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ABSTRACT

Modeling the dynamics of shell-and-tube heat-exchangers is an important step in developing dynamic system models of liquid chillers that are used for studying transient system performance. Existing literature on the subject is limited and much of what exists uses either a lumped parameter approach or a finite volume approach for the shell-and-tube heat-exchangers. The lumped parameter approach is simplistic and provides neither spatial detail nor sufficient accuracy in predicting exit conditions. The finite volume approach provides extensive spatial detail but at significant computational expense. A third alternative, known as the moving-boundary approach, has thus far only been used for refrigerant-in-tube coils. It has the potential for fast execution due to the reduced number of equations as compared to the finite-volume method, while retaining some spatial detail. This paper details the formulation of shell-and-tube evaporators and condensers using the moving-boundary approach and presents comparative results of model execution with a finite-volume approach. Both formulations are developed to capture start-up and load-change transients. The moving-boundary formulation has the ability to handle the discontinuities associated with phase-boundaries exiting and entering the heat-exchanger during transient operation. A significant saving in execution time is shown over the finite-volume approach with comparable accuracy.

1. INTRODUCTION

In the published literature on dynamic models of vapor compression equipment, several models exist for air-to-air systems. Liquid chiller modeling is however is less studied. A detailed review of the literature was compiled by Bendapudi and Braun [2002b]. Recent work includes Wang &Wang [2000], Grace & Tassou [2000] and Bendapudi et al [2002]. For the refrigerant-in-tube heat-exchangers used in air-to-air systems, two formulations have been used viz., the finite-volume (MacArthur & Grald [1987] and Rossi & Braun [1999]) and the moving-boundary (Grald & MacArthur [1992], He et al [1994] and Pettit et al [1998]). In the former, the heat-exchanger is divided into a series of time-invariant control volumes. Transient mass, energy, and if necessary momentum, conservation equations are discretized over these control volumes to yield a system of coupled, first-order, algebraic differential equations which are then solved. In the moving-boundary formulation, the heat-exchanger is divided into time-varying control volumes defined at any instant by the phase of the refrigerant. The heat-exchanger volume is divided into zones such that the zone-boundaries coincide with the saturated liquid and saturated vapor points. As operating conditions change, these boundaries move within the heat-exchanger and the control-volumes have to move, hence the name.

While these two formulations can also be applied to modeling the dynamics of shell-and-tube heat-exchangers, what little literature exists on the subject predominantly uses the finite-volume approach. The moving-boundary approach applied to flooded condensers is presented by Svennson [2000], albeit in a simplified form and only applied to load-change transients. This paper presents both formulations applied to shell-and-tube evaporators and condensers and presents simulation results comparing relative accuracies and execution speeds.

2. HEAT-EXCHANGER MODELING

Dynamic modeling of shell-and-tube heat-exchangers begins with the 3-dimensional transient forms of the mass, energy and momentum conservation applied to the refrigerant, and energy conservation applied to the tube-material and the secondary fluid. By assuming one-dimensional flow, neglecting refrigerant pressure drop, viscous dissipation, axial conduction in the refrigerant, tube or water and also neglecting conductive resistances, these can be simplified to the following:

$$\frac{\partial r}{\partial t} + \frac{\partial (r w)}{\partial z} = 0 \tag{1}$$

$$\frac{\partial (r h - P)}{\partial t} + \frac{\partial (r h w)}{\partial z} + d\dot{Q}_r = 0 \tag{2}$$

$$\left(C_{pt} r_t A_t\right) \frac{\partial T_t}{\partial t} + d\dot{Q}_w - d\dot{Q}_r = 0 \tag{3}$$

$$\left(C_{pw} r_w A_w\right) \frac{\partial T_w}{\partial t} + \left(C_{pw} r_w A_w w_w\right) \frac{\partial T_w}{\partial z} - d\dot{Q}_w = 0 \tag{4}$$

To discretize these equations for solution, the flow-arrangement in the heat-exchangers has to be identified. The refrigerant flow through the shell relative to the water flow is truly a combination of cross flow and counter flow. However, modeling such a combination is difficult and even unnecessary for the purposes envisaged here. Therefore, the flow-arrangement here is simplified to a pure tube-in-tube counter-flow, with the shell acting as the outer tube and the water tubes acting as the inner tube, as shown in Figs. 1 or 2. The subsequent development of the conservation equations depends upon the formulation, as described in the following sections.

3. FINITE-VOLUME FORMULATION

In the finite-volume formulation, the heat-exchanger is divided into several (typically identical) control volumes along the length as shown in Fig. 1. By integrating the above conservation equations over one (the k^{th}) control volume and simplifying (Rossi & Braun [1999]), the following coupled, linearized set of algebraic differential equations are obtained:

$$a_k \frac{dP}{dt} + b_k \frac{dh_k}{dt} = \dot{m}_{r,k-1} - \dot{m}_{r,k} \tag{5}$$

$$c_k \frac{dP}{dt} + d_k \frac{dh_k}{dt} = \dot{m}_{r,k-1} h_{k-1} - \dot{m}_{r,k} h_k - \dot{Q}_{r,k} \tag{6}$$

where the coefficients a_k, b_k, c_k and d_k are defined as:

$$a_k = V_k \left(\frac{\partial r_k}{\partial P}\right)_{h_k},\ b_k = V_k \left(\frac{\partial r_k}{\partial h}\right)_P,\ c_k = V_k \left[h_k \left(\frac{\partial r_k}{\partial P}\right)_{h_k} - 1\right],\ d_k = V_k \left[\left(\frac{\partial r_k}{\partial h}\right)_P + r_k\right]$$

By combining these equations over all the control volumes, a system of 2N equations is obtained in the 2N unknowns viz., the refrigerant pressure, N enthalpies and N-1 intermediate refrigerant mass-flow rates. The number of equations can be algebraically reduced (Bendapudi et al [2005]) to N+1 by recursively eliminating the N-1 intermediate refrigerant mass-flow rates. This allows the refrigerant side of the heat-exchanger to be modeled as a matrix equation $A_{N+1,N+1} \cdot \dot{\overrightarrow{X_{FV}}} = \overrightarrow{B_{FV}}$, where the forms of A, and $\overrightarrow{B_{FV}}$ are as given in appendix 1 and $\overrightarrow{X_{FV}} = \begin{bmatrix} P & h_1 & \cdots & h_N \end{bmatrix}^T$.

The discretized forms of the energy conservation on the tube material and water can be simplified to the following:

$$M_{t,k} C_{pt} \frac{dT_t}{dt} = \dot{Q}_{r,k} - \dot{Q}_{w,k} \tag{7}$$

$$M_{w,k} C_{pw} \frac{dT_{w,k}}{dt} = \dot{m}_w C_{pw} \left(T_{w,k-1} - T_{w,k}\right) V_k + \dot{Q}_{w,k} \tag{8}$$

With the specification of an initial condition, and the boundary conditions of inlet and outlet refrigerant flow-rate, inlet refrigerant enthalpy, and the flow-rate and temperature of water, the above constitutes a closed system of equations that can be solved for the state derivatives and integrated forward in time. The development of the finite-volume formulation makes no essential distinction between a condenser and an evaporator and can thus be used for either heat-exchanger. Differences between the two heat-exchangers appear in how the heat-transfer coefficients are computed to determine the heat-transfer rates, i.e. for condensation or for evaporation, and in the specific values of the boundary conditions. For a detailed description of the development and solution, please refer Bendapudi et al [2005].

4. MOVING-BOUNDARY FORMULATION

The development of the moving-boundary formulation is more involved and doesn't share the generality of the finite-volume formulation in being identical for both heat-exchangers. This is because phase-regions or zones occur along the flow-direction in different sequences in the evaporator and the condenser. In addition to this, during transient operation, zones may enter and leave the heat-exchanger. For example, the condenser may initially be completely superheated at start-up and as refrigerant is pumped into it a two-phase zone, and eventually a sub-cooled zone, develop. Such conditions require a discrete change in the equations being solved. It is therefore necessary to develop models for the heat-exchangers under different plausible combinations of zones. For the purpose of this work, the following combinations were developed:

One Zone – ETP:	Evaporator in fully two-phase condition
One Zone – CSH:	Condenser in fully superheated condition;
Two Zones – ETPSH:	Evaporator in two-phase at entry and superheat at exit
Two Zones – CSHTP:	Condenser with superheated vapor at entry and two-phase at exit and
Three Zones – CSHTPSC:	Condenser with superheated vapor at entry and sub-cooled at exit with two-phase in between.

Referring to Fig. 2, the conservation equations (1)-(4) are integrated between general time-varying limits Z_1 and Z_2 ($0 < Z_1 < Z_2 < L$) which represent the beginning and end of any zone, or the ends of the heat-exchanger as appropriate. Integration under such time-varying limits requires the use of Liebnitz's Rule according to which, if $f(t,z)$ is a well behaved bi-variate function in a domain bounded between $a_1(t)$ and $a_2(t)$, the following holds:

$$\int_{a_1(t)}^{a_2(t)} \frac{\partial f(t,z)}{\partial t} dz = \frac{d}{dt} \int_{a_1(t)}^{a_2(t)} f(t,z)\, dz + f\left(t, a_1(t)\right) \frac{da_1(t)}{dt} - f\left(t, a_2(t)\right) \frac{da_2(t)}{dt}$$

Application of this rule to the integration of equations (1)-(4), followed by some simplifying algebra, results in general forms of the conservation equations that can be applied to each zone. The general refrigerant mass balance applicable to any zone becomes:

$$A_r\left(\overline{r} - r_{Z_2}\right)\frac{dZ_2}{dt} - A_r\left(\overline{r} - r_{Z_1}\right)\frac{dZ_1}{dt} + A_r\left(Z_2 - Z_1\right)\frac{d\overline{r}}{dt} + \dot{m}_{r,Z_2} - \dot{m}_{r,Z_1} = 0 \tag{9}$$

where $\overline{r}$ is the mean density in that zone and computed at the heat-exchanger pressure and mean density $\overline{h}$. The refrigerant energy balance for any zone, similarly, becomes:

$$\begin{aligned} &A_r\left(Z_2 - Z_1\right)\overline{h}\frac{d\overline{r}}{dt} + A_r\left(Z_2 - Z_1\right)\overline{r}\frac{d\overline{h}}{dt} + A_r\left[\left(\overline{r}\overline{h}\right) - \left(r\,h\right)_{Z_2}\right]\frac{dZ_2}{dt} - A_r\left[\left(\overline{r}\overline{h}\right) - \left(r\,h\right)_{Z_1}\right]\frac{dZ_1}{dt} \\ &- A_r\left(Z_2 - Z_1\right)\frac{dP}{dt} + \left(\dot{m}_r h\right)_{Z_2} - \left(\dot{m}_r h\right)_{Z_1} + A_r\left(Z_2 - Z_1\right)\dot{Q}''' = 0 \end{aligned} \tag{10}$$

The mean enthalpies in each zone are evaluated by making profile assumptions for the refrigerant state over the zone. The effectiveness-NTU method of computing heat-transfer rates in the single phase zone yields an exponential temperature profile and an assumption of uniform heat-flux in the two-phase zones yields a linear quality profile; the latter is used in this paper. The energy balances on the tube material and water are obtained as:

$$\begin{aligned} &\left(C_{pt}\, r_t A_t\right)\left(Z_2 - Z_1\right)\frac{d\overline{T}_t}{dt} + \left(C_{pt}\, r_t A_t\right)\left(\overline{T}_t - T_{t,Z_2}\right)\frac{dZ_2}{dt} - \left(C_{pt}\, r_t A_t\right)\left(\overline{T}_t - T_{t,Z_1}\right)\frac{dZ_1}{dt} \\ &= a_i p D_i \left(Z_2 - Z_1\right)\left(\overline{T}_r - \overline{T}_t\right) - a_o p D_o \left(Z_2 - Z_1\right)\left(\overline{T}_t - \overline{T}_w\right) \end{aligned} \tag{11}$$

and

$$\left(C_{pw}\rho_w A_w\right)\left[\left(Z_2-Z_1\right)\frac{d\overline{T}_w}{dt}+\left(\overline{T}_w-T_{w,Z_2}\right)\frac{dZ_2}{dt}-\left(\overline{T}_w-T_{w,Z_1}\right)\frac{dZ_1}{dt}\right]+\left(\dot{m}_w C_{pw}\right)\left(T_{w,Z_2}-T_{w,Z_1}\right)$$
$$=\alpha_i p D_i\left(Z_2-Z_1\right)\left(\overline{T}_t-\overline{T}_w\right) \tag{12}$$

where $\overline{T}_t$ and $\overline{T}_w$ are the mean tube and water temperatures in the zone under consideration. By applying these to the super-heated, two-phase and sub-cooled zones of the heat-exchanger the system of DAEs can be obtained. Recognizing the state postulate and expressing the density as a function of pressure and enthalpy as

$$\frac{d\overline{\rho}}{dt}=\left(\frac{\partial\rho}{\partial P}\right)_{\overline{h}}\frac{dP}{dt}+\left(\frac{\partial\rho}{\partial h}\right)_{P}\frac{d\overline{h}}{dt}$$

further simplifies equations (9) - (12). These simplified equations can be applied to each of the cases mentioned above. As an example, Table 1 shows the substitutions of parameters for the CSHTPSC case. Such tables may be developed for the other cases also.

Table 1: Variables to be substituted for the CSHTPSC case.

Parameter	Superheated zone	Condensing zone	Sub-cooled zone
Z_1 =	0	L_1	L_2
Z_2 =	L_1	L_2	L
$\overline{h}$ =	$\left(h_{in}+h_v\right)/2$	$\left(h_v+h_l\right)/2$	$\left(h_l+h_{out}\right)/2$
$\dot{m}_{r,Z_1}$ =	$\dot{m}_{rin}$	$\dot{m}_{r,L1}$	$\dot{m}_{r,L2}$
$\dot{m}_{r,Z_2}$	$\dot{m}_{r,L1}$	$\dot{m}_{r,L2}$	$\dot{m}_{rout}$
$\overline{T}_t$ =	$T_{t,1}$	$T_{t,2}$	$T_{t,3}$
$\overline{T}_w$ =	$T_{w,1}$	$T_{w,2}$	$T_{w,3}$

Using such substitutions and simplifying algebraically to eliminate the inter-zone refrigerant flow-rates $\dot{m}_{r,L1}$ and $\dot{m}_{r,L2}$, the refrigerant can be modeled as matrix equations of the form $B_{g,g}\cdot\overrightarrow{X_{MB}}=\overrightarrow{D_{MB}}$, where g is 2, 3 or 4 depending upon the number of zones that co-exist in the heat-exchanger. Appendix 2 shows the fully developed equations for the condenser with all three phase regions, which is typical of normal condenser operation.

As mentioned earlier, during transient operation the number of zones in the heat-exchanger changes as phase boundaries enter or leave the heat-exchanger. When such an event occurs, it is necessary to discretely switch between sets of equations. This raises three issues – the first is of how to detect the occurrence of such an event, the second is the identification of which event it is and the last is of how to handle the change in equation set. From an understanding of the physical behavior of vapor compression equipment, it is believed that only the following transitions are possible under normal operation: ETP ↔ ETPSH, CSH ↔ CSHTP ↔ CSHTPSC. The occurrence of these events is detected by comparing the exit enthalpy with the saturated liquid or vapor enthalpy. Whenever the exit enthalpy crosses over the appropriate saturated enthalpy, an event has occurred. The specific event that has occurred is identified by the direction of cross-over. The third issue of handling the change in equation depends on the answers to the first two issues. If a zone has exited the heat-exchanger, some variables (such as L_2, $T_{t,2}$, $T_{w,2}$ etc., for a change from CSHTPSC→CSHTP) become defunct. This does not alter the simulation since the variables in the reduced equation set are a subset of the preceding set. The same applies during the transitions of ETPSH→ETP or CSHTP→CSH. However, when the reverse transitions occur, i.e. zones enter the heat-exchanger (ETP→ETPSH, CSH→CSHTP, CSHTP→CSHTPSC), new variables enter the equation set. If these are not properly initialized, the simulation can numerically collapse. In a physically meaningful sense, the entering variables may be initialized to the values in the zone preceding it. This prevents numerical discontinuities in the simulation. One variable, however that requires special attention is the initial length of the entering zone. If this starts exactly at 0, the refrigerant equations become singular. To avoid this, the entering zone length is initialized to a small, positive non-zero number. Some trial-and-error is involved in selecting this initial length to ensure that it is small enough to avoid discontinuities in the solution.

5. SIMULATION RESULTS AND DISCUSSION

The heat-exchanger models described above were implemented in C++ and tested under start-up and load-change conditions. Physical dimensions and geometrical parameters were used from an available 90-ton, centrifugal chiller test stand, details of which are described by Comstock [1999]. For both formulations, the differential equations for the finite-volume condenser and evaporator and the moving-boundary condenser were integrated using the 2^{nd} order modified Euler predictor-corrector method (Press et al [2002]). A 1^{st} order, explicit Euler method was used for the moving-boundary evaporator as no noticeable advantage in speed or accuracy was found in using the 2^{nd} order method. Using the results of the numerical study described by Bendapudi et al [2005], the finite volume condenser and evaporator were discretized into 15 nodes each and the integration step-sizes were chosen by interval-halving until consecutive results were identical. The integration step-sizes for the moving-boundary formulation were also similarly chosen.

The condenser and evaporator models were incorporated into separate simplified system models. The system model used for the condenser consisted of a lumped evaporator, an orifice for the expansion and a pair of black-box regression models for the compressor to predict mass flow rate and exit enthalpy. The regression models for the compressor were based on available data from a 90-ton centrifugal chiller. The throttling is modeled using the orifice equation and assuming isenthalpic expansion. The condenser is initialized to a condition of containing fully superheated refrigerant at thermal equilibrium with the ambient. The system model is then executed through start-up until it reaches steady state. During start-up, the load on the evaporator and water flow rate through the condenser are kept constant; the temperature of the water entering the condenser is raised gradually from 30^{o}C to 35^{o}C. At 1000s into the simulation the condenser is subjected to a step-drop of 2^{o}C in the entering water temperature, to simulate a transient caused by a drop in cooling-tower temperature. Fig. 3 shows the behavior of the moving-boundary and the finite-volume condensers under such a simulation. The condenser pressure, leaving water temperature and sub-cooling are presented. It is seen that with either formulation, the predictions are very closely matched during start-up as well as during a load-change. The difference in steady-state pressures is less than 2.5% and is primarily due to the coarser (linear) property approximations in the moving-boundary formulations. By employing a more exponential profile, as would result from an effectiveness-NTU approach, it is believed that even this already small difference can be made smaller.

The evaporator model is simulated within a system model consisting of a thermo-static expansion valve and the compressor model described by Bendapudi et al [2002]. This allowed the evaporator to be studied for capacity as well as superheat control. The high-pressure side was simplified to assume a constant pressure and sub-cooling at entry to the expansion valve. The evaporator was initialized to contain low-quality two-phase vapor at equilibrium with the water in the tubes. Start-up and a step-change in return water temperature are simulated as for the condenser. Fig. 4 shows the behavior of the two formulations in terms of evaporator pressure, leaving water temperature and superheat. While the pressure and leaving water temperature are predicted quite comparably, the superheat is found to be significantly different, with the finite-volume formulation predicting about 30% larger superheat at steady-state. The transient response is also seen to be markedly different in spite of the fact that steady-state is reached at the same time. It is believed that this difference is due to the fact that superheat is a highly sensitive parameter and responds more strongly to small changes in pressure and enthalpy. Therefore, the specific arrangement of the two evaporator formulations within a simulated test-bench causes small variations in driving conditions that magnify the superheat results. A fully detailed system model is expected to produce better results.

An important observation regarding the moving-boundary formulation is the smooth and continuous prediction as phase-regions enter the heat-exchanger during start-up. In terms of execution speeds, the moving boundary formulation was significantly faster than the finite-volume, by a factor of 10-15 in the evaporator and 2-4 in the condenser. In a system simulation, this can yield significant savings in computation time.

6. CONCLUSIONS

This paper presents and compares finite-volume and moving-boundary formulations for shell-and-tube heat-exchangers that can be used for liquid chiller system models where the refrigerant flows on the shell-side. A generalized set of equations are developed for the moving-boundary formulation that can be developed into sets of

equations for different combinations of phase-regions in either heat-exchanger. The set of equations for the condition where the condenser has all three phases is explicitly provided for demonstrating the method. Individual condenser and evaporator models were simulated within simplified system models, and both formulations were compared for numerical equivalence and execution speed under identical conditions. The moving boundary formulation was found to be significantly faster by at least a factor of 2 computationally for quite comparable results.

FIGURES

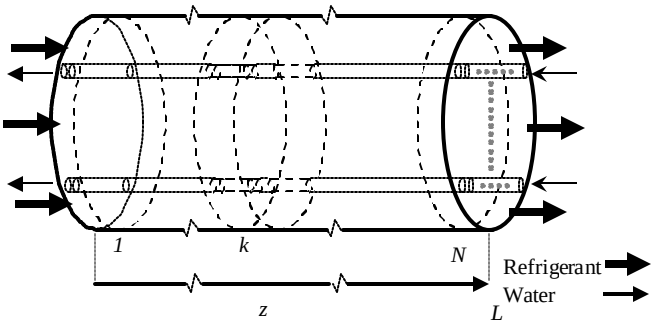

Fig.1: FV discretization

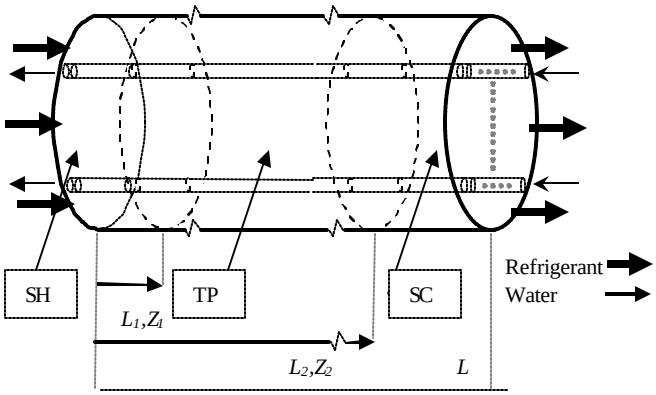

Fig. 2: MB discretization (CSHTPSC case)

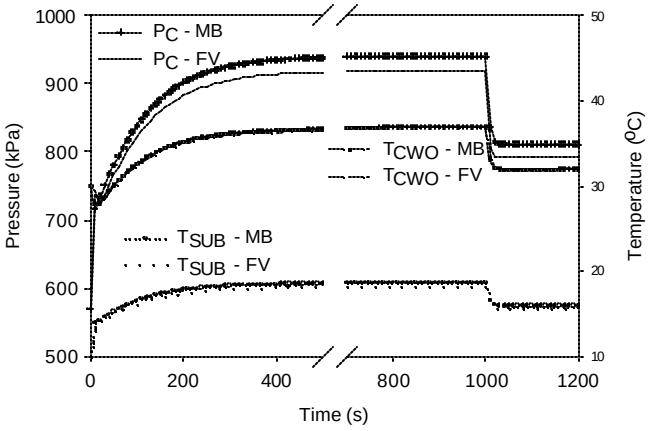

Fig. 3: Condenser simulations

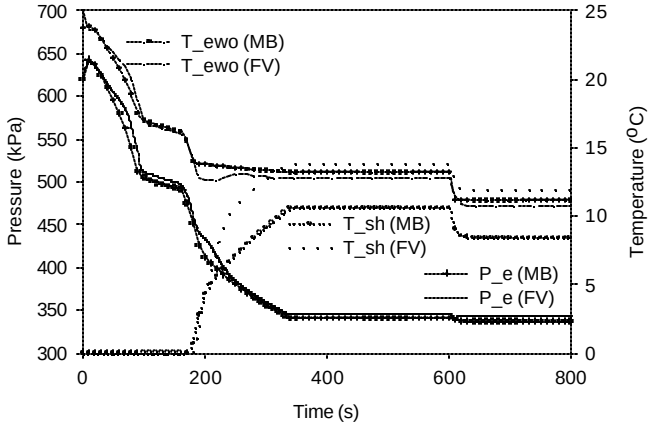

Fig. 4: Evaporator simulations

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NOMENCLATURE

Symbol	Description	Symbol	Description	Subscripts	Description
r	Density	$\dot{m}$	Mass flow-rate	w	Water
w	Velocity	V	Volume	r	Refrigerant
h	Enthalpy	M	Mass	Z_1	First phase-boundary
P	Pressure	D	Diameter	Z_2	Second phase-boundary
$\dot{Q}$	Heat transfer rate	$\dot{Q}'''$	Heat transfer per volume	t	Tube
C_p	Specific heat	a	Heat-transfer coefficient	1	First phase-zone
A	Cross-sectional area	Z	Distance from entry	2	Second phase-zone
T	Temperature	L	Total tube length	3	Third phase-zone
FV	Finite Volume	*MB*	Moving-Boundary	i	Inside tube
CSH	Condenser in fully superheated mode	*ETP*	Evaporator in fully two-phase mode	o	Outside tube
CSHTP	Condenser with superheat at entry & two-phase at exit	*ETPSH*	Evaporator with two-phase at entry & superheated at exit	*CSHTPSC*	Condenser with superheat at entry and sub-cooled at exit

APPENDIX 1

Refrigerant-side matrices for the Finite-Volume formulation

$$\vec{A}=\begin{bmatrix}
\sum_{j=1}^{N} a_j & b_1 & b_2 & b_3 & \cdot & \cdot & \cdot & \cdot & b_N \\
\sum_{j=1}^{N} c_j & d_1 & d_2 & d_3 & \cdot & \cdot & \cdot & \cdot & d_N \\
(c_1 - a_1 h_1) & d_1 - b_1 h_1 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\
(c_2 - a_2 h_2) - a_1 (h_2 - h_1) & b_1 (h_1 - h_2) & d_2 - b_2 h_2 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\
(c_3 - a_3 h_3) - (a_1 + a_2)(h_3 - h_2) & b_1 (h_2 - h_3) & b_2 (h_2 - h_3) & d_3 - b_3 h_3 & 0 & \cdot & \cdot & \cdot & 0 \\
(c_4 - a_4 h_4) - (a_1 + a_2 + a_3)(h_4 - h_3) & b_1 (h_3 - h_4) & b_2 (h_3 - h_4) & b_3 (h_3 - h_4) & d_4 - b_4 h_4 & 0 & \cdot & \cdot & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & & & \cdot & \cdot \\
(c_{N-1} - a_{N-1} h_{N-1}) - \sum_{j=1}^{N-2} a_j (h_{N-1} - h_{N-2}) & b_1 (h_{N-2} - h_{N-1}) & b_2 (h_{N-2} - h_{N-1}) & b_3 (h_{N-2} - h_{N-1}) & b_4 (h_{N-2} - h_{N-1}) & \cdot & \cdot & \cdot & d_{N-1} - b_{N-1} h_{N-1}
\end{bmatrix}$$

$$\vec{B}_{FV}=\begin{bmatrix}
\dot{m}_{r,in} - \dot{m}_{r,out} & \dot{m}_{r,in} h_{r,in} - \dot{m}_{r,out} h_N - \sum_{j=1}^{N} \dot{Q}_{r,j} & \dot{m}_{r,in} (h_{r,in} - h_1) - \dot{Q}_{r,1} & \dot{m}_{r,in} (h_1 - h_2) - \dot{Q}_{r,2} & \cdots & \dot{m}_{r,in} (h_{N-2} - h_{N-1}) - \dot{Q}_{r,N-1}
\end{bmatrix}$$

APPENDIX 2

Refrigerant-side mass-energy balances for the Moving-Boundary formulation

$$B_{4,4} = A_r \begin{Bmatrix}
b_1 & (\bar{r}_1 - \bar{r}_2) & (\bar{r}_2 - \bar{r}_3) & \frac{1}{2}(L - L_2)\left(\frac{\partial r}{\partial h_3}\right)_P \\
b_2 & (\bar{r}_1 \bar{h}_1 - \bar{r}_2 h_v) & h_v (\bar{r}_2 - \bar{r}_3) & h_v \frac{1}{2}(L - L_2)\left(\frac{\partial r}{\partial h_3}\right)_P \\
b_3 & -\bar{r}_2 (\bar{h}_2 - h_v) & \bar{r}_2 (\bar{h}_2 - h_v) + \bar{r}_3 (h_v - h_l) & -(h_v - h_l)\frac{1}{2}(L - L_2)\left(\frac{\partial r}{\partial h_3}\right)_P \\
b_4 & 0 & -\bar{r}_3 (\bar{h}_3 - h_l) & \frac{1}{2}(L - L_2)\left(\bar{r}_3 + (\bar{h}_3 - h_l)\left(\frac{\partial r}{\partial h_3}\right)_P\right)
\end{Bmatrix}$$

where

$$b_1 = L_1\left[\frac{1}{2}\left(\frac{\partial r}{\partial h_1}\right)_P \frac{dh_v}{dP} + \left(\frac{\partial r}{\partial P}\right)_{h_1}\right] + (L_2 - L_1)\left[\frac{1}{2}\left(\frac{\partial r}{\partial h_2}\right)_P \left(\frac{dh_v}{dP} + \frac{dh_l}{dP}\right) + \left(\frac{\partial r}{\partial P}\right)_{h_2}\right]$$
$$+ (L - L_2)\left[\frac{1}{2}\left(\frac{\partial r}{\partial h_3}\right)_P \frac{dh_l}{dP} + \left(\frac{\partial r}{\partial P}\right)_{h_3}\right]$$

$$b_2 = L_1\left[\frac{1}{2}\frac{dh_v}{dP}\left(\bar{r}_1 + \bar{h}_1\left(\frac{\partial r}{\partial h_1}\right)_P\right) + \bar{h}_1\left(\frac{\partial r}{\partial P}\right)_{h_1} - 1\right]$$
$$+h_v\left\{(L_2 - L_1)\left[\frac{1}{2}\left(\frac{\partial r}{\partial h_2}\right)_P\left(\frac{dh_v}{dP} + \frac{dh_l}{dP}\right) + \left(\frac{\partial r}{\partial P}\right)_{h2}\right] + (L - L_2)\left[\frac{1}{2}\left(\frac{\partial r}{\partial h_3}\right)_P\frac{dh_l}{dP} + \left(\frac{\partial r}{\partial P}\right)_{h_3}\right]\right\}$$

$$b_3 = (L_2 - L_1)\left[\frac{1}{2}\left(\frac{dh_v}{dP} + \frac{dh_l}{dP}\right)\left(\bar{r}_2 + \bar{h}_2\left(\frac{\partial r}{\partial h_2}\right)_P\right) + \bar{h}_2\left(\frac{\partial r}{\partial P}\right)_{h_2} - 1\right]$$
$$-h_v(L_2 - L_1)\left[\frac{1}{2}\left(\frac{\partial r}{\partial h_2}\right)_P\left(\frac{dh_v}{dP} + \frac{dh_l}{dP}\right) + \left(\frac{\partial r}{\partial P}\right)_{h2}\right] - (h_v - h_l)(L - L_2)\left[\frac{1}{2}\left(\frac{\partial r}{\partial h_3}\right)_P\frac{dh_l}{dP} + \left(\frac{\partial r}{\partial P}\right)_{h_3}\right]$$

$$b_4 = (L - L_2)\left[\frac{1}{2}\frac{dh_l}{dP}\left(\bar{r}_3 + \bar{h}_3\left(\frac{\partial r}{\partial h_3}\right)_P\right) + \bar{h}_3\left(\frac{\partial r}{\partial P}\right)_{h_3} - 1\right] - h_l(L - L_2)\left[\frac{1}{2}\left(\frac{\partial r}{\partial h_3}\right)_P\frac{dh_l}{dP} + \left(\frac{\partial r}{\partial P}\right)_{h_3}\right]$$

and

$$\overrightarrow{X_{MB}} = \begin{bmatrix} \frac{dP}{dt} & \frac{dL_1}{dt} & \frac{dL_2}{dt} & \frac{dh_{out}}{dt} \end{bmatrix}^T \quad \overrightarrow{D_{MB}} = \begin{bmatrix} \dot{m}_{r,in} - \dot{m}_{r,out} - \frac{1}{2}A_r L_1\left(\frac{\partial r}{\partial h_1}\right)_P\frac{dh_{in}}{dt} \\ \dot{m}_{r,in}h_{in} - \dot{m}_{r,out}h_{out} - \dot{Q}_{r,1} - \frac{1}{2}A_r L_1\left(\bar{r}_1 + \bar{h}_1\left(\frac{\partial r}{\partial h_1}\right)_P\right)\frac{dh_{in}}{dt} \\ \dot{m}_{r,out}(h_v - h_l) - \dot{Q}_{r,2} \\ \dot{m}_{r,out}(h_l - h_{out}) - \dot{Q}_{r,3} \end{bmatrix}$$

Tube-side energy balances

$$\frac{dT_{t,1}}{dt} = \frac{\dot{Q}_{r,1} - \dot{Q}_{w,1} + r_t A_t C_{pt}(T_{t,2} - T_{t,1})\frac{dL_1}{dt}}{r_t A_t C_{pt} L_1}$$

$$\frac{dT_{t,2}}{dt} = \frac{\dot{Q}_{r,2} - \dot{Q}_{w,2} - r_t A_t C_{pt}\left[(T_{t,2} - T_{t,1})\frac{dL_1}{dt} - (T_{t,3} - T_{t,2})\frac{dL_2}{dt}\right]}{r_t A_t C_{pt}(L_2 - L_1)}$$

$$\frac{dT_{t,3}}{dt} = \frac{\dot{Q}_{r,3} - \dot{Q}_{w,3} - r_t A_t C_{pt}(T_{t,3} - T_{t,2})\frac{dL_2}{dt}}{r_t A_t C_{pt}(L - L_2)}$$

Water-side energy balances

$$\frac{dT_{w,1}}{dt} = \frac{\dot{Q}_{w,1} + \dot{m}_w C_{pw}(T_{w,2} - T_{w,1}) + r_w A_w C_{pw}(T_{w,2} - T_{w,1})\frac{dL_1}{dt}}{r_w A_w C_{pw} L_1}$$

$$\frac{dT_{w,1}}{dt} = \frac{\dot{Q}_{w,2} + \dot{m}_w C_{pw}(T_{w,3} - T_{w,2}) - r_w A_w C_{pw}\left[(T_{w,2} - T_{w,1})\frac{dL_1}{dt} - (T_{w,3} - T_{w,2})\frac{dL_2}{dt}\right]}{r_w A_w C_{pw}(L_2 - L_1)}$$

$$\frac{dT_{w,1}}{dt} = \frac{\dot{Q}_{w,3} + \dot{m}_w C_{pw}(T_{w,in} - T_{w,3}) - r_w A_w C_{pw}(T_{w,3} - T_{w,2})\frac{dL_2}{dt}}{r_w A_w C_{pw}(L - L_2)}$$