# Scroll Compressor: Thrust Bearing Design with Rigid Body Dynamics of the Runner Plate

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SCROLL COMPRESSOR: THRUST BEARING DESIGN WITH RIGID BODY DYNAMICS OF THE RUNNER PLATE

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ABSTRACT

This paper presents the results of a theoretical study of the scroll compressor orbiting thrust bearing. The film thickness profile is computed from the rigid body dynamics of the scroll plate with six degrees of freedom. Euler's equations of motion are integrated to describe the angular velocities. Euler parameter representation is used to obtain the angular position of the rigid body. Translation equations are derived and integrated to obtain linear velocities and position. The Reynolds equation under incompressible laminar condition with constant viscosity is solved sequentially to the rigid body dynamics.

NOMENCLATURE

$\bar{A}_G$ = Acceleration of center of gravity

$B_i$ = Orthonormal basis; i = 0, 1, 3

C, D = Transformation matrices

2d = thickness of the scroll plate

E = Euler theorem axis of rotation

$E_i$ = Direction cosines of the axis E; i = 1, 2, 3

$\bar{F}$ = Resultant force acting on the scroll plate

$F_{AX}$ = Axial gas force on the scroll plate

$F_{OIL}$ = Oil film restoring force

$F_{RAD}$ = Radial gas force component on the scroll plate

g = Gravitational acceleration

G = Center of Gravity of the scroll plate

h = Local film thickness (Reynolds Equation)

$h_{41}$, $h_{42}$, $h_{43}$, $h_{44}$ = Film thickness at the outer radius

H = Radial gas force moment arm

$I_i$ = Principal moment of inertias; i = 1, 2, 3

m = Mass of the scroll plate

$M_i$ = Moments in $B_3$ basis; i = 1, 2, 3

O = Spherical joint support

$p_0$, $s_0$ = Restoring oil force location ($\bar{x}_3$, $\tilde{y}_3$) in $B_3$ basis

p = Pressure in clearance space (Reynolds Equation)

$p_1$ = Pressure at the inner radius of the bearing (Reynolds Equation)

$p_2$ = Pressure at the outer radius of the bearing (Reynolds Equation)

$q_i$ = Euler parameters or quaternions; i = 0, 1, 2, 3

$r$ = Radial coordinate (Reynolds Equation)

$r_G$ = Center of gravity of scroll plate

$R_1$ = Inside radius of bearing

$R_2$ = Outside radius of bearing

$R_B$ = Radial coordinate with shaft center as reference

$u$ = Radial velocity component (Reynolds Equation)

$v$ = Tangential velocity component (Reynolds Equation)

$w$ = Squeeze velocity component (Z direction - Reynolds Equation)

$w_s$ = Scroll plate local squeeze velocity component

X, Y, Z = Cartesian coordinate system

$X_i$ = Cartesian coordinate Systems; i = 1, 2, 3

$\bar{\omega}$ = Angular velocity vector

$\omega$ = Shaft angular velocity

$\omega_i$ = Angular velocity in $B_3$ basis; i = 1, 2, 3

$\theta$ = Tangential coordinate (Reynolds Equation)

$\mu$ = Lubricant viscosity (Reynolds Equation)

$\phi$ = Velocity phase angle (Reynolds Equation)

$\psi$ = Euler theorem rotation angle

## INTRODUCTION

The study of the dynamic process between an orbiting scroll plate and a stationary thrust bearing is of definite scientific and practical interest. The thrust bearing in the scroll compressor must be designed to support the axial gas force and at the same time resist the overturning moment.

Paper [1] treats this problem statically and uses a modification of Rumbarger's Theory [2]. In reference [3] the Reynolds equation is developed and solved for an orbiting thrust bearing. The pressure developed in the oil film is computed for a given film thickness distribution. This procedure also solves the thrust bearing problem statically.

In the present paper rigid body equations of motion of the scroll plate are considered in parallel with the Reynolds equation for the orbiting thrust bearing. The Reynolds equation for the bearing is briefly described here, which is fully developed in reference [3]. The restoring oil force and moment are obtained after numerical integration of the Reynolds equation, and are used in the rigid body equations of motion of the scroll plate. The rigid body equations of motion are integrated numerically. In turn, the rigid body dynamics time-dependent film thickness and squeeze velocity are used to solve the Reynolds equation.

## DYNAMIC ANALYSIS OF ORBITING SCROLL PLATE

The orbiting motion of the scroll plate can be defined by two successive coordinate transformations. Consider a perfectly rigid body in a gravitational field with a spherical joint at the point of support O translating in the z direction as shown in Figure 1. To derive the equations of motion, consider a Cartesian inertial coordinate system $X_0$ with orthonormal basis $B_0 = (\bar{i}, \bar{j}, \bar{k})$. The origin of the rotating Cartesian coordinate system $X_1$ is fixed to the shaft center-line. Let $B_1 = (\bar{x}_1, \bar{y}_1, \bar{z}_1)$ denote an orthonormal basis for this rotating coordinate system. The basis vectors $B_0$ and $B_1$ are related by a linear transformation D defined by $B_1^T = [D]\, B_0^T$. The origin of the rotating Cartesian coordinate system $X_3$ is fixed at the support point O of the rigid body. Let $B_3 = (\bar{x}_3, \bar{y}_3, \bar{z}_3)$ denote an orthonormal basis for this rotating coordinate system. The basis vectors $B_3$ and $B_1$ are related by a linear transformation C defined by $B_3^T = [C]\, B_1^T$. This gives, $B_3^T = [C]\,[D]\, B_0^T = [CD]\, B_0^T$. The linear transformations D and C are defined as follows:

$$D = \begin{bmatrix} \cos(\omega t) & \sin(\omega t) & 0 \\ -\sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} (q_1^2-q_2^2-q_3^2+q_0^2) & 2(q_1q_2+q_3q_0) & 2(q_1q_3-q_2q_0) \\ 2(q_1q_2-q_3q_0) & (-q_1^2+q_2^2-q_3^2+q_0^2) & 2(q_2q_3+q_1q_0) \\ 2(q_1q_3+q_2q_0) & 2(q_2q_3-q_1q_0) & (-q_1^2-q_2^2+q_3^2+q_0^2) \end{bmatrix}$$

where $\omega$ is the shaft angular velocity and $q = (q_0, q_1, q_2, q_3)$ denotes Euler parameters or quaternions with:

$$q_0 = \cos\left(\frac{\psi}{2}\right),\quad q_r = E_r \sin\left(\frac{\psi}{2}\right),\quad r = 1, 2, 3 \text{ and}$$

$$E = E_1\,\bar{x}_1 + E_2\,\bar{y}_1 + E_3\,\bar{z}_1 = E_1\,\bar{x}_3 + E_2\,\bar{y}_3 + E_3\,\bar{z}_3\,.$$

The Euler parameters are closely linked to Euler's Theorem, i.e.; any rotation of one coordinate system with respect to another system may be described by a single rotation through some finite angle about a fixed axis (E). The transformation C can also be defined with the use of Euler angles. By equating the terms of C(q) to those in C(Euler angles) one can convert quaternions into Euler angles and vice versa [4]. The concept of Euler parameters as rotational coordinates may appear as a mathematical tool without any physical meaning. However, careful study of these parameters will prove the contrary. Physical interpretation of Euler parameters is simple and is more natural to implement than any other set of rotational coordinates, such as Euler or Bryant angles [5].

The translational equation of motion of the center-of-gravity G of the orbiting scroll plate is:

$$\bar{F} = m\bar{A}_G \tag{1}$$

The position vector of G is:

$$\bar{R}_G = R_{OR}\,\bar{x}_1 + z_0\,\bar{k} + d\,\bar{z}_3 \tag{2}$$

The angular velocity of the scroll plate with respect to the inertial coordinate system is:

$$\overline{\omega} = \omega\ \overline{k} + \omega_1 \overline{x}_3 + \omega_2\ \overline{y}_3 - \omega\ \overline{z}_3 \tag{3}$$

The velocity and acceleration of G are:

$$\overline{V}_G = \dot{z}_0\ \overline{k} + R_{OR}\omega\ \overline{y}_1 + \omega d(CD_{31}\overline{j} - CD_{32}\overline{i})$$

$$- \omega_1\ d\ \overline{y}_3 + \omega_2\ d\ \overline{x}_3 \tag{4}$$

$$\overline{A}_G = \ddot{z}_0\ \overline{k} - R_{OR}\omega^2\ \overline{x}_1 + \omega^2 d(-CD_{31}\overline{i} - CD_{32}\overline{j})$$

$$- \dot{\omega}_1 d\ \overline{y}_3 + \dot{\omega}_2 d\ \overline{x}_3 + \omega\overline{k}X(-\omega_1 d\ \overline{y}_3 + \omega_2\ d\ \overline{x}_3)$$

$$-(\omega_1^2 + \omega_2^2)\ d\ \overline{z}_3 - \omega_1 \omega d\ \overline{x}_3 - \omega_2 \omega d\ \overline{y}_3 \tag{5}$$

[CD] is used to convert $\overline{x}_3$, $\overline{y}_3$, $\overline{z}_3$ in terms of $\overline{i}$, $\overline{j}$, $\overline{k}$. Combining the $\overline{i}$, $\overline{j}$, and $\overline{k}$ terms of acceleration of G along $\overline{i}$, $\overline{j}$, $\overline{k}$ yield respectively:

$$A_{Gx} = -\ \omega^2 d\ CD_{31} + \omega\omega_1 d\ CD_{22} - \omega\omega_2 d\ CD_{12}$$

$$+(\dot{\omega}_2 d - \omega_1 \omega d)\ CD_{11} + (-\dot{\omega}_1 d - \omega_2 \omega d)\ CD_{21}$$

$$+(-\omega_1^2 d - \omega_2^2 d)\ CD_{31} - R_{OR}\omega^2 COS(\omega t) \tag{6}$$

$$A_{Gy} = -\ \omega^2 d\ CD_{32} - \omega\omega_1 d\ CD_{21} + \omega\omega_2 d\ CD_{11}$$

$$+(\dot{\omega}_2 d - \omega_1 \omega d)\ CD_{12} + (-\dot{\omega}_1 d - \omega_2 \omega d)\ CD_{22}$$

$$+(-\omega_1^2 d - \omega_2^2 d)\ CD_{32} - R_{OR}\omega^2 SIN(\omega t) \tag{7}$$

$$A_{Gz} = \ddot{z}_0 + (\dot{\omega}_2 d - \omega_1 \omega d)\ CD_{13} + (-\dot{\omega}_1 d - \omega_2 \omega d)\ CD_{23}$$

$$+ (-\omega_1^2 d - \omega_2^2 d)\ CD_{33} \tag{8}$$

The resultant force acting on the scroll plate is:

$$\overline{F} = (-F_{AX} + F_{OIL})\ CD_{31}\overline{i} + (-F_{AX} + F_{OIL})\ CD_{32}\overline{j}$$

$$+ (-mg + (-F_{AX} + F_{OIL})\ CD_{33})\ \overline{k} \tag{9}$$

From equations (1) through (9) the acceleration terms $A_{Gx}$, $A_{Gy}$ and $A_{Gz}$ can be obtained.

The components of the angular velocity can be obtained by solving Euler's equations referred to the principal axes for a rigid body with center-of-gravity G as the moment center [6] as follows:

$$M_1 = I_1 \dot{\omega}_1 + (I_3 - I_2)\, \omega_2 \omega_3 \quad (10)$$

$$M_2 = I_2 \dot{\omega}_2 + (I_1 - I_3)\, \omega_3 \omega_1 \quad (11)$$

$$M_3 = I_3 \dot{\omega}_3 + (I_2 - I_1)\, \omega_1 \omega_2 \quad (12)$$

where $I_1$, $I_2$ and $I_3$ are the principal moments of inertia. The external moments $M_1$ and $M_2$ are functions of the axial and radial gas force components, and the thrust bearing restoring force and its location as indicated below:

$$M_1 = F_{AX} R_{OR} \frac{SIN(\omega t)}{2} + F_{RAD} H\ COS(\omega t) + s_O F_{OIL} \quad (13)$$

$$M_2 = F_{RAD} H\ SIN(\omega t) - F_{AX} R_{OR} \frac{COS(\omega t)}{2} - p_O F_{OIL} \quad (14)$$

For $\omega_3 = -\omega$ (constant) and $I_1 = I_2$, then:

$$M_3 = 0 \quad (15)$$

The Euler parameter equations are:

$$\dot{q}_0 = 0.5(-\omega_1 q_1 - \omega_2 q_2 - \omega_3 q_3) \quad (16)$$

$$\dot{q}_1 = 0.5(\ \omega_1 q_0 + \omega_3 q_2 - \omega_2 q_3) \quad (17)$$

$$\dot{q}_2 = 0.5(\ \omega_2 q_0 - \omega_3 q_1 + \omega_1 q_3) \quad (18)$$

$$\dot{q}_3 = 0.5(\ \omega_3 q_0 + \omega_2 q_1 - \omega_1 q_2) \quad (19)$$

where $\omega_3 = -\omega$ (constant).

## REYNOLDS EQUATION

The restoring oil force and its location for the orbiting thrust bearing is investigated in [3]. Therefore, only a very brief discussion of the formulation follows. For the laminar flow and constant viscosity with the usual assumptions in lubrication theory, the Reynolds equation for the orbiting thrust bearing (refer to Figure 2) is:

$$\frac{\partial}{\partial r}\left(\frac{r h^3}{12\mu}\frac{\partial p}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial \theta}\right)$$

$$= \frac{1}{2}\frac{\partial}{\partial \theta}(\omega\, r h + \omega\, R_B h \cos\phi) + \frac{\omega}{2}\frac{\partial}{\partial r}(r R_B \sin\phi\ h)$$

$$- R_B \omega \cos\phi \frac{\partial h}{\partial \theta} - r R_B \omega \sin\phi \frac{\partial h}{\partial r} - r w_s \quad (20)$$

The pressure boundary conditions are:

$$p = p_1 \text{ at } r = R_1,\ p = p_2 \text{ at } r = R_2 \quad (21)$$

The restoring oil force and its location, using the Sommerfield boundary condition, can be calculated as follows:

$$F_{OIL} = \int_0^{2\pi} \int_{R_1}^{R_2} pr\ dr\ d\theta \tag{22}$$

$$s_O = \frac{1}{F_{OIL}} \int_0^{2\pi} \int_{R_1}^{R_2} r^2 p \sin(\theta-\pi)\ dr\ d\theta \tag{23}$$

$$p_O = \frac{-1}{F_{OIL}} \int_0^{2\pi} \int_{R_1}^{R_2} r^2 p \cos(\theta-\pi)\ dr\ d\theta \tag{24}$$

The oil film thickness and squeeze velocity each vary at different points on the scroll plate, and are also time-dependent.

## METHOD OF SOLUTION

The rigid body equations of motion of the orbiting scroll plate can be integrated by using the fourth order Runge-Kutta method to give linear velocities and position, and angular velocities and position. The Reynolds equation (sequentially to the rigid body dynamics) is solved for the restoring pressure and its centroid. Note that at t = 0, the Reynolds equation is solved using initial conditions for the film thickness and squeeze velocity. The calculated restoring force and its location is then used in rigid body dynamics to estimate the film thickness and squeeze velocity, which are then transferred to the Reynolds equation. This procedure is repeated until $t = t_{final}$.

## NUMERICAL EXAMPLE

The following hypothetical data will be used as an example of the procedures explained above:

at t = 0, $z_O = h_O$ (constant) inch, $q_0 = 1$, all other initial conditions are zero.

$R_1 = 1.566$ inch, $R_2 = 2.969$ inch, $R_{OR} = .2701$ inch,

$H = 1.837$ inch, $d = 2.0$ inch,

$I_1 = I_2 = \frac{m}{4}\left(R_2^2 + \frac{4d^2}{3}\right)$, $I_3 = \frac{m}{2} R_2^2$,

shaft rpm = 3500, $m = 7.764\times10^{-3}$ slug,

$p_1 = 1.0$ PSI, $p_2 = 0.0$ PSI, $\mu = 10^{-6}\ \frac{lb_f\text{-sec}}{inch^2}$

The time dependent radial and axial gas force components are shown in Figure 3.

Figure 5 demonstrates angular velocities $\omega_1$ and $\omega_2$ as a function of time. In Figure 6 quaternions are represented for the example. Figure 7 shows the linear velocity and position of point O. The time-dependent film thickness of the four body-fixed points (defined in Figure 2) are shown in Figure 8 (A and B). The restoring oil force and its location in $B_3$ basis are represented in Figure 4 (A and B); the system develops a restoring force immediately and also the system behavior is stable.

## SUMMARY AND CONCLUSIONS

The rigid body equations of motion for the orbiting scroll plate are derived. Euler's equations and Euler parameter equations are used to represent angular velocity and angular position. The rigid body equations of motion are solved using the fourth order Runge-Kutta method. The Reynolds equation for the orbiting thrust bearing is integrated sequentially to the rigid body dynamics using finite difference methods.

The restoring oil force and its position portray a stable behavior. To support the axial gas load and the gas tilting moment for a given inner radius, the bearing outer radius can thus be estimated without any assumption of the film thickness profile and squeeze velocity.

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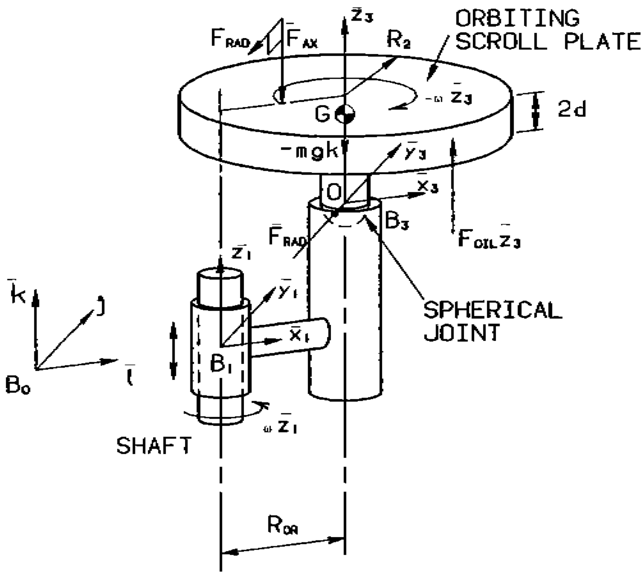



FIGURE 1.

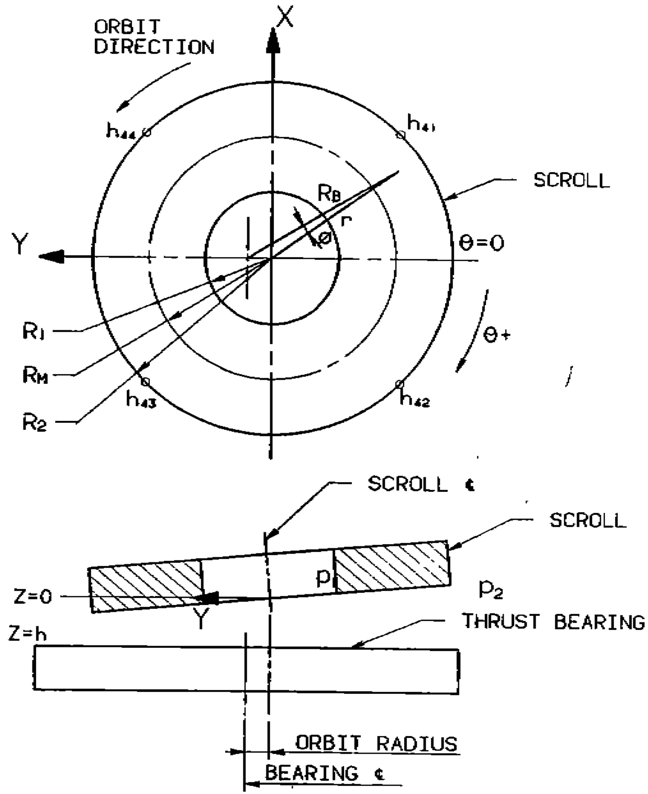
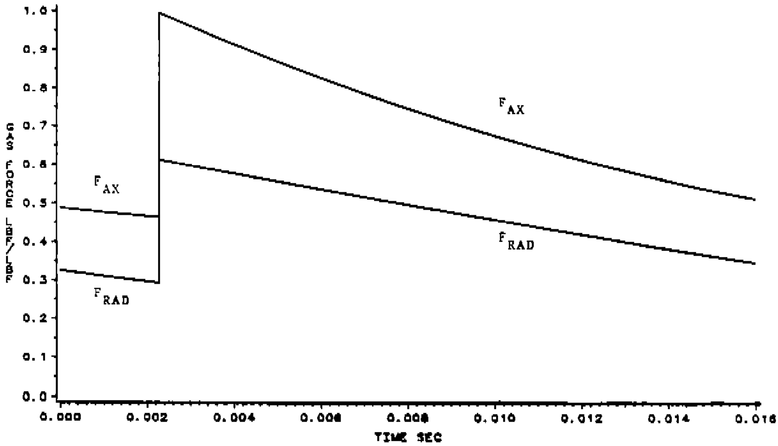



FIGURE 2.

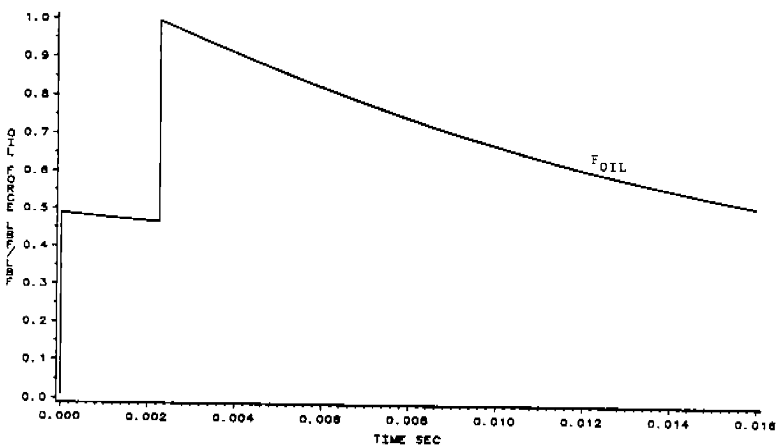
**ORBITING THRUST BEARING ANALYSIS**
VARIABLE GAS FORCE COMPONENTS




( $\mu$ =1E-06, RPM=3500)
(VARIABLE GAS FORCE) ($R_1$ =1.566, $R_2$=2.969, $R_{OR}$ =.2701, H=1.837)

Figure 3.

**ORBITING THRUST BEARING ANALYSIS**
RESTORING OIL FORCE




( $\mu$ =1E-06, RPM=3500)
(VARIABLE GAS FORCE) ($R_1$=1.566, $R_2$ =2.969, $R_{OR}$ =.2701, H=1.837)

Figure 4-A.

ORBITING THRUST BEARING ANALYSIS
RESTORING OIL FORCE LOCATION

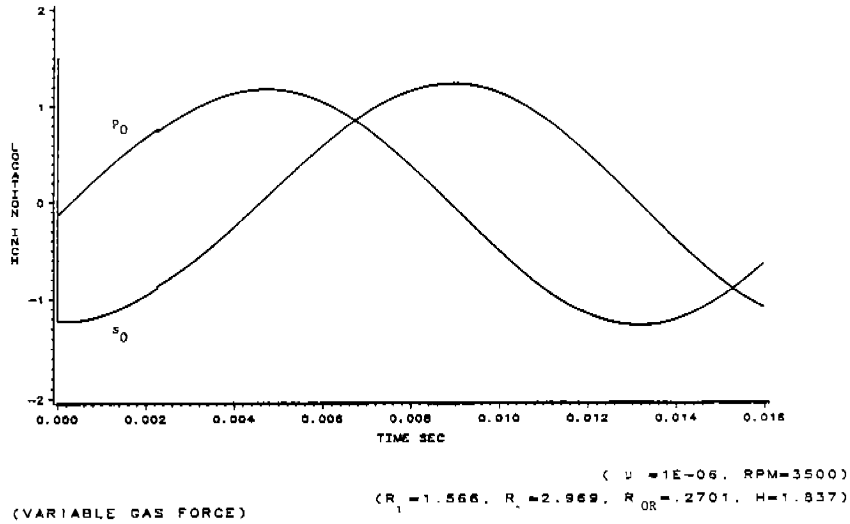



Figure 4-B.

ORBITING THRUST BEARING ANALYSIS
ANGULAR VELOCITY

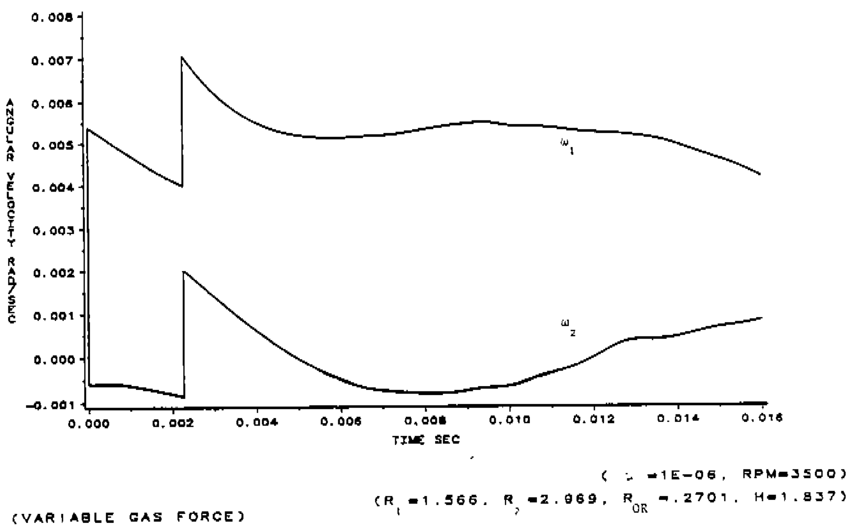



Figure 5.

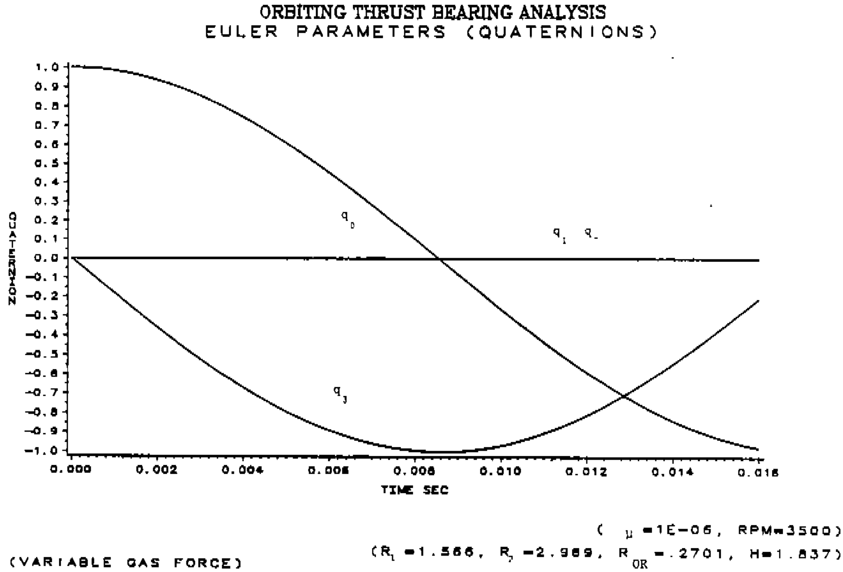



Figure 6.

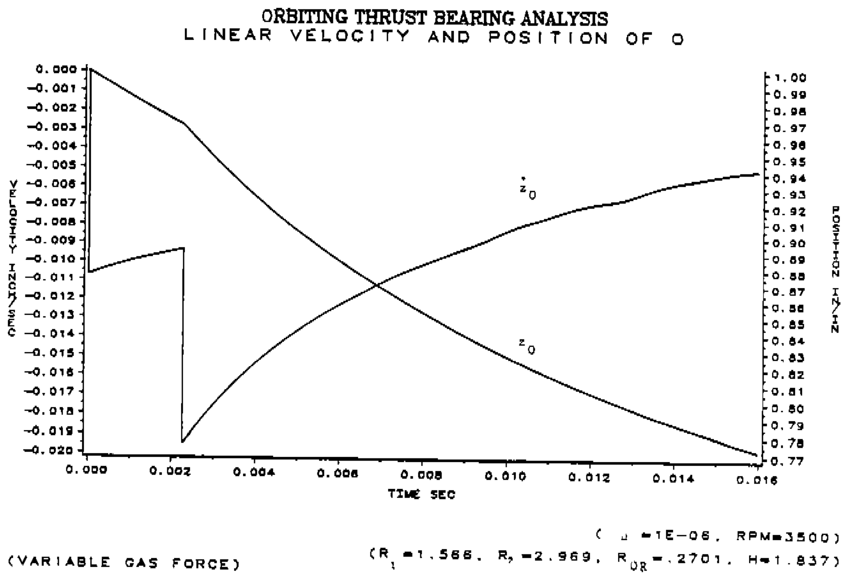
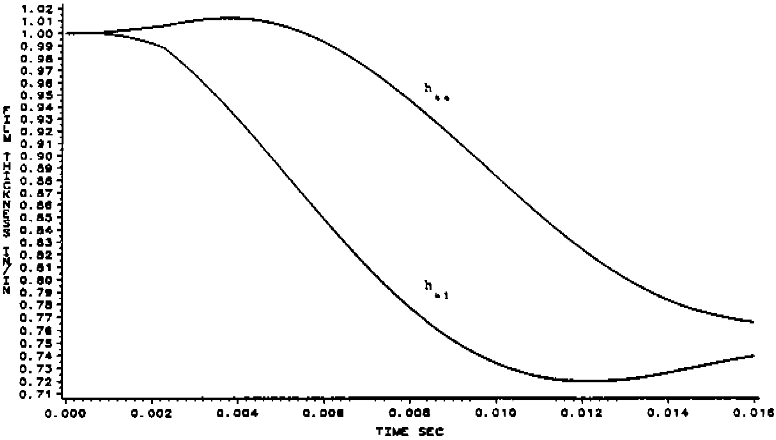



Figure 7.

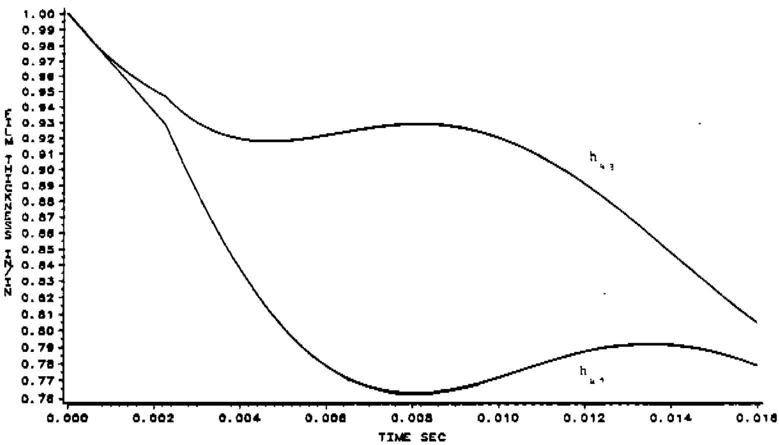
## ORBITING THRUST BEARING ANALYSIS
OIL FILM THICKNESS




( $\mu$ =1E-06, RPM=3500)
(VARIABLE GAS FORCE) ($R_1$ =1.566, $R_2$ =2.969, $R_{OR}$ =.2701, H=1.837)

Figure 8-A.

## ORBITING THRUST BEARING ANALYSIS
OIL FILM THICKNESS




( $\mu$ =1E-06, RPM=3500)
(VARIABLE GAS FORCE) ($R_1$ =1.566, $R_2$ =2.969, R =.2701, H=1.837)

Figure 8-B.