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ABSTRACT

A model for a low speed compressor system is proposed which is developed from an automatic control system viewpoint. The dynamic model is comprised of a low speed fan, a delivery duct, and a storage volume with exit nozzle. This model is similar to a HVAC system with variable exhaust areas and a desired steady-state positive system pressure. The system's simulations are used to demonstrate unstable operating conditions that could arise during normal operations. A Liapunov stability criterion test predicts the instability condition for this compressor system. The simulations are performed with a continuous system modeling program.

NOMENCLATURE

λ	eigenvalues of the Jacobian system
$(\ )'$	first derivative - w/r to time (1/sec)
X	n × 1 state vector
$\dot{m}_i$	i = 1,2 mass flow rate (lb_m/sec)
p_i	i = 1,2 pressure lb_f/ft^2
ρ	fluid density (lb_m/ft^3)
$\underline{u}$	fluid velocity vector (ft/sec)
$\bar{A}_i$	i = 1,2 area of duct or exit nozzle (ft^2)
^oR	degrees Rankine
R	universal gas constant (ft-lb_f)/(lb_m-^oR)
ℓ_1	length of supply duct (ft)
V_1	volume of compressible flow area (ft^3)
p_a	atmospheric pressure (lb_f/ft^2)
C_p	constant pressure specific heat (Btu/(lb_m-^oR))
C_v	constant volume specific heat (Btu/(lb_m-^oR)
κ	ratio of specific heats
T_i	i = 1,2 fluid temperature (^oR)
K_d	supply duct flow coefficient (lb_m-ft^2)/(lb_f-sec)
K_v	volume compressibility coefficient (lb_f/ft^2-sec))
K_o	nozzle discharge coefficient (lb_m/sec)$(ft^2/lb_f)^{1/2}$

I INTRODUCTION

A turbomachine is a rotating machine in which there is an exchange of energy between the rotating rotor and a fluid due to the dynamic behavior of the fluid. The flow rate of the fluid depends on the pressure difference across the rotor rather than a fixed displacement volume. The pressure increase is strongly dependent upon the rotational speed of the rotor. Two classes of turbomachines are centrifugal and axial-flow compressors. Either of these machines may be further classified as operating under compressible or incompressible flow conditions. Fans are usually classified as incompressible flow turbomachines which increase the pressure of a fluid (usually air) under nearly incompressible conditions.

Centrifugal fans are constructed with several blade styles, including forward slanting blades and backward slanting blades. Forward slanting blade fans are characterized by low cost, poor efficiency, and poor operating stability. Backward slanting blade fans are characterized by high cost, high efficiency, and good operating stability. High-flow volume fans are usually axial-flow fans which have high purchase cost and exhibit poor operating stability. Many of such turbomachines have been installed as the low-speed air supply machines for HVAC systems. All of these turbomachines can be further characterized by a particular shape of their "compressor map". The compressor map is a plot of the pressure rise across the rotor as a function of mass flow rate. Each of the compressor maps associated with these three fans will show an operating region labeled "unstable flow" or "surge condition". What is not obvious from an analysis of the operation of a turbomachinery system, is that this region can be modeled, simulated, and predicted by the use of a stability analysis from automatic control theory. This paper will then present the model of a dynamic turbomachine system as an automatic control system in which the stability of the compressor system will be determined by Liapunov's first method[34].

II MODEL OF DYNAMIC SYSTEM

An axial-flow fan system is modeled as a low-speed compressor supplying a large volume of fluid (air) through a supply duct, and this flow exhausts through a nozzle. The low-speed compressor is assumed to have a known compressor map that can be modeled as

$$p_1(m_1') = a_n(m_1')^n + \ldots + a_3(m_1')^3 + a_2(m_1')^2 + a_1(m_1')^1 + a_0 + p_a, \qquad (1)$$

where $p_1(m_1')$ is the pressure rise across the fan, m_1' is the mass flow-rate of the fan, and p_a is the atmospheric pressure. This nth order polynomial could be determined from previous experimental data.

The fan exhausts into a supply duct in which the flow is modeled as an incompressible, invisid, 1-dimensional flow. The sum of the forces in the x-direction on a control volume around the supply duct is

$$\Sigma F_x = \frac{\partial}{\partial t} \int_V \rho \underline{u} dV + \int_{cs} \rho \underline{u}^2 dA \qquad (2)$$

where ρ is the density of the fluid, $\underline{u}$ is the flow velocity, V is the duct volume, and A the area of the duct. For incompressible, fully developed flow the momentum flux (the second term on the right) is equal to zero. This equation may be written using the pressure forces on the control surface as

$$(p_1 - p_2)A = \frac{\partial}{\partial t} \int_V \rho \underline{u} dV. \qquad (3)$$

For an ideal gas the equation of state is

$$\rho = \frac{p}{RT} \qquad (4)$$

where R is the universal gas constant, and (p) is the pressure of the fluid. The volume of the supply duct is a function of the area and the length as

$$V = A\ell. \qquad (5)$$

Therefore, the forces on the duct flow may be written as

$$(p_1 - p_2)A = \ell \frac{\partial}{\partial t} \int_A \frac{p}{RT} \underline{u} dA. \qquad (6)$$

If uniform properties of the fluid are assumed, and for a constant area duct this equation (6) may be written as

$$(p_1 - p_2)A = \ell \frac{d}{dt} \frac{pA}{RT} u. \qquad (7)$$

The velocity of the fluid is a function of the density and mass flow-rate as

$$u = \frac{m' RT}{Ap}. \qquad (8)$$

and substituting this into (7),

$$(p_1-p_2)A = \ell \frac{d}{dt}(m') \quad (9)$$

the differential equation for the mass flow-rate is formed

$$\frac{d}{dt}(m') = (p_1-p_2)\frac{A}{\ell}. \quad (10)$$

The flow in the volume V_2 is modeled as an ideal gas that compresses isentropically. An energy balance of this system shows that the pressure in the volume is a function of the mass flow-rate from the supply duct and out of the exit orifice. For an ideal gas with constant specific heats C_p and C_v, it can be shown that the difference of the rate of energy in the system equals the rate of change of energy in the system as

$$C_p(m_1'T_1 - m_2'T_2) = \frac{d}{dt}(\rho_2 V_2 C_v T_2). \quad (11)$$

Dividing this equation by C_p and T_2, and making the appropriate substitution for ρ_2, this equation is simplified as

$$\frac{m_1'T_1}{T_2} - m_2' = \frac{d}{dt}\left(\frac{p_2 C_v}{R C_p}\right). \quad (12)$$

Since $\kappa = \frac{C_p}{C_v}$, and $\frac{T_1}{T_2} \simeq 1$

the rate of change of pressure in the volume V_2 can be written as

$$\frac{d}{dt}(p_2) = T_2 R\kappa(m_1' - m_2'). \quad (13)$$

The flow through the exit orifice can be modeled as an incompressible, invisid flow through a nozzle. From Bernoulli's equation, the mass flow-rate through a nozzle is

$$m_2' = C_d A_2 (2\rho_2)^{1/2} (p2 - p_a)^{1/2} \quad (14)$$

where C_d is the ratio of C_c and C_v, which are

$$C_c = \frac{\text{effective flow area}}{\text{geometric area}}$$

and $$C_v = \frac{\text{actual flow velocity}}{\text{ideal velocity}}.$$

These differential equations and algebraic solutions may be rewritten in state space form as

$$\frac{d}{dt}\begin{vmatrix} m_1' \\ p_2 \end{vmatrix} = \begin{vmatrix} K_d\{p_1(m_1') - p_2\} \\ Kv\{m_1' - K_o\sqrt{p_2-p_a}\} \end{vmatrix} \quad (15)$$

which is in the form of a system of nonlinear equations

$$X'(t) = F(X(t)). \quad (16)$$

where

$$K_d = \frac{A_1 g_c}{V_2}, \qquad K_v = \frac{\rho R\kappa T_2}{\ell_1}, \quad (17)$$

and $K_o = C_d A_2\sqrt{2\rho g_c}$.

III STABILITY ANALYSIS

A system is stable if trajectories leaving an initial state return to and remain within a specified region surrounding an equilibrium state. This stability is call "stability in the sense of Liapunov", and it permits limit cycles and vortices[2,3,4]. A system is asymptotically stable if the trajectories of a systems that is stable in the sense of Liapunov eventually converges to the equilibrium state. Further a system is locally stable (or stable in the small) if the system is stable only for initial states within a bounded region of state space. If the system is stable for all initial states within the entire state space it is globally stable (or stable in the large).

Obviously a test for globally, asymptotically stable system operating conditions would be of benefit to the operation of a compressor system. Krasovskii's method for the generation of a Liapunov function is such a method. This stability check begins by determining the Jacobian of the nonlinear system equations. This partial differentiation linearizes the system equations about each equilibrium state and forms the Jacobian matrix

$$J = \left| \frac{\partial f_i}{\partial x_j} \right| \qquad i = 1,2,..n;\ j=1,2,...n \tag{18}$$

$$J = \begin{vmatrix} K_d \dfrac{\partial p_1(m_1')}{\partial m_1} & -K_d \\ K_v & -\dfrac{K_v K_\rho}{2}(p_2 - p_a)^{1/2} \end{vmatrix}. \tag{19}$$

The eigenvalues of the Jacobian matrix are found by

$$\det(\lambda I - J) = \det \begin{vmatrix} \lambda - K_d \dfrac{\partial p_1(m_1')}{\partial m_1} & K_d \\ -K_v & \lambda + \dfrac{K_v K_\rho}{2}(p_2 - p_a)^{1/2} \end{vmatrix} \tag{20}$$

$$= (\lambda - \beta_1)(\lambda + \beta_2 + K_v K_d)$$

$$= \lambda^2 + (\beta_2 - \beta_1)\lambda + K_v K_d - \beta_1 \beta_2$$

where $$\beta_1 = K_d \frac{\partial p_1(m_1')}{\partial m_1} \tag{21}$$

$$\beta_2 = \frac{K_v K_\rho}{2}(p_2 - p_a)^{1/2}. \tag{22}$$

By the Liapunov stability criterion, the Jacobian characteristic equation must have eigenvalues with negative real parts for asymptotic global stability, therefore,

$$\beta_2 - \beta_1 > 0 \text{ for stability.} \tag{23}$$

Note that this stability test is a function of all the system parameters, including the area/length ratio of the supply duct and the magnitude of the compressible volume space V_2.

IV SIMULATION

A low-speed compressor was experimentally determined to have a compressor map equation of the form[5]

$$p_1(m_1') = a_4(m_1')^4 + a_3(m_1')^3 + a_2(m_1')^2 + a_1(m_1')^1 + a_0 + p_a, \tag{24}$$

where $a_4 = -0.474\ (lb_f/ft^2)^4$, $a_3 = -2.47\ (lb_f/ft^2)^3$
$a_2 = 0.0775\ (lb_f/ft^2)^2$, $a_1 = -10.7\ (lb_f/ft^2)$
$a_0 = 0.00$.

Figure (1) is a plot of the compressor map operating points simulated in this paper under stable and asymptotically stable conditions. For p_a= 2020 lb_f/ft^2, V= 64.0 ft^3, A_1= 2.00 ft^2, L= 4.0 ft, rho= 0.0765 lb_m/ft^3, T_2 = 530 R, and C_d = 0.97 the compressor system was simulated with several exit area nozzles sizes

$$A_2 = \{ 0.24, 0.26, 0.27, 0.28,\ldots,0.36\} \; ft^2. \qquad (25)$$

Figure (2) shows the variation of the nozzle area for the simulation time of 96.0 seconds. The period of 8.0 seconds between each step increase in A_2 was sufficient to allow an asymptotic response to occur if it was possible.

The area changes brought about a dramatic response in the mass flow-rate in the supply duct ($\dot{m}_1$) , figure (3), and the storage volume flow rate ($\dot{m}_2$) as shown in figure (4). Note that there are values of A_2 for which there is a asymptotically stable operation of the system, and that there is a large range of A_2 in which stable (in the sense of Liapunov) operation is possible. The limit-cycle response of the compressor system in the area range of $A_2 = \{0.27, 0.28, 0.29,\ldots,0.34\}$ ft^2 would obviously be undesirable operating points. Similarly, in figure (5), there is corresponding change in the pressure in the supply duct (p_1) and storage volume ($\dot{p}_2$) figure (6). One should remember that the actual state variables are ($\dot{m}_1$) and (p_2), and that ($\dot{m}_2$) and (p_1) are algebraic solutions.

A plot of the variable (β_2-β_1) is shown in figure (2) in an attempt to predict the onset of stable operation (with limit-cycles) from asymptotic stable operations. This figure (2) shows the stability criterion variable to be <u>always</u> positive in steady-state for <u>every</u> asymptotic operating point (in the sense of Liapunov) and <u>never always</u> positive in steady-state for <u>every</u> stable operating point (in the sense of Liapunov). For operations of increasing the nozzle area above A_2 = 0.35 ft^2 and operations below the area A_2 = 0.28 ft^2 the stability test (β_2-β_2) correctly predicts the operation of the compressor system. This compressor system is then found to be locally stable in the sense of Liapunov.

V CONCLUSION

It has been shown that a dynamic model of a low-speed compressor system can be formed from first principles. This model can then be used to check the stability of the modeled compressor systems with a Liapunov function. The stability criterion checks the positive definiteness of the Jacobian matrix of the compressor system. The stability criterion predicts the onset of the switch from an asymptotic stable operating point (in the sense of Liapunov) to a stable operating point with limit-cycle flow. This stability criterion could be extended to control the stability of the operating system or define limits of asymptotic operating conditions.

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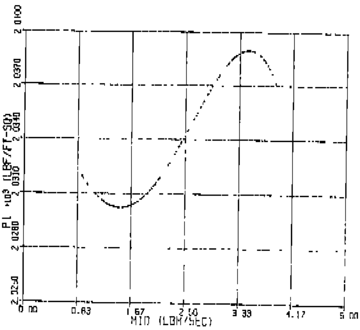

Figure (1), Compressor Map, Low Speed Fan

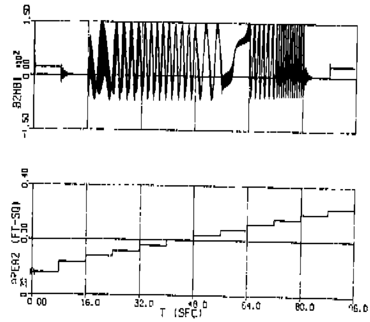

Figure (2), Exit Nozzle Area A_2 and Stability Criterion $(\beta_2-\beta_1)$

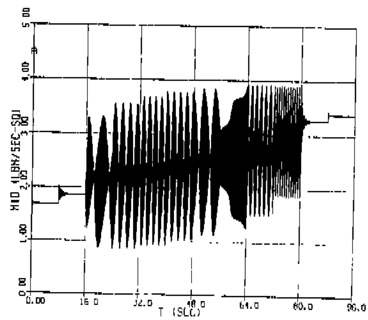

Figure (3), Mass Flow-Rate m_1' from Fan

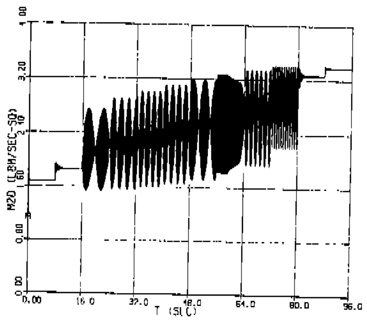

Figure (4), Mass Flow-Rate m_2' from Exit Nozzle

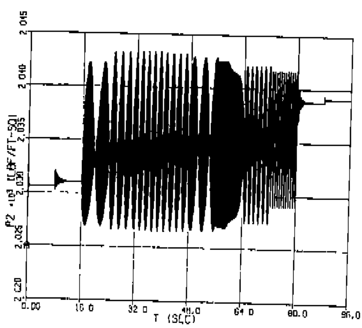

Figure (5), Pressure p_1 from Fan

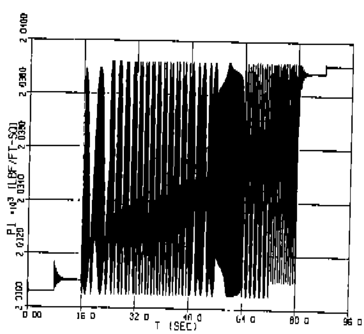

Figure (6), Pressure p_2 in Volume V_2