Gravity-induced birefringence within the framework of Poincare gauge theory

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Gauge theories of gravity provide an elegant and promising extension of general relativity. In this paper we show that the Poincaré gauge theory exhibits gravity-induced birefringence under the assumption of a specific gauge invariant nonminimal coupling between torsion and Maxwell's field. Furthermore we give for the first time an explicit expression for the induced phase shift between two orthogonal polarization modes within the Poincaré framework. Since such a phase shift can lead to a depolarization of light emitted from an extended source this effect is, in principle, observable. We use white dwarf polarimetric data to constrain the essential coupling constant responsible for this effect.

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I. INTRODUCTION

Almost 90 years after its formulation, Einstein's concept of gravity as a purely geometrical property of a four dimensional Riemannian manifold still provides a valid description of gravitational interactions. A major reason for this success is that the influence of matter is introduced solely by means of its energy-momentum tensor. It is clear that this phenomenological approach is justified as long as we are interested only in macroscopic events but obviously a more complete description of matter properties is achieved if we include also spin angular momentum besides energy momentum as an additional basic feature which determines the dynamics of matter on microscopic scales.

Currently, in this sense the most promising extensions of general relativity are given in the framework of gauge theories of gravity [1,2]. The description of fundamental interactions by means of gauge symmetries has become a cornerstone in modern theoretical physics. Especially Poincaré symmetry has been proven to play an important role in particle physics, and the analysis of the Colella-Overhauser-Werner experiment [3] suggests the emergence of a post-Riemannian spacetime structure which could be described by a Riemann-Cartan U_4 spacetime. A very natural alternative to general relativity [4] based on such an U_4 spacetime is given by Poincaré gauge theory (PGT) of gravity. PGT features torsion and curvature as gravitational gauge fields so that within this framework both mass and spin act as sources of the gravitational field.

In this paper we focus on consequences which arise from a possible nonminimal coupling between the torsion of PGT and the electromagnetic field. In contrast to the usual minimal coupling scheme where the propagation of electromagnetic waves is not affected by the presence of torsion, the direct coupling of the electromagnetic field with a gravitational gauge field leads to new physical effects like gravity-induced birefringence [5–9]. This nonminimal approach is motivated also by low-energy limits of string theories where torsion is identified with a massless antisymmetric second rank Kalb-Ramond (KR) field, present in most supergravity theories and as such in the massless sector of the most viable string theories [10]. Consequently, the covariant derivative of this KR field is a field of the same tensor type as the torsion field we consider and so can, in principle, couple to the electromagnetic field in the same ways that torsion can [11–13]. Recently, Lämmerzahl and Hehl investigated light propagation within a Finslerian geometry of spacetime [14]. They found that vanishing birefringence automatically yields a Riemannian structure and no Finslerian structure can occur.

In addition to the conventional Maxwell Lagrangian, the specific nonminimal coupling we employ is given by

$$L_{EM} = p^2 \star (T_\alpha \wedge F) T^\alpha \wedge F, \tag{1}$$

where $\star$ is the Hodge dual, T^α denotes the torsion 2-form and F the electromagnetic field strength 2-form, which is related in the usual way $F = dA$ to its potential A [7–9,15][1].

This addition requires a new coupling constant p with the dimension of [length]. p is supposed to be a new fundamental constant of nature, related to torsion and is not just a parameter of a special solution. From the theoretical side, there is no preferred choice for what the length scale defined by p might be. However, one should be careful in assigning a real physical length to it. Remember that also the cosmological constant Λ in general relativity with the dimension of $[\text{length}]^{-2}$ turns out to be possibly a measure of the vacuum energy density. Since we are using geometrized units throughout this article, p

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[1] Note that the wedge symbol of the exterior product between the 0-form (or scalar) $*(T_\alpha \wedge F)$ and the 4-form $(T_\alpha \wedge F)$ is omitted in accordance with the standard mathematical convention.

could also be related to a mass or an energy scale. Hence, p may be related to the restmass or the energy of the exchange particles of the torsion field [16]. These questions are going to be further investigated in an upcoming project. Nevertheless, what we can say is that p is related to the strength of a possible nonminimal coupling between the electromagnetic field and torsion, leading to birefringent spacetime (see also [17]).

This addition is gauge invariant and, so, compatible with charge conservation. In this context, it was later shown by Itin and Hehl [18] that (1) is a special case of a complete family of quadratic torsion Lagrangians which couple to Maxwell's field and which leads besides birefringence to an axion-induced optical activity of spacetime and, furthermore, to a torsion dependent speed of light:

$$L_{EM} = -\frac{1}{8} p \sum_{a,\ldots,q} (F_{ab} F_{cd} T_{klm} T_{npq}). \tag{2}$$

Here the summation is performed by contracting the indices by means of the metric tensor.

This paper is organized as follows: First, we give a brief recapitulation of the Poincaré gauge theory where the field strengths of the compensating gauge fields are identified as torsion and curvature. Then, using the Baekler-Lee solution for a spherically symmetric torsion, we show that the Lagrangian (1) leads to gravity-induced birefringence and give an explicit expression for the accumulated phase shift between orthogonal polarization modes in the gravitational field of a star. Since this effect leads to a depolarization of radiation, emitted from an extended source, we use white dwarf polarimetric data to set strong limits on the essential coupling constant. Finally, a discussion and conclusions are presented.

II. POINCARÉ GAUGE THEORY

Our subsequent brief summary of basic features of PGT mainly follows the notation of Blagojević [2]. Within the framework of Lagrangian based field theory, the equations of motion are given as the Euler-Lagrange equations of the action integral

$$S_M = \int d^4 x \mathcal{L}_M(\phi, \partial\phi), \tag{3}$$

with the matter field $\phi(x)$ as the dynamical variable in a Minkowski spacetime M_4. To ensure the conservation of energy momentum and angular momentum, one demands the invariance of (3) under global (rigid) Poincaré transformations

$$x'^{\mu} = x^{\mu} + \xi^{\mu}(x), \qquad \xi^{\mu} = \omega^{\mu}{}_{\nu} x^{\nu} + \epsilon^{\mu}, \tag{4}$$

where the Lorentz rotations $\omega^{\mu\nu} = -\omega^{\nu\mu}$ and translations ϵ^{μ} provide ten constant parameters. Here, Greek indices always refer to coordinate lines of the underlying Minkowski space M_4. As a consequence, matter fields $\phi(x)$ transform according to

$$x'^{i} = x^{i} + \omega^{i}{}_{j} x^{j} + \epsilon^{i}, \qquad \phi'(x') = \left(1 + \frac{1}{2}\omega^{ij}\Theta_{ij}\right)\phi(x), \tag{5}$$

where Θ_{ij} denotes the spin matrix, related to the multicomponent structure of $\phi(x)$. Latin indices refer to a local Lorentz frame, tangent to M_4. If one defines $\delta_0\phi(x) = \phi'(x) - \phi(x)$, the action (3) is invariant under the transformation $x' = x + \xi(x)$ if

$$\Delta\mathcal{L} \equiv \delta_0\mathcal{L} + \xi^{\mu}\partial_{\mu}\mathcal{L} + (\partial_{\mu}\xi^{\mu})\mathcal{L} = 0, \tag{6}$$

where $\delta_0\mathcal{L} = (\partial\mathcal{L}/\partial\phi)\delta_0\phi + (\partial\mathcal{L}/\partial\phi_{,k})\delta_0\phi_{,k}$ [19,20], so that Noether's theorem leads to conserved energy momentum and angular momentum tensors.

In a next step, the Poincaré transformations are generalized by replacing the ten constant group parameters with corresponding functions of spacetime points, i.e.

$$\omega^{ij} \rightarrow \omega^{ij}(x), \qquad \epsilon^{k} \rightarrow \epsilon^{k}(x). \tag{7}$$

Based on experience, e.g. from QED, it is then no surprise that the invariance condition (6) is now violated. However, this problem can be circumvented in the usual way by means of a covariant derivative $\nabla_k\phi$ of ϕ which is introduced in two steps:

$$\nabla_{\mu}\phi = (\partial_{\mu} + A_{\mu})\phi, \qquad A_{\mu} \equiv \frac{1}{2} A^{ij}{}_{\mu}\Theta_{ij}, \tag{8}$$

$$\nabla_k\phi = \delta^{\mu}_{k}\nabla_{\mu}\phi - A^{\mu}{}_{k}\nabla_{\mu}\phi \equiv h_k{}^{\mu}\nabla_{\mu}\phi, \tag{9}$$

with the new field $h_k{}^{\mu} = \delta^{\mu}_{k} - A^{\mu}{}_{k}$. In order to restore the local invariance of the theory, one introduces $\tilde{\mathcal{L}}_M = \Lambda\mathcal{L}_M(\phi, \nabla_k\phi)$, where Λ is a suitable function of the new fields. Then the invariance condition (6) is restored if $\delta_0\Lambda + \partial_{\mu}(\xi^{\mu}\Lambda) = 0$, which is given by $\Lambda = \det(b^{k}{}_{\mu}) \equiv b$, where $b^{k}{}_{\mu}$ is just the inverse of $h_k{}^{\mu}$: $b^{k}{}_{\mu} h_i{}^{\mu} = \delta^{k}_{i}$. Finally, the locally invariant Lagrangian for matter fields reads

$$\tilde{\mathcal{L}}_M = b\mathcal{L}_M(\phi, \nabla_k\phi). \tag{10}$$

The corresponding field strengths of the new compensating fields $b^{k}{}_{\mu}$ and $A^{ij}{}_{\mu}$ are given by the tensors

$$F^{ij}{}_{\mu\nu} \equiv \partial_{\mu}A^{ij}{}_{\nu} - \partial_{\nu}A^{ij}{}_{\mu} + A^{i}{}_{s\mu}A^{sj}{}_{\nu} - A^{i}{}_{s\nu}A^{sj}{}_{\mu} \tag{11}$$

$$F^{i}{}_{\mu\nu} \equiv \nabla_{\mu}b^{i}{}_{\nu} - \nabla_{\nu}b^{i}{}_{\mu}, \tag{12}$$

which are called the Lorentz and translation field strengths, respectively. From the structure of these tensors it is now easy to conclude that the translation field strength $F^{i}{}_{\mu\nu}$ is nothing but the torsion $T^{\lambda}{}_{\mu\nu}$, while the Lorentz field strength $F^{ij}{}_{\mu\nu}$ can be identified with the curvature $R^{\lambda}_{\tau\mu\nu}$ [2]. Therefore, it is evident that PGT possesses a Riemann-Cartan spacetime where both mass *and* spin are sources of the gravitational field.

III. BIREFRINGENCE ANALYSIS

The starting point for our analysis is the frequently discussed quadratic PGT Lagrangian proposed by von der Heyde [21]

$$L_{vdH} = \frac{1}{4\ell^2}(T^{ij}{}_{\alpha}T_{ji}{}^{\alpha} + 2T^{i\beta}{}_{\beta}T_{i\gamma}{}^{\gamma}) + \left(\frac{1}{4\kappa}\right)R^{ij}{}_{\alpha\beta}R_{ji}{}^{\alpha\beta} \tag{13}$$

where ℓ denotes the Planck length and κ is a dimensionless coupling constant for the Lorentz gauge bosons. Among the numerous solutions which have been found for this Lagrangian, the most transparent one is the Baekler-Lee solution [22,23] of Reissner-Nordström type with dynamic torsion.

Starting with the usual Schwarzschild tetrad

$$e^{\hat{t}} = \sqrt{2\Psi}\,dt, \qquad e^{\hat{r}} = dr/\sqrt{2\Psi} \tag{14}$$

$$e^{\hat{\theta}} = rd\theta, \qquad e^{\hat{\phi}} = r\sin\theta d\phi, \tag{15}$$

the Baekler-Lee solution is appreciably simplified by applying a suitable boost $\vartheta^{\alpha} = \Lambda^{\alpha}_{\beta}e^{\beta}$ so that the corresponding orthogonal coframe takes the form [1]

$$\begin{aligned}\vartheta^{\hat{t}} &= \frac{1}{2}\left((\Psi+1)dt + \left(1-\frac{1}{\Psi}\right)dr\right)\\ \vartheta^{\hat{r}} &= \frac{1}{2}\left((\Psi-1)dt + \left(1+\frac{1}{\Psi}\right)dr\right)\\ \vartheta^{\hat{\theta}} &= rd\theta\\ \vartheta^{\hat{\phi}} &= r\sin\theta d\phi,\end{aligned} \tag{16}$$

with the Reissner-Nordström function

$$\Psi := 1 - \frac{2(Mr-q^2)}{r^2} - \frac{\kappa}{4\ell^2}r^2, \tag{17}$$

which can be interpreted as a Newtonian plus a "confinement" type of potential as discussed in [24]. The "confinement" or sometimes called "cosmological" term is induced by the coupling constant κ of curvature square term in the Lagrangian (13) which means that the von der Heyde Lagrangian (13) itself does not carry a cosmological constant. M and q denote the gravitational mass and electric charge, respectively.

From (13) and the absence of an explicit cosmological term in the Lagrangian, one has to conclude that within Poincaré gauge theory basically two types of gravity exist: (i) Weak gravity which is the usual gravity of Newton-Einstein type. Weak gravity couples to mass and energy momentum with the gravitational constant as the relevant coupling constant. (ii) Strong gravity mediated by a short range potential with a dimensionless strong coupling constant κ. This potential is asymptotically free and confining, thus relating gravity to the physics of hadrons [22,24]. Consequently, this strong gravity would not be observable on macroscopic scales so that this term does not play any role in our further analysis. It is also obvious that the strong coupling constant κ is not related to the new coupling constant p in Eq. (1) which describes the coupling between torsion and electromagnetism.

The corresponding metric is then given by

$$ds^2 = -\Psi dt^2 + \frac{1}{\Psi}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{18}$$

The metric in Eq. (18) is of Scharzschild–de Sitter type. It describes a spacetime that is not asymptotically but conformally flat, i.e. it has an asymptotic curvature induced by the term $\kappa r^2/4\ell^2$ from Eq. (17) so that an asymptotically flat spacetime is given only for $\kappa \to 0$. By adding a bare cosmological constant to (13), one could shift its value by a suitable choice of κ which would asymptotically lead to a Minkowski space. Asymptotically flat solutions of PGT were also investigated by Hayashi and Shirafuji in [25].

Now, the torsion of the Baekler-Lee solution reads

$$\begin{aligned}T^{\hat{t}} = T^{\hat{r}} &= \frac{Mr-2q^2}{r^3}\vartheta^{\hat{t}}\wedge\vartheta^{\hat{r}},\\ T^{\hat{\theta}} &= \frac{Mr-q^2}{r^3}(\vartheta^{\hat{t}}\wedge\vartheta^{\hat{\theta}} - \vartheta^{\hat{r}}\wedge\vartheta^{\hat{\theta}}),\\ T^{\hat{\phi}} &= \frac{Mr-q^2}{r^3}(\vartheta^{\hat{t}}\wedge\vartheta^{\hat{\phi}} - \vartheta^{\hat{r}}\wedge\vartheta^{\hat{\phi}}).\end{aligned} \tag{19}$$

This solution is consistent with the most general static, spherically $O(3)$-symmetric form for a torsion field [26]

$$T^{\hat{t}} = \alpha(r)\theta^{\hat{t}}\wedge\theta^{\hat{r}} + \tilde{\alpha}(r)\theta^{\hat{\theta}}\wedge\theta^{\hat{\phi}}, \tag{20}$$

$$T^{\hat{r}} = \beta(r)\theta^{\hat{t}}\wedge\theta^{\hat{r}} + \tilde{\beta}(r)\theta^{\hat{\theta}}\wedge\theta^{\hat{\phi}}, \tag{21}$$

$$\begin{aligned}T^{\hat{\theta}} = {}&\gamma_{(1)}\theta^{\hat{t}}\wedge\theta^{\hat{\theta}} + \gamma_{(2)}\theta^{\hat{t}}\wedge\theta^{\hat{\phi}}\\ &+ \gamma_{(3)}\theta^{\hat{r}}\wedge\theta^{\hat{\theta}} + \gamma_{(4)}\theta^{\hat{r}}\wedge\theta^{\hat{\phi}},\end{aligned} \tag{22}$$

$$\begin{aligned}T^{\hat{\phi}} = {}&\gamma_{(1)}\theta^{\hat{t}}\wedge\theta^{\hat{\phi}} - \gamma_{(2)}\theta^{\hat{t}}\wedge\theta^{\hat{\theta}}\\ &+ \gamma_{(3)}\theta^{\hat{r}}\wedge\theta^{\hat{\phi}} - \gamma_{(4)}\theta^{\hat{r}}\wedge\theta^{\hat{\theta}}.\end{aligned} \tag{23}$$

The solution (19) is a special case having $\tilde{\alpha}(r) = \tilde{\beta}(r) = \gamma_{(2)} = \gamma_{(4)} = 0$. Plugging this general torsion field into the Lagrangian density (1) the coefficients of the magnetic and electric field components can be expressed in terms of SO(3)-symmetric tensors ξ^{ij}, ζ^{ij} and γ^{ij} which represent spatial anisotropy induced by the gravitational field. As shown in [7,8], the accumulated phase shift $\Delta\Phi$ which is due to the fractional difference $\delta c/c$ in the propagation speed of linear polarization states with frequency ω

$$\Delta\Phi = \omega\int\frac{\delta c}{c}dt, \tag{24}$$

can be expressed in terms of the spherical components of these SO(3) tensors by using the Haugan-Kauffmann formalism [27]. However, one has to be careful since [27] uses a $(+ - - -)$ metric, while the Baekler-Lee solution is based on a $(- + + +)$ metric so that either (19) or the Haugan-Kauffmann formalism has to be rewritten in terms of a different metric.

The general expression for $\delta c/c$ is then given by

$$\frac{\delta c}{c} = \sqrt{\frac{2}{3}} \sin^2\theta \sqrt{(\xi_0^{(2)} + \zeta_0^{(2)})^2 + 4(\gamma_0^{(2)})^2}, \qquad (25)$$

with

$$\xi_0^{(2)} = (\gamma_{(3)}^2 + \gamma_{(4)}^2) \qquad (26)$$

$$\zeta_0^{(2)} = -(\alpha^2 - \beta^2) + 2(\gamma_{(1)}^2 + \gamma_{(2)}^2) \qquad (27)$$

$$\gamma_0^{(2)} = (\gamma_{(1)}\gamma_{(4)} - \gamma_{(2)}\gamma_{(3)}). \qquad (28)$$

Comparing the coefficients of the general $O(3)$-symmetric Torsion with the Baekler-Lee solution we find

$$\xi_0^{(2)} = \gamma_{(3)}^2 = \frac{(Mr - q^2)^2}{r^6} \qquad (29)$$

$$\zeta_0^{(2)} = \gamma_{(1)}^2 = \frac{(Mr - q^2)^2}{r^6} \qquad (30)$$

$$\gamma_0^{(2)} = 0, \qquad (31)$$

which leads to

$$\frac{\delta c}{c} = \sqrt{\frac{2}{3}} \sin^2\theta \sqrt{\left(2\frac{(Mr - q^2)^2}{r^6}\right)^2} \qquad (32)$$

$$= 2\sqrt{\frac{2}{3}} M^2 \sin^2\theta \frac{1}{r^4}, \qquad (33)$$

in case of vanishing charge $q = 0$. Therefore, the total phase shift becomes

$$\Delta\Phi = 2\omega\sqrt{\frac{2}{3}} M^2 p^2 \int_{t_0}^{t_1} \frac{\sin^2\theta}{r^4} dt. \qquad (34)$$

The evaluation of this integral requires a ray parametrization $\mathbf{x}(t) = \mathbf{b} + \mathbf{k}_0 t$ where the unit vector $\mathbf{k}_0$ denotes the ray direction and $\mathbf{b}$ is the impact vector that connects the center of the star with the closest point on the ray. When $\mathbf{b}$ is smaller than the radius R of the star, the portion of the ray inside the object is, of course, of no interest. The integration of (34) is performed from the star's surface with $t_0 = (R^2 - b^2)^{1/2}$ along a straight line up to an observer at an infinite distance $t_1 = \infty$, which yields

$$\Delta\Phi = 2\sqrt{\frac{2}{3}} \omega M^2 p^2 R^2 (1 - \mu^2) \times \int_{t_0 = R\mu}^{\infty} \frac{dt}{(R^2(1 - \mu^2) + t^2)^3}, \qquad (35)$$

where μ denotes the cosine of the heliocentric angle θ between the ray's source and the center of the visible stellar disk. This integral is easily evaluated, so that we finally get the total phase shift which accumulates between two orthogonal polarization states within the framework of Poincaré gauge theory

$$\Delta\Phi = \sqrt{\frac{2}{3}} \frac{4\pi M^2 p^2}{\lambda R^3} \left(\frac{3\pi}{16(1 - \mu^2)^{3/2}} - \frac{\mu}{4} - \frac{3\mu}{8(1 - \mu^2)} - \frac{3}{8(1 - \mu^2)^{3/2}} \arcsin(\mu) \right) \qquad (36)$$

with the new Poincaré coupling constant p having the dimension of length and the mass M in geometrized units. It is interesting to see that (36) shows a remarkable similarity to the phase shift formula from Moffat's old version of nonsymmetric gravitational theory [28]. Nevertheless, while this nonvanishing birefringence was found on the basis of the special Baeckler-Lee solution, it is important to note that Rubilar *et al.* [29] proved that our nonminimal Lagrangian (1) leads to birefringence even for a general O(3)-symmetric torsion field.

IV. ASTROPHYSICAL CONSTRAINTS

The question how gravitational birefringence influences polarized radiation in the vicinity of a particular star depends, among others, on the properties of the emitting source. In case of a pointlike source of polarized radiation, all received light suffers the same phase shift $\Delta\Phi(\mu)$.

Polarized light is described by means of wavelength dependent Stokes parameters $I_\lambda, Q_\lambda, U_\lambda, V_\lambda$ [30], where Stokes Q represents the difference between linear polarization parallel and perpendicular to the local stellar limb. The effect of gravitational birefringence is to introduce a crosstalk between the linear polarization parameter Stokes U and the net circular polarization, V. This crosstalk is such that, although the observed values $U_{\rm obs}$ and $V_{\rm obs}$ differ from the values emitted by a point source, $U_{\rm src}$ and $V_{\rm src}$, the composite degree of polarization remains equal: $(U_{\rm obs}^2 + V_{\rm obs}^2)^{1/2} = (U_{\rm src}^2 + V_{\rm src}^2)^{1/2}$.

In case of an extended source covering a range of μ values, light emitted from different points suffers different phase shifts and, so, adds up to an incoherent superposition. Using the additive properties of Stokes parameters, summing over different contributions yields a reduction of the observed polarization relative to the light emitted from the source: $(U_{\rm obs}^2 + V_{\rm obs}^2)^{1/2} < (U_{\rm src}^2 + V_{\rm src}^2)^{1/2}$. Since the rotationally modulated polarization from magnetic white dwarfs can only be produced by an extended source [31], any observed (i.e. nonzero) degree of polarization provides a limit on the strength of birefringence induced by the star's gravitational field [6].

It is generally agreed that polarized radiation from white dwarfs is produced at the stellar surface as a result of the presence of ultrastrong (up to 10^5 T) magnetic fields [32]. Since the disk of a white dwarf is unresolved, only the total polarization from all surface elements is observable. Therefore, the flux of net circular polarization at wavelength λ emitted toward the observer can be written as

$$V_{\lambda,\mathrm{tot}}(p) = 2\pi \iint V_\lambda(\mu, B, \theta)\cos(\Delta\Phi)\mu d\theta d\phi. \quad (37)$$

Here, the Stokes parameter V_λ changes over the visible hemisphere and depends on the wavelength λ, the location μ (limb darkening), the total magnetic field strength B, the angle θ between the magnetic field and the line-of-sight component, and on the parameters of the stellar atmosphere influencing line formation. The influence of gravitational birefringence on the polarization is introduced by the term $\cos(\Delta\Phi)$ as a function of μ. The Stokes parameters can be calculated by solving the radiative transfer equations through a magnetized stellar atmosphere on a large number of surface elements on the visible hemisphere (e.g. [33]). If the star is rotating, the spectrum and polarization pattern changes according to the respective magnetic field distribution visible at a particular moment. The degree of circular polarization is obtained by dividing Eq. (37) by the total stellar flux $I_{\lambda,\mathrm{tot}}$ emitted to the observer at wavelength λ. Below we will calculate a maximum circular polarization $V_{\lambda,\max}/I_{\lambda,\mathrm{tot}}$ from radiative transfer calculations which is higher than the observed value $V_{\lambda,\mathrm{obs}}/I_{\lambda,\mathrm{obs}}$. Then we assume that the reduction from $V_{\lambda,\max}$ to $V_{\lambda,\mathrm{obs}}$ is entirely due to the factor $\cos(\Delta\Phi(p))$ in Eq. (37), thereby calculating the maximum value for p, i.e. our limit on p is reached as soon as $V_{\lambda,\mathrm{tot}}/I_{\lambda,\mathrm{tot}}$ in Eq. (37) becomes smaller than $V_{\lambda,\mathrm{obs}}/I_{\lambda,\mathrm{tot}}$ for a certain value of p.

RE J0317-853 is a highly unusual object within the class of isolated magnetic white dwarfs which sets several records: Besides being the most rapidly rotating star ($P = 725$ sec) of this type, it is also the most massive at $1.35M_\odot$, close to the Chandrasekhar limit [34] with a corresponding radius of only $0.0035R_\odot$. In [35] a degree $V_{\lambda,\mathrm{obs}}/I_{\lambda,\mathrm{tot}}$ of 20% at $\lambda = 576$ nm [35], RE J0317-853 is also the magnetic white dwarf with the highest known level of circular polarization. Because of its small radius and high degree of circular polarization, RE J0317-853 is a very suitable object for setting limits on gravitational birefringence. The analysis of time resolved UV flux spectra obtained with the Hubble space telescope has shown that the distribution of the field moduli is approximately that of an off-centered magnetic dipole oriented obliquely to the rotation axis with a polar field strength at the surface of $B_d = 3.63 \times 10^4$ T, leading to visible surface field strengths between 1.4×10^4 T and 7.3×10^4 T [36]. This model is not only able to describe the UV, but also the optical spectra (Jordan *et al.* [37]), which means that the distribution of the magnetic field moduli—but not necessarily of the longitudinal components—is correctly described. This result is completely independent of the magnitude of the gravitational birefringence, since it is obtained entirely from the intensity spectrum. From radiative transfer calculations it follows that, at the phase of rotation when the maximum value of 20% polarization at 576 nm is measured, almost the entire visible stellar surface is covered by magnetic fields between 1.4×10^4 and 2.0×10^4 T, with only a small tail extending to maximum field strengths of 5.3×10^4. This distribution is best reproduced at a rotational phase where the axis of the off-centered dipole is nearly perpendicular to the line of sight. Using this field geometry, we calculated a histogram distribution of the visible surface magnetic field strengths in order to set sharp limits on gravitational birefringence. For each field strength bin of the histogram, we calculated the maximum circular polarization from radiative transfer calculations by assuming that the field vector always points towards the observer. The total maximum polarization from the whole visible stellar disk without gravitational birefringence is then calculated by adding up the contributions from each field strength bin weighted with its relative frequency. This results in $V_{\lambda,\max}/I_{\lambda,\mathrm{tot}} = 26.5\%$. Assuming that the reduction to $V_{\lambda,\mathrm{obs}}/I_{\lambda,\mathrm{tot}} = 20\%$ is entirely due to gravity-induced depolarization—and not due to the fact that in reality not all field vectors point towards the observer—we find an upper limit for this effect of $p^2 \lesssim (0.9\ \mathrm{km})^2$. Since there is always a small uncertainty in determining the exact mass of a white dwarf, we also calculated an upper limit on p^2 assuming a lower mass of $1M_\odot$. This leads to $p^2 \lesssim (1.2\ \mathrm{km})^2$. An even more extreme assumption would be to take 100% emerging polarization, i.e. neglect the dipole model and make no reference to radiative transfer calculations. This leads to $p^2 \lesssim (2.125\ \mathrm{km})^2$.

V. DISCUSSION AND CONCLUSIONS

We have shown that the Poincaré gauge theory exhibits gravitational birefringence under the assumption of a specific nonminimal coupling and have given an explicit expression for the gravity-induced phase shift between orthogonal polarization states. Using spectropolarimetric observations of the massive white dwarf RE J0317-853 we imposed strong constraints on the birefringence of spacetime with an upper limit on the relevant coupling constant p^2 of $(0.9\ \mathrm{km})^2$ or $p^2 \lesssim (2.125\ \mathrm{km})^2$ for the most conservative assumptions. Since gravity-induced birefringence violates the Einstein equivalence principle, our analysis also provides a test of this foundation of general relativity. Tighter limits could be achieved either by observing more massive white dwarfs or by circular polarization measurements at significantly shorter wavelength, such as in the far ultraviolet (e.g. in the Lyα absorption features). In addition, a consistent model for the magnetic field geometry which reproduces the spectropolarimetric measurements in the optical would help.

The properties of the exchange particles of the torsion field within PGT, especially their masses, are currently not bound from the theoretical side and, therefore, the relevance for astrophysical observations still requires further work [16,38].

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