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Utilization of Volumetric Displacement in Reciprocating Compressors

J. A. McGovern
Trinity College

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J.A. McGovern
Department of Mechanical and Manufacturing Engineering
Trinity College, Dublin, Ireland

ABSTRACT

The main factors affecting utilization of volumetric displacement of reciprocating compressors are analysed. A coherent system of parameters to quantify these is formulated. Reference is made to conventional parameters and to the work of other authors. Some new parameters are introduced and interrelationships are described. The overall purpose of the theory is to apportion shortcomings in the volumetric efficiency of reciprocating compressors to specific causes, on the basis of test measurements. The measurement problems which would arise in applying the theory fully are not addressed.

THE PROCESSES OF THE RECIPROCATING COMPRESSOR CYCLE

Fig. 1 illustrates the cycle of a reciprocating compressor as a diagram of cylinder pressure versus volume. The horizontal lines through points 4 and 1 and through points 3 and 2 represent the mean values of the fluctuating pressures within the suction and discharge pipes respectively.

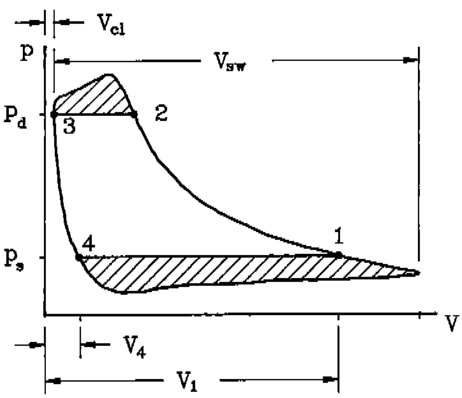

Fig. 1 An indicator diagram as might be measured experimentally for a reciprocating compressor, showing overpressure and underpressure caused by valve throttling.

VOLUMETRIC EFFICIENCY

The volumetric efficiency, which is also known as the displacement utilization efficiency, quantifies the utilization of the volumetric displacement of the piston and is defined as follows:

$$\eta_v = \frac{\dot{m} v_s}{N V_{sw}} \tag{1}$$

where,

η_v	=	volumetric efficiency, or, displacement utilization efficiency
$\dot{m}$	=	actual external mass flow rate produced by the compressor
N	=	number of cycles per unit time
	=	(compressor speed)×(number of cylinders)
V_{sw}	=	swept volume per cylinder
v_s	=	specific volume of the fluid at the suction condition

The volumetric efficiency can be determined experimentally from measurements of the mass flow rate and the compressor speed.

FACTORS WHICH AFFECT DISPLACEMENT UTILIZATION

Kleinert and Najork [1] presented an expression for the volumetric efficiency as the product of four efficiency terms which they described. These terms quantified different influences on the overall utilization of displacement. Pandeya and Soedel [2] had previously described an approach wherein the losses in mass flow rate were additive.

In the following sections the interrelationships between various influences on displacement utilization are re-examined, taking account of earlier work, and it is shown how the quantifying parameters can be combined to approximate the actual volumetric efficiency.

RE-EXPANSION OF THE CLEARANCE MASS

The fluid in the clearance volume when the piston is at the top dead centre position is re-expanded in the initial part of the induction stroke. The volume occupied by this 'clearance mass' of refrigerant at the suction pressure reduces the volume of fresh charge taken into the cylinder correspondingly.

The fundamental expression for the volumetric efficiency of a compressor which is ideal in every respect except that it has a clearance ratio greater than zero has the form

$$\lambda = 1 - r_c \left[\frac{v_4}{v_3} - 1 \right] \tag{2}$$

where

λ	=	clearance volumetric efficiency
r_c	=	clearance ratio $= \dfrac{\text{clearance volume}}{\text{swept volume}}$
v_4	=	specific volume of the fluid at the end of the re-expansion process
v_3	=	specific volume of the fluid at the start of the re-expansion process

For an actual compressor it is proposed that the clearance volumetric efficiency should be evaluated using the specific volume values corresponding to points 4 and 3 on the actual indicator diagram as represented in Fig. 1. It should be emphasized that the ratio of the specific volumes is not necessarily the same as the ratio of the actual volumes on the indicator diagram, due to leakage effects.

The clearance volumetric efficiency of an actual compressor may also be written as the following well known expression

$$\lambda = 1 - r_c \left[\left(\frac{p_d}{p_s} \right)^{1/n} - 1 \right] \tag{3}$$

where

p_s = mean pressure in suction pipe

p_d = mean pressure in discharge pipe

n = polytropic index for the re-expansion process

$$n = \frac{\log(p_3) - \log(p_4)}{\log(v_4) - \log(v_3)} \tag{4}$$

It should be noted that λ depends on the value of the polytropic index for the re-expansion process, n, and decreases with decreasing n. For an ideal gas a reversible adiabatic re-expansion process would have an index, n, equal to the ratio of the specific heats, γ. A value of unity would be expected for isothermal re-expansion: this would involve heat transfer to the gas during the process. The clearance volumetric efficiency, λ, thus includes the effect of heat transfer to the fluid during re-expansion.

Using the above expressions, (2), or (3) with (4), the clearance volumetric efficiency is precisely defined in terms of the actual specific volume values at points 3 and 4 on the indicator diagram.

THE INDUCTION PROCESS

The Indicated Volumetric Efficiency

This parameter is calculated from measurements on an indicator diagram such as that shown in Fig. 1.

$$\eta_{vi} = \frac{V_1 - V_4}{V_{sw}} \tag{5}$$

where

η_{vi} = indicated volumetric efficiency

V_1, V_4 = volumes at the points of suction pressure equalization, from the indicator diagram, Fig. 1.

The indicated volumetric efficiency is less than the clearance volumetric efficiency for two main reasons:

1. The volume at the end of the re-expansion stroke (point 4, Fig. 1) may be greater than that in the case of an ideal compressor with the same clearance ratio and polytropic index (based on pressure and specific volume), due to factors such as leakage or backflow through the discharge valve during re-expansion.

2. Due to underpressure within the cylinder when the piston reaches the bottom dead centre position, the induction process continues while the piston begins to move back up the cylinder and so the volume within the cylinder is less than the sum of the clearance and swept volumes when pressure equalization occurs (point 1, Fig. 1).

To quantify the first effect the 're-expansion loss ratio' is defined:

$$R_{rl} = \lambda - \frac{V_{sw} + V_{cl} - V_4}{V_{sw}} \tag{6}$$

where

V_{cl} = clearance volume per cylinder

This loss ratio quantifies the difference between the volume increase of the actual re-expansion process and that of an ideal compressor (having the same polytropic index for re-expansion, the same clearance ratio and the same value of λ) as a proportion of the swept volume. It quantifies part of the suction leakage effects which are described later.

To quantify the second effect described above, the 'under-pressure loss ratio' is defined:

$$R_{upl} = \frac{V_{sw} + V_{cl} - V_1}{V_{sw}} \tag{7}$$

The indicated volumetric efficiency is related to the clearance volumetric efficiency by the loss terms defined in equations (6) and (7).

$$\eta_{vi} = \lambda - R_{rl} - R_{upl} \tag{8}$$

Various other effects, some of which are described in this paper, cause the actual volumetric efficiency to be less than the indicated volumetric efficiency.

Throttling at the Suction Valve

As well as causing underpressure at the bottom dead centre position, as already mentioned, this reduces the suction fluid density. Costagliola analysed these effects [3 (1950)] and presented and explained the derivation of an expression equivalent to equation (9) for volumetric efficiency, based on an indicator diagram such as that in Fig. 1.

$$\eta_{vst} = \eta_{vi} - \frac{\gamma - 1}{\gamma} \int_{V_4}^{V_1} \left(1 - \frac{p}{p_s}\right) \frac{dV}{V_{sw}} \tag{9}$$

where

η_{vst} = volumetric efficiency taking account of suction throttling
γ = ratio of specific heats
p = pressure within cylinder
V = volume enclosed by the cylinder head, cylinder, and piston

Note: In Costagliola's original paper the terms in brackets in equation (9) were incorrectly printed in a form equivalent to (p/p_s-1).

The second term in Costagliola's equation (9) takes account of the irreversible suction throttling process which has the overall effect of increasing the temperature and reducing the density at point 1 in the diagram. The equation can also be written in the following form:

$$\eta_{vst} = \eta_{vi} - \frac{\gamma - 1}{\gamma} \frac{W_{si}}{V_{sw} p_s} \tag{10}$$

where

W_{si} = indicated suction work

= lower crosshatched area in Fig. 1.

The right hand term in equation (10) represents the loss in volumetric efficiency due to suction throttling:

$$R_{lst} = \frac{\gamma - 1}{\gamma} \frac{W_{si}}{V_{sw} p_s} \qquad (11)$$

where

R_{lst} = volumetric efficiency loss due to suction throttling.

In order to illustrate the likely significance of the right hand term in equation (10), typical values from compressor tests are given below.

For

compressor speed	=	600 r.p.m.
suction pressure	=	2.19 bara
ratio of specific heats (R-12)	=	1.18

typical test results are:

η_{vi}	=	0.865
W_{si}	=	6.30 J
$V_{sw} p_s$	=	36.33 J
R_{lst}	=	0.026
η_{vst}	=	0.865 - 0.026 = 0.839

Throttling at the suction valve depends not only on the geometry and flow area versus lift characteristics of the valve port and reed, but, also on the stiffness and dynamic characteristics of the reed. Furthermore, if there is a preload on the valve reed, holding it in position, or if there is any tendency for it to adhere to the seat, there may be a delay from the time when pressure equalization occurs across the valve to the moment when the reed begins to lift. This effect too influences the efficiency loss due to suction valve throttling.

Heat Transfer to the Suction Vapour

Heat transfer which occurs to the fluid after it leaves the suction pipe and before the suction valve closes on the induction stroke further reduces the fluid density at point 1 in Fig. 1. Gosney [4 (1951), 5 (1953)] extended the type of analysis carried out by Costagliola to include the effect of heat transfer to the suction fluid, equation (12). A full derivation of the equation, based on the approach he used, is given in [6].

$$\eta_{vstq} = \eta_{vi} - \frac{Q + W_{si}}{V_{sw} p_s} \frac{\gamma - 1}{\gamma} \qquad (12)$$

where

η_{vstq} = volumetric efficiency taking account of suction fluid throttling and heat pick up

Q = heat transfer to the fluid per cycle from the point where it leaves the suction pipe to closure of the suction valve

Equation (12) is the same as equation (10) in the special case where the amount of heat transfer is zero. The term representing the loss in volumetric efficiency due to suction heat transfer can be separated out as follows:

$$R_{lq} = \frac{Q}{V_{sw}p_s} \frac{\gamma - 1}{\gamma} \tag{13}$$

where

R_{lq} = volumetric efficiency loss due to suction heat transfer

In order to estimate η_{vstq}, using equation (12), the amount of heat transfer to the fluid must be determined. This could be done, if, in addition to the indicator diagram, the mean fluid temperature entering the suction valve and the mean fluid temperatures within the cylinder at points 1 and 4 in Fig. 1 were available.

Heat transfer occurs to the suction fluid in two stages. The first stage occurs within the suction plenum before the fluid enters the suction valve. It can be evaluated if the mean temperature of the fluid entering the suction valve is measured.

$$Q_{sp} = m_E(h_{sv} - h_s) \tag{14}$$

For an ideal gas,

$$Q_{sp} = m_E c_p(T_{sv} - T_s) \tag{15}$$

where

Q_{sp} = heat transfer to fluid in the suction plenum per cycle

m_E = the mass transferred externally per cycle

h = specific enthalpy

c_p = specific heat capacity of fluid at constant pressure

T = absolute temperature

subscripts:

s indicates a property mean value in the suction pipe

sv indicates a property mean value at entry to the suction valve

The second stage of heat transfer to the suction fluid occurs within the cylinder, from point 4 to point 1 of the cycle (Fig. 1).

$$Q = Q_{sp} + Q_{cyl} \tag{16}$$

where

Q_{cyl} = heat transfer to the fluid within the cylinder between points 4 and 1 of the cycle

Assuming T_1, T_4 and T_{sv} are known, the first step in the evaluation of Q_{cyl} is to determine the temperature, T_{1a}, which would exist at point 1 if there were no heat transfer during the induction process and if the same indicator diagram applied. The term 'adiabatic suction temperature after induction' is used to denote this quantity. Whereas the throttling process itself produces no temperature change, if ideal gas behaviour is assumed, a temperature increase would come about due to compression as the pressure of the induced charge increased adiabatically to the suction pressure at point 1. A derivation of equation (17) for the adiabatic suction temperature after induction is given in Appendix A.

$$T_{1a} = \frac{1}{1 + \frac{V_4}{V_1}\left(\frac{T_{sv}}{T_4} - 1\right) - \frac{\gamma - 1}{\gamma}\frac{W_{si}}{p_s V_1}} T_{sv} \qquad (17)$$

where

T_{1a} = adiabatic suction temperature after induction

The quantity of heat transfer to the fluid within the cylinder from the start of the induction process to the start of the compression process is given by equation (18) (a derivation is given in Appendix B).

$$Q_{cyl} = \frac{\gamma}{\gamma - 1} p_s V_1 T_{sv}\left(\frac{1}{T_{1a}} - \frac{1}{T_1}\right) \qquad (18)$$

Thus, a procedure has been presented whereby T_{1a}, Q and thence η_{vstq} can be determined from an indicator diagram with the corresponding mean gas temperatures at the points of suction pressure equalization (4 and 1 in Fig. 1) and the mean temperature of the fluid entering the suction valve.

BACKFLOW AND LEAKAGE

Due to late closure of the suction valve, fluid may flow back into the suction plenum. Also, leakage past the piston and past the suction valve when it is seated causes a loss in mass from the cylinder to the suction plenum during compression, during the discharge process, and during re-expansion.

$$m_E = m_I - m_{bls} \qquad (19)$$

where

m_{bls} = backflow through the suction valve and leakage from the cylinder to the suction plenum per cycle

m_I = mass induced

The mass induced is defined as follows:

$$m_I = m_1 - m_4 \qquad (20)$$

where

m_1 = mass of fluid present within the cylinder at the point of suction pressure equalization nearest bottom dead centre (point 1, Fig. 1)

m_4 = mass of fluid within the cylinder at the point of suction pressure equalization nearest top dead centre (point 4, Fig. 1)

The 'suction leakage and backflow ratio' can be defined as follows:

$$\nu_{bls} = \frac{m_{bls}}{m_I} \qquad (21)$$

The 'suction side sealing efficiency' can be defined as

$$\eta_{ss} = 1 - \nu_{bls} \qquad (22)$$

The volumetric efficiency allowing for valve throttling, heat transfer to the suction fluid and leakage is given by

$$\eta_{vstql} = \eta_{vstq}\eta_{ss} \quad (23)$$

where

η_{vstql} = volumetric efficiency taking account of suction fluid throttling, heat transfer and leakage

OTHER FACTORS

Throttling of the fluid entering the suction plenum chamber, from the inlet pipe where the suction pressure is measured, will reduce volumetric efficiency. This effect is included in the equations already listed.

In refrigerant compressors there may also be a loss of refrigerant from the cylinder with lubricating oil which bypasses the piston, as the solubility of refrigerant in the oil can be high at the conditions encountered within the cylinder. This is in fact another form of leakage and would be included in the ratio ν_{bls}.

The possibility that cyclic solubility of refrigerant in lubricating oil may reduce the volumetric efficiency of reciprocating refrigerant compressors has been suggested in references [1] and [6]. The expressions presented in this paper could be modified to take account of any such effect.

CONCLUSIONS

The processes which occur within a reciprocating compressor and particularly the factors which affect its displacement utilization have been described. These factors have been discussed, with reference to the literature, and parameters have been defined to assist in their quantification.

The expressions given in this paper which follow on from the work of Costagliola, are based on the assumption that the fluid can be regarded as an ideal gas. These expressions do not strictly apply to real fluids, particularly refrigerants, which have rather complicated empirical equations of state, or to refrigerant/oil mixtures. Equivalent expressions to those described in this paper can be formulated for real gases, vapours, or even refrigerant/oil mixtures, but, cannot all be written as relatively simple expressions in terms of a small number of measurable parameters. The ideal gas assumption is, however, a useful tool in identifying and quantifying, if a little crudely, the main factors which influence the utilization of volumetric displacement.

Inevitably, generalizations and approximations must be made in practice and in theory, due to the limitations of test measurements, and in order to separate out and roughly quantify effects which, in reality, overlap, interact and are not totally distinct. The theory is put forward as a set of tools for the detailed analysis of displacement utilization in reciprocating compressors.

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APPENDIX A

DERIVATION OF THE ADIABATIC SUCTION TEMPERATURE AFTER INDUCTION

In the definition of the adiabatic suction temperature after induction it is assumed that the actual indicator diagram applies for a hypothetical adiabatic induction process.

Application of the energy equation to the closed system bounded by the cylinder head, cylinder, and piston for the induction process between points 4 and 1 on the indicator diagram shown in Fig. 1 results in the following expression:

$$Q_{cyl} + W_{si} = m_1 c_p T_1 - m_4 c_p T_4 - (m_1 - m_4) c_p T_{sv} \tag{24}$$

where

- Q_{cyl} = heat transfer to the fluid within the cylinder during the induction process
- c_p = specific heat capacity at constant pressure
- T = absolute temperature
- T_{sv} = the mean absolute temperature of the fluid entering the suction valve

From equation (24), if there is no heat transfer, $Q_{cyl} = 0$ and

$$m_{1a} c_p T_{1a} - m_4 c_p T_4 = W_{si} + (m_{1a} - m_4) c_p T_{sv} \tag{25}$$

where

- T_{1a} = the adiabatic suction temperature after induction
- m_{1a} = the mass within the cylinder at point 1 on the indicator diagram after adiabatic induction

Note: Both T_{1a} and m_{1a} are hypothetical rather than actual quantities.

But m = pV/RT and so

$$p_s V_1 \frac{c_p}{R} - p_s V_4 \frac{c_p}{R} = W_{si} + p_s \left(\frac{c_p}{R} \frac{V_1}{T_{1a}} - \frac{V_4}{T_4} \right) T_{sv}$$

where

R = specific gas constant

Also $c_p/R = \gamma/(\gamma - 1)$ and therefore

$$p_s \frac{\gamma}{\gamma - 1} (V_1 - V_4) = W_{si} + p_s \frac{\gamma}{\gamma - 1} \left(\frac{V_1}{T_{1a}} - \frac{V_4}{T_4} \right) T_{sv}$$

$$(V_1 - V_4) \frac{1}{T_{sv}} - \frac{W_{si}}{T_{sv} p_s} \frac{\gamma - 1}{\gamma} = \frac{V_1}{T_{1a}} - \frac{V_4}{T_4}$$

$$\frac{V_1}{T_{1a}} = \frac{V_4}{T_4} + \frac{V_1 - V_4}{T_{sv}} - \frac{W_{si}}{T_{sv} p_s} \frac{\gamma - 1}{\gamma}$$

$$T_{1a} = \frac{V_1}{\dfrac{V_4}{T_4} + \dfrac{V_1 - V_4}{T_{sv}} - \dfrac{W_{si}}{T_{sv} p_s} \dfrac{\gamma - 1}{\gamma}}$$

$$= \frac{1}{1 + \dfrac{V_4}{V_1} \left(\dfrac{T_{sv}}{T_4} - 1 \right) - \dfrac{\gamma - 1}{\gamma} \dfrac{W_{si}}{p_s V_1}} T_{sv} \qquad (26)$$

APPENDIX B
EVALUATION OF THE HEAT TRANSFER TO THE FLUID WITHIN THE CYLINDER DURING INDUCTION

From equations (24) and (25)

$$Q_{cyl} = m_1 c_p T_1 - m_{1a} c_p T_{1a} - (m_1 - m_{1a}) c_p T_{sv}$$

But m = pV/RT and so

$$Q_{cyl} = p_1 V_1 \frac{c_p}{R} - p_1 V_1 \frac{c_p}{R} - \left(\frac{p_s V_1}{T_1} - \frac{p_s V_1}{T_{1a}} \right) \frac{c_p}{R} T_{sv}$$

$$= \frac{\gamma}{\gamma - 1} p_s V_1 T_{sv} \left(\frac{1}{T_{1a}} - \frac{1}{T_1} \right) \qquad (27)$$