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T. E. Clark S. T. Love

M. Nitta T. ter Veldhuis

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Gauging nonlinear supersymmetry

T. E. Clark,[1,*] S. T. Love,[1,†] Muneto Nitta,[2,‡] and T. ter Veldhuis[3,§]

[1]*Department of Physics, Purdue University, West Lafayette, Indiana 47907-1396, USA*
[2]*Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*
[3]*Department of Physics & Astronomy, Macalester College, Saint Paul, Minnesota 55105-1899, USA*
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Coset methods are used to construct the action describing the dynamics associated with the spontaneous breaking of the local supersymmetries. The resulting action is an invariant form of the Einstein-Hilbert action, which in addition to the gravitational vierbein, also includes a massive gravitino field. Invariant interactions with matter and gauge fields are also constructed. The effective Lagrangian describing processes involving the emission or absorption of a single light gravitino is analyzed.

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I. INTRODUCTION

The low energy dynamics of a theory with spontaneously broken global supersymmetry includes the Nambu-Goldstone fermion, the Goldstino, of the broken SUSY. The action for the Goldstino fields, denoted by Weyl spinors λ_α and $\bar{\lambda}_{\dot{\alpha}}$, with $\alpha = 1, 2$ and $\dot{\alpha} = 1, 2$, was found by Akulov and Volkov [1] to be

$$\begin{aligned}\Gamma &= -F^2 \int d^4x \det A \\ &= -F^2 \int d^4x \det(\delta_\mu{}^m - i\lambda \overleftrightarrow{\partial}_\mu \sigma^m \bar{\lambda}),\end{aligned} \tag{1.1}$$

where $A_\mu{}^m = (\delta_\mu{}^m - i\lambda \overleftrightarrow{\partial}_\mu \sigma^m \bar{\lambda})$ is the Akulov-Volkov vierbein and F^2 is the Goldstino decay constant, a scale set by the dynamics responsible for SUSY breaking. When the supersymmetry is realized as a local symmetry, the super-Higgs mechanism becomes operational and the Goldstino provides the spin $\frac{1}{2}$ components of the now massive spin $\frac{3}{2}$ gravitino fields, which are denoted by $\psi_{\mu\alpha}$ and $\bar{\psi}_{\mu\dot{\alpha}}$ with $\mu = 0, 1, 2, 3$. On the other hand, the space-time coordinate invariance remains unbroken and so the graviton, which is described by the vierbein $e_\mu{}^m$, with $m = 0, 1, 2, 3$ remains a massless spin 2 field [2–5]. The purpose of this paper is to derive the low energy action governing the dynamics of these degrees of freedom and then examine some of its consequences. This is achieved using the method of nonlinear realizations [6]. In Sec. II, this coset method is applied to the case of the local super-Poincaré group $\mathcal{SP}_4$ [2,3]. The generalized locally covariant Maurer-Cartan oneform is constructed. It includes the vierbein as well as the locally covariant derivatives of the Goldstino fields which involve the gravitino fields and the spin (and hence affine) connections. The general covariant derivatives of these covariant one-forms are used as the building blocks of the action [7]. In Sec. III, the invariant action for spontaneously broken supergravity is constructed from the above mentioned Maurer-Cartan one-forms and their derivatives. The action is transformed to the unitary gauge which more clearly reveals the physical content of the supergravity vierbein and massive gravitino. Note that the nonlinear realization of local symmetry which we construct is achieved using only the graviton and gravitino degrees of freedom. There is no need to include other degrees of freedom which appear in linear realizations of supergravity. Alternatively, the action for spontaneously broken supergravity was obtained using superspace coset methods in [8]. In Sec. IV, the couplings to matter and gauge fields are also determined using the covariant one-forms. In particular, the interactions of the gravitino with the standard model particles describing its single emission or absorption are delineated. These match those obtained previously [9] using the equivalence theorem [10] to describe high energy processes involving the helicity $\pm\frac{1}{2}$ components of the gravitino.

II. THE COSET CONSTRUCTION

In this section, we construct the nonlinear realization of the super-Poincaré group of transformations $\mathcal{SP}_4$ when it is spontaneously broken to the Poincaré subgroup $\mathcal{P}_4$. The method of nonlinear realizations begins with the construction of the coset element $\Omega \in \mathcal{SP}_4/SO(1,3)$

$$\Omega(x) = e^{ix^\mu P_\mu} e^{i[\lambda^\alpha(x) Q_\alpha + \bar{\lambda}_{\dot{\alpha}}(x) \bar{Q}^{\dot{\alpha}}]}, \tag{2.1}$$

where the $SO(1, 3)$ subgroup is the Lorentz stability group. The coset elements are labeled by the space-time coordinates x^μ and the superspace coordinates $\lambda_\alpha(x)$ and $\bar{\lambda}_{\dot{\alpha}}(x)$, which are the Weyl spinor Goldstino fields. The generators of $\mathcal{SP}_4$ are the energy-momentum operator P_μ, the supersymmetry Weyl spinor charges Q_α and $\bar{Q}_{\dot{\alpha}}$ and the angular momentum operator $M_{\mu\nu}$. They obey the usual SUSY algebra

*Electronic address: clark@physics.purdue.edu
†Electronic address: loves@physics.purdue.edu
‡Electronic address: nitta@th.phys.titech.ac.jp
§Electronic address: terveldhuis@macalester.edu

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$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} + \eta_{\nu\sigma}M_{\mu\rho} - \eta_{\nu\rho}M_{\mu\sigma})$$
$$[M_{\mu\nu}, P_\lambda] = i(P_\mu \eta_{\nu\lambda} - P_\nu \eta_{\mu\lambda})$$
$$[M_{\mu\nu}, Q_\alpha] = -\frac{1}{2}(\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta$$
$$[M_{\mu\nu}, \bar{Q}_{\dot\alpha}] = \frac{1}{2}(\bar\sigma^{\mu\nu})_{\dot\alpha}{}^{\dot\beta}\bar{Q}_{\dot\beta} \qquad \{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu, \tag{2.2}$$

with all remaining commutators vanishing. Here $\eta_{\mu\nu}$ is the Minkowski space metric tensor defined with signature $(+1, -1, -1, -1)$.

Left multiplication of Ω by an $\mathcal{SP}_4$ group element g characterized by the local infinitesimal parameters $\epsilon^\mu(x)$, $\xi_\alpha(x)$, $\bar\xi_{\dot\alpha}(x)$, $\alpha^{\mu\nu}(x)$ so that

$$g(x) = e^{i\epsilon^\mu(x)P_\mu} e^{i\xi^\alpha(x)Q_\alpha} e^{i\bar\xi_{\dot\alpha}(x)\bar{Q}^{\dot\alpha}} e^{(i/2)\alpha^{\mu\nu}(x)M_{\mu\nu}}, \tag{2.3}$$

results in transformations of the space-time coordinates and the Nambu-Goldstone fields according to the general form [6]

$$g(x)\Omega(x) = \Omega'(x')h(x). \tag{2.4}$$

The transformed coset element, Ω', is a function of the transformed space-time coordinates and the total variations of the fields so that

$$\Omega'(x') = e^{ix'^\mu P_\mu} e^{i[\lambda'^\alpha(x')Q_\alpha + \bar\lambda'_{\dot\alpha}(x')\bar{Q}^{\dot\alpha}]}. \tag{2.5}$$

In the $\mathcal{SP}_4$ case, h is field independent and is simply an element of the Lorentz subgroup $SO(1, 3)$ given by

$$h = e^{(i/2)\alpha^{\mu\nu}(x)M_{\mu\nu}}. \tag{2.6}$$

Exploiting the algebra of the $\mathcal{SP}_4$ charges displayed in Eq. (2.2), along with use of the Baker-Campbell-Hausdorff formulae, the local $\mathcal{SP}_4$ transformations are obtained as coordinate variations and total variations of the fields

$$\begin{aligned} x'^\mu &= x^\mu + \Delta x^\mu \\ &= x^\mu + \epsilon^\mu(x) + i[\xi(x)\sigma^\mu\bar\lambda(x) - \lambda(x)\sigma^\mu\bar\xi(x)] \\ &\quad - \alpha^{\mu\nu}(x)x_\nu \\ \lambda'_\alpha(x') &= \lambda_\alpha(x) + \Delta\lambda_\alpha(x) \\ &= \lambda_\alpha(x) + \xi_\alpha(x) + \frac{i}{4}\alpha_{\mu\nu}(x)(\sigma^{\mu\nu})_\alpha{}^\beta\lambda_\beta(x) \\ \bar\lambda_{\dot\alpha}(x') &= \bar\lambda_{\dot\alpha}(x) + \Delta\bar\lambda_{\dot\alpha}(x) \\ &= \bar\lambda_{\dot\alpha}(x) + \bar\xi_{\dot\alpha}(x) + \frac{i}{4}\alpha_{\mu\nu}(\bar\sigma^{\mu\nu})_{\dot\alpha\dot\beta}\bar\lambda^{\dot\beta}(x). \end{aligned} \tag{2.7}$$

The spontaneously broken SUSY transformations are nonlinearly realized as intrinsic variations of the fields, $\delta\lambda = \Delta\lambda - \Delta x^\mu\partial_\mu\lambda$ with analogous results for $\bar\lambda$. The Nambu-Goldstone fields λ_α and $\bar\lambda_{\dot\alpha}$ transform inhomogeneously under the broken local SUSY transformations Q_α and $\bar{Q}_{\dot\alpha}$, respectively. Thus these broken transformations can be used to transform to the unitary gauge in which both λ_α and $\bar\lambda_{\dot\alpha}$ vanish. This will be done in Sec. III in order to exhibit the physical degrees of freedom in a more transparent fashion.

The $\mathcal{SP}_4$ transformations induce a coordinate and field dependent general coordinate transformation of the space-time coordinates. From the x^μ coordinate transformation given above, the general coordinate Einstein transformation for the space-time coordinate differentials is given by

$$dx'^\mu = dx^\nu G_\nu{}^\mu(x), \tag{2.8}$$

with $G_\nu{}^\mu(x) = \partial x'^\mu/\partial x^\nu$. The $\mathcal{SP}_4$ invariant interval can be formed using the metric tensor $g_{\mu\nu}(x)$ so that $ds^2 = dx^\mu g_{\mu\nu}(x)dx^\nu = ds'^2 = dx'^\mu g'_{\mu\nu}(x')dx'^\nu$ where the metric tensor transforms as

$$g'_{\mu\nu}(x') = G_\mu^{-1\rho}(x)g_{\rho\sigma}(x)G_\nu^{-1\sigma}(x). \tag{2.9}$$

The form of the vierbein (and hence the metric tensor) as well as the locally covariant derivatives of the Goldstino fields and the spin connection can be extracted from the locally covariant Maurer-Cartan oneform, $\omega \equiv \Omega^{-1}D\Omega$, which can be expanded in terms of the generators as

$$\begin{aligned} \omega &= \Omega^{-1}D\Omega \equiv \Omega^{-1}(d + i\hat{E})\Omega \\ &= i\left[\omega^m P_m + \omega_Q^\alpha Q_\alpha + \bar\omega_{\bar{Q}\dot\alpha}\bar{Q}^{\dot\alpha} + \frac{1}{2}\omega_M^{mn}M_{mn}\right]. \end{aligned} \tag{2.10}$$

Here Latin indices $m, n = 0, 1, 2, 3$, are used to distinguish tangent space local Lorentz transformation properties from space-time Einstein transformation properties which are denoted using Greek indices. In what follows, Latin indices are raised and lowered using of the Minkowski metric tensors, η^{mn} and η_{mn}, while Greek indices are raised and lowered with use of the curved metric tensors, $g^{\mu\nu}$ and $g_{\mu\nu}$. Since the Nambu-Goldstone fields vanish in the unitary gauge it is useful to exhibit the oneform gravitational fields in terms of their translated form

$$\hat{E} = e^{ix^\mu P_\mu} E e^{-ix^\mu P_\mu}. \tag{2.11}$$

The oneform gravitational fields E have the expansion in terms of the charges as

$$E = E^m P_m + \psi^\alpha Q_\alpha + \bar\psi_{\dot\alpha}\bar{Q}^{\dot\alpha} + \frac{1}{2}\gamma^{mn}M_{mn}. \tag{2.12}$$

Similarly expanding $\hat{E}$ as

$$\hat{E} = \hat{E}^m P_m + \hat\psi^\alpha Q_\alpha + \hat{\bar\psi}_{\dot\alpha}\bar{Q}^{\dot\alpha} + \frac{1}{2}\hat\gamma^{mn}M_{mn}, \tag{2.13}$$

one finds the various fields are related according to

$$\begin{aligned} \hat{E} &= E^m + \gamma^{mn}x_n \qquad \hat\psi^\alpha = \psi^\alpha \\ \hat{\bar\psi}_{\dot\alpha} &= \bar\psi_{\dot\alpha} \qquad \hat\gamma^{mn} = \gamma^{mn}. \end{aligned} \tag{2.14}$$

Defining the oneform gravitational fields to transform as a gauge field so that

$$\hat{E}'(x') = g(x)\hat{E}(x)g^{-1}(x) - ig(x)dg^{-1}(x), \qquad (2.15)$$

the covariant Maurer-Cartan oneform transforms analogously to the way it varied for global transformations:

$$\omega'(x') = h(x)\omega(x)h^{-1}(x) + h(x)dh^{-1}(x), \qquad (2.16)$$

with $h = e^{(i/2)\alpha^{mn}(x)M_{mn}}$. Expanding in terms of the $\mathcal{SP}_4$ charges, the individual one-forms transform according to their local Lorentz nature as

$$\begin{aligned}
\omega'^m(x') &= \omega^n(x)\Lambda_n{}^m(\alpha(x)) \\
\omega'_{Q\alpha}(x') &= D_\alpha^{(1/2,0)\beta}(\alpha(x))\omega_{Q\beta} \\
\bar{\omega}'^{\dot{\alpha}}_{\bar{Q}}(x') &= D^{(0,1/2)\dot{\alpha}}{}_{\dot{\beta}}(\alpha(x))\bar{\omega}^{\dot{\beta}}_{\bar{Q}} \\
\omega'^{mn}_M(x') &= \omega^{rs}_M(x)\Lambda_r{}^m(\alpha(x))\Lambda_s{}^n(\alpha(x)) - d\alpha^{mn}(x).
\end{aligned} \qquad (2.17)$$

For infinitesimal transformations, the local Lorentz transformations are $\Lambda_n{}^m(\alpha(x)) = \delta_n{}^m + \alpha_n{}^m(x)$ and the spinor transformations are $D_\alpha^{(1/2,0)\beta}(\alpha(x)) = \delta_\alpha{}^\beta + \frac{i}{4}\alpha_{mn}(x)\times(\sigma^{mn})_\alpha{}^\beta$ and $D^{(0,1/2)\dot{\alpha}}{}_{\dot{\beta}}(\alpha(x)) = \delta^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{i}{4}\alpha_{mn}(\bar{\sigma}^{mn})^{\dot{\alpha}}{}_{\dot{\beta}}$, while the infinitesimal local $\mathcal{SP}_4$ transformations of the gravitational one-forms take the form

$$\begin{aligned}
\hat{E}'^m &= \hat{E}^m + \hat{\gamma}^{mn}\epsilon_n + 2i(\xi\sigma^m\bar{\hat{\psi}} - \hat{\psi}\sigma^m\bar{\xi}) - \alpha^{mn}\hat{E}_n - d\epsilon^m \\
\hat{\psi}'^\alpha &= \hat{\psi}^\alpha - \frac{i}{4}\alpha_{mn}(\hat{\psi}\sigma^{mn})^\alpha + \frac{i}{4}\hat{\gamma}_{mn}(\xi\sigma^{mn})^\alpha - d\xi^\alpha \\
\bar{\hat{\psi}}'_{\dot{\alpha}} &= \bar{\hat{\psi}}_{\dot{\alpha}} - \frac{i}{4}\alpha_{mn}(\bar{\hat{\psi}}\bar{\sigma}^{mn})_{\dot{\alpha}} + \frac{i}{4}\hat{\gamma}_{mn}(\bar{\xi}\bar{\sigma}^{mn})_{\dot{\alpha}} - d\bar{\xi}_{\dot{\alpha}} \\
\hat{\gamma}'^{mn} &= \hat{\gamma}^{mn} + (\alpha^{mr}\hat{\gamma}^n{}_r - \alpha^{nr}\hat{\gamma}^m{}_r) - d\alpha^{mn}.
\end{aligned} \qquad (2.18)$$

Using the Feynman formula for the variation of an exponential operator in conjunction with the Baker-Campbell-Hausdorff formulae, the individual one-forms appearing in the above decomposition of the covariant Maurer-Cartan oneform are secured as

$$\begin{aligned}
\omega^m &= dx^m - i[\lambda\sigma^m(d\bar{\lambda} + 2\bar{\psi}) - (d\lambda + 2\psi)\sigma^m\bar{\lambda}] \\
&\quad + E^m + \frac{1}{4}\gamma_{rs}\lambda(\sigma^m\bar{\sigma}^{rs} + \sigma^{rs}\sigma^m)\bar{\lambda} \\
\omega^\alpha_Q &= d\lambda^\alpha + \psi^\alpha - \frac{i}{4}\gamma_{mn}(\lambda\sigma^{mn})^\alpha \\
\bar{\omega}_{\bar{Q}\dot{\alpha}} &= d\bar{\lambda}_{\dot{\alpha}} + \bar{\psi}_{\dot{\alpha}} - \frac{i}{4}\gamma_{mn}(\bar{\lambda}\bar{\sigma}^{mn})_{\dot{\alpha}} \\
\omega^{mn}_M &= \gamma^{mn}.
\end{aligned} \qquad (2.19)$$

The covariant coordinate differential ω^m is related to the space-time coordinate differential dx^μ by the vierbein $e_\mu{}^m$ so that $\omega^m = dx^\mu e_\mu{}^m$

$$\begin{aligned}
e_\mu{}^m &= A_\mu{}^m + E_\mu{}^m - 2i(\lambda\sigma^m\bar{\psi}_\mu - \psi_\mu\sigma^m\bar{\lambda}) \\
&\quad + \frac{1}{4}\gamma^{rs}_\mu\lambda(\sigma^m\bar{\sigma}_{rs} + \sigma_{rs}\sigma^m)\bar{\lambda}.
\end{aligned} \qquad (2.20)$$

The one-forms and their covariant derivatives are the building blocks of the locally $\mathcal{SP}_4$ invariant action. Indeed a mth-rank contravariant local Lorentz and nth-rank covariant Einstein tensor, $T^{m_1\cdots m_m}_{\mu_1\cdots\mu_n}$, is defined to transform as [7]

$$\begin{aligned}
T'^{m'_1\cdots m'_m}_{\mu'_1\cdots\mu'_n}(x') &= G^{-1\mu_1}_{\mu'_1}(x)\cdots G^{-1\mu_n}_{\mu'_n}(x)T^{m_1\cdots m_m}_{\mu_1\cdots\mu_n}(x) \\
&\quad \times \Lambda_{m_1}{}^{m'_1}(\alpha(x))\cdots\Lambda_{m_m}{}^{m'_m}(\alpha(x)),
\end{aligned} \qquad (2.21)$$

while a local Lorentz (m, n) rank spinor transforms as

$$\begin{aligned}
\Psi'_{\alpha_1\cdots\alpha_m}(x') &= D^{(1/2,0)\beta_1}_{\alpha_1}(\alpha(x))\cdots D^{(1/2,0)\beta_n}_{\alpha_m}(\alpha(x)) \\
&\quad \times \Psi_{\beta_1\cdots\beta_m}{}^{\dot{\beta}_1\cdots\dot{\beta}_n}(x) \\
&\quad \times D^{(0,1/2)\dot{\alpha}_1}_{\dot{\beta}_1}(\alpha(x))\cdots D^{(0,1/2)\dot{\alpha}_n}_{\dot{\beta}_n}(\alpha(x)).
\end{aligned} \qquad (2.22)$$

A mixed tensor-spinor is defined to transform analogously using the above transformation properties of pure quantities. For example, the vierbein transforms as $e'^m_\mu(x') = G^{-1\nu}_\mu(x)e_\nu{}^n(x)\Lambda_n{}^m(\alpha(x))$ while the covariant derivative of the Goldstino transforms as $\omega'_{Q\mu\alpha}(x') = G^{-1\nu}_\mu(x)D^{(1/2,0)\beta}_\alpha(\alpha(x))\omega_{Q\nu\beta}(x)$. Hence, the vierbein and its inverse can be used to convert local Lorentz indices into space-time, that is, world indices and vice versa. Since the Minkowski metric, η_{mn}, is invariant under local Lorentz transformations the metric tensor $g_{\mu\nu}$

$$g_{\mu\nu} = e_\mu{}^m\eta_{mn}e_\nu{}^n, \qquad (2.23)$$

is a rank 2 Einstein tensor. It can be used to define covariant Einstein tensors given contravariant ones. Likewise, the Minkowski metric can be used to define covariant local Lorentz tensors given contravariant ones, while the antisymmetric 2-index symbol, $\epsilon_{\alpha\beta}$ and $\epsilon^{\alpha\beta}$ and analogously for the dotted indices, can be used to raise, lower and contract spinor indices in the usual fashion.

Since the $x^\mu \to x'^\mu$ transformation produces the volume element transformation

$$d^4x' = d^4x\,\det G, \qquad (2.24)$$

while $\det\Lambda = 1$, it follows that $d^4x'\det e'(x') = d^4x\det e(x)$. Thus an $\mathcal{SP}_4$ invariant action can be constructed as

$$\Gamma = \int d^4x\,\det e(x)\mathcal{L}(x), \qquad (2.25)$$

where $\mathcal{L}'(x') = \mathcal{L}(x)$ is any invariant Lagrangian. The invariants that make up such a Lagrangian can be found by contracting tensor indices with the appropriate vierbein, its inverse and the Minkowski metric and spinor indices with the appropriate epsilon symbols. For example $\omega_{Q\mu}^{\alpha} g^{\mu\nu} \epsilon_{\alpha\beta} \omega_{Q\nu}^{\beta}$ is an invariant term which can be used in the construction of the action.

Besides products of the covariant Maurer-Cartan one-forms, their covariant derivatives can also be used to construct invariant terms of the Lagrangian. The covariant derivative of a general tensor can be defined using the affine and related spin connections. Consider the covariant derivative of the Lorentz tensor T^{mn}

$$\nabla_\rho T^{mn} = \partial_\rho T^{mn} - \omega_{M\rho r}^{m} T^{rn} - \omega_{M\rho r}^{n} T^{mr}. \qquad (2.26)$$

Since the spin connection transforms inhomogeneously according to Eq. (2.17), the covariant derivative of T^{mn} transforms homogeneously again

$$(\nabla_\rho T^{mn})' = G_\rho^{-1\sigma} (\nabla_\sigma T^{rs}) \Lambda_r{}^m \Lambda_s{}^n. \qquad (2.27)$$

Converting the Lorentz index n to a space-time index ν using the vierbein, the covariant derivative for mixed tensors is obtained

$$\nabla_\rho T^{m\nu} \equiv e_n^{-1\nu} \nabla_\rho T^{mn} = \partial_\rho T^{m\nu} - \omega_{M\rho}^{mr} T_r{}^\nu + \Gamma_{\sigma\rho}^{\nu} T^{m\sigma}, \qquad (2.28)$$

where the spin connection $\omega_{M\rho}^{mn}$ and $\Gamma_{\sigma\rho}^{\nu}$ are related according to [7]

$$\Gamma_{\sigma\rho}^{\nu} = e_n^{-1\nu} \partial_\rho e_\sigma{}^n - e_n^{-1\nu} \omega_{M\rho}^{nr} e_\sigma{}^s \eta_{rs}. \qquad (2.29)$$

(Note that this relation as well follows from the requirement that the covariant derivative of the vielbein vanishes, $\nabla_\rho e_\mu{}^m = 0$). Applying the above to the Minkowski metric Lorentz 2-tensor yields the formula relating the affine connection $\Gamma_{\mu\nu}^{\rho}$ to derivatives of the metric

$$\begin{aligned}\nabla_\rho \eta^{mn} &= \partial_\rho \eta^{mn} - \omega_{M\rho r}^{m} \eta^{rn} - \omega_{M\rho r}^{n} \eta^{mr} \\ &= -\omega_{M\rho}^{mn} - \omega_{M\rho}^{nm} = 0 = e_\mu{}^m e_\nu{}^n \nabla_\rho g^{\mu\nu} \\ &= e_\mu{}^m e_\nu{}^n (\partial_\rho g^{\mu\nu} + \Gamma_{\sigma\rho}^{\mu} g^{\sigma\nu} + \Gamma_{\sigma\rho}^{\nu} g^{\mu\sigma}). \qquad (2.30)\end{aligned}$$

The solution to this equation yields the affine connection in terms of its independent antisymmetric part $B_{\mu\nu}^{\rho} = \frac{1}{2} \times (\Gamma_{\mu\nu}^{\rho} - \Gamma_{\nu\mu}^{\rho})$ in addition to the derivative of the metric [7]

$$\begin{aligned}\Gamma_{\mu\nu}^{\rho} = &\frac{1}{2} g^{\rho\sigma} [\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}] + B_{\mu\nu}^{\rho} \\ &- g^{\rho\sigma} g_{\mu\lambda} B_{\sigma\nu}^{\lambda} - g^{\rho\sigma} g_{\nu\lambda} B_{\sigma\mu}^{\lambda}. \qquad (2.31)\end{aligned}$$

Finally a covariant field strength curvature twoform can be constructed out of the inhomogeneously transforming spin connection $\omega_{M\mu}^{mn}$

$$R^{mn} = d\omega_M^{mn} + \eta_{rs} \omega_M^{mr} \wedge \omega_M^{ns}. \qquad (2.32)$$

Expanding the forms yields the field strength tensor

$$\begin{aligned}R_{\mu\nu}^{mn} = &\partial_\mu \omega_{M\nu}^{mn} - \partial_\nu \omega_{M\mu}^{mn} + \eta_{rs} \omega_{M\mu}^{mr} \omega_{M\nu}^{ns} \\ &- \eta_{rs} \omega_{M\nu}^{mr} \omega_{M\mu}^{ns}. \qquad (2.33)\end{aligned}$$

It can be shown that $R_{\mu\nu}^{mn} = e^{-1n\sigma} e_\rho{}^m R^\rho{}_{\sigma\mu\nu}$ where $R^\rho{}_{\sigma\mu\nu}$ is the Riemann curvature tensor

$$R^\rho{}_{\sigma\mu\nu} = \partial_\nu \Gamma_{\sigma\mu}^{\rho} - \partial_\mu \Gamma_{\sigma\nu}^{\rho} + \Gamma_{\sigma\mu}^{\lambda} \Gamma_{\lambda\nu}^{\rho} - \Gamma_{\sigma\nu}^{\lambda} \Gamma_{\lambda\mu}^{\rho}. \qquad (2.34)$$

The Ricci tensor is given by $R_{\mu\nu} = R_{\mu\nu\rho}^{\rho}$ and hence the scalar curvature is an invariant

$$R = g^{\mu\nu} R_{\mu\nu} = -e_m^{-1\mu} e_n^{-1\nu} R_{\mu\nu}^{mn}. \qquad (2.35)$$

In similar fashion the covariant derivatives of spinor one-forms $\Psi_{\mu\alpha}$ and $\bar{\Psi}_\mu{}^{\dot\alpha}$, for example, are defined as

$$\begin{aligned}\nabla_\rho \Psi_{\mu\alpha} &= \partial_\rho \Psi_{\mu\alpha} + \frac{i}{4} \omega_{M\rho}^{mn} (\sigma_{mn})_\alpha{}^\beta \Psi_{\mu\beta} - \Gamma_{\rho\mu}^{\nu} \Psi_{\nu\alpha} \\ \nabla_\rho \bar{\Psi}_\mu{}^{\dot\alpha} &= \partial_\rho \bar{\Psi}_\mu{}^{\dot\alpha} + \frac{i}{4} \omega_{M\rho}^{mn} (\bar\sigma_{mn})^{\dot\alpha}{}_{\dot\beta} \bar{\Psi}_\mu{}^{\dot\beta} - \Gamma_{\rho\mu}^{\nu} \bar{\Psi}_\nu{}^{\dot\alpha}.\end{aligned} \qquad (2.36)$$

III. THE INVARIANT ACTION

The covariant derivatives of the Maurer-Cartan one-forms provide additional building blocks out of which the invariant action is to be constructed. For example the covariant derivatives of $\omega_{Q\alpha} = dx^\nu \omega_{Q\nu\alpha}$ and $\bar\omega_{\bar Q}{}^{\dot\alpha} = dx^\nu \bar\omega_{\bar Q\nu}{}^{\dot\alpha}$ yield the mixed tensors

$$\begin{aligned}\nabla_\mu \omega_{Q\nu\alpha} &= \partial_\mu \omega_{Q\nu\alpha} + \frac{i}{4} \gamma_\mu^{mn} (\sigma_{mn})_\alpha{}^\beta \omega_{Q\nu\beta} - \Gamma_{\mu\nu}^{\rho} \omega_{Q\rho\alpha} \\ \nabla_\mu \bar\omega_{\bar Q\nu}{}^{\dot\alpha} &= \partial_\mu \bar\omega_{\bar Q\nu}{}^{\dot\alpha} + \frac{i}{4} \gamma_\mu^{mn} (\bar\sigma_{mn})^{\dot\alpha}{}_{\dot\beta} \bar\omega_{\bar Q\nu}{}^{\dot\beta} - \Gamma_{\mu\nu}^{\rho} \bar\omega_{\bar Q\rho}{}^{\dot\alpha}.\end{aligned} \qquad (3.1)$$

Thus the invariant action describing spontaneously broken supergravity has the general low energy form

$$\begin{aligned}\Gamma = \int d^4x \det e \bigg\{ &\Lambda - \frac{M_{\rm Pl}^2}{16\pi} R \\ &+ Z M_{\rm Pl}^2 \epsilon^{\mu\nu\rho\sigma} \omega_{Q\mu} (\sigma^s e_{s\sigma}^{-1}) \nabla_\rho \bar\omega_{\bar Q\nu} \\ &- \frac{i}{2} Z_m M_{\rm Pl} M_S^2 [\omega_{Q\mu}{}^\alpha \sigma_\alpha^{\mu\nu\beta} \omega_{Q\nu\beta} \\ &+ \bar\omega_{\bar Q\mu\dot\alpha} \bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta} \bar\omega_{\bar Q\nu}{}^{\dot\beta}] + M_S^2 \omega_{Q\mu} [i Z_1 g^{\mu\nu} \sigma^\rho \\ &+ i Z_2 g^{\mu\rho} \sigma^\nu + i Z_3 g^{\nu\rho} \sigma^\mu] \nabla_\rho \bar\omega_{\bar Q\nu} \\ &- \frac{1}{2} Z'_m M_S^3 [\omega_{Q\mu}{}^\alpha g^{\mu\nu} \omega_{Q\nu\alpha} + \bar\omega_{\bar Q\mu\dot\alpha} g^{\mu\nu} \bar\omega_{\bar Q\nu}{}^{\dot\alpha}] \bigg\},\end{aligned} \qquad (3.2)$$

where $M_{\rm Pl} = \frac{1}{\sqrt{G}}$ is the Planck mass scale and G is Newton's constant. The cosmological constant $\Lambda = c M_S^4$ has its scale set by the SUSY breaking scale, M_S, with a coefficient c which can be fine tuned to any desired value.

The last two terms on the first line, the graviton and gravitino kinetic terms, are present even in the absence of SUSY breaking and hence their scale is set by the Planck mass $M_{\rm Pl}$ with $Z \sim 1$. The gravitino mass term appearing on the second line contains no bilinear Goldstino contribution and as such its scale is chosen to be a combination of $M_{\rm Pl}$ and M_S. On the other hand, the terms in the last two lines are nonvanishing in the global limit and so are naturally suppressed relative to the previous terms. Their coefficients are given by powers of the SUSY breaking scale M_S. These terms are suppressed relative to the terms in the first two lines of the action and thus can be ignored. Moreover, it is necessary to set their coefficients to zero in order for the massive spin $3/2$ field to be ghost free [11]. Finally, higher dimension terms are also possible but are suppressed by additional powers of the Planck scale and/or the SUSY breaking scale according to their character. Included in such Planck scale suppressed terms are those arising when the effects of $B^{\rho}_{\mu\nu}$ are eliminated through its field equations.

Hence, the minimal model is constructed by retaining the first two lines only or equivalently setting the parameters Z'_m, Z_1, Z_2 and Z_3 to zero. On the other hand, due to the Higgs mechanism, the parameter Z_m cannot be zero. Up to this point the Goldstino and gravitino have been defined with mass dimension $-1/2$ and $+1/2$, respectively. Rescaling the Goldstino fields λ and $\bar{\lambda}$ with M_S^2 and the gravitino fields ψ_μ and $\bar{\psi}_\mu$ with $M_{\rm Pl}$ so that they have canonical dimension $3/2$, the minimal nonlinearly realized supergravity action is given by

$$\begin{aligned}\Gamma = \int d^4x \det e\bigg\{&\Lambda - \frac{M_{\rm Pl}^2}{16\pi}R \\ &+ ZM_{\rm Pl}^2\epsilon^{\mu\nu\rho\sigma}\omega_{Q\mu}(\sigma^s e_{s\sigma}^{-1})\nabla_\rho\bar{\omega}_{\bar{Q}\nu} \\ &- \frac{i}{2}Z_m M_{\rm Pl}M_S^2[\omega_{Q\mu}{}^{\alpha}\sigma^{\mu\nu\beta}_{\alpha}\omega_{Q\nu\beta} \\ &+ \bar{\omega}_{\bar{Q}\mu\dot{\alpha}}\bar{\sigma}^{\mu\nu\dot{\alpha}}{}_{\dot{\beta}}\bar{\omega}_{\bar{Q}\nu}{}^{\dot{\beta}}]\bigg\}. \end{aligned} \tag{3.3}$$

Since λ_α and $\bar{\lambda}_{\dot{\alpha}}$ transform inhomogeneously under the broken local SUSY transformations, we can now fix the unitary gauge defined by $\lambda_\alpha = 0 = \bar{\lambda}_{\dot{\alpha}}$. So doing, the covariant one-forms in terms of the rescaled gravitino fields take a simplified form

$$\begin{aligned}\omega^m &= dx^m + E^m = dx^\mu e_\mu{}^m \\ \omega_Q{}^\alpha &= \psi^\alpha = dx^\mu \psi_\mu{}^\alpha/M_{\rm Pl} \\ \omega_{\bar{Q}\dot{\alpha}} &= \bar{\psi}_{\dot{\alpha}} = dx^\mu\bar{\psi}_{\mu\dot{\alpha}}/M_{\rm Pl} \\ \omega_M^{mn} &= \gamma^{mn}.\end{aligned} \tag{3.4}$$

Note that the det$e$ gives no contribution to the gravitino mass even though it is the source of Goldstino kinetic term in the model with spontaneously broken global SUSY. Instead, the mass of the gravitino is given by $m_{3/2} = Z_m\frac{M_S^2}{M_{\rm Pl}}$. In addition, since Z_m is arbitrary, the value of the $m_{3/2}$ can effectively be considered as a new scale arising from an independent monomial [4]. This is reminiscent of what transpires when gauging the spontaneously broken isometries of AdS_5 space on an embedded AdS_4 manifold [12]. In that case, the spectrum contains a massive Abelian vector whose mass is an independent scale. On the other hand, this realization of the Higgs mechanism is strikingly different from what occurs when gauging internal symmetries. In that case, when the symmetry is made local, the Nambu-Goldstone boson kinetic term gets replaced by the square of the covariant derivative containing the vector connection. In unitary gauge, the Nambu-Goldstone field vanishes leaving the residual vector mass term whose scale is set by the Nambu-Goldstone decay constant, a scale already present in the global model, times a gauge coupling constant. Here the former Goldstino kinetic energy term becomes a cosmological constant term.

Hence, the cosmological constant, Λ, the gravitino mass scale, $m_{3/2}$, and the gravitational scale, $M_{\rm Pl}$, along with the SUSY breaking (cut-off) scale, M_S, can all be treated as phenomemologically independent scales [4]. Thus, in unitary gauge, the effective action describing the supergravitational physics below the SUSY breaking scale M_S, Eq. (3.3), reduces to that of a massive gravitino field coupled to a gravitational field with cosmological constant

$$\begin{aligned}\Gamma = \int d^4x \det e\bigg\{&\Lambda - \frac{M_{\rm Pl}^2}{16\pi}R + Z\epsilon^{\mu\nu\rho\sigma}\psi_\mu(\sigma^s e_{s\sigma}^{-1})\nabla_\rho\bar{\psi}_\nu \\ &- \frac{i}{2}m_{3/2}[\psi_\mu\sigma^{\mu\nu}\psi_\nu + \bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu]\bigg\}.\end{aligned} \tag{3.5}$$

IV. INVARIANT COUPLING TO MATTER

As discussed in Sec. II, matter fields can be characterized by their Lorentz group (with generators M^{mn}) transformation properties. Each matter field, $M(x)$, transforms under G as

$$M'(x') \equiv \tilde{h}M(x), \tag{4.1}$$

where $\tilde{h}$ is given by

$$\tilde{h} = e^{(i/2)\alpha_{mn}(x)\tilde{M}^{mn}}, \tag{4.2}$$

with $\tilde{M}^{mn}$ the matrix for the corresponding matter field representation of the Lorentz algebra. For example, a scalar field, $S(x)$, is in the trivial representation of the Lorentz group, $\tilde{M}^{mn} = 0$, while fermion fields, $\psi_\alpha(x)$ or $\bar{\psi}_{\dot{\alpha}}$, carry the $(1/2, 0)$ spinor representation, $(\tilde{M}^{mn})_\alpha{}^\beta = 1/2(\sigma^{mn})_\alpha{}^\beta$, or the $(0, 1/2)$ spinor representation, $(\tilde{M}^{mn})^{\dot{\alpha}}{}_{\dot{\beta}} = 1/2(\bar{\sigma}^{mn})^{\dot{\alpha}}{}_{\dot{\beta}}$. The covariant derivative for the matter field is defined using the Maurer-Cartan spin connection oneform (c.f. Eqs. (2.26) and (2.36)) as [13]

$$\nabla M \equiv \left(d + \frac{i}{2} \omega_M^{mn} \tilde{M}_{mn} \right) M \tag{4.3}$$

so that it has the same transformation properties as the matter field itself,

$$(\nabla M)'(x') = \tilde{h} \nabla M(x). \tag{4.4}$$

Expanding the covariant derivative oneform in terms of space-time coordinate differentials, dx^μ, the component form of the covariant derivative is given by

$$\nabla_\mu M = \left(\partial_\mu + \frac{i}{2} \gamma_\mu{}^{mn} \tilde{M}_{mn} \right) M \tag{4.5}$$

and exhibits the $\mathcal{SP}_4$ transformation law

$$(\nabla_\mu M)'(x') = \tilde{h} G_\mu^{-1\nu} \nabla_\nu M(x). \tag{4.6}$$

The definition of the covariant derivative can be extended when the matter fields also carry a representation of a local internal symmetry group $\mathcal{G}$, [13,14], so that

$$M'^a(x) = (U(\epsilon))^a{}_b M^b(x), \tag{4.7}$$

where the representation matrix

$$(U(\epsilon))^a{}_b = (e^{ig\epsilon^A(x)T^A})^a{}_b, \tag{4.8}$$

is given in terms of the local transformation parameters $\epsilon^A(x)$ and the gauge coupling constant g. The generator representation matrices $(T^A)^a{}_b$, $A = 1, 2, \ldots, \dim[\mathcal{G}]$ satisfy the associated Lie algebra $[T^A, T^B] = if^{ABC}T^C$, and are normalized so that $\mathrm{Tr}[T^A T^B] = 1/2\delta^{AB}$. In order to extend the invariance of the action to include gauge transformations, the Yang-Mills gauge potential oneform,

$$A(x) = dx^\mu A_\mu(x) = dx^\mu (iT^A A_\mu^A(x)). \tag{4.9}$$

must be introduced. Under $\mathcal{SP}_4$-transformations, this oneform is invariant: $A'(x') = A(x)$, and thus the gauge field transforms as a coordinate differential

$$A'_\mu(x') = G_\mu^{-1\nu} A_\nu(x), \tag{4.10}$$

while under $\mathcal{G}$-transformations the Yang-Mills field transforms as a gauge connection

$$A' = U(\epsilon) A U^{-1}(\epsilon) + \frac{1}{g} (dU(\epsilon)) U^{-1}(\epsilon). \tag{4.11}$$

Thus the gauge and super-Poincaré covariant derivative of the matter field is secured as

$$\nabla M = \left[d + \frac{i}{2} \omega_M^{mn} \tilde{M}_{mn} - gA \right] M \tag{4.12}$$

so that, under super-Poincaré transformations the covariant derivative transforms identically to M, $(\nabla M)'(x') = \tilde{h}(\nabla M)(x)$, while under gauge transformations the covariant derivative carries the same matter field representation of $\mathcal{G}$ as M:

$$(\nabla M)' = U(\epsilon)(\nabla M). \tag{4.13}$$

The matter field covariant derivative can be expanded in terms of the space-time coordinate differentials dx^μ giving

$$\nabla_\mu M = \left(\partial_\mu + \frac{i}{2} \gamma_\mu{}^{mn} \tilde{M}_{mn} - gA_\mu \right) M. \tag{4.14}$$

The fully covariant derivatives for the scalar, $S(x)$, and fermion, $\psi_\alpha(x)$ and $\bar{\psi}^{\dot{\alpha}}(x)$, matter fields have the explicit form

$$\begin{aligned}
(\nabla_\mu S)^a &= \partial_\mu S^a - igA_\mu^A (T^A)^a{}_b S^b \\
(\nabla_\mu \psi)^a_\alpha &= \partial_\mu \psi^a_\alpha + \frac{i}{4} \gamma_\mu{}^{mn} (\sigma_{mn})_\alpha{}^\beta \psi^a_\beta - igA_\mu^A (T^A)^a{}_b \psi^b_\alpha \\
(\nabla_\mu \bar{\psi})^{\dot{\alpha} a} &= \partial_\mu \bar{\psi}^{\dot{\alpha} a} + \frac{i}{4} \gamma_\mu{}^{mn} (\bar{\sigma}_{mn})^{\dot{\alpha}}{}_{\dot{\beta}} \bar{\psi}^{\dot{\beta} a} \\
&\quad - igA_\mu^A (T^A)^a{}_b \bar{\psi}^{\dot{\alpha} b}.
\end{aligned} \tag{4.15}$$

The Yang-Mills field strength twoform, F, is defined as $F \equiv dA + gA \wedge A$. As a twoform, F is invariant under $\mathcal{SP}_4$-transformations while under $\mathcal{G}$-transformations it is in the adjoint representation $F' = U(\epsilon) F U^{-1}(\epsilon)$. Expanding F in terms of the coordinate differential basis dx^μ, $F = \frac{1}{2} dx^\nu \wedge dx^\mu (iT^A F^A_{\mu\nu})$, the space-time index field strength tensor is obtained as

$$F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu + gf^{ABC} A^B_\mu A^C_\nu. \tag{4.16}$$

A generic nonlinearly realized supergravity and gauge invariant matter field action can be constructed as

$$\Gamma_{\mathrm{matter}} = \int d^4x \det e \mathcal{L}_{\mathrm{matter}}, \tag{4.17}$$

where the fully invariant matter field Lagrangian $\mathcal{L}_{\mathrm{matter}}$ takes the form

$$\begin{aligned}
\mathcal{L}_{\mathrm{matter}} &= \mathcal{L}_{\mathrm{matter}}(M, \nabla_\mu M, \omega_Q, \bar{\omega}_{\bar{Q}}, \nabla_\mu \omega_Q, \nabla_\mu \bar{\omega}_{\bar{Q}}, e_\mu{}^m, \\
&\qquad R_{\mu\nu\rho\sigma}, F^A_{\mu\nu}),
\end{aligned} \tag{4.18}$$

where $\mathcal{L}_{\mathrm{matter}}$ is any SUSY and gauge invariant function of the basic building blocks which consist of the vierbein, $e_\mu{}^m$, the fermionic Maurer-Cartan one forms, $\omega^\alpha_{Q\mu}$ and $\bar{\omega}_{\bar{Q}\mu\dot{\alpha}}$, their covariant derivatives as given in Eq. (3.1), the Riemann tensor, $R_{\mu\nu\rho\sigma}$, the matter fields, M, their covariant derivatives, $\nabla_\mu M$, the gauge field strength tensor, $F^A_{\mu\nu}$, and higher covariant derivatives of all quantities. Combined with the pure supergravity action of Eq. (3.3), the low energy effective action describing the dynamics of the light matter and gauge fields along with the graviton and massive Goldstino/gravitino fields is given by

$$\Gamma_{\mathrm{eff}} = \int d^4x \det e \mathcal{L}_{\mathrm{eff}}, \tag{4.19}$$

where again the fully SUSY and gauge invariant effective Lagrangian has the generic form

$$\begin{aligned}
\mathcal{L}_{\mathrm{eff}} &= \mathcal{L}_{\mathrm{eff}}(M, \nabla_\mu M, \omega_Q, \bar{\omega}_{\bar{Q}}, \nabla_\mu \omega_Q, \nabla_\mu \bar{\omega}_{\bar{Q}}, e_\mu{}^m, \\
&\qquad R_{\mu\nu\rho\sigma}, F^A_{\mu\nu}).
\end{aligned} \tag{4.20}$$

It proves convenient to catalog the terms in the effective Lagranian, $\mathcal{L}_{\text{eff}}$, by an expansion in the number of Goldstino/gravitino fields which appear after the Goldstino and gravitino fields are set to zero in the fully covariant derivatives and in the vierbein [15]. This is tantamount to counting the number of factors of the fermionic Mauer-Cartan one-forms in each expression. So doing, the effective action has the expansion

$$\mathcal{L}_{\text{eff}} = [\mathcal{L}_{(0)} + \mathcal{L}_{(1)} + \mathcal{L}_{(2)} + \cdots], \tag{4.21}$$

where the subscript n on $\mathcal{L}_{(n)}$ denotes that each independent invariant operator in that set begins with n factors of $\omega^{\alpha}_{Q\mu}$ and $\bar{\omega}_{\bar{Q}\mu\dot{\alpha}}$, or equivalently, with n Goldstino/gravitino fields.

$\mathcal{L}_{(0)}$ consists of all gauge and SUSY invariant operators made only from the vierbein and the light matter and gauge fields and their SUSY covariant derivatives. Thus any Goldstino/gravitino field appearing in $\mathcal{L}_{(0)}$ arises only from higher dimension terms in the matter covariant derivatives and/or the field strength tensor and vierbein. For instance, taking the gravitino to be the lightest supersymmetric partner, then $\mathcal{L}_{(0)}$ has the form

$$\mathcal{L}_{(0)} = \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{Y-M}}, \tag{4.22}$$

where the matter field action is given by

$$\begin{aligned}\mathcal{L}_{\text{matter}} = {}& \mathrm{Tr}[(\nabla_{\mu}S)^{\dagger} g^{\mu\nu}(\nabla_{\nu}S)] - V(S) \\ & + i\psi\sigma^{m} e_{m}^{-1\mu}\nabla_{\mu}\bar{\psi} - \psi m\psi \\ & - \bar{\psi}\,\bar{m}\,\bar{\psi} + Y(S, \psi\psi, \bar{\psi}\,\bar{\psi}),\end{aligned} \tag{4.23}$$

so that $\mathcal{L}_{\text{matter}}$ includes any possible globally $\mathcal{G}$-invariant scalar field potential $V(S)$, fermion mass terms $\psi m\psi$ and $\bar{\psi}\,\bar{m}\,\bar{\psi}$, and generalized Yukawa couplings $Y(S, \psi\psi, \bar{\psi}\,\bar{\psi})$. The fully invariant Yang-Mills Lagarangian $\mathcal{L}_{\text{Y-M}}$ is

$$\mathcal{L}_{\text{Y-M}} = -\frac{1}{2}\,\mathrm{Tr}[F_{\mu\nu} g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma}]. \tag{4.24}$$

Note that the coefficients of these terms are fixed by the normalization of the gauge and matter fields, their masses and self-couplings. That is, by the normalization of the Goldstino/gravitino independent Lagrangian. For the case where the gravitino is the only supersymmetric partner whose mass is below the electroweak scale, the matter and gauge field terms are just those of the standard model and the normalization of these terms is just that given by the standard model.

The $\mathcal{L}_{(1)}$ terms in the effective Lagrangian begin with the couplings of the non-Goldstino/gravitino fields to only a single Goldstino/gravitino. The general form of these terms is given by

$$\mathcal{L}_{(1)} = \frac{M_{\text{Pl}}}{M_S}[\omega^{\alpha}_{Q\mu} Q^{\mu}_{\alpha} + \bar{Q}^{\mu}_{\dot{\alpha}}\bar{\omega}^{\dot{\alpha}}_{\bar{Q}\mu}], \tag{4.25}$$

where Q^{μ}_{α} and $\bar{Q}^{\mu}_{\dot{\alpha}}$ contain the light field contributions to the conserved gauge invariant supersymmetry currents. That is, it is the term in the effective Lagrangian which involves linear coupling to the Goldstino/gravitino fields. The Lagrangian $\mathcal{L}_{(1)}$ describes processes involving the emission or absorption of a single gravitino. If the gravitino is the lightest supersymmetric partner, the lowest mass dimension $SU(3)\times SU(2)\times U(1)$ invariant terms contributing to $\mathcal{L}_{(1)}$ are the gravitino-lepton number and R parity conserving terms given by

$$\mathcal{L}_{(1)} = \sum_{f}\frac{c_f}{M_S}\omega^{\alpha}_{Q\mu}(\sigma^{\mu\nu})_{\alpha}{}^{\beta}[l^{a}_{f\beta}(\nabla_{\nu}\phi)^{b}\epsilon_{ab}] + \text{h.c.}, \tag{4.26}$$

where $l_{f\alpha} = (\nu_{e_f} e^{-}_{f})^{T}_{\alpha}$ is the lepton doublet for family f in the $(1, 2, -\frac{1}{2})$ representation of $SU(3)\times SU(2)\times U(1)$, $\phi = (\phi_{+}\phi_{0})^{T}$ is the Higgs doublet in the $(1, 2, +\frac{1}{2})$ representation and c_f are generation dependent effective coupling constants. In the nonlinearly realized global SUSY case, the processes controlled by such terms involve only the helicity $\pm\frac{1}{2}$ modes of the gravitino. By means of the equivalence theorem at high energy, they were found from the corresponding Goldstino amplitudes and were delineated and investigated in [9]. Expanding the Lagrangian, Eq. (4.26), in terms of the component fields and transforming to the unitary gauge for the rescaled gravitino fields and Higgs multiplet so that $\omega_{Q\mu} = \psi_{\mu}/M_{\text{Pl}}$, $\bar{\omega}_{\bar{Q}\mu} = \bar{\psi}_{\mu}/M_{\text{Pl}}$ and $\phi = (0\frac{1}{\sqrt{2}}(v + H))^{T}$, this now yields the interactions involving all helicities of the gravitino

$$\begin{aligned}\mathcal{L}_{(1)} = {}& -\sum_{f}\frac{c_f}{M_S}\psi_{\mu}\sigma^{\mu\nu}\bigg\{\bigg[\frac{1}{\sqrt{2}}\nu_{e_f}\partial_{\mu}H + \text{h.c.}\bigg] \\ & + \bigg[\frac{1}{\sqrt{2}}M_{Z}\nu_{e_f}Z_{\mu} + \text{h.c.}\bigg] + [iM_{W}e^{-}_{f}W^{+}_{\mu} + \text{h.c.}] \\ & + \bigg[\frac{i}{\sqrt{2}}\frac{e}{\sin 2\theta_{W}}\nu_{e_f}Z_{\mu}H + \text{h.c.}\bigg] \\ & + \bigg[i\frac{e}{2\sin\theta_{W}}e^{-}_{f}W^{+}_{\mu}H + \text{h.c.}\bigg]\bigg\}.\end{aligned} \tag{4.27}$$

The phenomenological consequences of such interaction terms were investigated [9] in the Goldstino/helicity $\pm\frac{1}{2}$ gravitino case.

Finally the remaining terms in the effective Lagrangian all contain two or more Goldstino/gravitino fields. In particular, $\mathcal{L}_{(2)}$ begins with the coupling of two Goldstino/gravitino fields to matter or gauge and gravitational fields. The lowest dimension such terms, bilinear in $\omega_{Q} - \bar{\omega}_{\bar{Q}}$, have the form

$$\begin{aligned}\mathcal{L}_{(2)} = {}& \frac{M^{2}_{\text{Pl}}}{M^{2}_{S}}\omega^{\alpha}_{Q\mu}\bar{\omega}^{\dot{\alpha}}_{\bar{Q}\nu}M^{\mu\nu}_{1\alpha\dot{\alpha}} + \frac{M^{2}_{\text{Pl}}}{M^{2}_{S}}\omega^{\alpha}_{Q\mu}\overleftrightarrow{\nabla}_{\rho}\bar{\omega}^{\dot{\alpha}}_{\bar{Q}\nu}M^{\mu\nu\rho}_{2\alpha\dot{\alpha}} \\ & + \frac{M^{2}_{\text{Pl}}}{M^{2}_{S}}\nabla_{\rho}[\omega^{\alpha}_{Q\mu}\bar{\omega}^{\dot{\alpha}}_{\bar{Q}\nu}]M^{\mu\nu\rho}_{3\alpha\dot{\alpha}},\end{aligned} \tag{4.28}$$

where the composite operators that contain matter, gauge and gravitational fields are denoted by the M_i. They can be enumerated by their operator dimension, Lorentz structure and field content. Additional discussion of these couplings in the global SUSY case can be found in Ref. [15].

There is another useful oneform basis in which to express the derivatives and gauge fields [14]. The basis consists of the fully covariant coordinate differentials $\omega^m = dx^\mu e_\mu{}^m$. The exterior derivative can be expanded in this basis as $d = dx^\mu \partial_\mu = \omega^m \mathcal{D}_m = \omega^m e_m^{-1\mu} \partial_\mu$, while the gauge field oneform has the analogous expansion

$$A = dx^\mu A_\mu = \omega^m A_m. \tag{4.29}$$

As previously noted, the covariant basis ω^m transforms according to the vector representation of the $D = 4$ (local) Lorentz structure group, $\omega'^{\,m} = \omega^n \Lambda_n{}^m$. So the covariant gauge field transforms analogously, $A'_m(x') = \Lambda_m^{-1n} A_n(x)$, with the covariant derivative transforming as $\mathcal{D}'_m = \Lambda_m^{-1n} \mathcal{D}_n$. The invariant interval can be expressed in each of these bases using the metric specific to each as

$$ds^2 = dx^\mu g_{\mu\nu} dx^\nu = \omega^m \eta_{mn} \omega^n, \tag{4.30}$$

with η_{mn} the flat tangent space Minkowski metric.

In the new basis, the matter field covariant derivatives take the form

$$\begin{aligned}
(\nabla_m S)^a &= \mathcal{D}_m S^a - igA_m^A (T^A)^a{}_b S^b \\
(\nabla_m \psi)^a_\alpha &= \mathcal{D}_m \psi^a_\alpha + \frac{i}{4}\gamma_m{}^{rs}(\sigma_{rs})_\alpha{}^\beta \psi^a_\beta - igA^A_m (T^A)^a{}_b \psi^b_\alpha \\
(\nabla_m \bar{\psi})^{\dot\alpha a} &= \mathcal{D}_m \bar{\psi}^{\dot\alpha a} + \frac{i}{4}\gamma_m{}^{rs}(\bar\sigma_{rs})^{\dot\alpha}{}_{\dot\beta} \bar\psi^{\dot\beta a} \\
&\quad - igA^A_m (T^A)^a{}_b \bar\psi^{\dot\alpha b},
\end{aligned} \tag{4.31}$$

with $\gamma_m^{rs} = e_m^{-1\mu} \gamma_\mu{}^{rs}$. With these replacements, the fully invariant matter field Lagrangian, Eq. (4.23), can be written as

$$\begin{aligned}
\mathcal{L}_{\text{matter}} &= \text{Tr}[(\nabla_m S)^\dagger \eta^{mn} (\nabla_n S)] - V(S) + i\psi\sigma^m \nabla_m \bar\psi \\
&\quad - \psi m \psi - \bar\psi \bar m \bar\psi + Y(S, \psi\psi, \bar\psi\bar\psi).
\end{aligned} \tag{4.32}$$

The Yang-Mills action can also be recast in this new basis. The Yang-Mills fields have the modified gauge variation

$$A'_m = U(\epsilon) A_m U^{-1}(\epsilon) + \frac{1}{g}(\mathcal{D}_m U(\epsilon)) U^{-1}(\epsilon) \tag{4.33}$$

while the field strength tensor takes the form

$$F^A_{mn} = \mathcal{D}_m A^A_n - \mathcal{D}_n A^A_m + g f^{ABC} A^B_m A^C_n. \tag{4.34}$$

Consequently, in this basis, the fully invariant Yang-Mills Lagrangian, Eq. (4.24), becomes

$$\mathcal{L}_{\text{Y-M}} = -\frac{1}{2}\text{Tr}[F_{mn}\eta^{mr}\eta^{ns}F_{rs}]. \tag{4.35}$$

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