2004

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ABSTRACT

This paper presents a semi-empirical model to predict the growth of frost formation on the cold cylinder surface. The model in this paper is composed of the correlations for frost properties including the various frosting parameters and local heat transfer coefficient. The effects of varying the correlations for local heat transfer coefficient on the frost growth are examined to establish the model. The numerical results are compared with the experimental data from the previous researchers. The results agree well with the experimental data within a maximum error of 13%. As the results, the frost thickness decreases with changing angular position from the front stagnation to separation point. Also the effects of air velocity on the frost growth are negligible, as compared to the other frosting parameters.

1. INTRODUCTION

When a moist air flows across the cooling surface maintained below $0\,^{\circ}C$, the porous frost layer is formed on the cooling surface. The thermal and flow resistance are increased simultaneously due to the growth of the frost layer on the surface of evaporator used in the refrigerator and air-conditioner under frosting condition. For the above reasons, the frost formation becomes one of the most important factors deteriorating the performance of the refrigerating system. To predict the frost behavior on the fin-tube heat exchanger that has been generally used in the field of cryogenic engineering or HVAC industry, the frosting behavior on the fin which occupies approximately 80 % of the total heat transfer area in the heat exchanger has been universally investigated by the previous researchers. These days, however, the interest for the new type of evaporator whose heat transfer area of the tube becomes almost a half of the whole heat exchanger has been raised for reducing the material cost and improving the efficiency. Consequently, the frost formation on the cylinder surface becomes a matter of concern.

The previous researches (Mago and Sherif, 2003, Padki *et al*., 1989, Raju and Sherif, 1993, Parish and Sepsy, 1972, Ismail *et al*., 1996) of the frost formation on the cylindrical surface are categorized into three large groups. At first, Mago and Sherif (2003) and Padki *et al*. (1989) used the correlations for heat transfer coefficient on the cylinder surface and the frost properties, and then estimated the frost formation. Secondly, Raju and Sherif (1993) employed the established empirical correlations for the frost side, and Parish and Sepsy (1972) used the mass diffusion equation together with the experimental correlations. They (Raju and Sherif, 1993, Parish and Sepsy, 1972) carried out the flow analysis, which means the numerical analysis for the momentum, energy and continuity equation on the air side, and showed the frosting behavior on the cold cylinder surface. Finally, Ismail *et al.* (1996) developed the frost growth model by performing the flow analysis for the air side and considering the porous medium for the frost side.

Although most of the previous researchers employed the flow analysis for the air side, they presented the less accurate results despite the long calculating time. Hence the study using the correlation of heat transfer coefficient has come to occupy an important position in these days because of two advantages (Mago and Sherif, 2003); one is reducing the calculating time and another is obtaining the acceptable results within the reasonable boundary. Mago and Sherif (2003) employed the correlation of the local heat transfer coefficient for air side in order to predict the frosting behavior on the cold cylinder surface. However, they ignored the density-increasing part of the mass flux which eventually means all of the mass transfer owing to the difference of humidity between the moist air and the frost surface increases the frost thickness.

The previous researchers employed primarily the empirical correlation for the frost properties proposed by Hayashi *et al.* (1977) and Yonko and Sepsy (1967). The correlations of the frost properties used in the previous researches depend on the frosting conditions, and are also derived from the limited experimental conditions. Therefore, there should be a limitation of the applicable frosting condition. For example, the correlation of the frost density presented by Hayashi *et al*. (1977) which has been used by most previous researchers has a limited applicable range. It might cause a considerable error due to the restrictive range. This correlation is expressed only as a function of the frost surface temperature, excluding the frosting factors as the velocity, the temperature, the humidity of the moist air and the cold surface temperature. Thereon, the correlation doesn't guarantee the exact estimation of the frost formation.

This study presents the frost model for predicting the frosting behavior with the correlations of the local heat transfer coefficient and the various factor-accounted frost properties. It also presents the numerical results on the thickness and the surface temperature of the frost layer with respect to the time and the angular position domain. Additionally, the possibility in applying the correlations of the heat transfer coefficient from various researchers was examined. Finally, this paper presents the influences of the various frosting parameters for the frost growth.

2. THEORY

In this study, a mathematical model that can predict the behavior of the frost growth formed on the cylindrical cooling surface is presented. The model is composed of the correlation of the local heat transfer coefficient and the correlations of the density and the thermal conductivity of the frost layer. Figure 1 shows the computational domain to investigate the effects of the principal factors; temperature, absolute humidity and velocity of the air, the cooling surface temperature and most of all the angular position of the cylinder surface.

2.1 Assumptions

The following assumptions have been made for the analysis:
(1) All the process of the frost formation is in the quasi-steady state.
(2) The heat transfer mechanism from the frost surface to the cold cylinder surface is a pure conduction.
(3) Radiation heat transfer is ignored.
(4) The frost layer surface exists under a saturated condition.

2.2 Air side Modeling

The energy balance on an infinitesimal area segment at any angular position (θ) along the cylinder surface can be written as follows:

$$\delta\dot{Q}_{sen,f\theta} + \delta\dot{Q}_{lat,f\theta} = \delta\dot{Q}_{cond,f\theta} \tag{1}$$

Total heat transfer for air side is composed of two terms; the latent heat owing to mass transfer and the sensible heat owing to convection from the moist air to the frost layer surface.

At first, the sensible heat transfer rate due to temperature difference between humid air and frost surface is given by

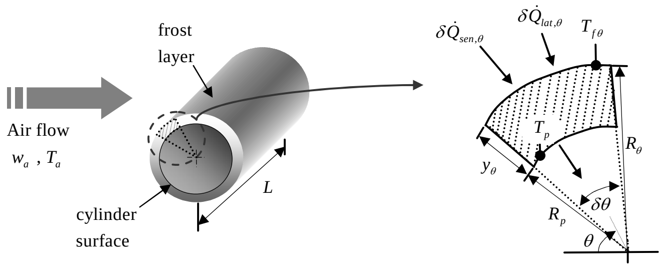


Figure 1: Schematic diagram of frosting formation on a cylinder surface

$$\delta \dot{Q}_{sen,f\theta} = h_{h,\theta} L R_{\theta} \delta\theta \left(T_a - T_{f\theta} \right) \tag{2}$$

In order to obtain the heat transfer coefficient, the following dimensionless correlation of the local Nusselt number introduced by Martinelli *et al*. (1943) was used in the paper:

$$\mathrm{Nu}_{\theta} = \frac{h_{h,\theta} d_{\theta}}{k_a} = 1.14\,\mathrm{Re}_{d_\theta}^{0.5}\,\mathrm{Pr}^{0.4} \left[1 - \left(\frac{\theta}{90} \right)^3 \right] \tag{3}$$

Because the applicable range of the local Nusselt number is the forward portion of the cylinder ($0^{\circ} \le \theta \le 80^{\circ}$), the analysis domain is limited to the front side.

Secondly, the latent heat transfer rate driven by humidity difference between the moist air and the saturated frost surface is written by:

$$\delta \dot{Q}_{lat,f\theta} = h_{m,\theta} L_{h,\theta} L R_{\theta} \delta\theta \left(w_a - w_{f\theta} \right) \tag{4}$$

where, $w_{f\theta}$ is the saturated absolute humidity at the frost surface temperature ($T_{f\theta}$).

The mass transfer coefficient due to the analogy between heat and mass transfer is defined by:

$$h_{m,\theta} = \frac{h_{h,\theta}}{C_{pa}\mathrm{Le}^{2/3}}$$

The Lewis number is a dimensionless number defined as:

$$\mathrm{Le} = \frac{\alpha_a}{D_a}$$

2.3 Frost side Modeling

The empirical correlations of the effective thermal conductivity and the frost density are employed in this study.

The temperature difference between the frost surface and the cooling surface during the frost formation is applied to the equation of Fourier's heat conduction, and this can be expressed as follows:

$$\delta \dot{Q}_{cond,f\theta} = \frac{k_{f\theta} L \delta\theta \left(T_{f\theta} - T_p \right)}{\log_e \left(R_{\theta} / R_p \right)} \tag{5}$$

The thermal conductivity of the frost in equation (5) is derived from the experiment results (Lee *et al*., 1994) under the various frosting conditions. Its correlation is given by:

$$k_{f\theta} = 0.132 + 3.13 \times 10^{-4} \rho_{f\theta} + 1.6 \times 10^{-7} \rho_{f\theta}^{\;2} \tag{6}$$

To determine the frost density used in equation (6), most previous researchers employed the empirical correlation of the frost density proposed by Hayashi *et al*. (1977). However, there occurs an inevitable error in applying the correlation under the various frosting conditions by two reasons; one is that this correlation is derived as only the function of the frost surface temperature, and another is the application ranges are restricted by the fixed humidity as follows:

$$-25^{\circ}\mathrm{C} \le T_{f\theta} \le 0^{\circ}\mathrm{C},\;\; 2\ \mathrm{m/s} \le U_a \le 6\ \mathrm{m/s},\;\;\; w_a = 0.0075\ \mathrm{kg/kg_{DA}}$$

Consequently, this study employs the modified correlation of the frost density by considering the various frosting parameters; the velocity, the temperature and the absolute humidity of the moist air and the frost surface temperature.

$$\frac{\rho_{f\theta}}{\rho_{ice}} = 797.534 \times 10^{-6} \left(\frac{U_a}{U_{\max}}\right)^{-0.0254} w_a^{-1.2643} \left[\exp\left(\frac{T_{f\theta} - T_{tp}}{T_a - T_p}\right)\right]^{3.934} \tag{7}$$

where, T_{tp} and $U_{\max}$ are the triple point temperature of water and the maximum value of air velocity, respectively.

The possible ranges of equation (7) is as follows:

$$-35^\circ\text{C} < T_p < -15^\circ\text{C}, \quad 5^\circ\text{C} < T_a < 15^\circ\text{C}, \quad 1\text{ m/s} < U_a < 2.5\text{ m/s},$$
$$0.00322\text{ kg/kg}_{\text{DA}} < w_a < 0.00847\text{ kg/kg}_{\text{DA}}$$

During the frost formation, some of the mass transfer causes the increase of frost layer thickness, and the rest causes the increase of frost density. The local mass flux owing to the humidity difference between the moist air and the frost surface can be written as follows:

$$m''_{f\theta} = h_{m,\theta}\left(w_a - w_{f\theta}\right) = m''_{f\theta,y} + m''_{f\theta,d} \tag{8}$$

where, $m''_{f\theta,y}$ and $m''_{f\theta,d}$ are the mass fluxes causing the increase in the frost thickness and density, respectively.

$$m''_{f\theta,y} = m''_{f\theta} - m''_{f\theta,d} = h_{m,\theta}\left(w_a - w_{f\theta}\right) - \frac{y_\theta}{\Delta\tau}\left(\rho_{f\theta}^{\tau+\Delta\tau} - \rho_{f\theta}^{\tau}\right) \tag{9}$$

where, $\Delta\tau$ and $\rho_{f\theta}{}^{\tau}$ are the time increment and the frost density at the previous time step, respectively.

The frost thickness at the present time step is computed as follows:

$$y_\theta = y_\theta + \frac{m''_{f\theta,y}}{\rho^{\tau}{}_{f\theta}}\Delta\tau \tag{10}$$

The frost surface temperature can be obtained by substituting equations (2), (4) and (5) into equation (1) as follows:

$$T_{f\theta} = \frac{h_{m,\theta}L_{h,\theta}R_\theta \log_e\left(R_\theta / R_p\right)\left(w_a - w_{f\theta}\right) + h_{h,\theta}R_\theta \log_e\left(R_\theta / R_p\right)T_a + k_{f\theta}T_p}{h_{h,\theta}R_\theta \log_e\left(R_\theta / R_p\right) + k_{f\theta}} \tag{11}$$

2.4 Numerical Procedure

(1) Input the data; air temperature, cylinder surface temperature, absolute humidity of the air, time and time step.
(2) Initialize frost temperature and thickness: $T_{f\theta} = T_p,\ R_p = R_\theta,\ y_\theta = 0$
(3) Figure out the air properties at the film temperature and calculate heat and mass transfer coefficients on the cylinder surface, Lewis number, the thermal conductivity of the frost, the mass deposition rate and the frost layer thickness, then finally compute the frost surface temperature.
(4) Recalculate the air properties at the film temperature using newly obtained frost surface temperature, and compute again the frost surface temperature with the modified heat and mass transfer coefficients on cylinder surface, Lewis number, the saturated absolute humidity of the cooling surface and the frost thermal conductivity.
(5) When the following convergence criterion is satisfied, the angular position for calculating is changed.

$$\left|T_{f\theta}^{new} - T_{f\theta}^{old}\right| \le 0.001$$

(6) When the calculation along with all given angular position is completed, the time step for calculation is increased.
(7) When the calculation for the total frosting time with all the angular position is finished, the numerical procedure is completed.

In this study, to determine the validity of the heat transfer coefficient correlation for the solution of the mathematical model, the following correlation is adopted additionally. The angular position (θ)-included correlation of the local heat transfer coefficient, presented by Galante and Churchill (1990), is given as follows:

$$\mathrm{Nu}_{\theta} = 2\left[\frac{(1+\cos\theta)\,\mathrm{Pe}_{D_\theta}}{\pi}\right]^{1/2} \tag{12}$$

In the equation (12), Nusselt number, based on the cylinder diameter, is applied for $\mathrm{Pe} > 8$.

Under the assumption that the frost grows uniformly on cylinder surface, the averaged Nusselt number presented by Churchill and Bernstein (1977) is suggested as follows:

$$\mathrm{Nu}_{avg} = 0.3 + \frac{0.62\,\mathrm{Re}_{d_\theta}^{\;1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(\dfrac{0.4}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_{d_\theta}}{282000}\right)^{5/8}\right]^{4/5} \tag{13}$$

The Nusselt number of equation (13) is recommended for $\mathrm{Re}_{d_\theta}\,\mathrm{Pr} > 0.2$.

3. Results and discussion

In this study, the model employed the correlations of the local heat transfer coefficient and the frost properties to predict the frost formation occurring during the moist air flow passing across the cylindrical cooling surface. Figure 2 shows the variations of the frost thickness along the angular position at the fixed time ($\tau = 180$ min) in order to validate correlations of the local heat transfer coefficient, equations (3) and (12). The frost layer thickness calculated by equation (12) doesn't vary along the angular coordinate, whereas the frost thickness by equation (3) decreases with the varying angular position from the front stagnation point to the separation point.

This phenomenon varying along the angular position is physically reasonable because the heat transfer coefficient on the cylinder surface is decreased from the front stagnation point ($\theta = 0°$) to the separation point ($\theta = 80°$) (Giedt, 1949), likewise the mass transfer coefficient is diminished due to the analogy between heat and mass transfer. For this reason, it can be inferred that there is a change in the frost formation along the angular coordinate. Consequently, this study employed equation (3) presenting more reasonable results than equation (12).

This study performed the analysis using three different time step; 1s, 5s and 10s, so as to validate the dependency of the time step on the numerical results. Because all the variations of the numerical results on the frost thickness for three time step are below 0.5%, this paper adopted the time step of 5 sec.

Figure 3 shows the comparisons between the experimental results (Lee and Ro, 2001, Schneider, 1978) and the numerical results on the frost thickness with the various frosting conditions. For CASE 1 and CASE 2, each case graph consists of one experimental result (Schneider, 1978) and two numerical results using different correlation, i.e., equations (3) and (13). The graph using the correlation, equation (3), uses the averaged value with the angular position of the frost thickness. And the correlation using equation (13) uses the averaged Nusselt number. Overall, the Figure 3 shows the results between the experimental data assuming the uniform growth of the frost and such numerical results.

Since the numerical results with using the correlation of the heat transfer coefficient presented by Churchill and Bernstein (1977) are in a good agreement with the experimental data (Schneider, 1978), this correlation can be used in the assumption that the frost grows uniformly along the angular position. The numerical results of CASE 3 is

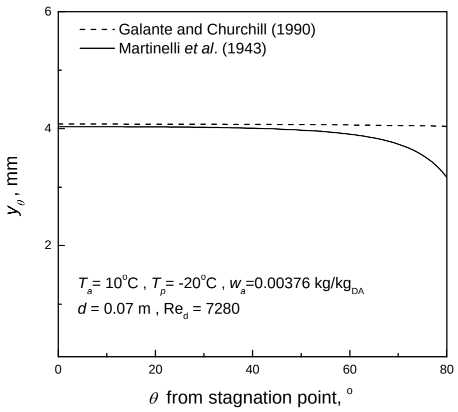


Figure 2: Variation of the frost thickness along the angular coordinate with correlations

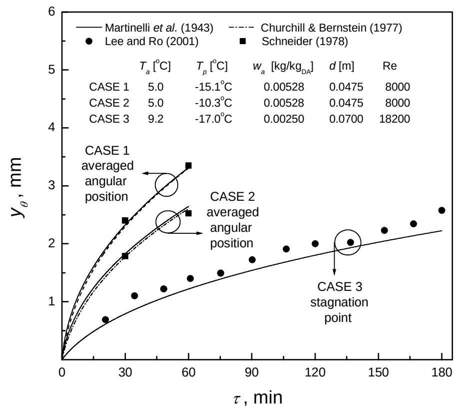
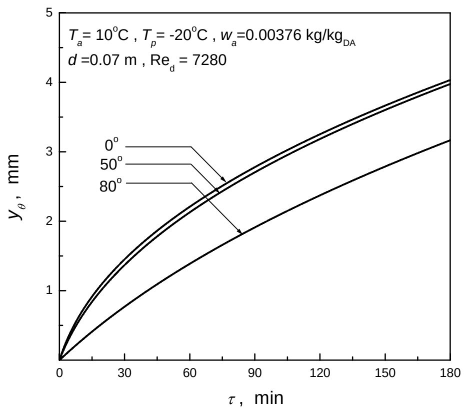
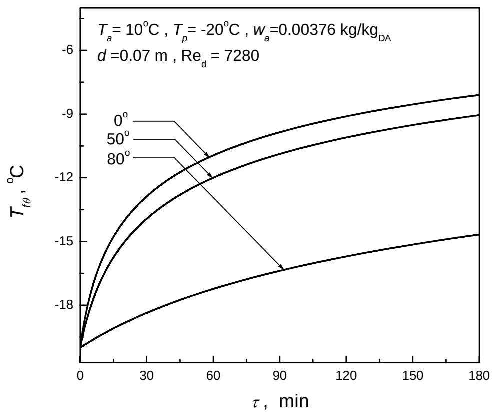


Figure 3: Comparisons of numerical results and experimental data on the frost thickness



(a) Frost thickness



(b) Frost surface temperature

Figure 4: Temporal variations of the frost thickness and surface temperature with angular positions

compared with the experimental data (Lee and Ro, 2001) on the temporal variation to the frost thickness at the front stagnation point. The results agree well with the experimental data (Lee and Ro, 2001) within a maximum error of 13%, except for the early stage of the frosting period.

Figure 4 shows the variations in the frost layer thickness and surface temperature with respect to the angular position. The frost thickness and surface temperature at all angular positions on cylinder surface increase during the frost formation. The frost thickness is thinner at the separation point than the front stagnation point due to decreasing of the mass transfer. The phenomenon causes the lower frost surface temperature at the separation point due to the reduction in the thermal resistance of the frost. This is because the heat and mass transfer is accelerated more at the stagnation point than at the separation point due to the generation of the thinner boundary layer at the front stagnation point.

Figure 5 shows the temporal variation of the frost thickness with respect to the angular position for the different time period; 20 min, 60 min, 100 min and 140 min. The frost grows uniformly with the angular position below 40° because the heat and mass transfer is not changed very much with the variation of the angular coordinate around the front side of the cylinder. The frost thickness decreases as approaching from the front stagnation point to the separation point because the boundary layer on the cylinder surface becomes thicker.

The numerical analysis with varying the frosting parameters (temperature, humidity and velocity of the moist air

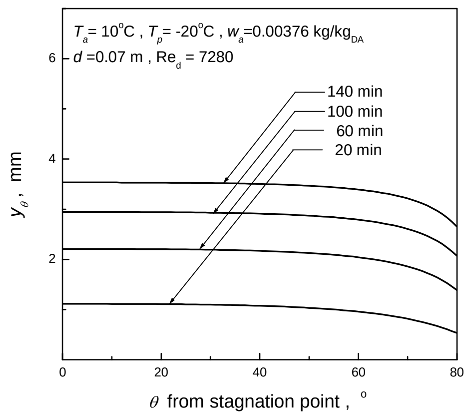


Figure 5: Variations of the frost thickness along the angular coordinate for different times

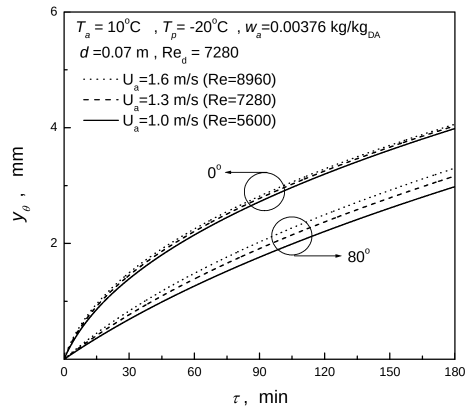


Figure 6: The effect of Reynolds number on the frost thickness at the stagnation and separation points

and the cold surface temperature) are performed to examine the effects of the frosting factors on the frost growth. As the results, the present study shows a consistent trend with the previous researchers. As a widely known fact, the higher air velocity (U_a), the lower cooling surface temperature (T_p), the lower air temperature (T_a) and the higher absolute humidity (w_a) of the air result in the thicker frost thickness.

Figure 6 shows the effects of Reynolds number on the frost thickness at the front stagnation and separation points. Although the frost growth slightly accelerates with increase in the air velocity, the effect of Reynolds number is relatively insignificant compared with other frosting parameters. However the effect of Reynolds number increases as approaching toward the separation point. Its influence is so small as compared with other parameters. The phenomena agree well with the previous experimental results (Schneider, 1978) that the Reynolds number has a little effect on the heat and mass transfer above its critical value.

4. Conclusions

In this paper, a mathematical model is presented to predict the frost formation on the cylindrical cooling surface with the angular position and time variations. In order to reduce the calculation time, this model employed the correlation of the local heat transfer coefficient on the cylinder surface and the empirical correlations of the frost properties that include the various frosting parameters. The influence of varying the correlations of the local heat transfer coefficient on the frost growth is examined to validate the numerical model. The numerical results are compared with the experimental data obtained by the previous researchers. The results agree well with a maximum error of 13%. As the results, the frost thickness becomes thinner as approaching from the front stagnation point to the separation point. Above 40° of the angular position, its trend increases with approaching the separation point. The effect of the air velocity on the frost growth is negligible compared to other frosting factors.

NOMENCLATURE

C_p	specific heat at constant pressure	(kJ/kg·K)	**Subscripts**	
d	cylinder diameter	(m)	a	air
D	mass diffusivity	(m^2/s)	f	frost surface
h_h	heat transfer coefficient	(W/m^2·K)	ice	ice
h_m	mass transfer coefficient	(kg/m^2·s)	max	maximum value
k	thermal conductivity	(W/m·K)	θ	specific angular position
L	length of cylinder	(m)		

Le	Lewis number	(-)
L_h	latent heat of sublimation of the frost	(kJ/kg)
m''_f	mass flux of the frost	($kg/m^2 \cdot s$)
Pe	Peclet number($= U_a d_\theta / \alpha$)	(-)
Pr	Prandtl number ($= \upsilon / \alpha$)	(-)
$\dot{Q}_{sen}$	sensible heat transfer rate	(W)
$\dot{Q}_{lat}$	latent heat transfer rate	(W)
$\dot{Q}_{cond}$	heat conduction rate	(W)
R	cylinder radius	(m)
$\mathrm{Re}_{d\theta}$	Reynolds number ($= U_a d_\theta / \upsilon$)	(-)
T	temperature	(^{o}C)
U	air velocity	(m/s)
w	absolute humidity	(kg/kg_{DA})
y	frost layer thickness	(m)

Greek symbols

α	thermal diffusivity	(m^2/s)
δ	infinitesimal change	(^{o}C)
θ	angular position from the front stagnation	(^{o})
ϕ	relative humidity	(-)
ρ	density	(kg/m^3)
υ	kinematic viscosity	(m^2/s)

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ACKNOWLEDGEMENT

This research was supported by The Center of Innovative Design Optimization Technology (iDOT), Korea Science and Engineering Foundation and the Brain Korea 21 Project.