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Kenji Tojo
*Hitachi Ltd.*

Kunihiko Takao
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Youzou Nakamura
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Kenichi Kawasima
*Hitachi Ltd.*

Yukio Takahashi
*Hitachi Ltd.*

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A Study on the Kinematics of a Variable Displacement Compressor for Automotive Air Conditioners

Kenji Tojo, Kunihiko Takao, Youzou Nakamura
Mechanical Engineering Research Laboratory, Hitachi Ltd.
Tsuchiura, Ibaraki, Japan

Kenichi Kawasima, Yukio Takahashi
Sawa Works, Hitachi Ltd.
Katsuta, Ibaraki, Japan

ABSTRACT

A variable displacement wobble plate compressor is developed to meet the demand for improvement in comfort, driveability standards and fuel economy. The compressor changes its displacement volume to exactly match the vehicle air conditioning requirements, allowing smooth continuous operation. Variable displacement is achieved by the kinematics of a nutating wobble plate, rotating journal, drive shaft assembly and sliding shaft sleeve.

This paper investigates the dynamics of the variable displacement mechanism and develops a mathematical model for evaluating stresses and bearing loads, and for optimizing inertia balance. The model gives detailed geometrical and kinematic information about the behavior of each element. These results of this can be used to estimate the effect of relevant parameters on compressor displacement control.

INTRODUCTION

With the popularization of automotive air conditioners, energy efficiency ratio and high compressor efficiency over a wide range of driving condition must be improved. An air conditioning system must keep a compartment comfortable, in which an automobile operates in various weather conditions. However, an air conditioning system inevitably consumes engine output power, which reduces fuel economy and driveability. To balance the comfortable environment against low fuel consumption, the concept of a variable capacity compressor was born and such type of compressors has been researched and developed.

Recently a new type of continuously variable displacement compressor has been developed and put into commercial production. The compressor has unique displacement control mechanism, which automatically changes its displacement volume to match the system cooling requirement. It controls the pressure differential between the crankcase pressure and the cylinder inlet pressure and uses the piston as the actuator to nutate the wobble plate/rotating journal.

This type of variable displacement compressor gives several advantages.

1) More comfortable environment and better driveability - because the smooth, continuous operation eliminates clutch on/off shock and outlet air temperature fluctuation. Compressor displacement can be controlled depending on engine output power.
2) Lower fuel consumption - since the compressor is controlled automatically to exactly match the vehicle air conditioning demand.
3) Higher reliability and durability - since a variable displacement compressor mostly operates with a small displacement and low pressure ratio. Therefore, the compressor operates at a lower rubbing speed and avoids excessively high discharge pressure and temperature.

This paper outlines an analytical model for evaluating the effect of relevant parameters on the compressor displacement control. The model gives detailed geometrical and kinematic information about the variable displacement compression mechanism.

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A cross-sectional view of the variable displacement compressor is shown in Fig. 1. The basic elements of the compression mechanism are the shaft-drive plate assembly, the sliding shaft sleeve, the rotating journal and the nutating wobble plate. The wobble plate is connected to each of the six pistons with a double ended ball connecting rod. The wobble plate translates the journal motion into linear reciprocation of the pistons.

Variable displacement is achieved by the kinematics of the journal, shaft sleeve and driving plate assembly. As the mechanism destrokes, the shaft sleeve slides up the shaft toward the cylinder head, and the journal angle and position are controlled by the sliding joint in the contour slot located in the drive plate. The contour and location of the slot are selected to give a displacement range from 10 to 100% while maintaining a constant piston head clearance at all stroke positions. Operating compressor efficiency is therefore maintained relatively high at all displacements.

A simple model of the displacement mechanism is shown in Fig. 2. Basic forces affecting compressor displacement control are the combined force, $\Sigma F_{c,s}$, acting on each piston head due to the gas compression in the cylinder, and the combined force, $\Sigma F_{c,c}$, acting on the back of each piston due to the crankcase pressure. Variable displacement is achieved by regulating one of these combined forces as follows:

1. Crankcase pressure control

The control valve is set in the passage between the crankcase and suction chamber. At full load, the valve will maintain a bleed from the crankcase to the suction chamber and there will be no crankcase-suction pressure differential. At partial load, the valve will respond in order to pressurize the crankcase by restricting the passage from the crankcase to suction. The valve will regulate the combined force, $\Sigma F_{c,c}$, and in turn, control resultant moment, $M_{c,e}$, on the wobble plate/journal about the journal pivot pin.
2. Cylinder inlet pressure control

The control valve is set in the passage between the suction chamber and the cylinder inlet as shown in Fig. 1. Bleed from the crankcase to the suction chamber is maintained at any displacement. At full load, the valve will open and there will be no crankcase-suction pressure differential. At partial load, the valve will respond in order to reduce the cylinder inlet pressure by restricting the passage from suction chamber to cylinder inlet and regulate the combined force, \( \Sigma P_{in} \), and in turn, control the resultant moment \( M_{tg} \).

**GEOMETRICAL RELATION**

To reveal the dynamic behaviour of the variable displacement mechanism, the equation of motion of the piston/connecting rod assembly, wobble plate and rotational journal must be derived. For this purpose, the orthogonal coordinates and variables are defined as shown in Fig. 3 and 4. The \( X, Y, Z \) coordinates are fixed on the cylinder block. The origin coincides with the journal pivot pin \( O \), at full stroke, the \( Y \) axis coincides with the opposite center line of the wobble plate anti-rotation mechanism and the \( Z \) axis coincides with the drive shaft center. The \( X, Y, Z \) coordinates are fixed to the journal and rotate with the drive plate. The \( X, Y, Z \) coordinates are fixed to each cylinder bore.

The main variable is the shaft rotating angle \( \phi \). The center of the wobble plate/connecting rod joint, \( Q_{w} \), in \( X, Y, Z \) coordinates is given by,

\[
\begin{align*}
x_{Q_w} &= R_{q} (\sin(\phi'+\theta)\cos\phi - \cos(\phi'+\theta)\cos\alpha\sin\phi) + e_{q}\sin\alpha\sin\phi \\
y_{Q_w} &= R_{q} (\sin(\phi'+\theta)\sin\phi + \cos(\phi'+\theta)\cos\alpha\cos\phi) - e_{q}\sin\alpha\cos\phi \\
z_{Q_w} &= R_{q} \cos(\phi'+\theta)\sin\alpha + e_{q}\cos\alpha
\end{align*}
\]  

where \( \phi' \) is the rotational angle to the wobble plate, \( \alpha \) is the rotation angle of the wobble plate and \( \theta \) gives the angular position of each cylinder bore center. Since the wobble plate is prevented from rotating by the guide arrangement at the periphery of the crankcase, rotational angle \( \phi' \), is defined as:

\[ \phi' = \tan^{-1}(\cos\phi\tan\alpha) \]  

The center of the piston/connecting rod joint, \( Q_{p} \), is given as:

\[
\begin{align*}
x_{Q_p} &= R_{p} \sin\theta \\
y_{Q_p} &= R_{p} \cos\theta \\
z_{Q_p} &= Z_{Q_p} + L_{p} \cdot (X_{Q_p} - X_{Q_w}) + (Y_{Q_p} - Y_{Q_w}) \end{align*}
\]  

When the fluctuation of the wobble plate/connecting rod joint in \( X \) direction is ignored, the piston stroke \( z_{Q_p} \) is given as:

\[
z_{Q_p} = R_{e}(\sin\alpha - \sin\alpha_{p}) + e_{q}(\cos\alpha - \cos\alpha_{p}) + L_{p}(\cos\beta - \cos\beta_{p}) \]  

where \( \beta = \sin^{-1}\left\{ \left( R_{e}(\cos\alpha - e_{q}\sin\alpha) \right) / L_{p} \right\} \) \( a_{p} = \tan^{-1}(\cos(\phi+\theta)\tan\alpha) \), \( \beta_{p} = \sin^{-1}\left\{ \left( R_{e}(\cos\alpha - e_{q}\sin\alpha) \right) / L_{p} \right\} \)

As the mechanism destrokes, the journal angle and shaft sleeve position are controlled by the sliding joint to maintain a constant piston head clearance. The center, \( Q_{b}(0, y_{b}, z_{b}) \), of the instantaneous rotation of the journal is given as:

\[
\begin{align*}
y_{b} &= -(R_{e}\cos\alpha - e_{q}\sin\alpha) \\
z_{b} &= R_{e}(\sin\alpha_{b} - \sin\alpha) + e_{q}(\cos\alpha_{b} - \cos\alpha) + L_{p}(\cos\beta_{b} - \cos\beta) \end{align*}
\]

The location of the sliding joint, \( Q_{b}(0, y_{b}, z_{b}) \), in the contour slot is obtained by

\[
\begin{align*}
y_{b} &= y_{b}'\cos\alpha + y_{b}'\sin\alpha \\
z_{b} &= z_{b}'\cos\alpha - z_{b}'\sin\alpha
\end{align*}
\]
The modeling of the dynamics of the variable displacement compression mechanism consists of the determination of the forces acting along with a force balance on the each compressor element. To simplify the analytical model, we assume that all members of the compression mechanism are solid, and friction forces acting on the moving elements are negligible.

**Piston/connecting rod**

Figure 5 shows the forces acting on the piston/connecting rod assembly. The \( X_i, Y_i, Z_i \) coordinates are fixed to each cylinder bore. Gas force \( F_{g,i} \) acts on the front of the piston due to the gas compression in the cylinder and \( F_{p,i} \) acts on the back of the piston due to crankcase pressure. From the equilibrium equation of the forces and moment exerted on the each element, the resulting forces, \( F_{x,i}, F_{y,i}, \) and \( F_{z,i} \) having \( x, y, \) and \( z \) components, are derived.

**Wobble plate**

Each of the six piston/connecting rod assemblies is connected to the wobble plate. At the connecting rod joint, the resultant forces \( F_{x,1}, F_{y,1}, \) and \( F_{z,1} \) act on the wobble plate as shown in Fig. 6. A ball bearing pressed into the inside diameter of the wobble plate carries the resultant radial force, \( F_r \), and a needle bearing carries the resultant thrust load, \( F_t \), through the wobble plate to the journal. A unique guide arrangement prevents the wobble plate from rotating. Point \( O \) shows the center of the guide/shoe arrangement, where reaction force \( F_w \) acts on the wobble plate. From the equilibrium equation, each force acting on the wobble plate and the coordinates of the thrust center, \( O_x, (l_x, l_y) \), are obtained.

**Rotating journal**

As Fig. 7 shows, the driving torque acts on the guide rib of the journal slide joint. On the other hand, reaction force \( F_s \) acts on the journal pivot pin at the shaft sleeve. Considering all forces acting on the rotating journal, the equilibrium equations in the \( x, y, \) and \( z \) directions are given by

\[
\begin{align*}
F_{sx} + F_{ty} - F_{ex} + F_{cx} &= 0 \\
(F_{ty} + F_{by}) - F_{ty} \cos \alpha + F_{sy} \sin \alpha + F_{ex} \sin \gamma + F_{cy} &= 0 \\
(F_{ty} + F_{by}) - F_{ty} \sin \alpha - F_{sy} \cos \alpha + F_{ex} \cos \gamma &= 0
\end{align*}
\]

where \( F_{sx} = m_x \omega^2_i, \) \( F_{sy} = m_y \omega^2_i \)

Moreover, considering the equilibrium of the moment about the instantaneous nutation center, \( O_x, y, \) and \( z \) axis, the following equations can be derived.

\[
\begin{align*}
F_{sx} \sin \gamma + F_{ex} \sin \gamma + F_{cy} &= 0 \\
(F_{cy} - F_{cy}) 1 + F_{sy} \cos \alpha + F_{ex} \cos \gamma + F_{cy} &= 0 \\
F_{sx} \sin \gamma + F_{ex} \cos \gamma + F_{cy} &= 0
\end{align*}
\]

where \( \gamma = \tan^{-1}\left(\frac{(y-y_o)/z_o}{z_o-z_e}\right) \)

From equations (7)~(9), the resultant forces \( F_{x,1}, F_{y,1}, F_{z,1}, F_{x,2}, \) and \( F_{z,2} \) are given by the following matrix.

\[
\begin{bmatrix}
F_{x,2} \\
F_{y,2} \\
F_{z,2}
\end{bmatrix} = [A]^{-1}
\begin{bmatrix}
F_{x,1} - F_{ex} \\
F_{cy} \cos \alpha - F_{sy} \sin \alpha - F_{cy} \\
F_{cy} \sin \alpha + F_{ex} \cos \alpha \\
F_{cy} \sin \gamma + F_{cy} \cos \gamma
\end{bmatrix}
\]

\[ \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (10) \]
\[ [A] = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ \sin \gamma & 0 & 0 & 1 & 0 \\ \cos \gamma & 0 & 0 & 0 & 1 \\ -(\cos \gamma \sin \rho + \sin \gamma \sin \rho) & 0 & 0 & 0 & 0 \\ \cos \gamma \sin \rho & -1 & 0 & 0 & 1 \\ \sin \gamma \cos \rho & 0 & 0 & 0 & 0 \end{bmatrix} \] 

Then the driving torque \( T_v \) is then given as:

\[ T_v = F_e \lambda \gamma \]  \hspace{1cm} (12)

At full stroke, the reacting force \( (F_{e+} + F_{e-}) \) on the journal pivot pin is positive. When the reacting force becomes negative, the mechanism reduces its displacement until the reacting force becomes zero.

Computer program outline

The program requires the physical characteristics of the mechanism and the operating parameters as inputs. The geometrical properties of the compression mechanism are first calculated. Then, the cylinder pressure and gas forces acting on the pistons are calculated. Finally, forces acting on each element, and the shaft torque are calculated at each shaft rotation. After each cycle has been completed, the crankcase-cylinder inlet pressure differential required for displacement control is determined.

RESULTS AND DISCUSSION

Figure 8 shows the behaviour of the wobble plate for sample variable displacement compressor characterized in Table 1. As the wobble plate is prevented from rotating by the guide/shoe arrangement at the periphery on the \( Y, Z \) plane, the center of the wobble plate/connecting rod joint traces an elliptical orbit on the \( X, Y \) plane twice in every shaft rotation. The fluctuation caused by the wobbling motion of the joint maximizes the amplitude at the chosen angular position of \( \pi/2 \).

The forces acting on the piston are shown in Fig 9. The solid line shows the net gas force and the dotted line shows reacting force in the \( X, Y \) directions. As a result of the elliptical wobbling motion of the wobble plate, the reaction force acts on the piston not only in the \( Y \) direction, but also in the \( X \) direction, during the latter half of the compression process of each cylinder.

The thrust center, \( O_{t1} \), on the rotating journal is shown in Fig 10. The point \( O_{t1} \) slightly changes its position six times while the shaft rotates once. For stable operation of the wobble plate,

\[ R_s \geq \sqrt{l_x^2 + l_y^2} \]  \hspace{1cm} (13)

is necessary, where \( R_s \) is the center radius of thrust bearing.

The crankcase-cylinder inlet pressure differential required for displacement control is shown in Fig. 11 and Fig. 12. In these figures, the pressure differential in the cylinder inlet pressure control system at rotation angle of 20 deg and discharge pressure of 1.5 Mpa is taken as 100 %.

Depending on the operating conditions of the compressor, the control valve regulates the pressure differential and in turn controls the resultant moment on the wobble plate/journal about the journal pivot pin, to move the mechanism to the required displacement. The dotted line shows the pressure differential in the cylinder inlet pressure control system, which requires a slightly larger pressure differential.

These figures clearly show that, the smaller the the rotating angle of the wobble plate/journal is, and higher the operating discharge pressure is, the larger the required pressure differential is.
CONCLUSIONS

An analytical model for evaluating the compressor displacement control mechanism was developed. The model calculates the compressor’s geometry, analyses the forces and moments acting on each element of the compression mechanism and determines the pressure differential required for displacement control.

The result of calculations on a sample variable displacement compressor proved:

1) Because the wobble plate is prevented from rotating by the guide-shoe arrangement, the wobble plate/connecting rod joint traces an elliptical orbit and side forces act on the piston.
2) Both the crankcase pressure control system and cylinder inlet pressure control system can regulate the compressor displacement at the required position. The crankcase pressure control system requires a slightly larger pressure differential.
3) The increase in operating discharge pressure and the decrease in mutating angle of the wobble plate/journal require larger pressure differentials for displacement control.

NOMENCLATURE

- $F_{x}, F_{y}, F_{z}$ : resultant forces on journal pivot pin
- $F_{x}, F_{y}, F_{z}$ : resultant forces on drive plate
- $F_{x}, F_{y}, F_{z}$ : resultant forces on piston/connecting rod joint
- $F_{x}, F_{y}, F_{z}$ : resultant forces on wobble plate/connecting rod joint
- $F_{x}$ : thrust force on wobble plate/rotating journal
- $F_{x}$ : radial forces on wobble plate/rotating journal
- $F_{x}$ : resultant forces on anti rotation mechanism
- $L_{p}$ : connecting rod length
- $O_{d}$ : instantaneous nutation center
- $O_{d}$ : location of sliding joint
- $O_{t}$ : thrust center on the journal
- $O_{w}$ : center of piston/connecting rod joint
- $O_{w}$ : center of wobble plate/connecting rod joint
- $O_{p}$ : center of journal pivot pin
- $P_{c}$ : crankcase pressure
- $P_{s}$ : suction pressure
- $P_{d}$ : cylinder inlet pressure
- $P_{s}$ : discharge pressure
- $R_{c}$ : arranged radius of cylinder bore center
- $R_{r}$ : arranged radius of wobble plate/connecting rod joint center
- $R_{t}$ : center radius of thrust bearing
- $X_{w}, Y_{w}, Z_{w}$ : orthogonal coordinate
- $X_{w}, Y_{w}, Z_{w}$ : orthogonal coordinate fixed to journal
- $Z_{w}$ : orthogonal coordinate fixed to each cylinder bore center
- $Z_{w}$ : piston stroke
- $\alpha$ : mutating angle
- $\gamma$ : angular position of each cylinder bore center
- $\phi$ : shaft rotational angle
- $\phi$ : rotational angle on wobble plate

REFERENCES

2) Y. Nishimura, et al, "Development of Two-Stage Variable Displacement Compressor for Automotive Air Conditioning", SAE Paper No. 850039, 1985
5) J. F. Below, "Dynamics of the Swash Plate Mechanism", Proc. of the 1984 Purdue Compressor Technology Conference
Table 1 Variable displacement compressor specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement volume (cc/rev)</td>
<td>158.0</td>
</tr>
<tr>
<td>Variable range (%)</td>
<td>10-100</td>
</tr>
<tr>
<td>Number of pistons</td>
<td>6</td>
</tr>
<tr>
<td>Cylinder bore (mm)</td>
<td>33.6</td>
</tr>
<tr>
<td>Piston stroke (mm)</td>
<td>29.0</td>
</tr>
<tr>
<td>Nutation angle ((\alpha)) (deg)</td>
<td>22.5</td>
</tr>
</tbody>
</table>

Fig. 3 Coordinates and variables

Fig. 4 Variable displacement mechanism
Fig. 5 Forces acting on the piston/connecting rod

Fig. 6 Forces acting on the wobble plate

Fig. 7 Forces acting on the rotating journal
Pig. 8 Behaviour of the wobble plate

Fig. 9 Resultant forces acting on the piston

Fig. 10 Thrust center \( O_r, (I_x, I_y) \) on rotating journal

Fig. 11 Nutation angle vs. crankcase-cylinder inlet pressure differential required for displacement control

Fig. 12 Discharge pressure vs. crankcase-cylinder inlet pressure differential required for displacement control

Pressure differential in cylinder inlet pressure control system at nutation angle of 20 deg and discharge pressure of 1.5 Mpa is taken as 100%.