8-1-1990

Comparison of Drive Strategies for Brushless DC Motors

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TR-EE 90-45
August 1990

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COMPARISON OF DRIVE STRATEGIES FOR BRUSHLESS DC MOTORS

A Project
Submitted to the Faculty
of
Purdue University
by

Zaker M. Azizi

In Partial Fulfillment of the Requirements
for the Degree
of
Master of Science in Electrical Engineering

August 1990

This is dedicated to my mother

Acknowledgments

I would like to thank my major professor Oleg Wasynczuk for his guidance and assistance throughout the course of this project. I also thank Linda Stovall for help in the preparation of this project.

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ABSTRACT

The brushless dc motor is a permanent-magnet synchronous machine controlled in such a way that the frequency of the applied voltages is proportional to the rotor speed.

This type of motor is becoming increasingly popular and is replacing conventional types of dc machines and synchronous machines in servo and variable speed applications.

One type of inverter, the 120 degree inverter, does not require rotor position sensing hardware. A second type of inverter is the 180 degree inverter. In this project, the difference between these two inverters is explained and a numerical example is used to show the differences between these two types of drives.

CHAPTER 1

INTRODUCTION

The brushless dc machine is actually a permanent-magnet synchronous machine. The torque versus speed characteristics of a brushless dc machine resemble those of a dc machine when the frequency of the applied voltages is made to correspond to the rotor speed. In order to get this, it is necessary to determine the rotor position. One way to measure the rotor position is by using Hall-effect sensors. Using the sensed rotor position, the necessary stator voltages can be applied by the 180 degree inverter. A second way is to use the 120 degree inverter to determine the position of the rotor from the line to ground voltages of the machine. In this project, a description, the operation, and the wave forms of these two types of inverter are studied. Also, for the purpose comparing these two types of inverters, an example is used in Sect. 2-7.

An analysis of 180 degree inverter strategies is given in Section 2-2 and the inverter output voltage operations and switching logic are discussed in Section 2-3. The voltage equations for stationary reference frames are stated in Section 2-6.

CHAPTER 2
DESCRIPTION OF THE BRUSHLESS DC MACHINE AND INVERTER

2-1 Description of the Brushless DC Drive System:

The brushless dc machine is a permanent-magnetic synchronous machine such as the one depicted in Fig. 1. The basic difference between the brushless dc machine and the permanent magnetic synchronous machine is in the form of the applied voltages. The B-DC (brushless dc) motor is so called, because it exhibits torque versus speed characteristics which are similar to those of an armature controlled dc machine. That is, the torque versus speed graph is linear.

Usually the three phase synchronous machines with three stator windings are connected in Y(wye), see Figure 1. The stator windings are identical windings displaced by 120°, each with resistance r_s with equivalent number of turns.

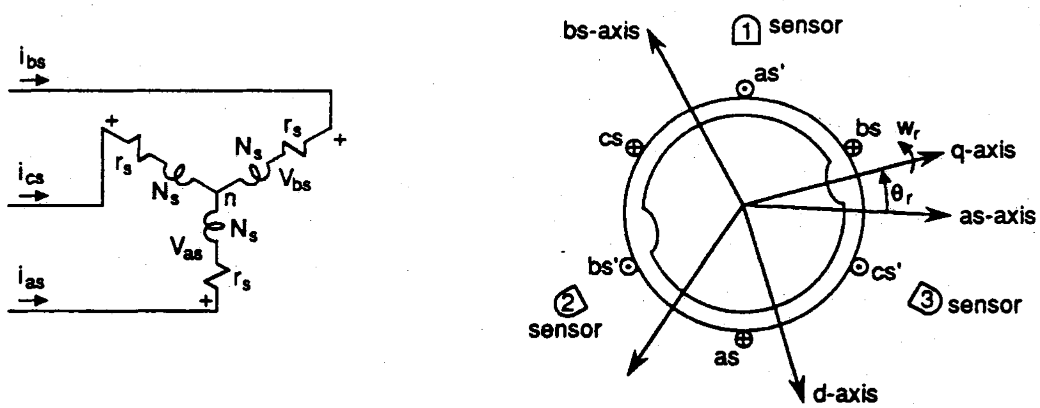

Fig 1. Three phase, two pole Brushless dc machine, stator winding connected as Y

The brushless dc machine is supplied by phase voltages whose frequency is equal to the rotor speed. It could be done by measuring the rotor position and by applying voltages with frequency according to the rotor speed. The rotor position is typically determined using Hall-effect devices which are located on the stator near the rotor magnets. The position of the Hall-effect devices is indicated in Fig. 1. The Hall-effect device signals and controls the polarity of the voltages applied to the three phases of the machine. This switching is done by an inverter, switched at a frequency corresponding to the rotor speed.

2-2 Analysis and Description of Inverter Strategies:

As stated in the previous section, the brushless dc machine is a three phase permanent-magnet synchronous machine and is supplied by an inverter. A typical rectifier-inverter drive system is shown in Fig. 2. The rectifier is generally a three phase, line-commutated, full converter. The parameters of the filter are selected to reduce the harmonics of the voltage applied to the inverter V_I. The inverter is to supply the brushless dc machine with variable-frequency voltage.

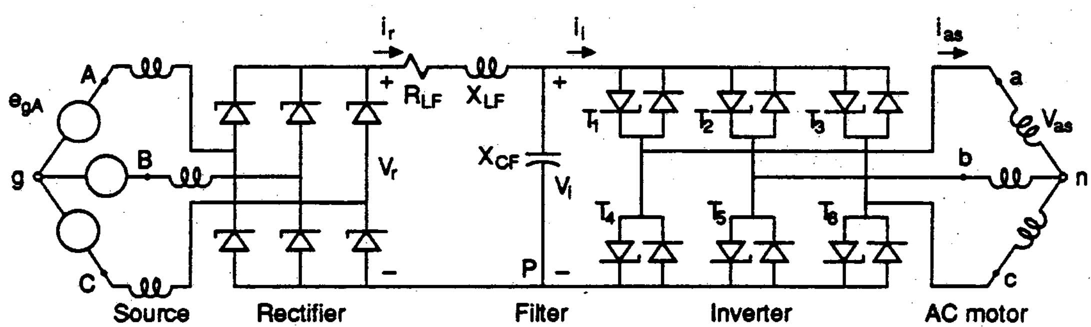

Fig 2. Rectifier - Inverter drive system

The three phase, six step inverter as shown in Fig. 3 is used to establish the required synchronization between the rotor speed and stator-voltage frequency.

As we see in Fig. 3, the inverter requires a dc voltage source ($V_{dc} = V_I$) and six transistors (switches) from $T_1 - T_6$, which switch (open and close) the circuit for variable intervals to modulate the amount of direct current supplied from the voltage source.

It is assumed that the brushless dc machine is wye connected and that all transistors and diodes are ideal; therefore, there is no voltage drop across the conducting device. In the circuit (Fig. 3), the common (neutral) terminal is denoted as "n" and the voltage source common (ground) is denoted as "g". The n and g terminals not connected electrically. The phase-to-neutral voltages V_{an}, V_{bn} and V_{cn} are equal to V_{as}, V_{bs} and V_{cs}, which are the output of the inverter.

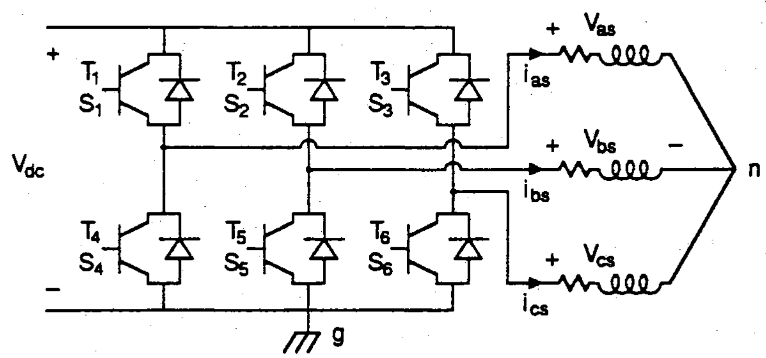

Fig 3. Three phase, six step voltage inverter and machine

The inverter switches (opens and closes) the circuit for variable intervals to modulate the amount of direct current supplied from the voltage source. There will be low power losses when the switch is fully on or fully off, and if fast switching is applied, the losses are small providing a very high efficiency.

For better switching logic, the voltage and the phase current could be controlled by applying pulse-width-modulation (PWM). There are two ways in controlling the phase current. One is by applying pulse-width-modulation to the voltage inverter with a fixed frequency and fixed duty cycle. A second way is by varying the duty cycle according to the error in the phase current, which is the phase-current subtracted from the present reference current.

In the PWM of the voltage-fed inverter, rather than having the transistors or switches continuously on for 180 degrees, they are switched on or off at a fixed frequency to reduce the fundamental component of the output voltages of the bridge inverter. As a result, this PWM voltage control technique represents a means of controlling the fundamental amplitude of the stator voltages in which the switching signals of the inverter are modulated and the dc source voltage V_{dc} is kept fixed.

2-3 180 Degree Inverter Output Voltage:

The switching signals for the inverter consist of a sequence of pulses whose width is modulated in a specific fashion. For example, in Fig. 3, S_1 is turned on for 180 degrees and then turned off for 180 degrees (the same holds for S_2, S_3). Therefore, it is called 180 degree conduction mode. Also, it follows from Fig. 3 that the phase-to-neutral voltages V_{ag}, V_{bg} and V_{cg} depend on the state of the six switches. For example, if S_1 is closed and S_4 is open, then $V_{ag} = V_{dc}$. Otherwise, if S_1 is open and S_4 is closed, then $V_{ag} = 0$. The same holds for V_{bg} and V_{cg}. By using Kirchoff's voltage law (KVL), the phase-to-neutral voltages V_{an}, V_{bn} and V_{cn} are expressed as

$$V_{an} = V_{ag} - V_{ng} \tag{1}$$

$$V_{bn} = V_{bg} - V_{ng} \tag{2}$$

$$V_{cn} = V_{cg} - V_{ng} \tag{3}$$

Adding the above three equations, yields an expression for V_{ng} (the neutral to ground voltage).

$$V_{ng} = \frac{1}{3}\left[V_{ag} + V_{bg} + V_{cg}\right] \tag{4}$$

where $V_{an} + V_{bn} + V_{cn} = 0$ for a balanced system, and V_{ng} is the voltage between n and g.

Substituting equation (4) into equations (1), (2), and (3) yields

$$V_{as} = \frac{2}{3} V_{ag} - \frac{1}{3} V_{bg} - \frac{1}{3} V_{cg} \tag{5}$$

$$V_{bs} = \frac{2}{3} V_{bg} - \frac{1}{3} V_{ag} - \frac{1}{3} V_{cg} \tag{6}$$

$$V_{cs} = \frac{2}{3} V_{cg} - \frac{1}{3} V_{ag} - \frac{1}{3} V_{bg} \tag{7}$$

The line-to-line voltage can be expressed as

$$V_{ab} = V_{ag} - V_{bg} = V_{an} - V_{bn} \tag{8}$$

$$V_{bc} = V_{bg} - V_{cg} = V_{bn} - V_{cn} \tag{9}$$

$$V_{ca} = V_{cg} - V_{ag} = V_{cn} - V_{an} \tag{10}$$

Using the above equations, the switching logic and output voltage waveforms are as in Fig. 4:

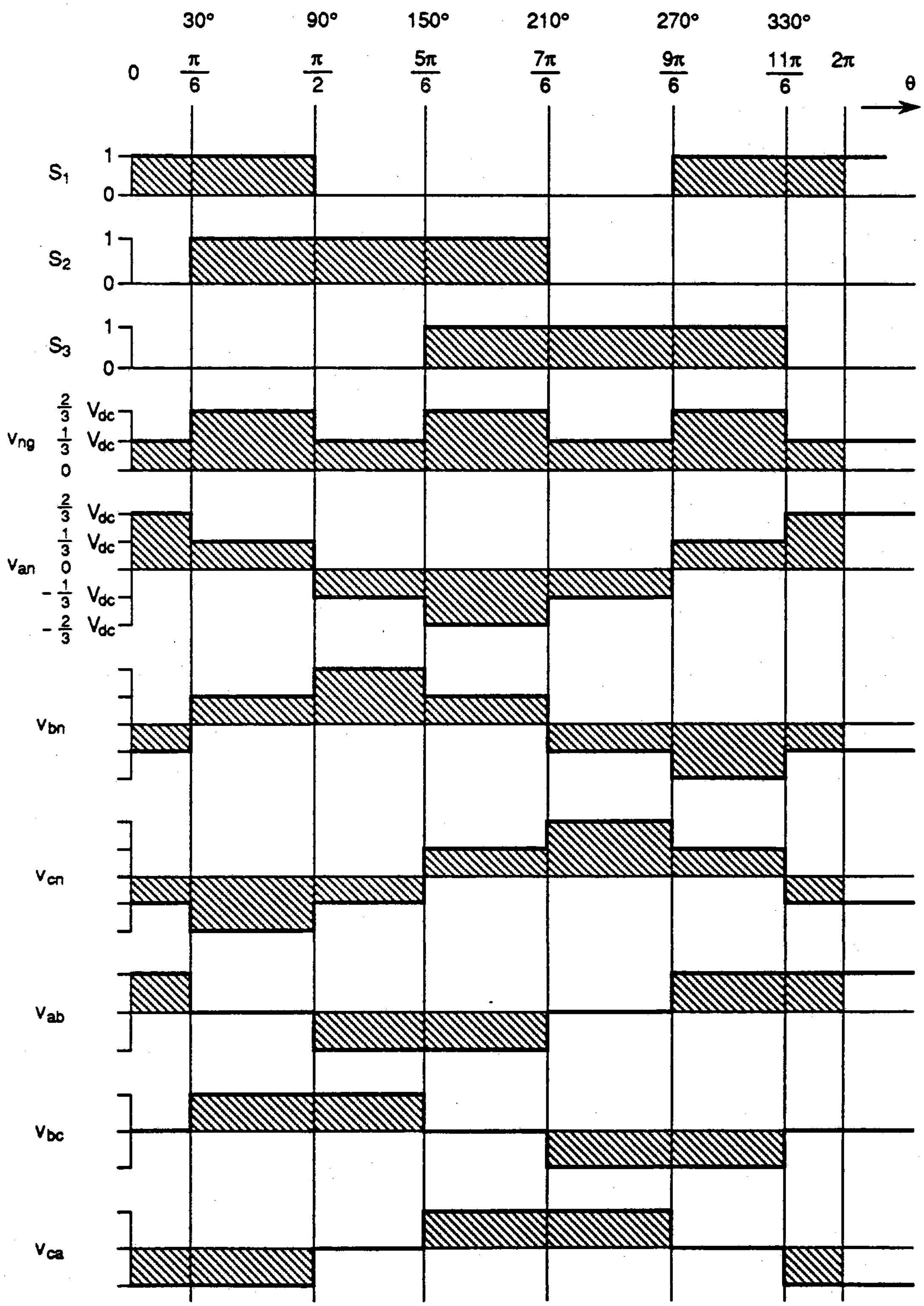

Fig 4. Switching logic and output voltage for 180 degree inverter

If we look at Fig. 4, we notice that V_{ng} is determined from the state of the switches. The remaining voltages are calculated by using V_{ng}. For example, the line-to-line voltage can be calculated as

$$
\begin{array}{llll}
V_{ac} = +V_{dc} & \text{if} & S_1 - S_6 & \text{closed} \\
V_{ac} = -V_{dc} & \text{"} & S_3 - S_4 & \text{"} \\
V_{ac} = 0 & \text{"} & S_1 - S_3 & \text{"}
\end{array}
$$

The other line-to-line voltages can be calculated similarly. The sum of the line-to-line voltages is zero. For the purpose of simplicity, we can express (from Fig. 4) the line-to-neutral and line-to-ground voltages as given in Table 1-1.

Table 1-1 Table for 180 Degree Inverter Operation

BSI	θ_r	Transistors On	v_{ag}	v_{bg}	v_{cg}	v_{as}	v_{bs}	v_{cs}
I	0°	1,2,6	V_{dc}	0	0	$\frac{2}{3} V_{dc}$	$-\frac{1}{3} V_{dc}$	$-\frac{1}{3} V_{dc}$
II	60°	1,2,3	V_{dc}	V_{dc}	0	$\frac{1}{3} V_{dc}$	$\frac{1}{3} V_{dc}$	$-\frac{2}{3} V_{dc}$
III	120°	2,3,4	0	V_{dc}	0	$-\frac{1}{3} V_{dc}$	$\frac{2}{3} V_{dc}$	$-\frac{1}{3} V_{dc}$
IV	180°	3,4,5	0	V_{dc}	V_{dc}	$-\frac{2}{3} V_{dc}$	$\frac{1}{3} V_{dc}$	$\frac{1}{3} V_{dc}$
V	240°	4,5,6	0	0	V_{dc}	$-\frac{1}{3} V_{dc}$	$-\frac{1}{3} V_{dc}$	$\frac{2}{3} V_{dc}$
VI	300°	1,5,6	V_{dc}	0	V_{dc}	$\frac{1}{3} V_{dc}$	$-\frac{2}{3} V_{dc}$	$\frac{1}{3} V_{dc}$

where in Table 1-1, the BSI means basic switching intervals.

2-4 Description of Current Flow in the Inverter Switching Strategy:

The purpose of the diodes is to allow the current to circulate back to the source. As soon as the switches open, diodes D_3, D_4 start conducting. The motor acts as an inductor and has energy stored in it after the switches are closed (see Figure 5).

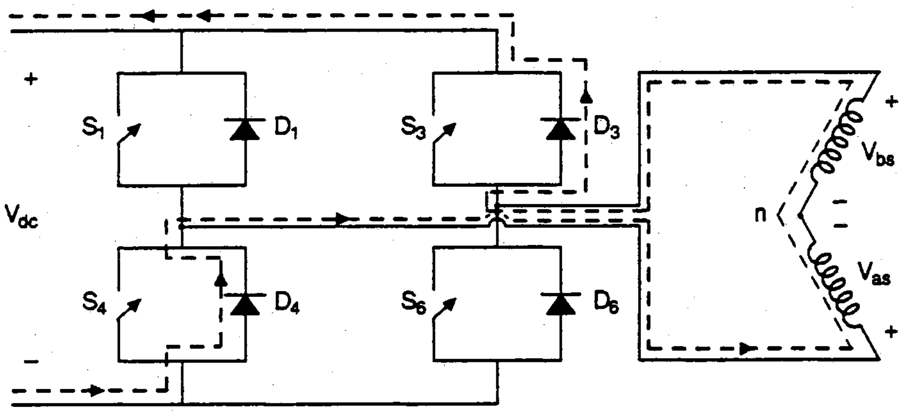

Fig 5. Dashlines indicate the direction of current after S_1 and S_6 opening

2-5 120 Degree Inverter:

The rotor position is measured using three Hall-effect sensors which determine if the rotor angle θr falls into one of three overlapping intervals. The outputs of the three position sensors generate the switching signals for T_1 through T_6 (Fig. 6). Each logic signal is high for 120 degrees and low for the next 240 degrees. Therefore, it is called 120 degree conduction mode.

The rotor position is divided into six equal intervals (see Fig. 6). The operation of a 120 degree inverter is almost the same as a 180 degree inverter except that only two transistors are gated at any given instant of time. For the 120 degree inverter, the equations (8), (9), and (10) yield

$$V_{as} = \frac{2}{3} V_{ag} - \frac{1}{3} V_{bg} - \frac{1}{3} V_{cg} \quad (11)$$

$$V_{bs} = \frac{1}{3} V_{ag} + \frac{2}{3} V_{bg} - \frac{1}{3} V_{cg} \quad (12)$$

$$V_{cs} = -\frac{1}{3} V_{ag} - \frac{1}{3} V_{bg} + \frac{2}{3} V_{cg} \quad (13)$$

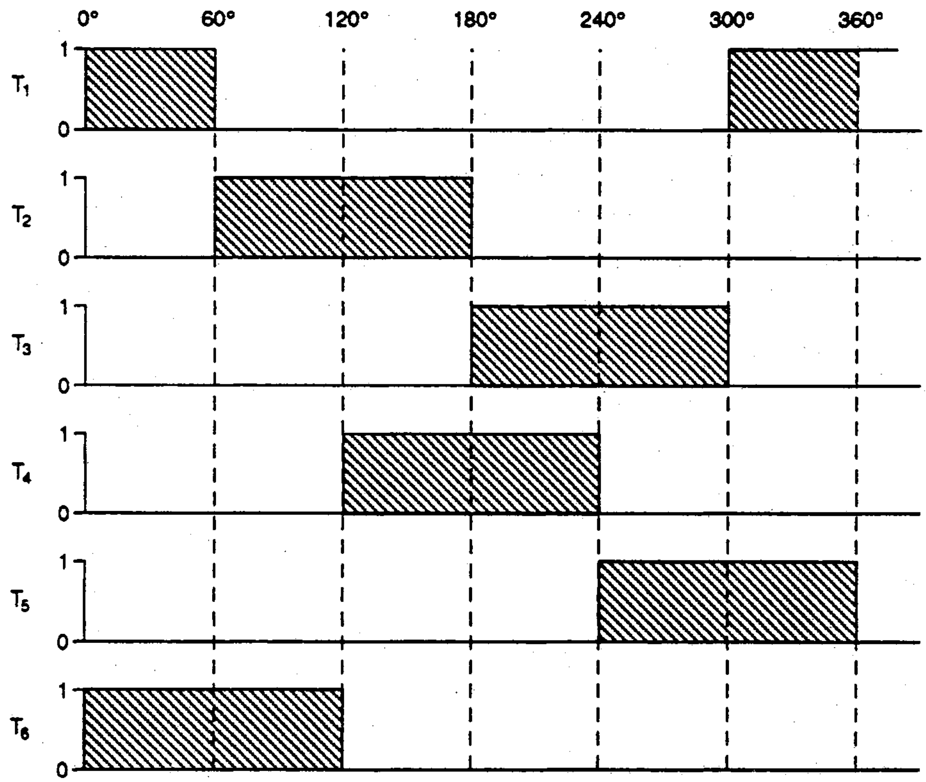

Fig 6. Switching logic for 120 degree conduction mode

2-6 Voltage Equation for a Stationary Reference Frame:

Considerable insight may be gained by making a change of variables which transforms all machine variables (abcs) to the rotor reference frame variables (qdos).

$$\vec{V}_{abcs} = r_s \vec{i}_{abcs} + \vec{p}\lambda_{abcs}, \; p = \frac{d}{dt} \quad (14)$$

where $r_s = \text{diag}\,(r_s \;\; r_s \;\; r_s)$. The flux linkage vector $\vec{\lambda}_{abcs}$ is given by

$$\vec{\lambda}_{abcs} = \begin{bmatrix} L & -M & -M \\ -M & L & -M \\ -M & -M & L \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \lambda_m' \begin{bmatrix} \sin\theta_r \\ \sin(\theta_r - 120^\circ) \\ \sin(\theta_r + 120^\circ) \end{bmatrix}$$

where L_s = self inductance and λ_m' = amplitude of flux linkage

θ_r = rotor displacement, $L_s = L + M$.

The transformation from abcs variables to qdos variables is given by

$$\boxed{\vec{f}^{\,r}_{qdos} = \bar{\bar{K}}^r_s \, \vec{f}_{abcs}} \quad \text{(rotor reference frame)} \tag{15}$$

where $\bar{\bar{K}}^r_s$ is the transformation matrix given by

$$\bar{\bar{K}}^r_s = \frac{2}{3} \begin{bmatrix} \cos\theta_r & \cos(\theta_r - 120^\circ) & \cos(\theta_r + 120^\circ) \\ \sin\theta_r & \sin(\theta_r - 120^\circ) & \sin(\theta_r + 120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\vec{f}^{\,r}_{qdos} = \begin{bmatrix} f^r_{qs} \\ f^r_{ds} \\ f^r_{os} \end{bmatrix} \text{ and } \vec{f}_{abcs} = \begin{bmatrix} f_{as} \\ f_{bs} \\ f_{cs} \end{bmatrix}$$

Here f could be voltage, current, flux, etc. Next, equation (14) can be written in the rotor reference frame using Park's transformation, where the stator variables (abc) are transformed to the rotor reference frame. The resulting voltage equations in matrix form are

$$\vec{V}^r_{qdos} = r_s \, \vec{i}^{\,r}_{qdos} + \omega_r \, \vec{\lambda}^r_{qdos} + p\vec{\lambda}^r_{qdos} \tag{16}$$

Equation (16), can be expanded as

$$V^r_{qs} = (r_s + pL_q)i^r_{ds} + \omega_r \, L_d \, i^i_{ds} + \omega_r \, \lambda'^r_m \tag{17}$$

$$V^r_{ds} = (r_s + pL_d)i^r_{ds} - \omega_r L_q i^r_{qs} \tag{18}$$

$$V^r_{os} = (r_s + pL_{ls})i^r_{os} \tag{19}$$

The instantaneous torque produced by the motor expressed in rotor reference frame variables is

$$T_e = (\frac{2}{3})(\frac{P}{2})(\lambda^r_{ds}\, i^r_{qs} - \lambda^r_{ds}\, i^r_{ds}) = (\frac{3}{2})(\frac{P}{2})\,[\lambda^r_{ds}\, i^r_{qs} - (L_d - L_q)\, i^r_{qs}\, i^r_{ds}]$$

where P is the number of poles and

$$\begin{cases} L_q = L_{ls} + L_{mq} \\ L_d = L_{ls} + L_{md} \end{cases}$$

In order to compare the 180 degree and the 120 degree drives, we can look at the following example.

2-7 Example

In this example, we would like to use the 180 degree inverter and then the 120 degree inverter as sources of voltage for a brushless dc motor, and compare the torque, efficiency, and power loss in each case.

A three phase, brushless dc machine is used as a motor in a computer disc drive. The motor parameters are given below:

# of poles = P = 4
Stator resistance = $r_s = 3.4\ \Omega$
Stator time constant = $\tau_s = 700\ \mu$sec
Steady-state speed = 3600 r/min
$\lambda'^r_m = 0.0677$ V–sec/rad

The calculation and comparison of T_e, stator phase voltage, current, instantaneous power (P_e), mechanical power (P_m), P_{losses} and efficiency for the cases of the 180 degree and the 120 degree inverter are shown as follows.

Solution:

$$W_r = (\frac{p}{2})(3600\ r/min)(\frac{1\,min}{60\,sec})(\frac{2\pi\,rad}{1\,rad}) = 754\ rad/sec$$

$$L_{ss} = \tau_s r_s = \text{stator winding self inductance} = 3.78\ mH$$

I. The 180 Degree Inverter Machine

Let $V_{dc} = V_{Inverter} = 99$ volts. Taking the Fourier series of the abc voltages in Table 1-1, neglecting the harmonics, and transforming to the rotor reference frame yields

$$V_{qs}^{r} = \frac{2V_{dc}}{\pi}\cos\phi \tag{20}$$

$$V_{ds}^{r} = -\frac{2V_{dc}}{\pi}\sin\phi \tag{21}$$

Assuming that we are in steady state or common mode operation, then $\phi = 0^\circ$ (for 180 degree inverter only), equations (20) and (21) yield

$$\left\{\begin{array}{l} V_{qs}^{r} = \dfrac{2V_{dc}}{\pi} = 63.0\ V \\ V_{ds}^{r} = 0 \end{array}\right\} \tag{22}$$

To establish the machine voltages and currents, we can use the equations (17)-(18) of the brushless dc machine derived previously ($p = \frac{d}{dt} = 0$, for steady-state operation). Repeating these here for convenience,

$$V_{qs}^{r} = r_s\, I_{qs}^{r} + \omega_r\, L_{ss}\, I_{ds}^{r} + \omega_r\, \lambda_m^{\prime r} \tag{23}$$

$$V_{ds}^{r} = r_s\, I_{ds}^{r} - \omega_r\, L_{ss}\, I_{qs}^{r} \tag{24}$$

Since we calculated V_{qs}^{r} and V_{ds}^{r} in equation (22), we can plug these values into equations (23) and (24), and solve these equations simultaneously for

I^r_{qs} and I^r_{ds}

$$\begin{Bmatrix} I^r_{qs} = 1.737 \text{ A} \\ I^r_{ds} = 0.917 \text{ A} \end{Bmatrix} \tag{25}$$

In the steady state, the electrical torque is given as

$$T_e = (\frac{3}{2})(\frac{P}{2})(\lambda'^r_m)(I^r_{qs}) \tag{26}$$

Using the value of I^r_{qs} from (25), and plug it in (26) yield

$$Te = (\frac{3}{2})\frac{(4}{2})(0.0677)(1.737) = \underline{0.3528 \text{ N-m}}$$

In order to calculate the stator voltages, we know in steady-state operation that $\phi = 0$. Thus equations (20)-(21) yield

$$V^r_{qs} = \sqrt{2}\, V_s = \frac{2V_{dc}}{\pi} = 63.0 \text{ V} \tag{27}$$

$$V^r_{ds} = 0 \tag{28}$$

Transforming V^r_{qs}, V^r_{ds} back into abc components with ϕ and V^r_{ds} taken as zero

$$V_{as} = \sqrt{2}\, V_s \cos\theta_r = 63.0 \cos\theta_r \quad \text{V} \tag{29}$$

$$V_{bs} = 63.0 \cos(\theta_r - 120^\circ) \quad \text{V} \tag{30}$$

$$V_{cs} = 63.0 \cos(\theta_r + 120^\circ) \quad \text{V} \tag{31}$$

The stator currents, I_{as}, I_{bs} and I_{cs} may be obtained by transforming I^r_{qs}, I^r_{ds} back into abc components with $I_{os} = 0$

$$I_{as} = I_{qs}^r \cos\theta_r + I_{ds}^r \sin\theta_r = 1.737 \cos\theta_r + 0.917 \sin\theta_r$$
$$= 1.964 \cos(\theta_r - 27.83^\circ)\ \text{A} \qquad (32)$$

Taking advantage of three phase symmetry, we can express the currents, in the remaining phases as

$$I_{bs} = 1.964 \cos(\theta_r - 27.83^\circ - 120^\circ)\ \text{A} \qquad (33)$$

$$I_{cs} = 1.964 \cos(\theta_r - 27.83 + 120^\circ)\ \text{A} \qquad (34)$$

The instantaneous power delivered to the motor may be expressed as

$$P_e = V_{as}I_{as} + V_{bs}I_{bs} + V_{cs}I_{cs} = \frac{3}{2}(V_{qs}^r I_{qs}^r + V_{ds}^r I_{ds}^r) \qquad (35)$$

For balanced, steady-state operation, the instantaneous power is constant and equal to the average power and can be expressed as

$$P_e = P_{e\ \text{average}} = 3V_s I_s \cos(\phi_v - \phi_i)$$

where V_s, I_s represent the rms amplitude of the stator voltages and currents, respectively, and $(\phi_v - \phi_i)$ is the angle by which the voltage leads the current. From (29), $\phi_v = 0^\circ$ and from (32), $\phi_i = 27.78$ degrees. Thus

$$P_e = 3\left[\frac{63}{\sqrt{2}}\right]\left[\frac{1.964}{\sqrt{2}}\right]\cos(27.78^\circ)$$
$$= 164.1\ \text{W}$$

The mechanical power, P_m, is equal to the speed times the torque.

$$P_m = T_e\,\omega_{rm} = (0.3528)(377) = 133.0\ \text{W}$$

Thus, the losses, P_{Losses}, are

$$P_{Losses} = P_e - P_m = 164.1 - 133.0 = 31.1\ \text{W}$$

Now, the efficiency, η, is

$$\eta = 1 - \frac{P_{loss}}{P_{in}} = \frac{P_{out}}{P_{in}} = 81.048\%$$

II. The 120 Degree Inverter Machine

Now, if we use the same input ($V_{dc} = V_I = 99$ volt) for the 120 degree inverter, we can calculate the Te, Pe, Pm, and η. Then, we can compare these values with 180 degree inverter.

Voltage Equation for the 120 Degree Inverter

During normal operation, ϕ is on the order of 30° (in the case of 180 degree inverter, $\phi = 0°$). Transforming (11) - (13) into the rotor reference frame by using (15) yields

$$V_{qs}^{r} = \frac{\sqrt{3}}{\pi} \cos(\phi - 30°) V_{dc} + \frac{1}{2} [1 - \frac{3\sqrt{3}}{2\pi} \cos(2\phi - 60°)] \quad (36)$$

$$V_{ds}^{r} = \frac{-\sqrt{3}}{\pi} \sin(\phi - 30°) V_{dc} + \frac{3\sqrt{3}}{4\pi} \sin(2\phi - 60°) \omega_r \lambda_m'^{r} \quad (37)$$

By plugging the values of $\phi = 30°$, and $V_{dc} = 99$ V in (36) and (37) yields

$$\boxed{\begin{aligned} V_{qs}^{r} &= 59.0 \text{ V} \\ V_{ds}^{r} &= 0 \end{aligned}} \quad (38)$$

Next, equation (14) can be written in the rotor reference frame using the transformation where the stator variables (abcs) are transformed to the rotor reference frame variables (which is the same as for the 180 degree inverter)

$$V_{qs}^{r} = (r_s + pL_s) I_{qs}^{r} + \omega_r L_s I_{ds}^{r} + \omega_r \lambda_m'^{r} \quad (39)$$

$$V_{qs}^{r} = (r_s + pL_s)\, I_{ds}^{r} - \omega_r\, L_s\, I_{qs}^{r} \tag{40}$$

where $p = \dfrac{d}{dt}$, and in steady-state operation $p = 0$. We can put the values V_{qs}^r and V_{ds}^r from (38) into (39) and (40), and solve for I_{qs}^r and I_{ds}^r, which yield

$$\boxed{\begin{aligned} I_{ds}^{r} &= 0.605 \text{ Amp} \\ I_{qs}^{r} &= 1.147 \text{ Amp} \end{aligned}} \tag{41}$$

Transforming the V_{qs}^r and V_{ds}^r back into abc components by using the inverse transformation, the stator voltages yield

$$V_{as} = \sqrt{2}\, V_s \cos\theta_r$$

$$V_{as} = 59 \cos\theta_r \tag{42}$$

Using three phase symmetry, we can express the voltage in the remaining phases as

$$V_{bs} = 59 \cos(\theta_r - 120^{o}) \tag{43}$$

$$V_{cs} = 59 \cos(\theta_r + 120^{o}) \tag{44}$$

The stator currents, I_{as}, I_{bs} and I_{cs} may be obtained by transforming I_{qs}^r, I_{ds}^r back into abc components with $I_{os} = 0$

$$I_{as} = I_{qs}^{r} \cos\theta_r + I_{ds}^{r} \operatorname{Sin}\theta_r \tag{45}$$

$$= 1.147 \cos\theta_r + 0.605 \sin\theta_r = 1.2967 \cos(\theta_r - 27.83^{\circ}) \tag{46}$$

Utilizing three phase symmetry, we can express the currents in the remaining phases as

$$I_{bs} = 1.2967 \cos(\theta_r - 27.83 - 120^{o}) \tag{47}$$

$$I_{cs} = 1.2967 \cos(\theta_r - 27.83 + 120°) \qquad (48)$$

The electrical torque can be obtained by using (26) and plugging in the value of I^r_{qs}. We get

$$T_e = \frac{P}{2} \left(\frac{3}{2}\right) \lambda'^r_m \, I^r_{qs} = \underline{0.2329 \text{ N--m}}$$

The instantaneous power (Pe) delivered to the motor may be expressed by using (35). Repeating this here for convenience

$$P_e = \frac{3}{2} \left[V^r_{qs} \, I^r_{qs} + V^r_{ds} \, I^r_{ds}\right]$$

Thus, from (38) and (41)

$$Pe = 101.45 \text{ watt} .$$

The mechanical power, P_m, is equal to the speed times the torque

$$\begin{aligned} Pm &= Te \, \omega_{rm} \\ &= (0.2329)(377) \\ &= 87.8 \text{ watts} \end{aligned}$$

Thus, the losses, P_{Losses}, are

$$\begin{aligned} P_{Losses} &= Pe - Pm \\ &= 101.45 - 87.8 = 13.65 \text{ watt} \end{aligned}$$

Now, the efficiency, η, is

$$\eta = 1 - \frac{P_{Losses}}{Pe} = 86.54\%$$

The above values are summarized in Table 1-2 for the purpose of comparison.

Table 1-2 Same Input Voltages for the 120 Degrees and 180 Degrees Inverter

	180° Inv. ($\phi = 0$)	120° Inv. ($\phi = 30°$)	Difference
Inverter Voltage (Volt)	99.0	99.0	0.0
Inverter Current (Amp)	1.657	1.025	0.632
Stator Voltage (Volt)	63.0	59.0	4
Stator Current (Amp)	1.964	1.297	0.667
T_e (N-m)	0.3528	0.2329	0.1199
Motor Losses (Watt)	31.1	13.65	17.45
Efficiency (%)	81.048	86.54	5.5

We can see from column 4 of Table 1-2 that the values are different for the 180 degree and 120 degree inverters. In order to get equal torque and efficiency in both cases, we should increase the input voltage V_{dc} for the 120 degree inverter by factor of 7.07%, and repeat all calculations, then we get the same torque and efficiency as the 180 degree inverter, see Table 1-3.

Table 1-3 Different Input Voltages for the 120 Degree and 180 Degree Inverter

	180° Inv. ($\phi = 0$)	120° Inv. ($\phi = 30°$)	Difference
Inverter voltage (Volt)	99	106.0	7.07
Inverter Current (Amp)	1.657	1.548	0.108
Stator Voltage (Volt)	63.0	63.0	0
Stator Current (Amp)	1.964	1.964	0
T_e (N-m)	0.3528	0.3528	0
Motor Losses (Watt)	31.1	31.1	0
Efficiency (%)	81.048	81.048	0

As we can see from Table 1-3, there are no differences between the 180 degree and 120 degree inverter if we increase V_{dc} by 7.07% for the 120 degree inverter.

2-8 Some advantages of the Brushless DC Machine:

1) Torque versus speed characteristic can be made linear (which is not the case for synchronous or induction machines).

2) Mechanical commutation is avoided (which is a large advantage over conventional dc machines).

3) Dynamic behavior is simple.

4) The 120 degree inverter may be operated without the Hall-effect sensors, and at the same time, it allows for greater switching tolerances than the 180 degree inverter.

Disadvantage:

For the case of the 120 degree inverter, until recently no one has been able to develop a transfer function between the inverter input voltage and the average torque produced by the machine.

SUMMARY

The 120 degree inverter system offers advantages for certain applications due to the larger timing tolerances which reduce the likelihood of inverter malfunction due to "shoot through". Also, with the 120 degree inverter, it is possible to program the switching of the inverter using the back emf of the machine as a reference. This avoids the use of Hall-effect devices whereupon the electronics necessary to develop the position information can be located with the inverter, away from the actual machine. In the 120 degree case, each transistor is on 120 out of 360 electrical degrees. But, in the 180 degree case, each transistor is on 180 out of 360 electrical degrees. The operation of a 120 degree inverter is almost the same as for the 180 degree inverter except that only two transistors are gated at any given instant (for the 180 degree case, three transistors are gated). For the 120 degree inverter the advance angle $\phi = 30^\circ$ but, for the 180 degree inverter, $\phi = 0^\circ$ with referencing used herein.

In order to get equal torque and efficiency in both cases (120 and 180 degree inverters), it is required that we increase the input voltage V_{dc} for the 120 degree inverter by a factor of 7.07%.

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