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A Pulsation Calculation Algorithm Well Suited for Use with Valve Dynamics Calculations

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A PULSATION CALCULATION ALGORITHM WELL SUITED FOR USE WITH VALVE DYNAMICS CALCULATIONS

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ABSTRACT

The importance of the influence of piping pulsations on the dynamics of compressor valves and on the compressor performance and reliability are well recognized and numerous programs have been written to calculate these effects. This paper outlines a calculation scheme for piping pulsations that is different from those normally used and may offer advantages in some situations. The technique is in the time domain and based directly on the method of characteristics. It avoids the interpolation and consequent smoothing inherent in mesh based methods.

NOMENCLATURE

a	Speed of Sound
a ref	Reference Speed of Sound
A = a/a ref	Dimensionless Speed of Sound
F_1, F_2, F_3	Pipe Cross Sectional Area for Pipes 1, 2 and 3
K	Ratio of Specific Heats (Gas is assumed ideal.)
p	Pressure
p ref	Reference Pressure
t	Time
u	Gas Velocity
U = u/a ref	Dimensionless Gas Velocity
x	Distance Along Pipe
z	Fraction of Time Step at Which Characteristic Reaches End of Pipe
$\lambda = A + \frac{K-1}{2} U$	Riemann Variable - Strength of Wave
λ_I, λ_{II}	Value of λ for Wave Travelling in Each Direction
λ_{in}	Value of λ for Wave Incident on Boundary at Pipe End
λ_{out}	Value of λ for Wave Reflected From Boundary at Pipe End

INTRODUCTION

Two methods are commonly used for the calculation of pressure pulsations in reciprocating compressor piping systems using digital methods. Frequency domain methods are well suited to the approximate calculation of pulsations in large piping systems as they require less computation time than time domain methods. In a large piping system with long pipe sections, a time domain solution will take many cycles to converge, i.e. for the start up transients to decay, and the calculation will be very slow. Conversely, the frequency domain method is not well suited for calculations in which non linear interactions such as flow through orifices, valve dynamics and cylinder conditions are to be treated accurately. The frequency domain solution requires that an iterative approach be used in this case and it will then be slow. For cases with long piping systems and non linear effects, neither method is good and either can be used. For cases with short pipes and important non linear effects, a time domain solution is the obvious choice.

This paper is concerned with the calculation of the effects of piping pulsations on compressor performance and valve dynamics. This is a case where non linear effects are critical and in which the piping of major interest is that close to the compressor, typically the cylinder passages and the nozzle connecting the cylinder to the pulsation bottle. This is an obvious case for a time domain solution. Most time domain solutions proceed by dividing the pipes into short sections, or meshes, and using either finite difference or finite element methods to advance the solution for a series of time increments. These calculations are interfaced with the boundary condition representing the valves, pipe junctions, closed or open pipe ends, orifices, etc. The accuracy of the methods and the highest frequency pulsation that can be accurately calculated depends on the number of meshes employed and the magnitude of the time step. For best accuracy, at least in the more simple integration schemes, the time step depends on the mesh size as given by the Courant-Friedrich condition. Thus, for equal time steps, equal length meshes must be used and the accurate representation of systems including several short pipes may require the use of many small meshes and corresponding small time steps.

The method described here can be considered a technique in which the size and number of meshes varies as the calculation proceeds depending on the rate of change of fluid conditions at each point in the piping system and to which the Courant-Friedrich condition does not apply. Alternatively it can be considered a computerized version of the graphical calculation by the method of characteristics. This leads to an accurate, rapid calculation at the expense of some programming complexity.

ASSUMPTION

It is assumed that the flow is one-dimensional homentropic flow of a perfect gas. However, a correction is applied for the effects of pipe friction and real gas effects can be considered for the flow through the valves and orifices.

BASIC THEORY

The basic equations for unsteady flow with the above assumptions and their solution by the method of characteristics are given in many places (eg. Ref 3)

The solution is that waves travel in each direction in a pipe, independent of each other and that for each wave,

a) The speed of travel of the wave $\frac{dx}{dt} = u + a$ . . . (1)

b) Conditions along the wave are given by

$$\lambda = A + \frac{K-1}{2} U = \text{constant} \quad \ldots (2)$$

where x, and hence u, are taken positive in the direction of wave travel.

Let λ_I be the values of λ for the waves travelling in one direction and λ_{II} the values for waves travelling in the opposite direction.

The pressure and velocity at any point can be found from the values of λ_I and λ_{II} at that point.

$$\left(p/p_{REF}\right)^{\frac{K-1}{2K}} = a/a_{REF} = A = \frac{\lambda_I + \lambda_{II}}{2} \quad \ldots (3)$$

$$u/a_{REF} = U = \frac{\lambda_I - \lambda_{II}}{K-1} \quad \ldots (4)$$

U positive in the direction of waves λ_I

Hence, the wave speed $$\frac{dx}{dt} = \left(\frac{K+1}{2(K-1)}\lambda_I - \frac{3-K}{2(K-1)}\lambda_{II}\right) a_{REF} \quad \ldots(5)$$

The effects of friction can be approximated by calculating a change of λ with time that is dependent on the instantaneous gas velocity

CALCULATION METHOD

Internal to Pipe

A variable number of characteristics of each type (λ_I and λ_{II}) are stored for each pipe. For each characteristic the position (x) and the strength (λ) of the waves are stored. Whenever needed during the calculation, linear interpolation between characteristics is used. It is arranged that the characteristics are spaced to give the least possible error in this interpolation. To allow interpolation, it is arranged that there is always a characteristic of each type at each end of each pipe. The direction of x positive is taken to be the direction of wave travel, so the positions of waves of each type are measure from opposite ends of the pipe. (Fig. 1). The solution algorithm is then identical for each family of characteristics.

METHOD OF CHARACTERISTICS

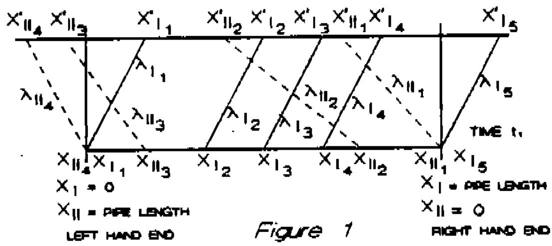

Figure 1

The solution consists simply of updating the position of each characteristic for the time step. The value of λ is also updated if friction losses are considered. It is assumed that the slope of the characteristics is constant for the time step and equal to the value at the beginning of the step. To calculate the slope (Equn. 5) the values of λ_I and λ_{II} are required. The value for the characteristics whose position is being updated is known and the value for the other family is obtained by linear interpolation between characteristics.

Then
$$x_{NEW} = x_{OLD} + \frac{dx}{dt}\Delta t$$

$$= x_{OLD} + \left(\frac{K+1}{2(K-1)}\lambda_I - \frac{3-K}{2(K-1)}\lambda_{II}\right) a_{REF}\,\Delta t$$

This calculation is repeated for each characteristic of each family.

Boundary Conditions

Two approaches have been used for the boundary conditions.

i) Single Boundary Condition Method (Fig 2).

In the more simple method, the boundary condition calculations are done only at the end of each time step. The value of λ_{in} at the end of the time step, i.e., the value of λ at the end of the pipe after the time step, is calculated by interpolation between the characteristics immediately inside and outside the pipe. All characteristics that have left the pipe are then removed. Thus a value of λ_{in} is obtained for each end of each pipe. The boundary condition equations are then applied as follows:

SINGLE BOUNDARY CONDITION METHOD

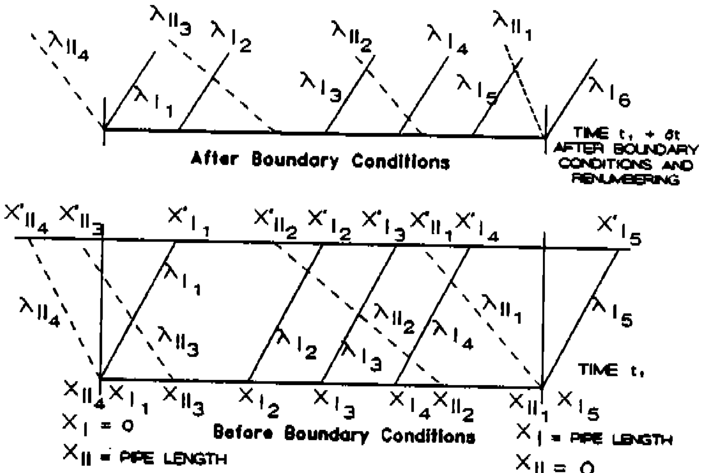

Figure 2

Open End $\lambda_{out} = 2 - \lambda_{in}$

Entry or Exit loss factors can easily be applied if needed

Closed End $\lambda_{out} = \lambda_{in}$

Infinite Pipe $\lambda_{out} = 1.0$

Cylinder Boundary. Pressure and flow conditions for the cylinder, valve dynamics, and valve flow losses are used to calculate λ_{out} from λ_{in}

Junction of 2 or 3 pipes. The present program assumes that the pressures are the same in all pipes at the junction. Then from Ref. 1

$$\lambda_{out_1} = \frac{F_1 - F_2 - F_3}{F_1 + F_2 + F_3}\lambda_{in_1} + \frac{2F_2}{F_1 + F_2 + F_3}\lambda_{in_2} + \frac{2F_3}{F_1 + F_2 + F_3}\lambda_{in_3}$$

$$\lambda_{out_2} = \lambda_{out_1} + \lambda_{in_1} - \lambda_{in_2}$$

$$\lambda_{out_3} = \lambda_{out_1} + \lambda_{in_1} - \lambda_{in_3}$$

Methods to allow for losses at the junction are given in Ref. 1.

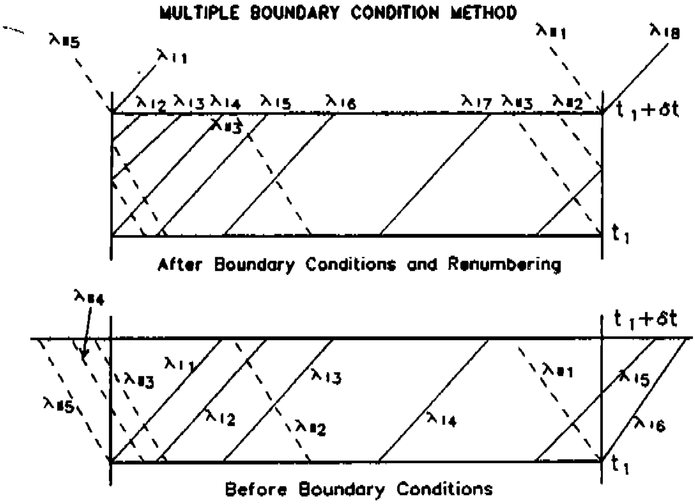

Figure 3

Orifice between 2 Pipes The usual compressible flow orifice equations are used. An iterative procedure is necessary to calculate λ_{out_1} and λ_{out_2} from λ_{in_1} and λ_{in_2}

The values of λ_{out} for each end of each pipe are then stored as new characteristics in each pipe at position $x = 0$. As all characteristics are stored in order of their position from $x = 0$ to $x =$ pipe length, it is necessary to move all the characteristics down one place to make room for the new value at $x = 0$.

ii) Multiple Boundary Condition Method (Fig 3)

The above method wastes valuable high frequency information carried by the characteristics whenever more than one characteristic leaves the pipe during a time step. The goal is to get the best possible accuracy with large time steps and it is therefore, frequently best to calculate the boundary conditions for each characteristic that reaches the end of a pipe. (Fig 3.) This is fairly straightforward for single pipe boundaries such as closed or open ends. The time at which the characteristic reaches the end of the step is given by z expressed as a fraction of the step ($0 < z < 1$). This is calculated from the position of the characteristic inside the pipe before the time step and its position outside the pipe after the time step. The value of λ_{out} is then calculated from the known λ_{in} as given above. The slope of the reflected characteristic is calculated from λ_{in} and λ_{out} using equation (5) and then the position of the reflected characteristic at the end of the step is calculated

$$x = \left(\frac{K+1}{2(K-1)} \lambda_{OUT} - \frac{3-K}{2(K-1)} \lambda_{IN}\right) a_{REF} \, \Delta t \, (1-z)$$

This is repeated for each characteristic and finally, as in the simple boundary condition method, new incident and reflected characteristics are introduced at the end of the pipe at the end of the time step.

For multi pipe boundaries such as orifices or junctions, it is necessary to calculate the z values for all characteristics that reach the boundary in any pipe at that boundary. These values are then put in order and the boundary condition calculated for each z. Linear interpolation is used to obtain the value of λ_{in} when necessary (Fig 4). Values of λ_{out} at the correct position are stored in each pipe each time the boundary condition is calculated. Thus, on average more characteristics will be added to each pipe than leave it.

The cylinder, or valve, boundary condition poses a dilemma. We want to correctly allow for any high frequency effects generated in the pipes, but we probably do not want to integrate for the cylinder conditions over what may be extremely small time intervals between adjacent characteristics. The solution chosen in the existing program is to integrate for the cylinder conditions over a complete time step and to interpolate for intermediate values of the reflected characteristics. Work will continue to investigate possible benefits of improving this technique.

Shocks

The formation of a shock is indicated by one characteristic overtaking another so that the x value for a characteristic is greater than that of a characteristic expected to be ahead of it. Any shocks formed are assumed to be weak and the x values of any characteristics in the shock are replaced by the average value of x for the first and last characteristics in the shock.

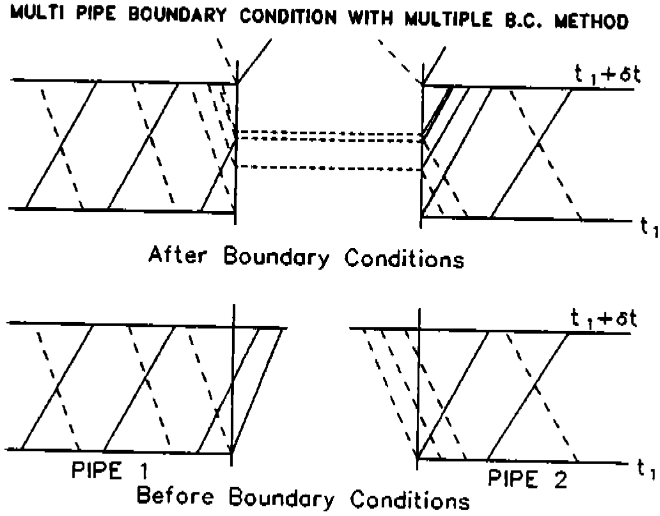

Figure 4

Control of Number of Characteristics

With the methods outlined above, it is possible for the number of characteristics in a pipe to become large. This increases the storage required and reduces the speed of the calculations, especially if the multiple boundary condition method is used and if complex boundary conditions such as orifices are used. It is therefore, necessary to reduce the number of characteristics. This is done in two phases.

i) Any characteristics that do not increase the accuracy of linear interpolation are removed. This will include characteristics inside a shock, identical values at the same position or values in a stagnant section of pipe.

ii) The user has the option of specifying the maximum number of characteristics they will allow in each pipe. If the actual number of characteristics exceeds this value, the program removes the characteristic that will have the least effect on the accuracy of linear interpolation. The existing program allows up to 20 characteristics of each family in each pipe. Experience has shown that three to five characteristics will often give excellent results. The optimum number depends on the pipe length, the time step used and the highest order harmonic of interest.

As presently implemented, the program will not allow the time step to be such that a characteristic passes completely through a pipe in less than one time step. However, with the multiple boundary condition method, this is not an inherent limitation. It does, however, simplify the algorithm significantly.

RESULTS

Space does not allow the presentation of comparative results. The method described here was chosen for a "cycle" program that calculates the complete compressor cycle and has been used to evaluate the effects of different losses on compressor performance (Ref. 2) and the effects of pulsations on valve dynamics (Ref. 4). Typical results are shown in these publications.

One illustration of the importance of pulsations on the compression cycle is given by the calculated curves of figure 5. It should be noted that here the valves are taken as ideal. That is, they have no dynamic or spring related losses.

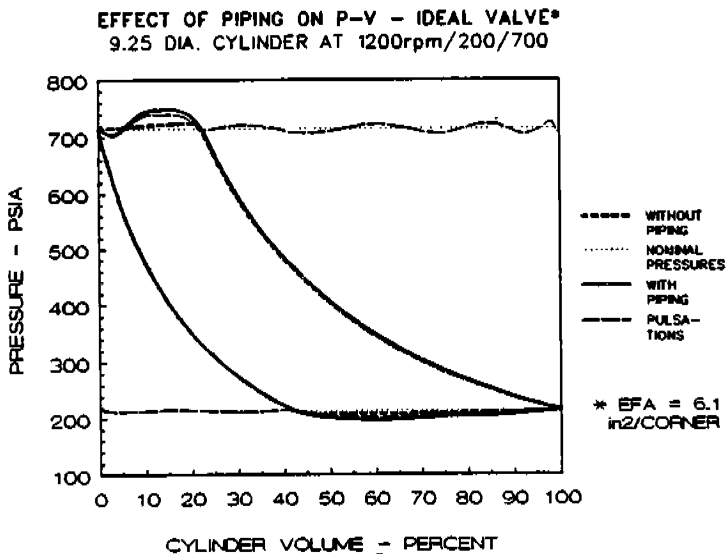

Figure 5

CONCLUSIONS

The method for calculating pulsations in compressor piping systems described here is conceptually very simple and efficient although the program to implement it can get complex. Many calculations have shown the method to be effective for predicting the effects of the piping near the compressor on the compressor performance.

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