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Xiaotang Zhou
Purdue University

James E. Braun
Purdue University

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Xiaotang ZHOU*, James E. BRAUN

Ray W. Herrick Laboratories
Purdue University
West Lafayette, IN, USA

*Author for correspondence
Phone: (765) 496-7515; Fax: (765) 494-0787
Email: zhou1@purdue.edu

ABSTRACT

Chilled water cooling coils are important components in commercial cooling systems. Transient modeling of chilled water cooling coils can be useful in the development of feedback controller and fault detection and diagnostic algorithms. In the initial design and validation of these algorithms, it is much more cost effective to use computer models than to run real time experiments. Relatively little work has been published on dynamic modeling of cooling coils, especially when considering dehumidification. This paper describes a physical model that involves the solution of transient energy balances expressed with partial differential equations. Energy storage within fin and tube material and water are considered. Temperature distributions in the direction of water flow are handled by using a finite-volume approximation to the partial differential equations. A fin efficiency method is utilized to characterize the temperature distribution in fins in the air flowdirection. The model performance is investigated by comparing model prediction with test data for an 8-row coil.

1. INTRODUCTION

Chilled water cooling coils are important components in cooling systems. Cooling and dehumidification of moist air are achieved by circulating chilled water through coils and driving moist air over coils. Transient models of cooling coils are useful in developing component and system control strategies, and in exploring fault detection and diagnostic algorithms. In the last 40 years or so, a few models have been developed, and different solution techniques have been adopted in the models.

For cooling coils without dehumidification, Gartner and Harrison (1963) developed an analytical model suitable for control engineering purposes. They derived three partial differential equations, which describe the energy storage in air, fin and tube material, and water. Frequency response transfer functions were obtained to relate the ratios of the changes of air and water outlet temperatures to the variations of air and water inlet temperatures. This model has become the basis of many other transient cooling coil models that have been developed including Gartner and Harrison (1965), Gartner and Daane (1969), Tamm (1969), Tamm and Green (1973), Bhargava et al. (1975).

For cooling and dehumidifying coils, McCullagh *et al*. (1969) developed a row-by-row model considering the dynamic behavior. The energy balance applied to the control volume of moist air, tube, fin, and water leads to five partial differential equations for each row. By finite difference numerical solution, the transient response to a step change in water inlet temperature was obtained. Shekar and Green (1970) used this model to determine the dynamic characteristics of cooling and dehumidifying coils. Clark (1985) developed a model of circular or continuous finned serpentine coils with four or more rows in counter cross-flow configuration. The model is based on the steady-state model presented by Elmahdy and Mitalas (1977). A single time constant is added artificially to the steady state model to predict the dynamics. Ding *et al*. (1990) presented a lumped parameter model based on the effectiveness model developed by Braun *et al.* (1989). A first order model is used to reproduce the responses of coils due to changes in water inlet temperature or water flow rate. These two models are solved by explicit time step difference.

The above models of cooling and dehumidifying coils all assume a complete mixing of air between rows, but in fact, a complete mixing is not possible. More details are required for the coil model, especially along the water flow direction. Numerical solutions are necessary to obtain transient responses of air outlet temperature and humidity, and water outlet temperature, and their spatial and time distributions. In this paper, a detailed physical model governed by partial differential equations is developed and the upwind space difference scheme and the implicit time difference scheme of the finite volume method (Patankar, 1980) are utilized to solve these equations.

2. TEST FACILITY

The Purdue Air Coil Test (PACT) facility, which is located at the Ray W. Herrick Laboratories at Purdue University, was used to perform various experiments for transient model validation. Figure 1 illustrates the PACT facility.

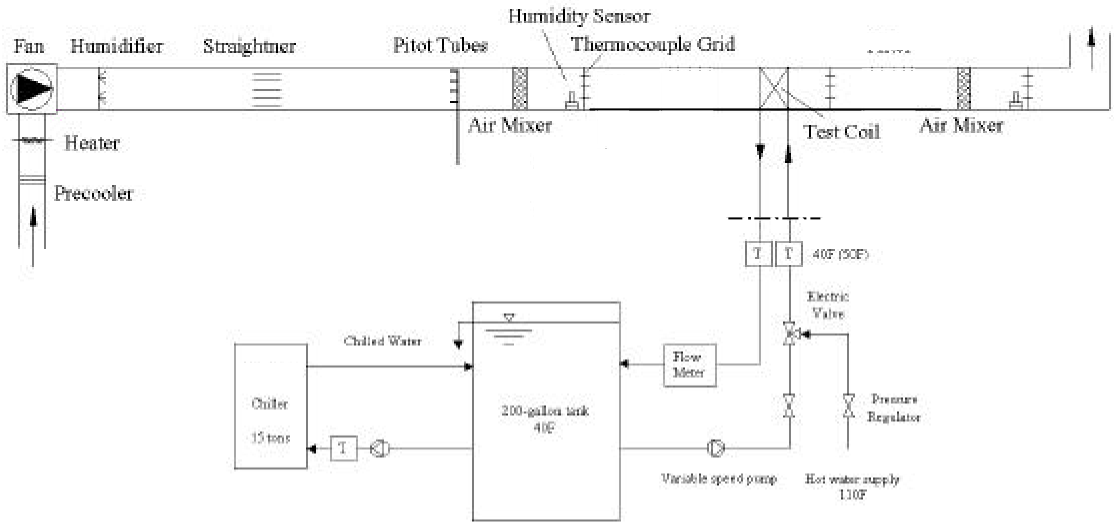

Figure 1: Schematic of PACT facility

Room air is drawn by the fan through the precooler and the electric heater, then passes the steam humidifier and the air velocity measurement section, which consists of the air straightener and the Pitot tube arrays. After that, air enters the insulated test section with the test coil between the upstream and downstream air mixers, thermocouple grids, and humidity sensors. At the outlet, air is discharged to the outdoors.

A tank is utilized to store chilled water coming from the chiller, and to supply water to the test coil. To adjust the water temperature entering the coil, the building hot water is introduced into the waterloop through a 3-way mixing valve. The thermocouples and coriolis-type flow meter are mounted for measurements.

3. MODEL DESCRIPTION

Chilled water cooling coils consist of multiple single cross-flow finned tubes in different kinds of arrangements. A single cross-flow finned tube is the basic element of cooling coil. A transient model of such a tube with the geometry described in Figure 2 is developed in this section.

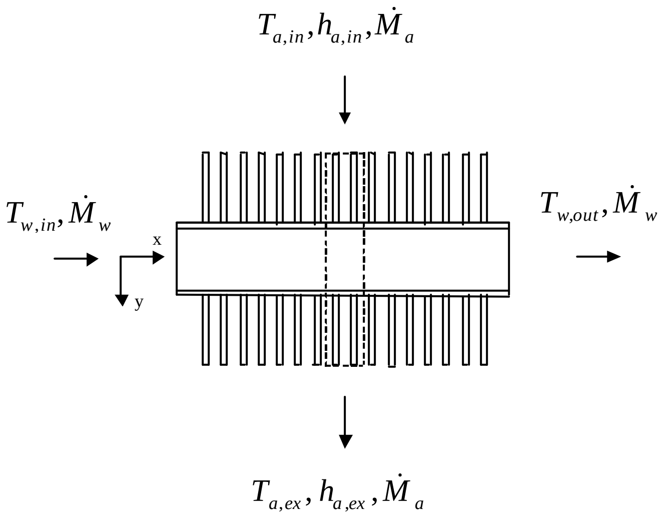

Figure 2: Schematic of single cross-flow finned tube

In this geometry, air flows over finned tube and is cooled and possibly dehumidified due to contact with cold surfaces. The dimension x is measured in the direction of water flow, whereas the dimension y is measured normal to the water flow direction and in the direction of the air flow. The model includes the following assumptions: air and water flows are steady; water is incompressible; ideal gas mixture for air and water vapor; constant specific heats for air and water; negligible conduction in flow directions for the fluids; negligible conduction in the x direction for the fin and tube material; water and air velocities are uniform along the flow directions; uniform air velocity in the x direction; negligible energy storage within the air, the temperature profile in the y direction (within the water, tube material, and fin material follows the steady-state profile.

The transient model of single finned tube without dehumidification is considered first. The governing energy equations in the domain are given in Equation (1) and (2).

$$C_w' \frac{\partial T_w}{\partial t} + \dot{C}_w \frac{\partial T_w}{\partial x} + \frac{1}{R_w'}\left(T_w - T_c\right) = 0 \tag{1}$$

$$C_c' \frac{\partial T_c}{\partial t} + \frac{1}{R_a'}\left(T_c - T_{a,in}\right) + \frac{1}{R_w'}\left(T_c - T_w\right) = 0 \tag{2}$$

Here, T_w and T_c are the local "bulk" temperatures of water and coil material; and $T_{a,in}$ is the air inlet temperature. C'_w and C'_c are the thermal capacitances of water and coil material per unit length in the water flow direction, respectively. C_w is the capacitance rate associated with the water flow. R'_w and R'_a are the thermal resistances for heat transfer per unit length between water and coil material, and coil material and air, respectively. An approach that is consistent with the use of the air inlet temperature in equation (2) is to use the effectiveness-NTU method for the air side. With this approach, the thermal resistances become:

$$R_w' = \frac{1}{h_w P} \text{ and } R_a' = \frac{1}{e_a \dot{C}_a'} \tag{3}$$

where h_w is the convection heat transfer coefficient for water flow through the tube, P is the tube perimeter, e_a is the air side heat transfer effectiveness, and C'_a is the capacitance rate of the air stream per unit length. Furthermore, the air side heat transfer effectiveness is expressed as:

$$e_a = 1 - e^{-NTU_a} \tag{4}$$

where the air side NTU_a is calculated by:

$$NTU_a = \frac{h_a h_a A_a}{\dot{C}_a} \tag{5}$$

and where h_a is the overall fin efficiency for heat transfer, h_a is the convection coefficient for air side heat transfer, A_a is the air side surface area, and C_a is the capacitance rate associated with the air flow. The local outlet air temperature may be written as:

$$T_{a,out} = T_{a,in} + e_a\left(T_c - T_{a,in}\right) \tag{6}$$

Dehumidification adds a significant complication. When a surface is wet, the driving potential for heat and mass transfer is primarily the difference between the air enthalpy and saturated air enthalpy at the surface temperature (Braun *et al*., 1989). With the assumption of Lewis Number of unity, Equation (2) is replaced for the wet case by:

$$C_c^{'}\frac{\partial T_c}{\partial t} + \frac{1}{R_a^{*'}}\left(h_{s,c} - h_{a,in}\right) + \frac{1}{R_w^{'}}\left(T_c - T_w\right) = 0 \tag{7}$$

while Equation (1) remains unchanged. In Equation (7), $h_{a,in}$ is the inlet air enthalpy, $h_{s,c}$ is the saturated air enthalpy at T_c, and $R^{*'}_a$ is a resistance for heat and mass transfer per unit length between coil material and air, which can be expressed as:

$$R_a^{*'} = \frac{1}{e_a^* \dot{M}_a^{'}} \tag{8}$$

where in this case e^*_a is an effectiveness for combined heat and mass transfer determined from:

$$e_a^* = 1 - e^{-NTU_a^*} \tag{9}$$

and where $\dot{M}_a^{'}$ is the air flow rate per unit length and

$$NTU_a^* = \frac{h_a^* h_a^* A_a}{\dot{C}_a} \tag{10}$$

Here, h^*_a is the overall fin efficiency, and h^*_a is the overall coefficient for both air side heat and mass transfer (different from those for heat transfer only). The local outlet air enthalpy may be written as:

$$h_{a,out} = h_{a,in} + e_a^*\left(h_{s,c} - h_{a,in}\right) \tag{11}$$

Equation (6) would still be used to determine the outlet air temperature for a wet coil. The outlet air humidity ratio can be obtained from these two parameters of temperature and enthalpy.

4. SOLUTION TECHNIQUE

A numerical method is utilized to solve the transient model to get the spatial and time distributions of the air outlet temperature and humidity ratio, and water outlet temperature. In order to handle partially dry and partially wet coils, it is most appropriate to discretize the partial differential equations spatially. For each control volume, it is necessary to evaluate whether moisture condenses. This is accomplished by comparing the value of coil material temperature to the dew point temperature of the inlet air. If the coil material temperature is below the dew point temperature, the heat and mass transfer equations apply; otherwise the heat transfer equations apply. For discretizations, the upwind scheme in space and the implicit scheme in time are used. The boundary conditions are the air inlet temperature and humidity, and the water inlet temperature. The initial coil states are provided by solving the governing equations without energy storage terms for the initial inlet conditions. Since a cooling coil is composed of a bunch of single finned tubes, the overall coil model involves using the coil circuiting and the tube element models to determine the water inlet and air inlet states for other tube elements. The air inlet states for each tube section are determined individually by discretization, instead of a fully mixed assumption between rows. For a multi-row coil, the geometry is cross-counterflow and the resulting model has to be solved using iteration. Here, the Jacobi Iterative Algorithm (Murthy and Mathur, 1998) is applied.

5. RESULTS AND DISCUSSION

The transient model can predict various transient responses of coil outlet conditions caused by the change of coil inlet air and water conditions. In order to investigate model performance, it was applied to an 8-row counter cross-flow cooling coil with corrugated fins. The arrangement of the coil is described in Figure 3 and the basic characteristics are listed in Table 1.

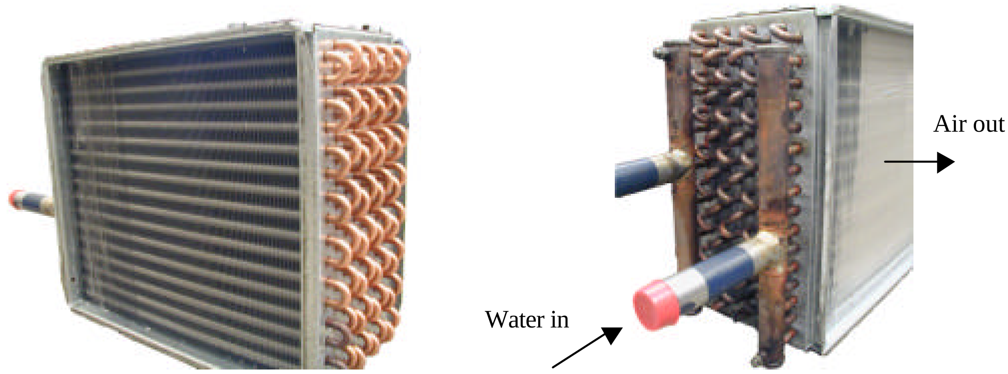

Figure 3: Arrangement of the 8-row coil

Table 1: Characteristics of the finned tube coil

Coil physical parameter	8-row coil
Coil face width	0.6096 m
Coil face height	0.6096 m
Coil depth	0.264 m
Tube material	Copper
Tube outer diameter	0.0127 m
Tube inner diameter	0.0119 m
Tube face spacing	0.0381 m
Tube row spacing	0.033 m
Fin material	Aluminum
Fin thickness	0.0002 m
No. fins on face width	192

Two sets of transient results are introduced to demonstrate the performance of the model, together with the comparisons to the corresponding test results. One case is the transients associated with the step change of air flow rate for the dry coil; the other case is the transients coming along the step change of water flow rate for the wet coil. Table 2 provides the coil inlet conditions of air side and water side for both cases, and also the changes of air and water flow rates.

Table 2: Inlet conditions of the coil

Case	$T_{a,in}$ (°C)	$RH_{a,in}$ (%)	$\dot{M}_a$ (kg/s)	$T_{w,in}$ (°C)	$\dot{M}_w$ (kg/s)
No. 1	23	34.9	1.3 – 0.67	2	0.4
No. 2	27.3	62.1	1.31	2.3	0.44 - 0.24

For Case No. 1, the air flow rate changed from 1.3 kg/s to 0.67 kg/s, while the other inlet conditions did not change. Since the coil was totally dry during the change, the transient responses of the air and water outlet temperatures are shown, not including the air outlet humidity. Figure 4 shows the comparisons of model prediction and test results for air and water outlet temperatures. The y-coordinate is expressed in the dimensionless form of

$$\Theta_{out} = \frac{T_{out} - T_{out,\min}}{T_{out,\max} - T_{out,\min}} \tag{12}$$

where T_{min} and T_{max} are the minimum and maximum temperatures of air and water during the transient process. The non-dimensional parameters are given in Table 3.

Table 3: Non-dimensional parameters in Case No. 1

Case No. 1	$T_{a,out,min}$ (°C)	$T_{a,out,max}$ (° C)	$T_{w,out,min}$ (°C)	$T_{w,out,max}$ (° C)
TEST	6.8	10.1	9.5	12.2
MODEL	6.4	10.5	9.1	12

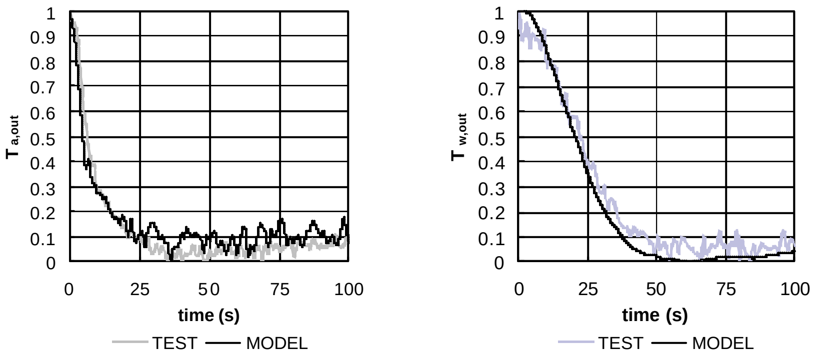

Figure 4: Comparisons of the outlet conditions in Case No. 1

It is seen in Figure 4 that the dimensionless transient responses of air and water outlet temperatures match well between model prediction and experimental data, neglecting the small fluctuation caused by the test setup itself. Also, the non-dimensional parameters in Table 4 agree with each other in a reasonable range. The transient model does a good job in simulating the dynamics of a dry coil.

Dehumidification occurred for Case 2, so the transient responses for the air and water outlet temperatures, and also the air outlet humidity are given. Figure 5 shows the comparisons of model prediction and experimental data for all the outlet parameters. The non-dimensional parameters are giv en in Table 4. Here,

$$\Psi_{out} = \frac{W_{out} - W_{out,\min}}{W_{out,\max} - W_{out,\min}} \tag{13}$$

is used to non-dimensionalize the air outlet humidity ratio.

Table 4: Non-dimensional parameters in Case No. 2

Case No. 2	$T_{a,out,min}$ (°C)	$T_{a,out,max}$ (° C)	$W_{a,out,min}$ (g/kg)	$W_{a,out,max}$ (g/kg)	$T_{w,out,min}$ (°C)	$T_{w,out,max}$ (°C)
TEST	15.8	18.5	10.5	12.8	16.6	18.8
MODEL	15.3	18.3	11.4	13	16.8	18.4

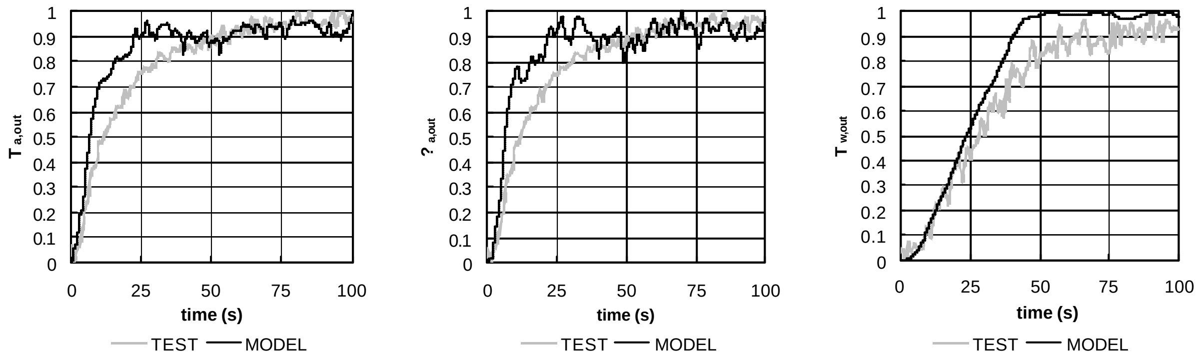

Figure 5: Comparisons of the outlet conditions in Case No. 2

Figure 5 demonstrates the transient responses of a wet coil. It shows that the model predictions react faster than the experimental data for all outlet parameters. The assumption of negligible energy storage of condensate water on the coil surface could be the reason for the discrepancy. Apparently the model needs some improvement to handle the dehumidifying case.

6. CONCLUSIONS

A transient model of chilled water cooling coils was developed from first principals and the performance was investigated for an 8-row coil. The model was found to predict the dynamics of a dry coil well, but for a wet coil, the model simulation responds faster than the test result. Further work is expected to improve the model performance when dehumidification occurs.

NOMENCLATURE

A	surface area	(m^2)
C	thermal capacity	(J/K)
C	capacitance rate	(W/K)
h	convection coefficient	(W/ m^2·K)
	specific enthalpy	(J/kg)
$\dot{M}$	mass flow rate	(kg/s)
NTU	number of heat transfer unit	(-)
P	tube perimeter	(m)
R	thermal resistance	(K/W)
t	time variable	(s)
T	temperature	(K)
W	humidity ratio	(kg/kg)
x	space variable	(m)
e	coil effectiveness	(-)
?	overall fin efficiency	(-)

Subscripts

a	air
c	coil material
w	water
in	inlet
out	outlet
s	saturation

Superscripts

’	per unit length (/m)
*	heat and mass transfer

REFERENCES

Bhargava, S.C., McQuiston F.C., Zirkle L.D., 1975, Transfer Functions for Crossflow Multirow Heat Exchangers, *ASHRAE Transactions*, vol. 81, part 2: p. 294-314.

Braun, J.E., Klein S.A., Mitchell J.W., 1989, Effectiveness Models for Cooling Towers and Cooling Coils, *ASHRAE Transactions*, vol. 95, part 2: p. 164-174.

Chandra Shekar, S., Green G.H., 1970, Dynamic Study of a Chill Water Cooling and Dehumidifying Coil, *ASHRAE Transactions*, vol. 76, part 2: p. 36-51.

Clark, D.R., 1985, Type 12: Cooling or Dehumidifying Coil, *HVACSIM+ Reference Manual*, National Bureau of Standards: p. 63-68.

Ding, X., Eppe J.P., Lebrun J., Wasacz M., 1990, Cooling Coil Model to be Used in Transient and /or Wet Regimes. Theoretical Analysis and Experimental Validation. *Proceedings of the third international conference on System Simulation in Building*, Liège, Belgium: p. 405-441.

Elmahdy, A.H., Mitalas G.P., 1977, A Simple Model for Cooling and Dehumidifying Coils for Use in Calculating Energy Requirements for Buildings, *ASHRAE Transactions*, vol. 83, part 2: p. 103-117.

Gartner, J.R., Harrison H.L., 1963, Frequency Response Transfer Function for a Tube in Crossflow, *ASHRAE Transactions*, vol. 69: p. 323-330.

Garnter, J.R., Harrison H.L., 1965, Dynamic Characteristics of Water-to-Air Crossflow Heat Exchangers, *ASHRAE Transactions*, vol. 71: p. 212-224.

Gartner, J.R., Daane L.E., 1969, Dynamic Response Relations for a Serpentine Crossflow Heat Exchanger with Water Velocity Disturbance, *ASHRAE Transactions*, vol. 75, part 2: p. 53-68.

McCullagh, K.R., Green G.H., Chandra Shekar S., 1969, An Analysis of Chill Water Cooling Dehumidifying Coils Using Dynamic Relationships, *ASHRAE Transactions*, vol. 75, part 2: p. 200-209.

Murthy, J.Y., Mathur, S.R., 1998, *Numerical Methods in Heat, Mass, and Momentum Transfer*.

Patankar, S.V., 1980, *Numerical Heat Transfer and Fluid Flow*, Hemisphere Pub. Corp., Washington, McGraw-Hill, New York.

Tamm, H., 1969, Dynamic Response Relations for Multi-Row Crossflow Heat Exchangers, *ASHRAE Transactions*, vol. 75, part 1: p. 69-80.

Tamm, H., Green G.H., 1973, Experimental Multi-Row Crossflow Heat Exchanger Dynamics, *ASHRAE Transactions*, vol. 79, part 2: p. 9-18.

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