is a dimensionless parameter which measures mainly the heat transfer to the ambient. Increasing such a parameter eases the flow of energy absorbed by the solid wall, to the outside. It is clear from this curve that there is a critical value for R_1 beyond which there is no detectable enhancement in the amount of dissipated heat. The reason for that is due to the fact that as the heat transfer coefficient between the solid and ambient increases, both solid and ambient at the interface region will attain the same temperature. This implies that the internal heat transfer convective resistance between the solid and ambient is absent. Also it is clear from Figure 5 that the dimensionless group R_3, which measure the heat transfer interaction between fluid and solid has a great role in improving the cooler performance. It measures the amount of heat lost from the flowing fluid to the solid domain. At some value of R_3 the outlet fluid temperature can be closest to the ambient temperature. Really, that value can not be attained. However, the value of R_3 can be improved for higher heat transfer coefficient between fluid and solid domains. At the other hand, low fluid heat capacity and mass flow rate tends to improve the cooler performance.

The effect of dimensionless group R_2 on the cooler performance is also presented in Figure 5. R_2 indicates the balance between the absorbed heat by the solid domain and the heat lost to the ambient which is measured by R_1, so for higher values of R_2 there will be a temperature equivalent between fluid and solid, and this will remove the convective resistance between fluid and solid domains. This effect can be noticed also from Figure 4. Increasing R_2 will keep the fluid and solid at high temperature which is closest to the inlet fluid temperature and no heat transfer enhancement will be attained.

4. CONCLUSIONS

The dynamic behavior of a micro channel cooler is investigated. Two coupled partial differential equations are solved exactly using Laplace transformation technique. The effect of different design parameters on thermal behavior of the cooler are investigated where it is found that there are mainly three design parameters affecting the performance of the cooler. One of the parameters measures the rejected energy to the ambient, the second measures the heat transfer interaction between fluid and solid domains, and the last parameter indicates the balance between the absorbed energy by the solid domain and the lost energy to the ambient.

NOMENCLATURE

A_f	cross sectional area of fluid passages	(m^2)
A_l	heat transfer area between fluid and solid	(m^2)

A_s	cross sectional area of solid domain	(m^2)
A_u	heat transfer area between solid and ambient	(m^2)
b	width of the micro channel cooler	(m)
h	height of the micro channel cooler	(m)
C_f	specific heat of fluid at constant pressure	(J/kg K)
C_s	specific heat of solid	(J/kg K)
D_i	micro channel tube diameter	(m)
L	length of micro channel cooler	(m)
$\dot{m}_f$	mass flow rate of fluid	(kg/m^3)
N	number of micro tubes	-
N_x	number of micro tubes in x - direction	-
N_y	number of micro tubes in y - direction	-
S	Laplacian domain	-
t	time	(s)
T_f	fluid temperature	(^{o}K)
T_s	solid temperature	(^{o}K)
T_∞	ambient temperature	(^{o}K)
U_l	heat transfer coefficient from fluid to solid	(W/m^2 K)
U_u	heat transfer coefficient from solid to ambient	(W/m^2 K)
x	axial direction	(m)
X	dimensionless axial distance, x/L	-
y	transverse direction	(m)

Greek letters

θ_f	dimensionless fluid temperature, $\left(T_f - T_\infty\right)/T_\infty$	
θ_s	dimensionless fluid temperature, $\left(T_s - T_\infty\right)/T_\infty$	
θ_o	dimensionless inlet fluid temperature, $\left(T_f(t,0) - T_\infty\right)/T_\infty$	
ρ_f	fluid density	
ρ_s	solid density	
τ	dimensionless time, $\left(\dot{m}_f t / \rho_f A_f L\right)$	

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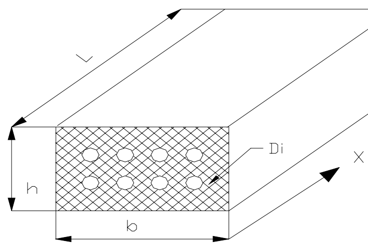
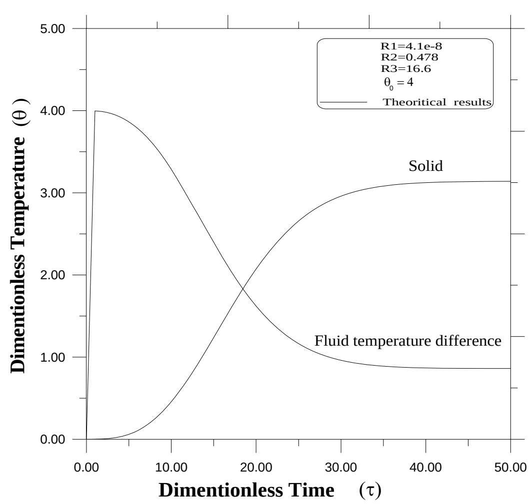


Figure 1: Micro channel tube cooler



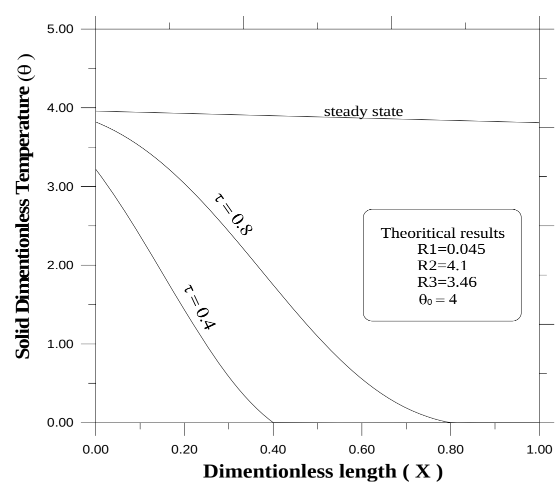


Figure 2: Fluid and solid temperature distributions with time at the exit of the micro channel heat

Figure 3: Solid temperature distributions the micro channel heat exchanger length at various times.

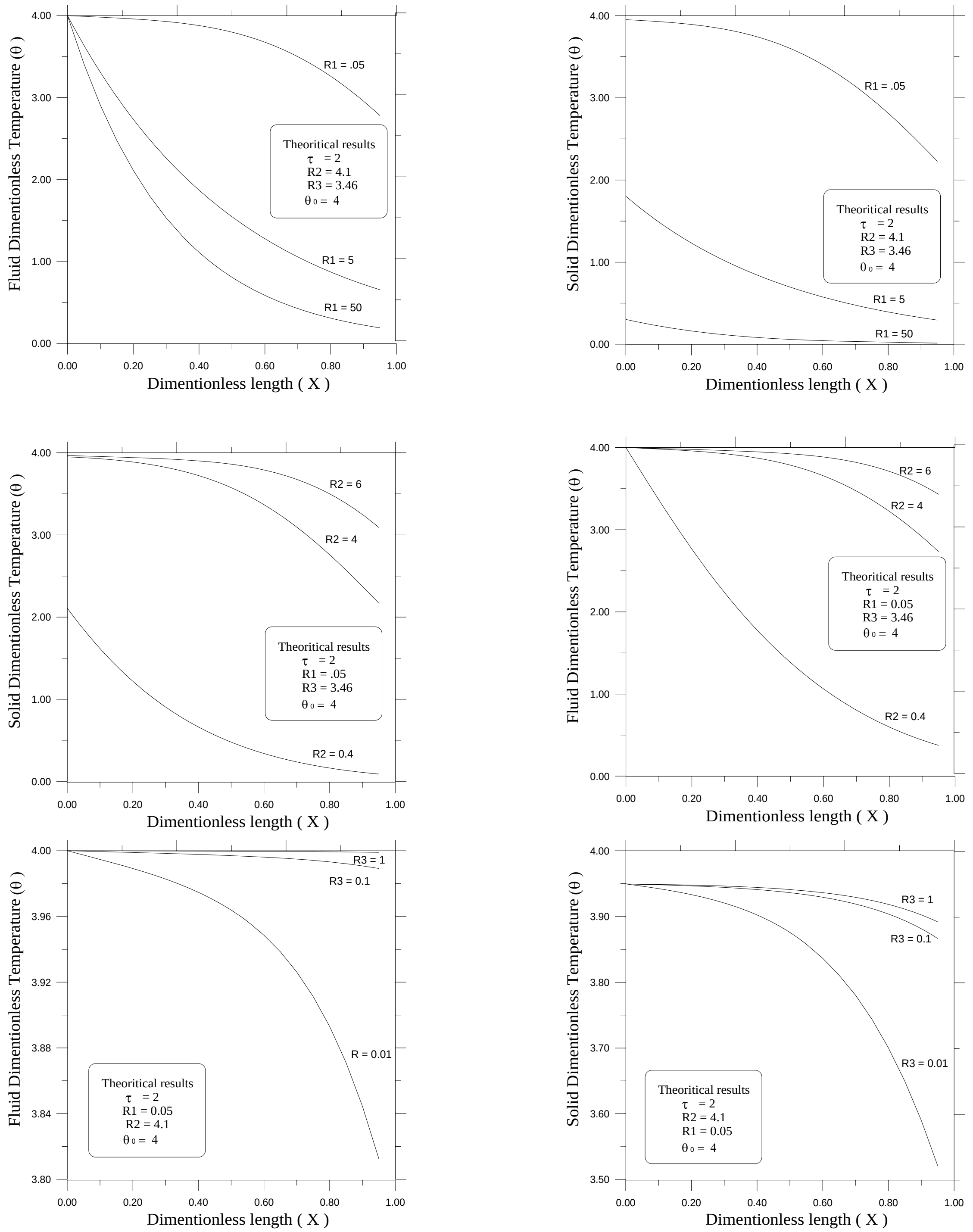


Figure 4: Effects of dimensionless groups on solid and fluid temperature distribution through the micro channel heat exchanger length.

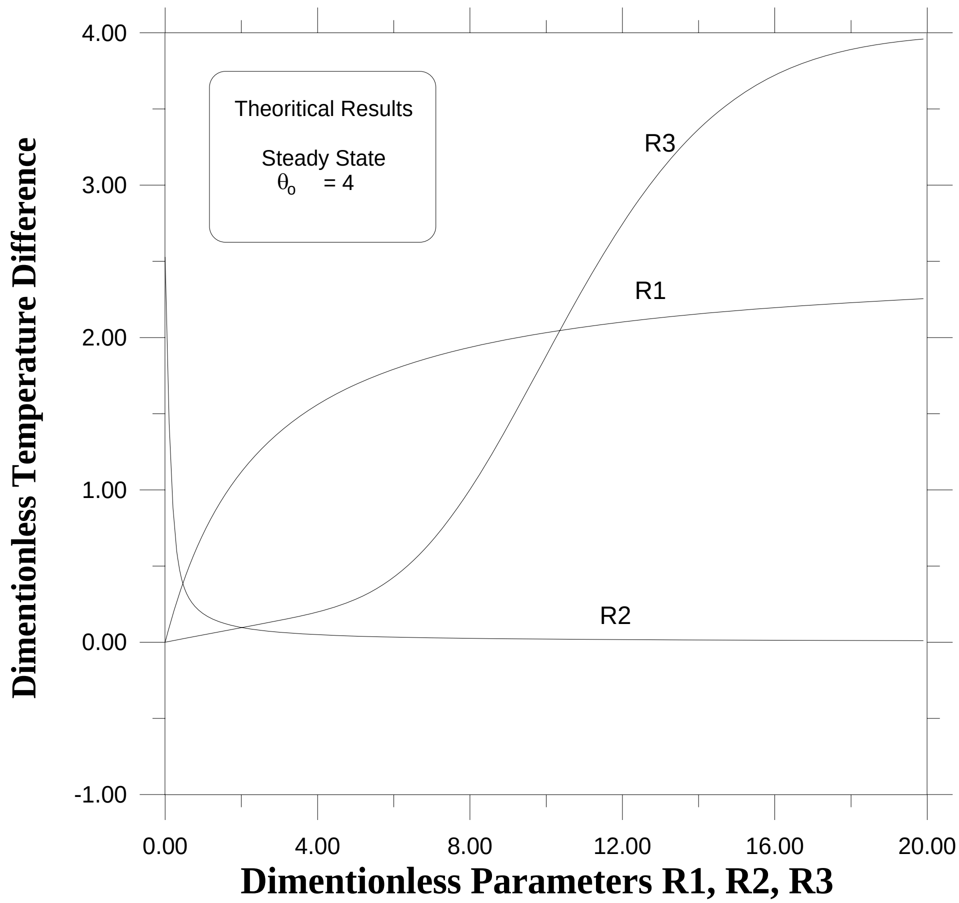


Figure 5: Effect of dimensionless parameters on micro channel tube thermal performance