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ABSTRACT

A mathematical model of a reciprocating compressor is developed. The following processes are used hereby: gas state in the control volume, real properties of the gas state, heat transfer between gas and chamber walls, pressure oscillation in suction and discharge chambers as also dynamics of driving mechanism of compressor. The differential equation system is numerically solved by multi-time integration according to the Runge-Kutta method of the fourth order.

INTRODUCTION

During operation, small household refrigerating systems with built-in capillary, which is a very stiff damphing element, are influenced by changing external conditions. Hence, the operation of such a refrigerating cycle cannot be studied at stationary but time variable conditions. For this purpose we have developed dynamic models of compressor, evaporator and condenser to combine them into a dynamic model of the whole refrigerating cycle. This work represents a mathematical model of reciprocating compressor with a crank and glide mechanism. To collect all influences of compressor operation, the system of algebraic equations cannot be used but system of unlinear differential equations.

MATHEMATICAL MODEL

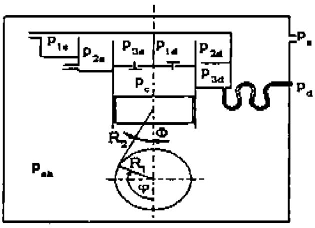

Fig. 1 Scheme of compressor with suction and discharge chambers

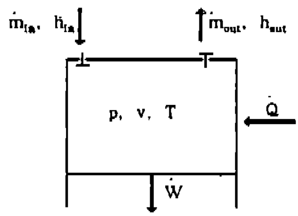

Fig 2. Scheme of control volume of cylinder

The whole compressor has been divided into eight control volumes (Fig 1): shell, three suction chambers, cylinder and three discharge chambers. The gas in the control volume is described by the mass law for an open system by

$$\frac{dm}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out} \tag{1}$$

and by the energy law for the same system by

$$\frac{d(mu)}{dt} = \frac{dQ}{dt} + \sum \dot{m}_{in} h_{in} - \frac{dW}{dt} - \sum \dot{m}_{out} h_{out}, \tag{2}$$

where the change of inner energy according to the time (Eq. 2) depends upon heat transfer, energy flows due to inlet and outlet flow as also upon work done in the control volume. The heat transfer in the cylinder was calculated according to Adair et al /1/, in suction and discharge chambers by /2/ The solution is obtained in the form:

$$\frac{dT}{dt} = \frac{\frac{dQ}{dt} + \dot{m}_{in}(h_{in} - h) - m\frac{dv}{dt}\left[\left(\frac{\partial h}{\partial v}\right)_T - \left(\frac{\partial p}{\partial v}\right)v\right]}{m\left[\left(\frac{\partial h}{\partial T}\right)_v - \left(\frac{\partial p}{\partial T}\right)v\right]} \tag{3}$$

The assumption was made that real gas properties were equal in the volume at the difinite time step, whereas the influence of lubricating oil was neglected. There were also neglected kinetic and potential energies being too small compared to the inner energy. Real gas properties were described by Martin-Downing equation /3/. Partial temperature and specific volume derivatives of pressure and enthalpy are gas properties and therefore determined by Martin-Downing equation. The time derivative of specific volume is Time derivative of the cylinder control volume is obtained by derivation of the equation of working volume /4/, whereas the time derivative of suction and discharge chamber volumes equals zero, hence the first term of Eq. 4 is irrelevant.

The dynamics of the suction and discharge valve is taken from /5/:

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = \frac{A c_p \Delta p}{m_v} \qquad (5)$$

where ζ and ω_0 are damphing coefficient and natural valve oscillation frequency calculated from the valve dimensions. The surface and pressure drag coefficient were determined by /6/. The pressure difference is obtained from pressure before and after valve. The Eq. 5 is general and available for a suction and discharge valve.

The dynamics of the driving mechanism was obtained from the dynamic force equilibrium on the piston and from moments on the shaft. The result is obtained in the form of rotatinal speed changing by time:

$$\ddot{\varphi}(t) = \frac{\eta_m M - \dfrac{m_e R_1 A \dot{\varphi}(t)^2 + F_p}{\cos\Phi} R_1 \sin[\varphi(t) - \Phi]}{I_e + m_e R_1^2 B \sin[\varphi(t) - \Phi]} \qquad (6)$$

The friction was considered by mechanical efficiency η_m. The electromotor moment was calculated by /7/. m_e and I_e as equivalent oscillating mass and equivalent mass moment of inertia were determined by /8/. The gas force F_p was calculated from the piston surface and the pressure difference in cylinder and compressor shell. Φ always equals zero at the glide mechanism. The constants A and B are functions of driving mechanism and time variables.

The heat accumulation in the walls was taken from /9/. The pressure drop in the discharge pipe was described according to /10/ and heat transfer by /11/.

NUMERICAL SOLUTION

The model developed is a combination of differential and algebraic equations as a link hereto. The compressor was divided into eight control volumes, where refrigerant state was modelled. These control volumes are: compressor shell, three suction chambers, cylinder and three discharge chambers. Two differential equations of the first order were used to describe the gas state in control volume. Suction and discharge valves were modelled by differential equations of the second order of damphing oscillation of the spring-mass system. For the heat accumulation in the walls of the compressor shell, cylinder and electromotor the differential equations of the first order were used. The change of the rotational speed was described by the differential equation of the second order. The whole system of the differential equations is consisting of 19 first order equations and three second order equations. Since there is a mutual dependency between variables, a single equation cannot be solved but as a whole system simultaneously. An analytic solution is not possible due to intertwinement of variables, therefore the numerical solution according to the Runge-Kutta method of the fourth order was chosen.

The integrating step depends upon rapidity of change of differential equations. Generally, each differential equation may have a different integration step changing also in single time intervals. In the case of the differential equation system such a step is chosen that a stability of all equations is ensured. Yet, some of these equations could be stable at a much larger step too. In that case also intermediate values are calculated, but they do not affect the accuracy of the results and therefore are to be considered as superfluent. Therefore, the required time step of single differential equation was analized, where the smallest step required is to be found in case of the differential equation of discharge valve motion and the state in the cylinder and the largest at the equation of the shell wall. If all differential equations are to be calculated by the step being the most convenient, there should arise great problems. A decision was made to integrate the equation system in four time intervals. Thereby the calculation of the compressor is nearly three times faster than in a case that there is equal time integration step at all differential equations. All equations are stable in spite of step changing.

RESULTS

The model was checked by simulation of the reciprocating compressors DANFOSS PW 5.5 K11 and TL5A and values compared to those given by the producer /12, 13/. The main specification is shown in Table 1. Besides basis data also values for suction and discharge chambers, dimensions of driving mechanism, discharge pipe and electromotor characteristics were needed.

The producer gives the calculated cooling and electric power at stationary conditions at following conditions:

- condensation temperature 55°C
- gas inlet temperature 32°C
- undercooling temperature of condensate 55°C
- refrigerant R 12.

Fig. 3 shows the comparison between calculated and measured cooling power giving by the producer. In Fig. 4 the comparison of electric power is shown.

Table 1. Main values for **PW5.5 K11** and **TL5A**

Datum \ Compressor	PW5.5 K11		TL5A	
D (m)	0.021		0.017	
R_1 (m)	0.008		0.0112	
R_2 (m)	0.0 (gliding mech.)		0.03396	
V_s (% V_{max})	4.0		3.0	
P_{el} (W)	170		125	
Valves	Suction	Discharge	Suction	Discharge
d (m)	0.0046	0.0044	0.005	0.003
x_{max} (m)	0.001	0.0005	0.0008	0.0005
m_v (kg)	1.56 10^{-4}	1.97 10^{-4}	7.93 10^{-5}	3.95 10^{-5}
ω_0 (s^{-1})	2836.3	7203.8	3372.0	4119.0

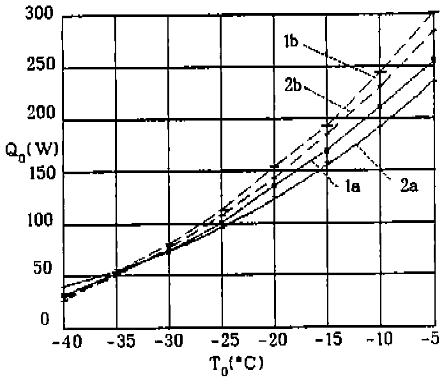

Fig 3. Comparison between cooling capacities of the producer and model results
1 - PW 5.5 K11, 2 - TL5A, a - producer, b - model

The calculated values of cooling power are in good agreement with the values given by the producer being within 15%. To describe the cooling capacity more precisely, the valve dynamics has to be examined experimentally as it affects the cooling capacity very strongly. The delay at opening affects the pressure drop through the valve, whereas the closing affects reverse flow in the cylinder through the discharge valve or from it through the suction valve. Besides the valves also the rotational speed dependant on electric power affects the cooling power directly. The values for electric power as obtained by the model are from 20 to 40% smaller than those given by the producer if no friction is considered in the model. If a mechanical efficiency smaller than 1 is taken in account, then better results also for electric power are obtained (Fig. 4). The mechanical efficiency for small reciprocating compressors is to be found between 85 and 90% /14/.

Since the electromotor load is intensified by evaporating pressure increase, the rotational speed is in decrease (Fig. 5). The changes are nearly the same for both compressors, hence we have shown it for PW 5.5 K11 only.

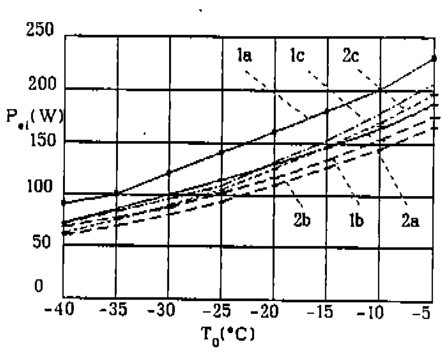

Fig. 4. Comparison between electric powers of the producer and model results
1 - PW 5.5 K11, 2 - TL5A, a - producer, b - model η_m = 1.0,
c - model η_m = 0.85

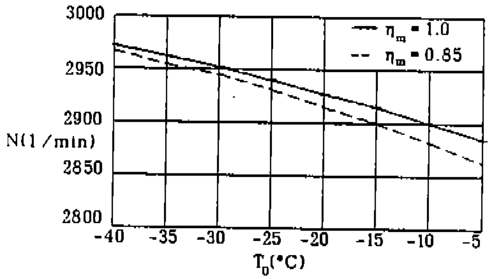

Fig. 5. Rotatinal speed in dependency on evaporating temperature

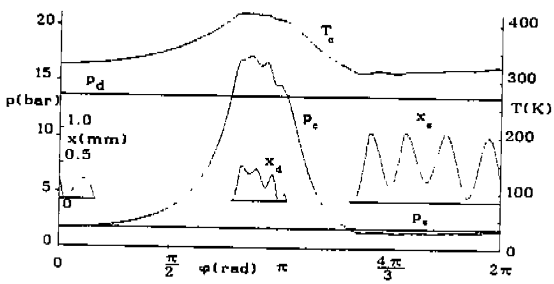

Fig. 6. Diagram of temperature and pressure in the cylinder and valves shifts of PW 5.5 K11 compressor

Figures 3, 4 and 5 show the results of stationary functioning of compressor giving us no information about processes in the compressor alone. Despite stationary external conditions temperatures, pressures and valve deviations are changing by time. Fig. 6 shows the changing of pressure and temperature in the cylinder and the suction and discharge valve lift as to the crank angle. The pressure and temperature values are dependent upon the gas mass in the cylinder, cylinder volume and flowing through valves. The valve dynamics is dependent upon the cylinder pressure. Delayed closing of valves means the loss of the working cycle whereby the refrigerating capacity of compressor is diminished. The temperature is slightly increasing during compression, slightly falling during discharge and expansion and slightly oscillating during suction.

To diminish the oscillating of suction and discharge pressures outside of the compressor, the inlet and outlet of gas is provided through chambers (Fig. 1). In chambers the pressure losses are occuring. The pressure differences between the pressure in the discharge chambers and discharge pressure and between the pressure in the suction chambers and suction pressure are shown in Fig. 7.

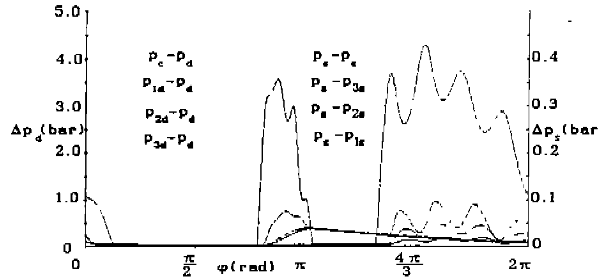

Fig. 7. Pressure differences in suction and discharge chambers

Changing of pressure in the valve affects also the power directed to the piston whereby the rotational speed is influenced too. The electric power depends upon rotational speed. The changing of the rotational speed and electric power of the compressor in dependency on crank angle is shown in Fig. 8, as also the changing of heat transfer coefficient in the cylinder.

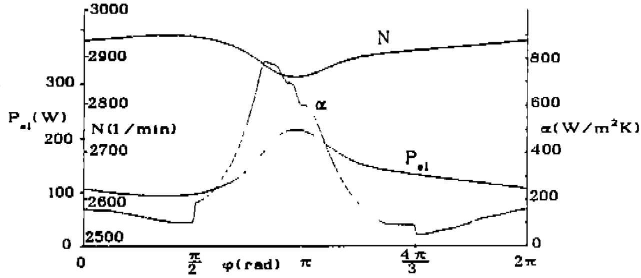

Fig. 8. Electric power, rotational speed and heat transfer coefficient in the cylinder of the working cycle of the compressor PW 5.5 K11

The simulation results in Fig. 6, 7 and 8 are shown at evaporating pressure of 1.83 bar (evaporating temperature -15 C) and condensating pressure of 13.67 bar (55 C). The figures are the result of compressor dynamics simulation only. The measurements of these properties are not available, they are only to be compared to values given by literature attained. Similar results as to time dependency as shown above are to be found in works published by scientists.

CONCLUSION

The dynamic model of reciprocating compressor is shown as a combination of differential and algebraic equations. It includes also modelling of the real gas in suction and discharge chambers. The dynamics of the driving mechanism has been also taken into consideration. The model has been solved by the multi-time integration according to the Runge-Kutta method to speed up the calculation for nearly three times. The results have been compared to the values given by the producer. The pressures in the control volumes, electric power and rotational speed as to the crank angle have been demonstrating.

The comparison to producer's values is rather rough - there is no information about the errors in the single process, so for better description, some processes should be checked experimentally.

NOMENCLATURE

A......... area (m^2)
B.........constant (-)
c_p......... pressure drag coefficient (-)
D.........cylinder diameter (m)
d......... valve bore diameter (m)
F.........force (N)
h.........specific entalpy (J/kg)
I..........mass moment of inertia (kgm)
M.........motor torque (Nm)
m.........mass (kg)
$\dot{m}$.........mass flow (kg/s)
N.........rotational speed (min^{-1})
P.........power (W)
p..........pressure (N/m)
Q......... heat transfer (J)
$\dot{Q}$.........heat transfer flow (W)
R_1.........crank (m)
R_2.........connecting rod length (m)
T. ....temperature (K)
t..........time (s)
u.........specific internal energy (J/kg)
V.........volume (m)
V_0.........clearance volume (% V_{max})
v......... specific volume (m /kg)
W........work (J) m^2K)
x..........valve lift (m)
α.........heat transfer coefficient (W/
Φ......... angle (rad)
$\varphi(t)$.....crank angle (rad) 1/s)
η ....efficiency (-)
ω_0....... natural frequency of valve (
ζ......... damphing coefficient (-)

SUBSCRIPTS

c......... cylinder
d......... discharge
e......... equivalent
el.........elektromotor
in........ inlet
max.....maximal
p......... pressure
s......... suction
out...... outlet
sh........shell
T......... constant temperature
v..........constant volume
valve
0......... refrigeration

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