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ANANLYSIS OF CONVECTIVE INSTABILITIES OF BINARY NANOFLUIDS

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ABSTRACT

In this paper the thermal convective instabilities of a binary nanofluid is theoretically investigated. In order to analyze a binary nanofluid, an addition factor is proposed under linear stability theory which is applied to the stability analysis of normal fluid. The factor describes the effect of nanoparticles on the convective instability of a basefluid. The Soret effect which represents the thermal effect on mass diffusion in a binary mixture is also considered. The results show that the convective motion in a binary nanofluid sets in easily as the separation ratio (the influence of the Soret effect) increases and the initial concentration of solute decreases. It is interesting that there is a critical condition to make the binary nanofluid most unstable depending on the volume fraction of nanoparticle.

1. INTRODUCTION

When little amount of certain nano-sized particles (nanoparticle) is added in a basefluid, thermal conductivity of the mixture greatly increases (Choi, 1995). Such a new fluid is called nanofluid where nanoparticles are stably and uniformly distributed. In recent, the enhancement of thermal conductivity of nanofluids has been investigated theoretically (Wang *et al*.,2003) and experimentally (Choi *et al*., 2001). But a few researches on convective heat transfer characteristics (Xuan and Li, 2003) and phase change of nanofluids (Das *et al*., 2003) have been conducted. In quite recent, Kim *et al*. (2004) reported a new method to analyze the effect of nanoparticle on the convective instability and convective heat transfer characteristics of nanofluids by using an addition factor.

For special purpose, it is possible to use binary mixture instead of a pure liquid as a basefluid i.e. working fluids in absorption refrigeration, solution used in electro or electroless plating and biofluids, etc. In binary mixture, it is well known that the thermodiffusion effect induces mass diffusion, which is called the Soret effect (Turner, 1973). Generally, the dimensionless separation ratio ψ represents the influence of the Soret effect on convective instability. If $\psi < 0$, an oscillatory instability occurs. On the other hand if $\psi > 0$ the effect is reversed and the onset of stationary instability is induced at a low Rayleigh number as compared to the basefluid value $\mathrm{Ra}_c = 1708$ (Ryskin *et al*., 2003). The objective of the present study is to investigate the thermal convective instability of binary nanofluids by the parametric analysis of an addition factor with respect to the volume fraction of nanoparticles

2. MATHEMATICAL FORMULATION

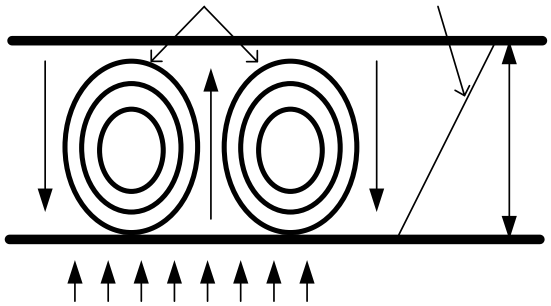

Figure 1: Schematic diagram

When a quiescent, horizontal binary nanofluid layer is heated from below, the buoyancy-driven convection, so-called Rayleigh-Bénard convection will set in with exceeding a certain thermal gradient as shown in Fig.1. The stable concentration is to reduce or enhance the thermal effect on the convective instability depending on the sign of the separation ratio. Based on the Boussinesq approximation and linear theory (Hort et al., 1992), the resulting vertical velocity component, the temperature and the concentration fields are represented in dimensionless forms by

$$\left(\frac{1}{\mathrm{Sc}}\frac{\partial}{\partial\tau}-\nabla^2\right)\nabla^2 w_1=-\mathrm{LeRa}\nabla_1^2\theta_1+\mathrm{Ra_s}\nabla_1^2\phi_1 \tag{1}$$

$$\frac{\partial}{\partial\tau}\theta_1+w_1\frac{\partial\theta_0}{\partial z}=\mathrm{Le}\nabla^2\theta_1 \tag{2}$$

$$\frac{\partial}{\partial\tau}\phi_1+w_1\frac{\partial\phi_0}{\partial z}=\nabla^2\phi_1+\frac{\mathrm{Ra}}{\mathrm{Ra_s}}\mathrm{Le}\,\psi\nabla^2\theta_1 \tag{3}$$

If the temperature profile is linear as shown in Fig. 1, the temperature and concentration profiles are represented as follow:

$$\theta_0=1-z \tag{4}$$

$$\phi_0=1-\beta\Delta T\left(1-z\right) \tag{5}$$

Equation (5) means that the local volume of a basefluid is related with temperature.

With normal mode approximation and the principle of the exchange of stability, the following stability equation is obtained

$$\left(\frac{\partial^2}{\partial z^2}-a^2\right)^3 w^*=\left(1+\psi+\frac{\alpha}{D}\beta_s C_u\right)\mathrm{Ra}a^2 w^* \tag{6}$$

If $\overline{\mathrm{Ra}}=\left(1+\psi+\alpha\beta_s C_u/D\right)\mathrm{Ra}$, Equation (6) is equal to Rayleigh results for stability condition (Chandrasekhar, 1961). Therefore the stability condition for both rigid boundary conditions is given by

$$\left(1+\psi+\alpha\beta_s C_u/D\right)\mathrm{Ra}=1708 \tag{7}$$

The value of right-hand-side in Equation (7) is obtained depending on the boundary conditions.

3. ANALYSIS OF BINARY NANOFLUIDS

There have been attempts that the properties of binary mixture are obtained directly from those of pure components using so-called "mixing rule". Though the deviation between experimental data and theoretical ones is somewhat large, the concept of mixing rule is widely used to estimate the properties of binary mixture due to its convenience for mathematical modeling. So we investigate the properties of nanofluids using only those of a basefluid and nanoparticles with respect to the volume fraction of nanoparticles based on the concept of mixing rule.

The relationship of the density of a nanofluid ρ_{nf} is obtained from the mass conservation as follow:

$$\frac{\rho_{nf}}{\rho_f} = (1-\phi) + \phi\delta_1 \tag{8}$$

If the nanofluid is in a thermal equilibrium state, the heat capacitance relationship is given by

$$\frac{(\rho C_p)_{nf}}{(\rho C_p)_f} = (1-\phi) + \phi\delta_2 \tag{9}$$

It is interesting that ρ_{nf} and $(\rho C_p)_{nf}$ are linearly related with the volume fraction of nanoparticles ϕ. For the viscosity μ, the Brinkman model has been recommended (Xuan and Roetzel, 2000).

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\phi)^{2.5}} \tag{10}$$

For the colloidal suspension, we proposed the following relationship for the thermal conductivity of nanofluid k_{nf} modifying Bruggeman model where the mean field approach is applied (Kim *et al*.,2004).

$$\frac{k_{nf}}{k_f} = \frac{(3\lambda\phi-1)\gamma + \{3(1-\lambda\phi)-1\} + \sqrt{\Delta_B}}{4} \tag{11}$$

where

$$\Delta_B = \left[(3\lambda\phi-1)\gamma + \{3(1-\lambda\phi)-1\}\right]^2 + 8\gamma \tag{12}$$

The effective volume λ means physically the increasing volume quantity by the chaotic motion of nanoparticles such as Brownian motion. By differentiating Equation (8) with respect to the temperature T for a constant volume fraction, the expression of the thermal expansion coefficient β_{nf} is produced as follow:

$$\beta_{nf} = (1-\phi)\beta_f + \phi\beta_p \tag{13}$$

Generally, the thermal expansion coefficient of a liquid β_f is much larger than that of solid particle β_p. So Equation (13) can be simplified as follow:

$$\beta_{nf} = (1-\phi)\beta_f \tag{14}$$

Table 1: Properties of a basefluid (water) and nanoparticles (copper and silver) (Çengel, 1998)

	ρ (kg/m^3)	C_p (J/kgK)	k (W/mK)	γ	δ_1	δ_2
H_2O	997	4,180	0.607			
Cu	8,933	385	401	661	8.96	0.83
Ag	10,500	235	429	714	10.5	0.59

By the same way, the diffusivity D_{nf}, the solutal expansion coefficient $(\beta_s)_{nf}$, and the initial concentration $(C_u)_{nf}$ can be expressed as follows:

$$D_{nf} = (1-\phi)D_f \tag{15}$$

$$(\beta_s)_{nf} = (1-\phi)(\beta_s)_f \tag{16}$$

$$(C_u)_{nf} = (1-\phi)(C_u)_f \tag{17}$$

Using the above mentioned simple relationships, the convective instability and the heat transfer characteristics of a binary nanofluid can be analyzed as a function of the properties of a binary basefluid and nanoparticles, and the volume fraction of nanoparticles. The Rayleigh number for a binary nanofluid which is most important parameter to measure the stability is expressed as follow:

$$\overline{\mathrm{Ra}}_{nf} = g\ \overline{\mathrm{Ra}}_f \tag{18}$$

Here the addition factor g is defined as

$$g = \frac{1+\psi_{nf}+\dfrac{\alpha_{nf}}{D_{nf}}(\beta s)_{nf}(C_u)_{nf}}{1+\psi_f+\dfrac{\alpha_f}{D_f}(\beta s)_f(C_u)_f}\left(\frac{\mathrm{Ra}_{nf}}{\mathrm{Ra}_f}\right) = \frac{1+\psi_{nf}+\dfrac{\alpha_{nf}}{D_{nf}}(\beta s)_{nf}(C_u)_{nf}}{1+\psi_f+\dfrac{\alpha_f}{D_f}(\beta s)_f(C_u)_f} f \tag{19}$$

An addition factor f is defined for a normal basefluid (Kim *et al.*, 2004). On the other hand an addition factor g represents a binary basefluid one. They play the key role to measure the effect of nanoparticles on the convective instability of binary nanofluids. For example, if g has the value of 2, Rayleigh-Bénard convection in a binary nanofluid sets in more easily than that of a binary basefluid by a half of temperature difference.

4. RESULTS AND DISCUSSIONS

The values used in the calculation of g and f are summarized in Table 1. A binary basefluid and nanoparticles are NH_3/H_2O, and silver and copper, respectively. Figure 2 shows the effect of the separation ratio ψ on the addition factor g with respect to the volume fraction of nanoparticles ϕ. As can be seen, g increases up to a certain maximum value and then decreases gradually to zero with respect to ϕ for all ψ. The maximum value indicates an optimal volume fraction at which binary nanofluids become most unstable. And the critical value of g decreases with an increase of ψ. It means that the Soret effect makes binary nanofluids stable. This result is a new feature only in binary nanofluids with difference from normal nanofluids. Consequently, the thermodiffusional property of binary basefluid plays an important role in the analysis of the instability of binary nanofluids. Figure 3 shows the effect of the initial concentration of binary basefluid C_u on the addition factor g. The results show that the addition factor g increases with an increase of the initial concentration C_u. It means that binary nanofluids become easily unstable in a high initial concentration of solute. Addition factors g and f are compared between silver based binary nanofluid and copper based one as shown in Fig. 4. The results show that the binary addition factor g is always higher than the normal addition factor f. It means that binary effect due to the Soret effect and the initial concentration makes binary nanofluids more unstable than normal nanofluids. Furthermore, the profiles in case of silver are always higher than those in case of copper. This is because the thermal conductivity of silver is higher than that of copper.

5. CONCLUSIONS

In the present study, the convective instability of binary nanofluids have been investigated by introducing new addition factors g and f. From the present study, the following conclusions are drawn:

- While the separation ratio ψ acts as a stabilizer to a binary nanofluid, the initial concentration C_u acts as a destabilizer.
- There is a critical condition to make binary nanofluids most unstable depending on the volume fraction of nanoparticles.

Recently various models are being developed to explain the anomalous enhancement of the thermal conductivity of a nanofluid. It is expected that the convective instability and the heat transfer enhancement of a nanofluid can be evaluated with easy adapting these proper models into the present method.

NOMENCLATURE

a	dimensionless wave number	(–)		**Greeks**	
C	concentration	(mol/m^3)	α	thermal diffusivity	(m^2/s)
C_p	heat capacity	(W/kgK)	β	temperature gradient	(K^{-1})
D	diffusivity	(m^2/s)	δ_1	ratio of density, ρ_p / ρ_f	(–)
f, g	addition factor	(–)	δ_2	ratio of capacity, $\left(\rho C_p\right)_p / \left(\rho C_p\right)_f$	(–)
k	thermal conductivity	(W/m^{2}K)	ϕ	volume fraction	(–)
Le	Lewis number, α / D	(–)	ϕ_1	dimensionless concentration	(–)
Ra	Rayleigh number, $g\beta\Delta T d^3 / \alpha\nu$	(–)	γ	ratio of conductivity, k_p / k_f	(–)
Ra$_s$	Rayleigh number, $g\beta_s C_u d^3 / D\nu$	(–)	λ	effective volume	(–)
Sc	Schmidt number, ν / D	(–)	θ_1	dimensionless temperature	(–)
T	temperature	(K)	ρ	density	(kg/m^3)
w	dimensionless vertical velocity	(–)	τ	dimensionless time	(–)
z	dimensionless coordinate	(–)	ψ	separation ratio	(–)

Subscripts			**Superscript**
0	basic state	*	amplitude
b	bottom		
f	basefluid		
nf	nanofluid		
p	nanoparticle		
s	solutal quantity		
u	upper		

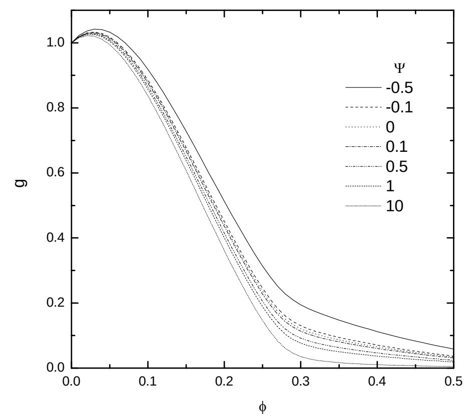

Figure 2: The addition factor g versus ϕ for various ψ

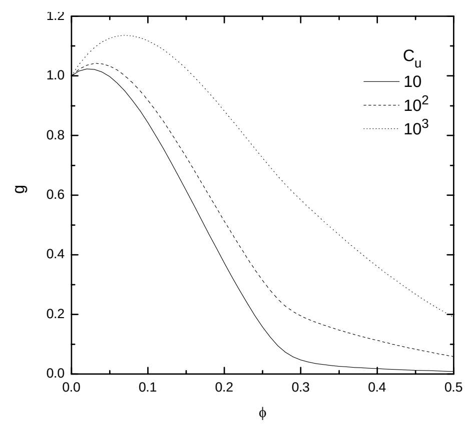

Figure 3: The addition factor g versus ϕ for various C_u

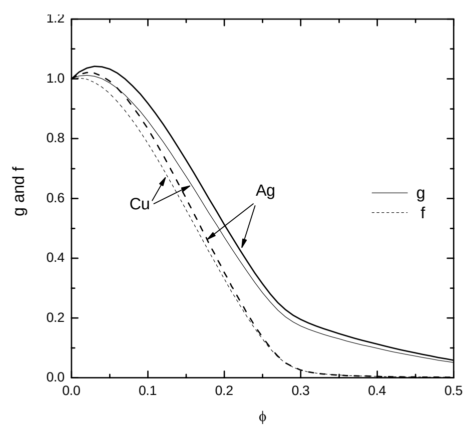

Figure 4: Comparison of addition factors between silver nanofluids and copper ones

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