6-1-1990

Adaptive Control of Flexible Joint Robots Derived from Arm Energy Considerations

Shaheen Ahmad
Purdue University

Fouad Mrad
Purdue University

Adaptive Control of Flexible Joint Robots Derived from Arm Energy Considerations

Shaheen Ahmad
Fouad Mrad

TR-EE 90-46
June 1990

School of Electrical Engineering
Purdue University
West Lafayette, Indiana 47907

To appear as a book chapter "Lecture Notes in Control" edited by Jan Skowronski, et al., Springer Verlag 1990.

ADAPTIVE CONTROL OF FLEXIBLE JOINT ROBOTS DERIVED FROM ARM ENERGY CONSIDERATIONS

Shaheen AHMAD, Fouad MRAD

Real-time Robot Control Laboratory
School of Electrical Engineering
Purdue University
West Lafayette, IN 47907 - 0501
U S A

1. Abstract *Almost all industrial robots exhibit joint flexibility due to mechanical compliance of their gear boxes. In this paper we outline the design of three controllers for flexible joint robots. Two of the three controllers are suitable for parameter adaptation, the candidate Lyapunov functions for these two controllers are derived from arm energy considerations.*

The desired actuator trajectory in a flexible joint robot is dependent not only on the desired kinematic trajectory of the link but also on the link dynamics. Unfortunately, link dynamic parameters are unknown in most cases, as a result the desired actuator trajectory is also unknown. To overcome this difficulty, a number of control schemes require the use of link acceleration and link jerk feedback. In this paper we describe three control schemes for flexible joint robots which do not use link jerk or acceleration. One of the controllers is suitable for trajectory tracking when the robot parameters are known in advance. The other two control laws are derived from candidate Lyapunov functions which resemble the energy of the arm deviating from the desired trajectory. Trajectory tracking and adaptation of robot arm parameters are possible with two of the controllers described in this paper. Our control schemes do not require the numerical differentiation of the velocity signal, or the inversion of the inertial matrices. Simulations are presented to verify the validity of the control scheme. The superiority of the proposed scheme over existing rigid robot adaptive schemes is also illustrated through simulation.

2. Introduction

Many of today's rigid robots are driven by actuators with high gear ratios, the load due to the arm at the actuator is reduced by a factor of n_g, where, $n_g > 1$, is the gear ratio. In fact, inertia of the arm experienced by the actuator is reduced by $(1/n_g^2)$, and as the actuator acceleration is n_g times the joint acceleration, the overall load is reduced by $(1/n_g)$. Thus the load experienced by robots with high gear ratios are dominated by actuator dynamics, link dynamics are secondary. Recent trend is towards high-technology direct-drive robots. Here, the actuators are directly connected to the links and the lack of high gear ratios and increasing demand for high-speed operation, requires the control system to compensate for the dominant nonlinear link dynamics. Thus the presence of high gear ratios reduces the effective load experienced at the motors but at slower robot operations, and the absence of gearing adds to the complexity of the control problem. Robots which move fast (apparently with reasonable manufacturing cycle times) and or carry large loads have additional problems. It is experimentally found that most gearing systems are compliant, as a result, actuators are connected to the robot links through effectively flexible shafts. Experimental evidence also indicates that joint flexibility should be accounted for in both modeling and control of manipulators [1] [14] [4]. The presence of joint flexibility in the direct-drive high-speed actuators can be modeled by a "linear" torsional spring. This flexibility may be attractive in some practical applications when the robot must make contact with an unknown surface.

Numerous techniques to control Flexible Joint Robots have been suggested [14], [2], [3], [6], [12], [4]. One approach is based on the idea of feedback linearization, which requires the measurement of joint acceleration and jerk to be used in the feedback loop [2], [12]. Another method is based on the concept of reduced order system and requires the restriction of the system to a suitable integral manifold in the state space [6].

We derive three controllers which drives the FJR to track a desired trajectory . Similar to the work on rigidly jointed robots [10],[11], [7], our controller design starts by selecting a candidate Lyapunov function which is similar to the energy of the FJR. Our control scheme does not require link jerk, or acceleration feedback or the inversion of the inertia matrix, in addition parameter adaptation is easily accommodated for two of the three controllers.

At this time, the only adaptive control scheme for flexible joint robots that we are aware of that uses position and velocity feedback is the one derived from singular perturbation arguments by Ghorbel, Spong and Hung [4]. In order to derive an adaptive scheme from a singular perturbation argument, several assumptions are necessary, these include sufficient joint stiffness and that it is possible to ignore the higher order terms in the singular perturbation expansion. Assumptions such as these are not necessary in our derivations.

An important problem in adaptive control is that of parameter convergence, providing a sufficiently rich tracking signal has sometimes been assumed to be adequate conditions for parameter convergence. However tracking a persistently exciting trajectory does not mean that all of the unknown parameters of a certain manipulator can be estimated. In general, the maximum number of parameters that may be estimated depends on the trajectory used for estimation and on the kinematic structure of the manipulator. These unknown parameters could be categorized as uniquely identifiable, identifiable in linear combinations only, or unidentifiable. Typically, only those dynamic parameters that affect the force/torque equations of at least one joint can be identified.

The organization of the remainder of this paper is now described. Section #3 and #4 summarize the dynamics and trajectory model of the manipulator. A trajectory tracking controller which does not use link jerk and acceleration is described in section #5, it is assumed that the manipulator parameters are known in advance. Section #6 explores the use of the arm's energy as possible Lyapunov function candidates. The energy based Lyapunov functions derived in section #6 are used to derive adaptive control schemes in section #7. Simulation results are given in section #8, conclusions are presented in section #9.

3. Manipulator Models

Experimental investigations of industrial robots with harmonic drive transmission and other forms of gearing indicate that joint flexibility contributes significantly to the overall dynamics of the system [1], [13]. The dynamic equations of the flexible joint robots are given as:

$$\tau = D_m \ddot{q}_m + B_m \dot{q}_m + K_s(q_m - q) \qquad (1)$$

$$0 = D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + K_s(q - q_m) \qquad (2)$$

where, an n-link manipulator becomes a 2n-degrees of freedom system:

D_m : Diagonal motor inertia matrix $\in \mathbb{R}^{n \times n}$
B_m : Diagonal motor damping matrix $\in \mathbb{R}^{n \times n}$
K_s : Diagonal drive shaft stiffness matrix $\in \mathbb{R}^{n \times n}$
q_m : Vector of sensed motor angles $\in \mathbb{R}^{n \times 1}$
$D(q)$: Link inertia matrix $\in \mathbb{R}^{n \times n}$
$C(q,\dot{q})$: Centrifugal and coriolis terms matrix $\in \mathbb{R}^{n \times n}$
$g(q)$: Gravitational vector term $\in \mathbb{R}^{n \times 1}$
q : Vector of link joint angles $\in \mathbb{R}^{n \times 1}$

Matrices D_m, B_m, K_s, are all positive definite matrices. Further, $D(q)$ is symmetric, positive definite and both $D(q)$ and $D^{-1}(q)$ are both bounded as function of q [13], [4]. When K_s tends toward infinity, the robot is considered to have rigid joints (i.e. $q=q_m$). The dynamic equations which represent the rigidly jointed robot, with the same inertial and coriolis matrices as the FJR defined above, are:

$$\tau = [D(q)+D_m]\ddot{q} + [C(q,\dot{q})+B_m]\dot{q} + g(q) \qquad (3)$$

Some properties of the rigid model concerning the inertia matrix, coriolis and centrifugal force matrix were discussed by Koditschek. Those properties remain valid for the flexible model [4]. The first most important property shows that $D(q)$ and $C(q,\dot{q})$ are not independent, but the matrix $(\dot{D}-2C)$ is skew symmetric, this can be easily derived from the Lagrangian formulation of the manipulator dynamics (see Appendix A). The second property confirms that the individual terms of the right hand side of equation (2), excluding the $K_s(q-q_m)$ term, could be represented by a linear relationship between a suitably selected set of unknown manipulator and load parameters [11], [4], [13], in other words equation (2) could be rewritten as:

$$0 = Y(\ddot{q},\dot{q},q)\, P + K_s\,(q - q_m) \qquad (4)$$

where $Y(\ddot{q},\dot{q},q) \in \mathbb{R}^{n \times r}$, is called the regressor matrix of known functions, and $P \in \mathbb{R}^{r \times 1}$ is a vector of unknown parameters.

4. Trajectory Model

Let $q_d(t) \in C^4$ denote a desired link trajectory in which case $q_d(t), \dot{q}_d(t), \ddot{q}_d(t), \dddot{q}_d(t)$ are all bounded and continuously differentiable. The set of desired motor trajectory can be derived using equation (4). The diagonal stiffness matrix, $K_s \in \mathbb{R}^{n\times n}$, can be written as $K_s = \mathrm{Diag}[k_{si}]$, where $k_{si} > 0$, for $i = 1,2,\ldots,n$, represents the spring constant of the i^{th} drive shaft. Since all of these constants are positive and K_s is a diagonal matrix, as a result matrix K_s is invertible and positive definite.

We assume the link parameters and the load handled by the end effector are time invariant, i.e.

$$P = \text{Constant vector, thus, } \dot{P} = \ddot{P} = 0 \tag{5}$$

The above assumption is valid in a large class of applications. The desired motor trajectory may now be computed as follows:

$$q_{md}(t) = K_s^{-1} Y(\ddot{q}_d, \dot{q}_d, q_d)P + q_d(t) \tag{6}$$

$$\dot{q}_{md}(t) = K_s^{-1} \dot{Y}(\ddot{q}_d, \dot{q}_d, q_d)P + \dot{q}_d(t) \tag{7}$$

$$\ddot{q}_{md}(t) = K_s^{-1} \ddot{Y}(\ddot{q}_d, \dot{q}_d, q_d)P + \ddot{q}_d(t) \tag{8}$$

The subscript "d" is used to denote the desired trajectory.

Notice that the desired motor trajectory $q_{md}(t)$, $\dot{q}_{md}(t)$ and $\ddot{q}_{md}(t)$ are dependent on the desired link trajectory $q_d(t)$, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ and also on the unknown parameters P and the link dynamics represented by Y_d, $\dot{Y}_d$ and $\ddot{Y}_d$. This makes it difficult to design a control law which utilizes the desired motor position and velocity.

Using equations (6), (7) and (8), removing subscripts d, and using equation (1) and (2), we can rewrite equation (1) in-link coordinates q as:

$$\tau = D_m K_S^{-1} D(q) q^{(4)} + N(q, q^{(1)}, q^{(2)}, q^{(3)}) \tag{8a}$$

$$= Y^*(q, q^{(1)}, q^{(2)}, q^{(3)}, q^{(4)}) P^* \tag{8b}$$

where, $N(.,.,.,.) \in \mathbb{R}^n$, is a nonlinear function and $q^{(i)}$ is the i^{th} time derivative of q. From the structure of equation (8a) we can see that the FJR can be stabilized by feeding back a nonlinear function of the link position, velocity, acceleration and jerk. Notice that the fourth order dynamics in the link coordinates can also be written in the regressor matrix form in terms of some suitably selected vector of unknown parameters P^*.

5. Control of the Flexible Joint Robot when the Arm Parameters are Known

An adaptive controller for the FJR can be derived if measurements of q, $q^{(1)}$, $q^{(2)}$ and $q^{(3)}$ are available. Generally it is difficult to measure acceleration and jerk and it is desirable to design control schemes which only require the use of link position and velocity and motor position and velocity. In this section we will show that we can derive such a controller from

Lyapunov's second method if the arm's dynamic parameters are known in advance. First notice that if the arm parameters are known, then the acceleration and jerk of the joints can be obtained directly from the link and motor position and velocity measurements.

$$q^{(2)} = D^{-1}\{K_s(q_m - q) - C(q, q^{(1)})q^{(1)} - g(q)\} \tag{9a}$$

$$\text{and } q^{(3)} = \frac{d}{dt}q^{(2)} = \mathscr{F}(q, q^{(1)}, q_m, q_m^{(1)}) \tag{9b}$$

Having shown that $q^{(2)}$ and $q^{(3)}$ can be obtained from velocity and position feedback, we will now assume that they are available.

Given, $q_d(t) \in C^4$, is the desired link trajectory, we can define tracking error $e(t) = q_d(t) - q(t)$, and a composite error vector $\eta(t) \in \mathbb{R}^n$, such that $\eta(t) = e^{(3)} + \Lambda_2 e^{(2)} + \Lambda_1 e^{(1)} + \Lambda_0 e$, where $\Lambda_0, \Lambda_1, \Lambda_2 \in \mathbb{R}^{n \times n}$ are gain matrices. We will also define, $\mu(t) = \Lambda_2 e^{(3)} + \Lambda_1 e^{(2)} + \Lambda_0 e^{(1)}$.

Theorem #1

The following control torque applied to the dynamical system given in (8a):

$$\tau = N(q, q^{(1)}, q^{(2)}, q^{(3)}) + \beta C(q, q^{(1)})\eta + K_D \eta + \beta D(\mu + q_d^{(4)}) \tag{10}$$

ensures $e(t) \rightarrow 0$ as $t \rightarrow \infty$ for an appropriate choice of matrices $\Lambda_0, \Lambda_1, \Lambda_2$ and $K_D \in \mathbb{R}^{n \times n}$, given $\beta = D_m K_s^{-1}$. Furthermore, $\tau = \tau(q, q^{(1)}, q_m, q_m^{(1)})$.

Proof of Theorem #1:

Consider the Lyapunov function

$$V = \frac{1}{2}\eta^t D \eta$$

Then using the fact $(\dot{D} - 2C)$ is skew symmetric (see Appendix A), we obtain,

$$\begin{aligned} \dot{V} &= \eta^t(D\dot{\eta} + \frac{1}{2}\dot{D}\eta) - \frac{1}{2}\eta^t(\dot{D} - 2C(q, q^{(1)}))\eta \\ &= \eta^t \beta^{-1}(\beta D(q_d^{(4)} + \mu) + N + \beta C(q, q^{(1)})\eta - \tau) \end{aligned} \tag{12}$$

If we set τ as given in the above (10), we have:

$$\dot{V} = -\eta^t \beta^{-1} K_D \eta < 0 \tag{13}$$

As $\beta = D_m K_s^{-1}$ is a positive definite diagonal matrix, in which case we can set K_D as a positive definite diagonal matrix such that $\beta^{-1}K_D$ is positive definite. This ensures that $\eta(t) \rightarrow 0$ as $t \rightarrow \infty$, therefore for an appropriate choice of the matrices $\Lambda_0, \Lambda_1, \Lambda_2$ such that the eigenvalues of $(s^3 I + \Lambda_2 s^2 + \Lambda_1 s + \Lambda_0)$ are in the LHP, ensures that, $e(t) \rightarrow 0$, as, $t \rightarrow \infty$.

Notice that as $q^{(3)}$ and $q^{(2)}$ can be expressed in terms of link and motor positions and velocities, as shown in equations (9a) and (9b), $\tau(q, q^{(1)}, q^{(2)}, q^{(3)}) = \tau(q, q_m^{(1)}, q, q^{(1)})$. ###

Notice when the arm parameters are uncertain, we cannot calculate the link acceleration and jerk (as in (9a) and (9b)). As a result, this scheme is not suitable for adaptation. In the next two sections of this paper we derive adaptive controllers based on the arm energy consideration.

6. Selection of an Energy-based Lyapunov function

The total energy of the robot arm is E, it is the sum of the kinetic and potential energies of the actuator and linkages:

$$E=\frac{1}{2}\dot{q}_m^t D_m \dot{q}_m+\frac{1}{2}\dot{q}^t D\dot{q}+\frac{1}{2}(q-q_m)^t K_s(q-q_m)+\Phi(q) \tag{14}$$

where, $\Phi(q)$ is the gravitational potential energy of the linkage. Then the power input to the FJR is through the actuator and is given as:

$$\frac{dE}{dt}=(\tau_m-B_m\dot{q}_m)^t\dot{q}_m \tag{15}$$

Notice that when $\Phi(q)=0$, $E(\dot{q}_m,\dot{q},q_m,q)$ becomes a quadratic in q, $\dot{q}$, q_m, and $\dot{q}_m$. Notice also, if we set $\tau_m=B_m\dot{q}_m-\Omega\dot{q}_m$, then

$$\frac{dE}{dt}=-\dot{q}_m^t\Omega\dot{q}_m \le 0 \tag{16}$$

where, $q_m \in \mathbb{R}^n$, and $\Omega \in \mathbb{R}^{n\times n} > 0$ is a positive definite matrix .

We can conclude that, with an appropriate rate feedback, we may track a static joint trajectory. This exposition shows why most FJR with appropriate velocity feedback can be stabilized. This exposition indicates that we may select a Lyapunov function similar to E given in (14), and we may stabilize the FJR along a nonstatic link trajectory by suitable position and velocity feedback.

Excluding the potential energy of the FJR, the energy of the robot arm along a prespecified trajectory is:

$$E(t)=\frac{1}{2}\dot{q}_d^t D\dot{q}_d+\frac{1}{2}(q_d-q_{md})^t K_s(q_d-q_{md})+\frac{1}{2}\dot{q}_{md}^t D_m\dot{q}_{md} \tag{17}$$

Likewise, the energy of the FJR which causes the deviation from the desired trajectory is given as:

$$V(t)=\frac{1}{2}\dot{e}^t D\dot{e}+\frac{1}{2}(e-e_m)^t K_s(e-e_m)+\frac{1}{2}\dot{e}_m^t D_m\dot{e}_m \tag{18}$$

where, we defined the error terms as: $e=(q_d - q)$ and $e_m=(q_{md}-q_m)$.
Throughout the trajectory it is desired to have $\frac{dV}{dt}<0$, furthermore $\dot{V}(t)$ and $V(t)$ should be dependent on e and e_m as well as $\dot{e}_m$ and $\dot{e}$. We can make $V(t)$ dependent on $\dot{e}$, e, $\dot{e}_m$ and e_m by selecting:

$$V(t)=\frac{1}{2}\dot{e}_m^t D_m\dot{e}_m+\frac{1}{2}\dot{e}^t D\dot{e}+\frac{1}{2}(e-e_m)^t K_s(e-e_m)$$
$$+\frac{1}{2}e^t K_p e+\frac{1}{2}e_m^t K_{pm} e_m \tag{19}$$

where, $K_p \in \mathbb{R}^{n\times n}$, $K_{pm} \in \mathbb{R}^{n\times n}$ are some positive definite gain matrices. The derivation of τ_m to make $\dot{V}(t)<0$ and quadratically dependent on the variables e, e_m, $\dot{e}$ and $\dot{e}_m$ will be addressed in the next section.

7. Control and Adaptation Law Design

As the dynamic parameters of the arm are unknown and assumed to be time invariant, we can define the parameter error vector as $e_p = \tilde{P} - P$, where $\tilde{P}$ is the estimated parameter vector. Notice $\dot{e}_p = \dot{\tilde{P}}$, as $\dot{P} = 0$. Based on the estimated value of the parameter vector $\tilde{P}$, we obtain an estimate of the desired motor position as $\tilde{q}_{md}$ using equation (6). Similarly, we can compute the estimated motor velocity and acceleration. We can define the following motor error as $\tilde{e}_m = (\tilde{q}_{md} - q_m)$. Similar terms for $\dot{\tilde{e}}_m$ and $\ddot{\tilde{e}}_m$ can be defined. Based on the above Lyapunov function (19), we can find the energy of the trajectory deviating from the desired trajectory as:

$$V(t) = \tfrac{1}{2}\dot{\tilde{e}}_m^t D_m \dot{\tilde{e}}_m + \tfrac{1}{2}\dot{e}^t D \dot{e} + \tfrac{1}{2}(e - \tilde{e}_m)^t K_s (e - \tilde{e}_m)$$

$$+ \tfrac{1}{2} e^t K_p e + \tfrac{1}{2}\tilde{e}_m^t K_{pm} \tilde{e}_m + \tfrac{1}{2} e_p^t M e_p \qquad (20)$$

The last term in (20) is added to account for parameter adaptation, where $K_p, K_{pm} \in \mathbb{R}^{n \times n}$, and $M \in \mathbb{R}^{r \times r}$ are some positive gain matrices.
For convenience let us define:

$$D(q)\ddot{q}_d + C(q,\dot{q})\dot{q}_d + g(q) = \Psi(\ddot{q}_d, \dot{q}_d, \dot{q}, q)P \qquad (21)$$

where, $\Psi \in \mathbb{R}^{n \times r}$, and

$$Y_d = Y(\ddot{q}_d, \dot{q}_d, q_d) \qquad (22)$$

where, $Y_d \in \mathbb{R}^{n \times r}$.

Theorem #2:

If the control torque is bounded such that $\|\tau\| < \tau_{max}$. The system given by the dynamical model (1) and (2), subjected to the below control and adaptation laws, results in bounded trajectory tracking error.

$$\tau = D_m \ddot{\tilde{q}}_{md} + B_m \dot{q}_m + K_s(\tilde{q}_{md} - q_d) + K_{pm}\tilde{e}_m + K_{dm}\dot{\tilde{e}}_m$$

$$+ \frac{\dot{\tilde{e}}_m}{\|\dot{\tilde{e}}_m\|^2}\left[\dot{e}^t((\Psi - Y_d)\tilde{P} + K_p e) + \dot{e}^t K_d \dot{e} + e^t K_p e + \dot{\tilde{e}}_m^t K_{pm} \tilde{e}_m\right] \qquad (23)$$

where, K_{dm}, $K_d \in \mathbb{R}^{n \times n}$ are some positive gain matrices. The parameter adaptation law is given by:

$$\dot{\tilde{P}}(t) = M^{-1}\Psi^t(\ddot{q}_d, \dot{q}_d, \dot{q}, q)\,\dot{e} \qquad (24)$$

The trajectory tracking error is bounded in a set which is given by:

$$\alpha_1 \|\tilde{e}_m\| + \|\dot{e}\|(\alpha_2 \|e\| + \alpha_3(t))/\|\dot{\tilde{e}}_m\| + \alpha_4(t) < \tau_{max} \qquad (25)$$

where α_1, α_2, α_3, α_4 are some positive scalars, and $\|\cdot\|$ is the Euclidean vector norm.

Proof of Theorem 2:

Differentiation of the positive Lyapunov function candidate V(t) in equation (20) yields the following:

$$\dot{V}(t)=\dot{\tilde{e}}_m^t[D_m\ddot{\tilde{e}}_m+K_s(\tilde{e}_m-e)+K_{pm}\tilde{e}_m]$$
$$+\dot{e}^t[D\ddot{e}+\tfrac{1}{2}\dot{D}\dot{e}+K_pe+K_s(e-\tilde{e}_m)]+e_p^tM\dot{e}_p-\dot{e}^t(\tfrac{1}{2}\dot{D}-C)\dot{e} \quad (26)$$

In order to simplify equation (26), we have subtracted the term $\dot{e}^t(\frac{1}{2}\dot{D}-C)\dot{e}=0$, see (Appendix A). Simplifying equation (26) and substituting the dynamic equation of the FJR given by (1) and (2), we have

$$\dot{V}(t)=\dot{\tilde{e}}_m^t\{D_m\ddot{\tilde{q}}_{md}+K_s(\tilde{q}_{md}-q_d)+K_{pm}\tilde{e}_m-[D_m\ddot{q}_m+K_s(q_m-q)]\}$$
$$+\dot{e}^t\{D\ddot{q}_d+C\dot{q}_d+K_s(q_d-\tilde{q}_{md})-[D\ddot{q}+C\dot{q}+K_s(q-q_m)+g(q)]$$
$$+K_pe+g(q)\}+e_p^tM\dot{e}_p$$

$$=\dot{\tilde{e}}_m^t[D_m\ddot{\tilde{q}}_{md}+K_s(\tilde{q}_{md}-q_d)+K_{pm}\tilde{e}_m+B_m\dot{q}_m-\tau]$$
$$+\dot{e}^t[D\ddot{q}_d+C\dot{q}_d+K_s(q_d-\tilde{q}_{md})+K_pe+g(q)]+e_p^tM\dot{e}_p \quad (27)$$

Then by substituting the controller (23) into (27), using the definition of Ψ given by equation (21), and by using the fact that, $K_s(q_d-\tilde{q}_{md})=-Y_d\tilde{P}$, derived from (6), we get:

$$\dot{V}(t)=\dot{\tilde{e}}_m^t\{-K_{dm}\dot{\tilde{e}}_m-\frac{\dot{\tilde{e}}_m}{\|\dot{\tilde{e}}_m\|^2}[\dot{e}^t((\Psi-Y_d)\tilde{P}+K_pe)+\dot{e}^tK_d\dot{e}+e^tK_pe+\tilde{e}_m^tK_{pm}\tilde{e}_m]\}$$
$$+\dot{e}^t[\Psi P-Y_d\tilde{P}+K_pe]+e_p^tM\dot{e}_p$$
$$=-\dot{\tilde{e}}_m^tK_{dm}\dot{\tilde{e}}_m-\dot{e}^tK_d\dot{e}-e^tK_pe-\tilde{e}_m^tK_{pm}\tilde{e}_m-\dot{e}^t\Psi e_p+e_p^tM\dot{e}_p$$
$$=-\dot{\tilde{e}}_m^tK_{dm}\dot{\tilde{e}}_m-\dot{e}^tK_d\dot{e}-e^tK_pe-\tilde{e}_m^tK_{pm}\tilde{e}_m+e_p^t[M\dot{e}_p-\Psi^t\dot{e}] \quad (28)$$

Since, $\dot{e}_p=\dot{\tilde{P}}-\dot{P}$, and as, $\dot{P}=0$ (robot arm parameters are time invariant), we can substitute the adaptation law (24) into (28) and the final expression for the derivative of the Lyapunov function is given as:

$$\dot{V}(t)=-\dot{\tilde{e}}_m^tK_{dm}\dot{\tilde{e}}_m-\dot{e}^tK_d\dot{e}-e^tK_pe-\tilde{e}_m^tK_{pm}\tilde{e}_m<0 \quad (29)$$

Which guarantees the convergence of $\dot{\tilde{e}}_m$, $\dot{e}$, $\tilde{e}_m$, and e as time goes to infinity.

The problem that can arise with this controller is that as $\|\dot{\tilde{e}}_m\|\rightarrow 0$, large torques are required to maintain the manipulator along the desired trajectory . As this is impractical , let us assume that the available joint torques are bounded , i.e. $\|\tau\|<\tau_{max}$, in which case we have $\dot{V}(t)<0$, if:

$$\|\gamma(t)\|+\|K_{pm}\|\|\tilde{e}_m\|+\frac{\|\dot{e}\|}{\|\dot{\tilde{e}}_m\|}[\|\rho(t)\|+\|K_p\|\|e\|] < \tau_{max} \quad (30a)$$

$$\text{where, } \gamma(t)=D_m\ddot{\tilde{q}}_{md}+B_m\dot{q}_m+K_s(\tilde{q}_{md}-q_d) \quad (30b)$$

$$\rho(t)=\Psi(\ddot{q}_d,\dot{q}_d,\dot{q},q)\tilde{P}-K_s(\tilde{q}_{md}-q_d) \quad (30c)$$

Let us define the following positive constants: $\alpha_1=\|K_{pm}\|$, $\alpha_2=\|K_p\|$, and positive scalars $\alpha_3(t)=\|\rho(t)\|$, and $\alpha_4(t)=\|\gamma(t)\|$.
The ultimate bound on the trajectory errors is then given by the set S defined as:

$$S=\{\, e(t),\dot{e}(t),\tilde{e}_m(t),\dot{\tilde{e}}_m(t) \,|\, \alpha_1\|\tilde{e}_m\|+\|\dot{e}\|(\alpha_2\|e\|+\alpha_3(t))/\|\dot{\tilde{e}}_m\|+\alpha_4(t) = \tau_{max} \,\} \quad (31)$$

###

Notice that for a particular trajectory an upperbound on $\alpha_3(t)$ and $\alpha_4(t)$ can be found, this allows one to find an upperbound on trajectory error for all time.

In order to reduce excessive torque demands as $\|\dot{\tilde{e}}_m\|\to 0$, we make use of the structural reduction in the system to propose a secondary controller. Let $\Lambda \in \mathbb{R}^{n\times n}$ be some positive diagonal matrix, then we let

$$\Gamma(\ddot{q}_d,\dot{q}_d,\dot{q},q)P=D(q)[\ddot{q}_d+\Lambda\dot{e}]+C(q,\dot{q})[\dot{q}_d+\Lambda e]+g(q) \quad (32)$$

where, $\Gamma \in \mathbb{R}^{n\times r}$. Furthermore let us define the following variables:
$s \in \mathbb{R}^n$, $s=(\dot{e}+\Lambda e)$. Let us also define a region where, $\ddot{e}_{mi} = \ddot{\tilde{q}}_{mdi}-\ddot{q}_{mi}$, as:

$$\mu_{min}(i) \le \ddot{e}_{mi} \le \mu_{max}(i) \quad \text{for } i=1,2,\ldots,n \quad (33)$$

where, $\mu_{min}(i)$, and $\mu_{max}(i)$ are real scalars. Let us also set vector $\lambda=(\lambda_1,\ldots\lambda_n)^t \in \mathbb{R}^n$ be defined such that:

$$\lambda_i = \tfrac{1}{2}D_{mi}\{Sgn(s_i)[\mu_{min}(i)-\mu_{max}(i)]+\mu_{min}(i)+\mu_{max}(i)\} \quad (34)$$

for $i=1,2,\ldots,n$.

$$\text{where, } Sgn(s_i)=\begin{cases}+1 & \text{if } s_i>0\\ -1 & \text{if } s_i<0\\ 0 & \text{if } s_i=0\end{cases} \quad (35)$$

Theorem #3:

If, $\|\dot{\tilde{e}}_m\|^2 < \epsilon > 0$, we employ the below control law (36) and adaptation law (37) in addition (when $\|\dot{\tilde{e}}_m\|^2 > \epsilon$), we employ the control and adaptation law described in theorem 2, then the norm of the system tracking error will decrease to the order of $O(\sqrt{\epsilon})$. The second stage control law is given by:

$$\tau=\Gamma(\ddot{q}_d,\dot{q}_d,\dot{q},q)\tilde{P}+D_m\ddot{\tilde{q}}_{md}+B_m\dot{q}_m+K_ds-\lambda \quad (36)$$

where, K_{dm}, $K_d \in \mathbb{R}^{n\times n}$ are some positive definite gain matrices. The second stage adaptation law is given by

$$\dot{\hat{P}}(t)=M^{-1}\Gamma^{t}(\ddot{q}_d,\dot{q}_d,\dot{q},q)[\dot{e}+\Lambda e] \tag{37}$$

Proof of theorem 3:

Let us consider the system when $\|\dot{e}_m\|^2 \leq \epsilon$. If ϵ is suitably small then at this stage the motor is tracking the estimated actuator trajectory in velocity, but a steady state error may exist between the actual and desired motor position.

Notice now as $\|\dot{e}_m\|^2 \rightarrow 0$, **a structural reduction in the system** is apparent as the "Lyapunov" function V(t) in (20) resembles that of a rigid robot, as the first term is approximately zero. We exploit this property in the secondary control. The dynamic equations (1) and (2) can be added to obtain a single system equation:

$$\tau = D(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)+D_m\ddot{q}_m+B_m\dot{q}_m \tag{38}$$

Consider now the Lyapunov function candidate W(t):

$$W(t)=\tfrac{1}{2}(\dot{e}+\Lambda e)^t D(\dot{e}+\Lambda e) + \tfrac{1}{2}e_p^t M e_p \tag{39}$$

Differentiating W(t) with respect to time, substituting Γ given by (32), and using the dynamic equation (38) leads us to:

$$\begin{aligned}\dot{W}(t)&=s^t[D(\ddot{e}+\Lambda\dot{e})+\tfrac{1}{2}\dot{D}(\dot{e}+\Lambda e)] + e_p^t M\dot{e}_p\\&=s^t[D\ddot{q}_d+D\Lambda\dot{e}+C(q,\dot{q})(\dot{q}_d+\Lambda e)-(D\ddot{q}+C(q,\dot{q})\dot{q})] + e_p^t M\dot{e}_p\\&=s^t[D(\ddot{q}_d+\Lambda\dot{e})+C(q,\dot{q})(\dot{q}_d+\Lambda e)+g(q)\\&+D_m\ddot{q}_m+B_m\dot{q}_m-\tau] + e_p^t M\dot{e}_p\\&=s^t[\Gamma P + D_m\ddot{\hat{q}}_{md}-D_m\ddot{\hat{e}}_m+B_m\dot{q}_m-\tau] + e_p^t M\dot{e}_p\end{aligned} \tag{40}$$

Substituting for τ from equation (36) into (40) yields,

$$\begin{aligned}\dot{W}(t)&=s^t[-\Gamma e_p - K_d s - D_m\ddot{\hat{e}}_m+\lambda] + e_p^t M\dot{e}_p\\&=-s^t\Gamma e_p - s^t K_d s - s^t[D_m\ddot{\hat{e}}_m-\lambda]+ e_p^t M\dot{e}_p\\&= -s^t K_d s - s^t[D_m\ddot{\hat{e}}_m-\lambda]+e_p^t[M\dot{e}_p-\Gamma^t s]\end{aligned} \tag{41}$$

As, $\dot{e}_p(t) = \dot{\hat{P}}(t)$, since $\dot{P}=0$, now let us substitute $\dot{\hat{P}}(t)$ given by equation (37) into (41), it yields:

$$\begin{aligned}\dot{W}(t)&= -s^t K_d s - s^t[D_m\ddot{\hat{e}}_m-\lambda]+ e_p^t[MM^{-1}\Gamma^t s-\Gamma^t s]\\&= -s^t K_d s-\sum_{i=1}^{n}s_i(D_{mi}\ddot{\hat{e}}_{mi}-\lambda_i) < 0\end{aligned} \tag{42}$$

A substitution for the values of λ_i's from equation (34) guarantees that $\dot{W}(t)$ is upper bounded by zero and decreases for any nonzero ($s=\dot{e}+\Lambda e$), and s converges to zero with time going to infinity for positive definite gain matrices Λ, and K_d. Consequently, this implies that both $\dot{e}(t)$ and $e(t)$ decreases to zero as time goes to infinity providing the controller stays in the secondary controller. If however the controller switches back to the primary controller the link and motor errors decrease towards zero until $\|\dot{\tilde{e}}_m\|^2=\epsilon$. Therefore the dual control scheme will result in the norm of the tracking error ($\sqrt{\|e\|^2+\|\tilde{e}_m\|^2+\|\dot{\tilde{e}}_m\|^2+\|\dot{e}\|^2}$) to decrease to the order of $O(\sqrt{\epsilon})$. ###

8. Simulation Results For A Two-Link Planar FJR.

We now describe the computer simulation for a two-link planar manipulator with revolute joints (see Figure 1). The linkage are composed of two identically uniform beams which are infinitely rigid, with actuators mounted at the joints (see Figure 2). We assume that the load carried by the end-effector is a part of the second link. From equations (1) and (2), the dynamic equations of the two link manipulator are given as:

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}=\begin{bmatrix}d_{m1} & 0\\ 0 & d_{m2}\end{bmatrix}\begin{bmatrix}\ddot{q}_{m1}\\ \ddot{q}_{m2}\end{bmatrix}+\begin{bmatrix}b_{m1} & 0\\ 0 & b_{m2}\end{bmatrix}\begin{bmatrix}\dot{q}_{m1}\\ \dot{q}_{m2}\end{bmatrix}+\begin{bmatrix}k_{s1} & 0\\ 0 & k_{s2}\end{bmatrix}\begin{bmatrix}q_{m1}-q_1\\ q_{m2}-q_2\end{bmatrix} \tag{43}$$

and,

$$\begin{bmatrix}0\\ 0\end{bmatrix}=\begin{bmatrix}d_{11} & d_{12}\\ d_{21} & d_{22}\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}+\begin{bmatrix}c_{11} & c_{12}\\ c_{21} & c_{22}\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}+\begin{bmatrix}g_1(q)\\ g_2(q)\end{bmatrix}+\begin{bmatrix}k_{s1} & 0\\ 0 & k_{s2}\end{bmatrix}\begin{bmatrix}q_1-q_{m1}\\ q_2-q_{m2}\end{bmatrix} \tag{44}$$

where the coefficients can be derived from the Lagragian formulation (similar to that in Paul's book, [8]). Notice that $g \in \mathbb{R}^1$ is the gravitational acceleration and it is assumed to be $9.81\ ms^{-2}$.

$d_{11}=I_1+I_2+(m_1+4m_2)l_1^2+m_2l_2^2+4m_2l_1l_2\cos(q_2)$

$d_{12}=d_{21}=I_2+m_2l_2^2+2m_2l_1l_2\cos(q_2)$

$d_{22}=I_2+m_2l_2^2$

$c_{11}=-4m_2l_1l_2\dot{q}_2\sin(q_2)$

$c_{12}=-2m_2l_1l_2\dot{q}_2\sin(q_2)$

$c_{21}=2m_2l_1l_2\dot{q}_1\sin(q_2)$

$c_{22}=0$, and $I_j=.33m_jl_j^2+.01m_jl_j^2$ for $j=1,2$

$g_1(q)=g[(m_1+2m_2)l_1\cos(q_1)+m_2l_2\cos(q_1+q_2)]$

$$g_2(q)=gm_2l_2\cos(q_1+q_2) \tag{45}$$

For notational convenience let us define

$$S_1=\sin q_1 \quad C_1=\cos q_1 \quad C_{12}=\cos(q_1+q_2)$$

$$S_2=\sin q_2 \quad C_2=\cos q_2 \quad S_{12}=\sin(q_1+q_2)$$

We can rewrite the manipulator dynamics in the regressor form with the unknown parameters appearing linearly as:

$$\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}\ddot{q}_1 & (\ddot{q}_1+\ddot{q}_2) & (\alpha C_2-\beta S_2) & gC_1 & gC_{12}\\ 0 & (\ddot{q}_1+\ddot{q}_2) & (\ddot{q}_1 C_2+\dot{q}_1^2 S_2) & 0 & gC_{12}\end{bmatrix}\begin{bmatrix}p_1\\p_2\\p_3\\p_4\\p_5\end{bmatrix}$$

$$+\begin{bmatrix}k_{s1} & 0\\ 0 & k_{s2}\end{bmatrix}\begin{bmatrix}q_1-q_{m1}\\ q_2-q_{m2}\end{bmatrix} \tag{46}$$

where, $\alpha = 2\ddot{q}_1 + \ddot{q}_2$ and $\beta = 2\dot{q}_1\dot{q}_2 + \dot{q}_2^2$ (47)

Furthermore, the unknown parameter vector P is given as:

$$P=\begin{bmatrix}p_1\\p_2\\p_3\\p_4\\p_5\end{bmatrix}=\begin{bmatrix}I_1+m_1 l_1^2+4m_2 l_1^2\\ I_2+m_2 l_2^2\\ 2m_2 l_1 l_2\\ l_1(m_1+2m_2)\\ m_2 l_2\end{bmatrix} \tag{48}$$

Therefore the vector functions of unknown parameters $P \in \mathbb{R}^5$ and the regressor matrix $Y(\ddot{q},\dot{q},q) \in \mathbb{R}^{2\times 5}$ are well defined. After choosing the desired links trajectory, we use equations (6), (7) and (8) to derive the desired motor trajectory.

The control law given in equation (23) and (36) and the adaptation laws given (24) and (37) were used with the following definitions of Ψ and Γ:

$$\Psi=\begin{bmatrix}\ddot{q}_{1d} & (\ddot{q}_{1d}+\ddot{q}_{2d}) & (\alpha_1 C_2-\beta_1 S_2) & gC_1 & gC_{12}\\ 0 & (\ddot{q}_{1d}+\ddot{q}_{2d}) & (\ddot{q}_{1d}C_2+\dot{q}_{1d}\dot{q}_1 S_2) & 0 & gC_{12}\end{bmatrix} \tag{49}$$

where, $\alpha_1 = 2\ddot{q}_{1d} + \ddot{q}_{2d}$ and $\beta_1 = 2\dot{q}_{1d}\dot{q}_2 + \dot{q}_{2d}\dot{q}_2$ (50)

and

$$\Gamma=\begin{bmatrix}(\ddot{q}_{1d}+a) & \gamma_1 & (\alpha_2 C_2-\beta_2 S_2) & gC_1 & gC_{12}\\ 0 & \gamma_2 & (\ddot{q}_{1d}+a)C_2+(\dot{q}_{1d}+c)\dot{q}_1 S_2 & 0 & gC_{12}\end{bmatrix} \tag{51}$$

where, $\alpha_2 = 2(\ddot{q}_{1d}+a)+\ddot{q}_{2d}+b$, $\beta_2 = 2(\dot{q}_{1d}+c)\dot{q}_2 + (\dot{q}_{2d}+d)\dot{q}_2$

and $\gamma_1=(\ddot{q}_{1d}+\ddot{q}_{2d}+a+b)$, $\gamma_2=(\ddot{q}_{1d}+\ddot{q}_{2d}+a+b)$ (52)

assuming a, b, c, and d are derived from:

$$\Lambda\dot{e}=\begin{bmatrix}a\\b\end{bmatrix} \text{ and } \Lambda e=\begin{bmatrix}c\\d\end{bmatrix} \tag{53}$$

We selected a sampling period of 10×10^{-3} seconds corresponding to a servo rate of 100 Hz. We selected, $\epsilon = 0.5$, and the second controller (36) was activated when, $\|\dot{e}_m\|^2 < \epsilon$. The value of ϵ is quite large and it was selected to ensure small torque demand in the first level controller

(23). The following bounds were used in the definition of λ_i's $-2 \leq \|\ddot{e}_{mi}\| \leq 2$, for $i = 1,2$. Table S-1 shows the numerical values of the parameter vector P, and the known motors parameters.

parameter	value
$K_{s1}=k_{s2}$	50 N m rad^{-1}
$D_{m1}=d_{m2}$	.05 kg m^2
$D_{m1}=b_{m2}$	.05 N m sec rad^{-1}
P_1	1.66
P_2	0.42
P_3	0.63
P_4	3.75
P_5	1.25
$l_1=l_2$	.25 m
$m_1=m_2$	5 kg

Table S-1 : Actual parameters values

Four different cases were simulated to show the improvement obtained over current adaptive control schemes for robotic manipulators. The need for adaptive control is also illustrated through simulations. Simulations for the controller described in equation(10), when parameters of the arm are known are not presented in this paper.

As seen from table (S-1), the robot considered here has extremely flexible joints. A load of 5kg, when the arm is fully extended and parallel to the horizontal plane, results in the inner joint q_1 to deflect by 1rad (or 57.3°). Current industrial robots have joint stiffness in excess of several hundred $Nmrad^{-1}$. Notice also, this manipulator is not light and each link has a weight of 5kg.

In the below simulations, we assume the manipulator is initially at rest with $q_1 = -90°$, and $q_2 = 0°$. The desired trajectory is given by:

$$q_{1d}(t) = [-\frac{\pi}{2} + 0.3\sin(\pi t)]\text{rad}. \quad (54)$$

$$q_{2d}(t) = [-0.3 + 0.3\cos(\pi t)]\text{rad}. \quad (55)$$

Case #1:

In order to show that current rigid robot adaptive schemes are ineffective when applied to FJR, we applied the elegant adaptive control schemes suggested by Slotine and Li [11] to the FJR described in table (S-1). As this controller was derived on the assumption that the joints are rigid, equation (3) was used for the rigid robot model and the rigid robot control law was:

$$\tau = \Gamma(\ddot{q}_d, \dot{q}_d, \dot{q}, q)\tilde{P} + D_m(\ddot{q}_d + \Lambda e) + B_m\dot{q} + K_d(\dot{e} + \Lambda e) \quad (56)$$

and the adaptation law was:

$$\dot{\tilde{P}}(t) = M^{-1}\Gamma^t(\ddot{q}_d, \dot{q}_d, \dot{q}, q)[\dot{e} + \Lambda e] \quad (57)$$

The controller gains were found to be

$$K_p = K_{pm} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix} \quad K_d = K_{dm} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{58}$$

$$\Lambda = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix} \quad M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{59}$$

The response of the manipulator to Slotine and Li's adaptive control law is shown in Figures (3a, 3b, and 3c). Figure 3a shows the link angle responses, the motor responses are shown in Figure 3b, and parameter $\tilde{P}_1$ is shown in Figure 3c. Notice all the parameters behave similarly to $\tilde{P}_1$, shown in Figure 3c. From Figures 3a and 3b, it can be seen that unacceptable link and motor responses are obtained before the system goes unstable. Figure 3c shows that the parameters vary wildly before diverging.

We expect that all other rigid robot adaptive control schemes would also produce unstable responses when applied to control FJR's with such low joint stiffness. These simulations indicate clearly the need to develop new adaptive control schemes for the FJR. Note that the rigid control law (56) gives acceptable responses for very large joint stiffness, eg with $K_{si}=6000 \text{Nmrad}^{-1}$.

Case #2:

In order to show the effectiveness of the adaptive controller given by equations(23) and (24) we applied the control scheme to the FJR described in table S-1. Here we set $|\tau_1| < 25$Nm and $|\tau_2| < 25$Nm. The controller gains were selected as shown below:

$$K_p = K_{pm} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \quad K_d = K_{dm} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \tag{60}$$

$$\Lambda = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad M = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \tag{61}$$

The response of the motors, joint angles and parameters are shown in Figures 4a, 4b and 4c respectively. Notice that stable responses are obtained , parameters do not diverge and small steady state errors in joint#1 and in joint#2 are observed. This one level adaptive scheme is clearly more effective than the rigid control schemes.

Case #3:

In order to show the need for adaptation and the effectiveness of the dual level control scheme, we applied the control scheme given by equations (23) and (36). We assumed the parameter vector $\tilde{P}=[2,1,1,3.5,1]$. The parameters are different from their actual values given in table S-1. The response of the FJR to the control scheme without the adaptation is given in Figures (5a, and 5b). We can see the tracking errors of the links in Figure 5a, and the tracking errors of the motors in Figure 5b. Notice that the scheme given in equations (23) and (36) is more effective in tracking the FJR trajectory than Slotine and Li's rigid adaptive scheme, which gave unstable responses. The controller gains used were the same as those given in Case #2. We can see a significant steady state errors develop, clearly this is undesirable in many applications. In order to compensate for the steady state tracking error, it is desirable to employ an adaptive control scheme. Notice, even if $\tilde{P}$ was determined such that, $\tilde{P}-P=0$, the need for adaptation is not eliminated as the robot may pick up unknown loads and therefore alter the P vector. This would once again result in steady state tracking error.

Case #4:

In order to show the effectiveness of the adaptive control scheme given by equations (23) through (36), we applied our two level control scheme to the FJR. The response is given in Figures (6a, 6b, and 6c). Figure 6a shows the responses for the links, while Figure 6b shows the responses of the motors, and Figure 6c shows the estimates of the parameters. We can see that the motor and the link tracking errors go to zero. The parameters also do not diverge, although they do not converge to their exact actual values, they oscillate about their true values. The controller gain matrices given in equations (60) and (61) were used for this case. Clearly, the response of the manipulator to the adaptive FJR scheme described in this paper is significantly better than applying rigid robot adaptive schemes as seen in case #1. Notice also the the two level adaptive scheme has superior performance over the non-adaptive control law simulated in case #3 and the single level adaptive controller simulated in case #2. The non-adaptive controller developed significant steady state errors while the single level adaptive controller with torque constraints developed some steady state tracking errors. Extensive simulations show that the steady state tracking error developed by this two level controller is quite small and the error is mainly in the motor coordinates. Notice also the behavior of the parameters, they vary slowly about their nominal values with the two-level scheme, whereas they do not appear to track their nominal values as accurately in the one-level scheme for the given simulation time.

9. Conclusion

In this paper we have presented several schemes to control flexible joint robots. Two of them are adaptive control schemes, they were derived without employing linearization techniques, link acceleration or jerk measurements, and inertia matrix inversion. Adaptive controllers for the FJR were derived using Lyapunov's second method. From the simulation results, it is clear that the improvement in the tracking and parameter estimation is significant over rigid robot adaptive schemes. Therefore it is necessary to account for joint flexibility effects when deriving control schemes for industrial robots with such form of compliance.

It is obvious that some correction scheme could be added to the derived adaptation law to improve the robustness of our controller in the presence of bounded disturbances or unmodeled dynamics [5]. Experimental work will be necessary to demonstrate the practicality of our scheme.

It is important to point out that most industrial robots use feedback sensors mounted on the actuator and in order to compensate for joint flexibility additional sensors must be mounted to measure the joint angles and velocities.

10. Acknowledgement

We would like to thank Professor's Martin Corless and Stanislaw Żak and Dr.Mehdi Madani-Esfahani for initial expository discussions on the area of flexibly jointed systems and Lyapunov stability theory.

10. Appendix A : To show $[\dot{D} - 2C]$ is Skew Symmetric.

Here we will show that D(q) and $C(q,\dot{q})$ are not independent, but the matrix $(\dot{D}-2C)$ is skew symmetric [10], [4], [13]. This can be easily derived from the Lagrangian formulation of the manipulator dynamics. In order for a square matrix W to be skew-symmetric, we need $W^t = - W$. From equation (2), we can represent the $(kj)^{th}$ element of $C(q,\dot{q})$ by

$$c_{kj} = \sum_{i=1}^{n} [\frac{\partial d_{kj}}{\partial q_i} - ½\frac{\partial d_{ij}}{\partial q_k}]\, \dot{q}_i \qquad (A1)$$

where, d_{kj} is the $(kj)^{th}$ element of the inertia matrix D(q). Now, by interchanging the (i,j) indices and using the symmetry property of D (q), we note:

$$\sum_{i,j}^{n} \frac{\partial d_{kj}}{\partial q_i} = ½\sum_{i,j}^{n} [\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j}] \qquad (A2)$$

Therefore, we can substitute (A2) into (A1), and:

$$c_{kj} = \sum_{i=1}^{n} ½[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}]\, \dot{q}_i \qquad (A3)$$

Let, $W(q,\dot{q})=[\dot{D}(q)-2C(q,\dot{q})]$, then the $(jk)^{th}$ element of W is:

$$w_{kj}=\dot{d}_{kj} - 2c_{kj}$$

$$=\sum_{i=1}^{n} [\frac{\partial d_{kj}}{\partial q_i} - (\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k})]\, \dot{q}_i$$

$$=\sum_{i=1}^{n} [\frac{\partial d_{ij}}{\partial q_k} - \frac{\partial d_{ki}}{\partial q_j}]\, \dot{q}_i \qquad (A4)$$

Since D(q) is symmetric, it is clear from (A4) that, $w_{kj} = -w_{jk}$. Therefore, $W(q,\dot{q})$ is skew symmetric, furthermore the diagonal entries of W are zero as:

$$w_{jj} = \sum_{i=1}^{n} [\frac{\partial d_{ij}}{\partial q_j} - \frac{\partial d_{ji}}{\partial q_j}]\, \dot{q}_i=0 \qquad (A5)$$

Again by the symmetry property of D(q), (A5) is straight forward.

Now we can conclude that $W(q,\dot{q})=[\dot{D}(q)-2C(q,\dot{q})]$ is skew symmetric with zero diagonal entries, which yields

$$\dot{q}^t\ [\dot{D}(q) - 2C(q,\dot{q})]\ \dot{q} = 0 \,. \qquad (A6)$$

12. References

[1] S. Ahmad, "Analysis of Robot Drive Train Errors, their Static Effects and their Compensations," *IEEE Journal of Robotics and Automation,* Vol. 4, No. 2, April 1988.

[2] A. De-Luca, "Dynamic Control of Robots with Joint Elasticity," *Proceedings of 1988 IEEE International Conference on Robotics and Automation,* Philadelphia, PA, pp. 152-158, April 1988.

[3] L. Fu and K. Chen, "Nonlinear Adaptive Motion Control for a Manipulator with Flexible Joints," *Proceedings of 1989 IEEE International Conference on Robotics and Automation,* Phoenix, AZ, pp. 1201-1206, May 1989.

[4] F. Ghorbel, W. Spong and J. Hung, "Adaptive Control of Flexible Joint Manipulators," *Proceedings of 1989 IEEE International Conference on Robotics and Automation,* Phoenix, AZ, pp. 1188-1193, May 1989.

[5] P. Ioannou, "Robust Adaptive Controller with Zero Residual Tracking Errors," *IEEE Trans. Automatic Control,* Vol. 31, No. 8, pp. 773-776, Aug. 1986.

[6] K. Khorasani, "Robust Adaptive Stabilization of Flexible Joint Manipulators," *Proceedings of 1989 IEEE International Conference on Robotics and Automation,* Phoenix, AZ, pp. 1194-1199, May 1989.

[7] D. Koditschek, "Lyapunov Analysis of Robot Motion," *Tutorial Workshop of the 1987 IEEE International Conference on Robotics and Automation,* Raleigh, NC, pp. 1B-3-1 to 1B-3-56, March 1987.

[8] R. Paul, *Robot Manipulators: Mathematics, Programming and Control,* MIT Press, 1981.

[9] S. Sastry, P. Hsu, M. Bodson and B. Paden, "Adaptive Identification and Control for Manipulators without Using Joint Accelerations," *Proceedings of 1987 IEEE International Conference on Robotics and Automation,* Raleigh, NC, pp. 1210-1215, March 1987.

[10] J. Slotine and W. Li, "Adaptive Manipulator Control: A Case Study," *IEEE Trans. Automatic Control,* Vol. 33, No. 11, pp. 995-1003, Nov. 1988.

[11] J. Slotine and W. Li, "On the Adaptive Control of Robot Manipulators," *Int. J. Robotics Research,* Vol. 6, No. 3, pp. 49-59, Fall 1987.

[12] M. Spong, K. Khorasani and P. Kokotovic, "An Integral Manifold Approach to the Feedback Control of Flexible Joint Robots," *IEEE Journal of Robotics and Automation,* Vol. RA-3, No. 4, pp. 291-300, Aug. 1987.

[13] M. Spong, "Modeling and Control of Elastic Joint Robots," *Transactions of the ASME,* Vol. 109, pp. 310-319, Dec. 1987.

[14] G. Widmann and S. Ahmad, "Control of Robots with Flexible Joints," *Proceedings of 1987 IEEE International Conference on Robotics and Automation,* Raleigh, NC, pp. 1561-1566, March 1987.

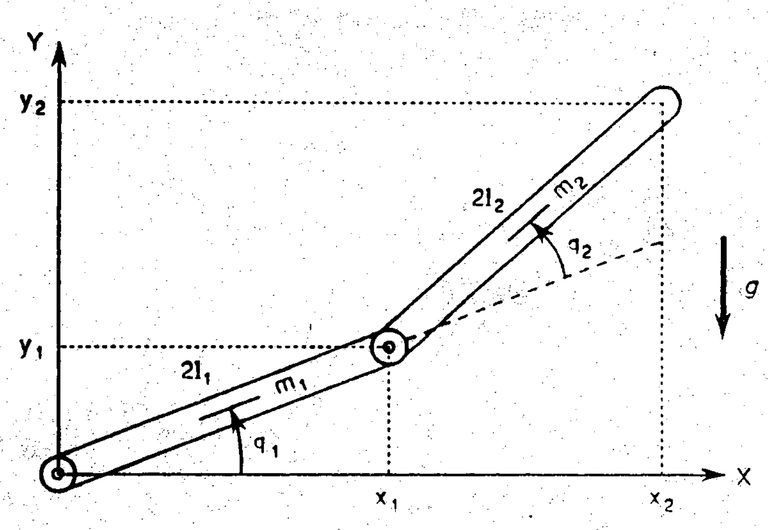


Figure 1. The Two Link Planar Manipulator.

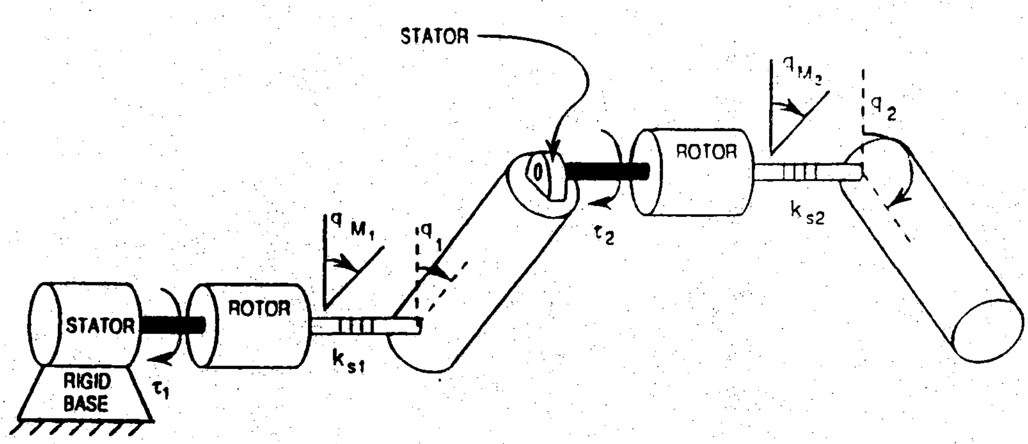
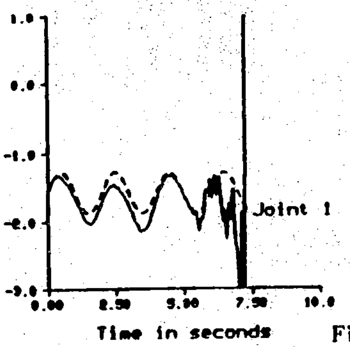


Figure 2. Conceptual Diagram of Two Link Compliant Manipulator.



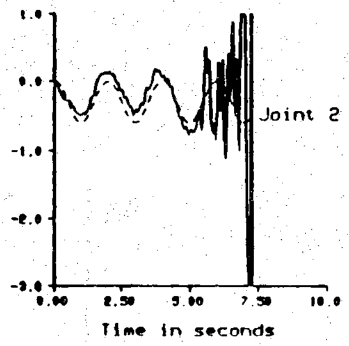
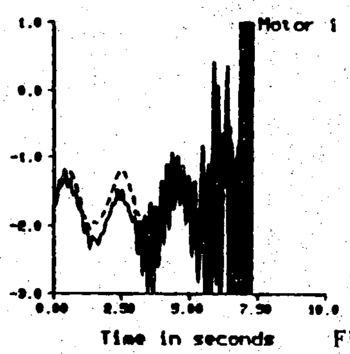


Figure 3a



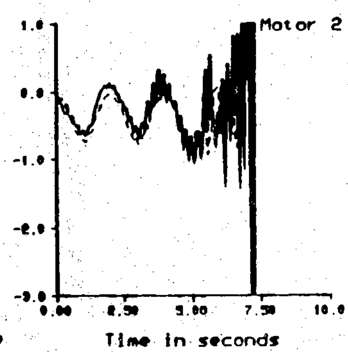


Figure 3b

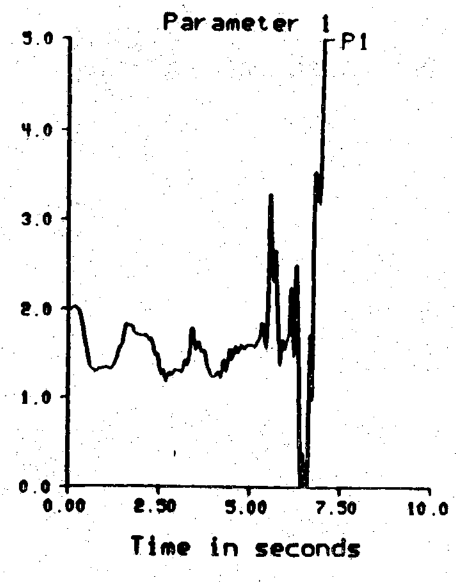


Figure 3c

Case#1: Current rigid robot adaptive scheme applied to FJR.

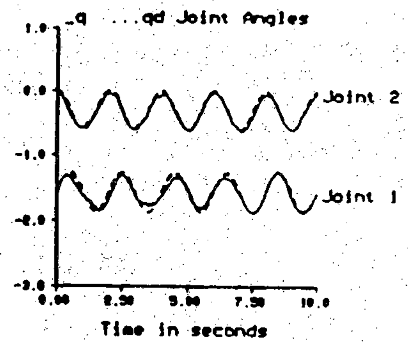


Figure 4a

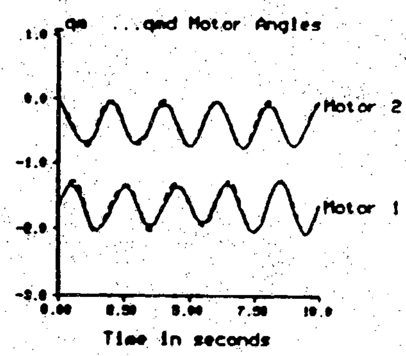


Figure 4b

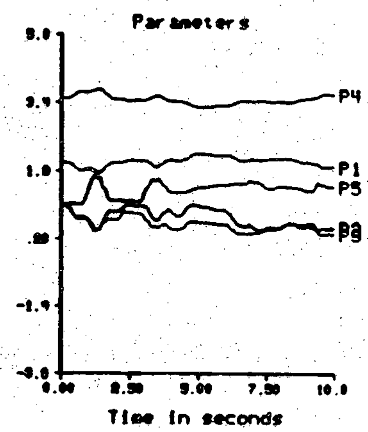


Figure 4c

Case#2: Derived one-level FJR adaptive control scheme.

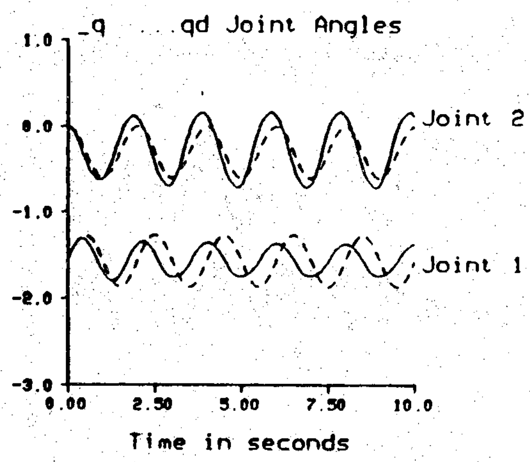


Figure 5a

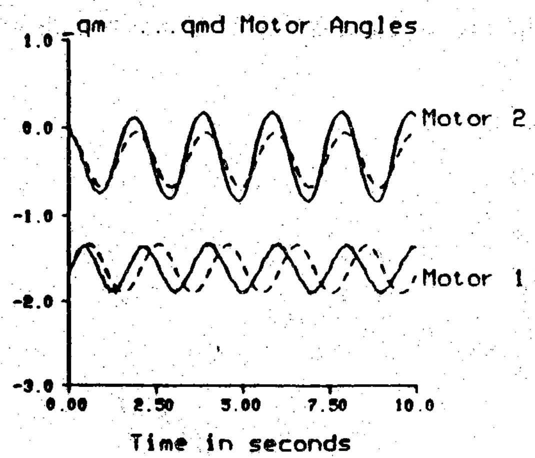


Figure 5b

Case#3: Derived two-level FJR control scheme without adaptation.

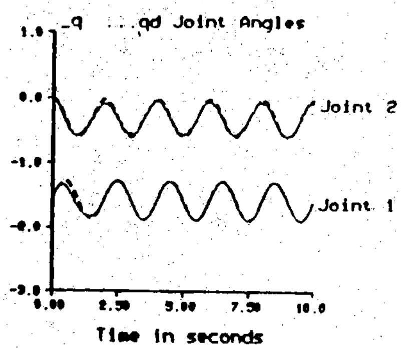


Figure 6a

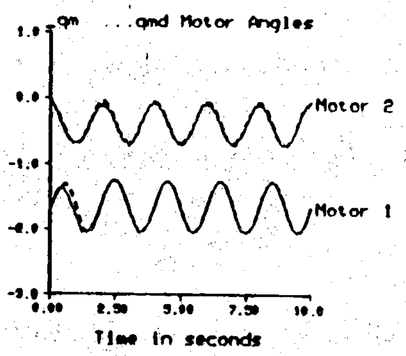


Figure 6b

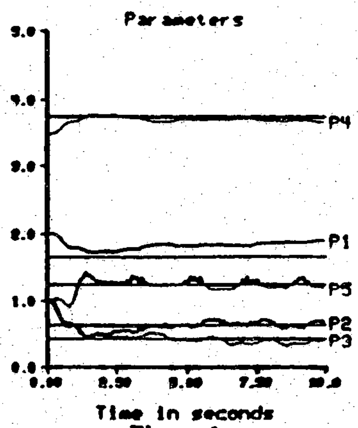


Figure 6c

Case#4: Derived two-level FJR adaptive control scheme.