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FOUR POLE PARAMETERS OF SHELL CAVITY AND APPLICATION TO GAS PULSATION MODELING

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ABSTRACT

Gas pulsations in compressors are important because of their influence on compressor performance and noise. Typically, only one dimensional or lumped acoustic models were used in compressor simulations. In this paper, the compressor shell cavity is modeled as an annular cylindrical geometry and its four pole parameters are formulated so that the cavity can be included in the overall simulation model of the compressor. As an example, this formulation was applied to the analysis of a shell cavity connected to an annechoic pipe with a single and double input.

INTRODUCTION

Gas pulsations in the hermetic shell cavity were observed as one of the major noise sources in compressors by many investigators[1,2,3]. Gas pulsations in the shell cavity should be analyzed in conjunction with the overall compressor system model. Four pole parameters are a very useful concept for the analysis of a composite acoustic system because of computational efficiency and flexibility. If four pole parameters of the shell cavity are known, the effect of gas pulsations in the cavity can be combined with the overall system analysis.

If the acoustic system has large dimensions compared to the wave length of interest, the pressure responses which are necessary to formulate four pole parameters have to be obtained by solving the three dimensional continuous wave equation of the system[4]. In this paper, an eigenfunction expansion method was employed to obtain four pole parameters of the shell cavity by idealizing it as an annular cylindrical geometry.

As an application example, this formulation was applied to a cavity connected to an anechoic pipe, with a single and double input. Application examples were chosen in such a way that they contribute to the understanding of pressure pulsations inside a shell enclosing a refrigeration compressor, with the acoustic input being the suction port of the compressor and the output being the evaporator pipe.

DERIVATION OF FOUR POLES OF AN ANNULAR CYLINDRICAL CAVITY BY NORMAL MODE EXPANSION METHOD

Figure 1 shows a typical hermetic compressor shell which is modeled as an annular cavity bounded by two concentric cylindrical walls connected to an anechoic pipe. Four poles of a linear acoustic system can be formulated in terms of forced response solutions of the system[5].

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$$A = \frac{f_2(\underline{r}_2,\omega)}{f_1(\underline{r}_2,\omega)} \quad (1)$$

$$B = \frac{1}{f_1(\underline{r}_2,\omega)} \quad (2)$$

$$C = -f_2(\underline{r}_1,\omega) + \frac{f_1(\underline{r}_1,\omega)}{f_1(\underline{r}_2,\omega)} \cdot f_2(\underline{r}_2,\omega) \quad (3)$$

$$D = \frac{f_1(\underline{r}_1,\omega)}{f_1(\underline{r}_2,\omega)} \quad (4)$$

where, $\underline{r}_1 = (r_1,\theta_1,z_1)$ and $\underline{r}_2 = (r_2,\theta_2,z_2)$ are the input point and output point of the system, and $f_j(\underline{r}_i,\omega)$ is the pressure response at $\underline{r}_i$ due to a unit volume flow input at $\underline{r}_j$.

Therefore, four poles of an acoustic system can be obtained if a general pressure response solution of the system is known. The pressure response solution of an annular cylindrical cavity which is shown in Figure 1 was obtained as follows[5].

$$f_j(\underline{r}_i,\omega) = j\omega\rho_o C_o^2 \sum_{l=1}^{2} \sum_{q=o}^{\infty} \sum_{n=o}^{\infty} \sum_{m=o}^{\infty} \frac{P_{(mnq)l}(\underline{r}_i)\, P_{(mnq)l}(\underline{r}_j)}{N_{mnq}\,[(\omega_{mnq}^2-\omega^2) + 2j\,\omega\,\omega_{mnq}\,\xi_{mnq}]} \quad (5)$$

In equation (5), ξ_{mnq} are modal damping coefficients, and $P_{(mnq)l}(\underline{r})$, $l = 1,2$ are two orthogonal natural modes of the annular cavity in terms of pressure, which are

$$P_{(mnq)1}(\underline{r}) = [\, J_n(\kappa_{nq}r) + C_{nq}\, Y_n(\kappa_{nq}r)\,] \cos n\theta \cos \frac{m\pi z}{h} \quad (6)$$

$$P_{(mnq)2}(\underline{r}) = [\, J_n(\kappa_{nq}r) + C_{nq}\, Y_n(\kappa_{nq}r)\,] \sin n\theta \cos \frac{m\pi z}{h} \quad (7)$$

where,

$$C_{nq} = \frac{n\, J_n(\kappa_{nq}a) - \kappa_{nq}a\, J_{n+1}(\kappa_{nq}a)}{n\, Y_n(\kappa_{nq}a) - \kappa_{nq}a\, Y_{n+1}(\kappa_{nq}a)} \quad (8)$$

and κ_{nq} is the q^{th} root of the characteristic equation of the system;

$$J_n'(\kappa a)\, Y_n'(\kappa b) - J_n'(\kappa b)\, Y_n'(\kappa a) = 0 \quad (9)$$

where primes denote differentiation with respect to r. Also, ω_{mnq} are given by

$$\omega_{mnq} = C_o \sqrt{\kappa_{nq}^2 + \left(\frac{m\pi}{h}\right)^2} \quad (10)$$

where, C_o is the sound velocity in the medium. N_{mnq} in equation(5) are defined as

$$N_{mnq} = \int_o^h \int_o^{2\pi} \int_a^b r\, P_{(mnq)1}^2(r,\theta,z)\, dr\, d\theta\, dz \quad (11)$$

It shoud be noticed that a constant pressure mode should be included by taking $P_{(o,o,o)1} = 1$, $P_{(o,o,o)2} = 0$, and $\omega_{ooo} = 0$. Detail derivation is given in reference[5].

For a specific case of $a = 0.06$ m, $b = 0.08$ m, $h = 0.185$ m, and $C_o = 162.9$ m/sec, natural frequencies for various m,n,q are shown in Figure 2. From Figure 2, it can be seen that the lower natural frequencies correspond to q = 1, therefore lower natural modes are predominantly circumferential or vertical modes.

APPLICATION TO SYSTEM PROBLEMS

As an example of an application of four pole parameters, consider an annular cavity which is connected to an anechoic pipe and subjected to a harmonic input volume flow source as shown in Figure 1. The relationship between the volume velocity and the pressure of the pipe entrance is

$$Q_E = \frac{S_{an}}{\rho_o C_o} P_E \tag{12}$$

Therefore the four pole equation relating input Q_i, P_i to output Q_E becomes

$$\begin{Bmatrix} Q_i \\ P_i \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} 1 \\ \dfrac{\rho_o C_o}{S_{an}} \end{Bmatrix} Q_E \tag{13}$$

where, ρ_o is average density of the gas in the cavity.

From equation (13), the driving point impedance is obtained as

$$\frac{P_i}{Q_i} = \frac{C \dfrac{S_{an}}{\rho_o C_o} + D}{A \dfrac{S_{an}}{\rho_o C_o} + B} \tag{14}$$

Also, transfer functions are

$$T_Q(\omega) = \frac{Q_E}{Q_i} = \frac{\dfrac{S_{an}}{\rho_o C_o}}{A \dfrac{S_{an}}{\rho_o C_o} + B} \tag{15}$$

$$T_P(\omega) = \frac{P_E}{Q_i} = \frac{1}{A \dfrac{S_{an}}{\rho_o C_o} + B} \tag{16}$$

Volume flow transfer functions are shown in Figure 3 for four types of input/output arrangements. The cavity has the same dimension as that of Figure 2, while $\rho_o = 6.04$ Kg/ m^3 and $\xi_{mnq} = 0.01$. The input position is taken as $\underline{r}_1 = (0.065\text{m}, 0^\circ, 0.1321\text{m})$, and the ouput ports are taken as $\underline{r}_2 = (0.075\text{m}, 180^\circ, 0.1321\text{m})$ for case C_1, $(0.075\text{m}, 180^\circ, 0.0925\text{m})$ for case C_2, $(0.075\text{m}, 0^\circ, 0.1321\text{m})$ for case C_3, $(0.075\text{m}, 0^\circ, 0.0925\text{m})$ for case C_4. The cross sectional area of the anechoic pipe S_{an} was taken as 0.3 Cm^2.

Figure 4 shows the volume flow transfer functions at the first, the second and the third lowest natural frequencies as function of the position of output port. It is expected that the transfer function variation resembles each corresponding mode. As a practical application, if a certain frequency components are found to be important to be controlled as the result of sound measurement, the input port position can be changed to minimize the transfer function.

If there are two inputs whose frequencies are slightly different, a beating phenomenon can be observed. The pressure response of the cavity to two input flows are,

$$P(t) = T_{P1}\, Q_1\, e^{j\omega_1 t} + T_{P2}\, Q_2\, e^{j\omega_2 t} \tag{17}$$

where, T_{P1} and T_{P2} are transfer functions between input point and the output point which are defined by equation (16). Time history of the pressure at the output point is shown in Figure 5, for the case when $\omega_1 = 315$ Hz, $\omega_2 = 285$ Hz. Two inputs are at $\underline{r}_1$ = (0.065m, 0^o, 0.13214m) and $\underline{r}_2$ = (0.065m, 0^o, 0.0925m) and all other conditions are the same as those of Figure 3.

ACKNOWLEDGEMENT

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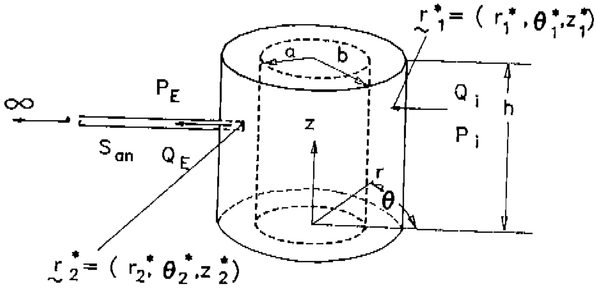

Figure 1. Annular cylindrical cavity

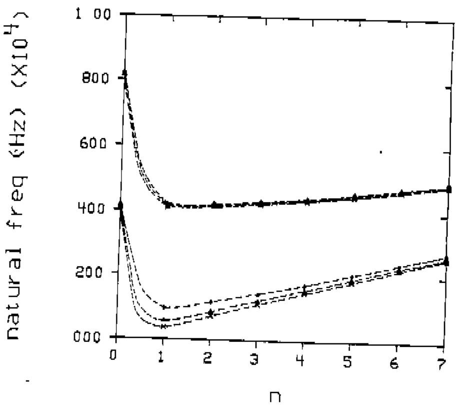

Figure 2. Natural frequencies, m = 0 --x--, m = 1 --☆--, m = 2 --◊--, q = 1 ; lower curve, q = 2 ; upper curve.

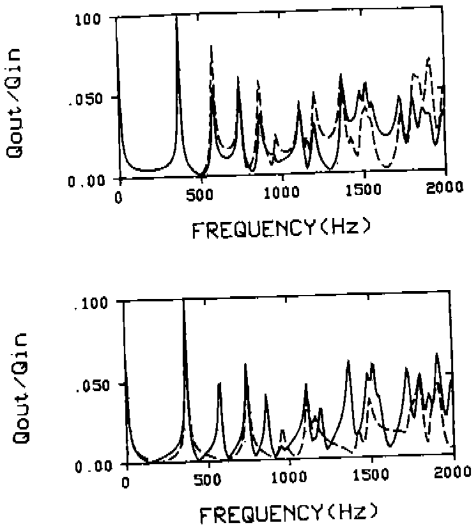

Figure 3. Volume flow transfer function for different input-output geometry case C1, ——— (top), case C2, ---- (top), case C3, ———(bottom) case C4, ----- (bottom)

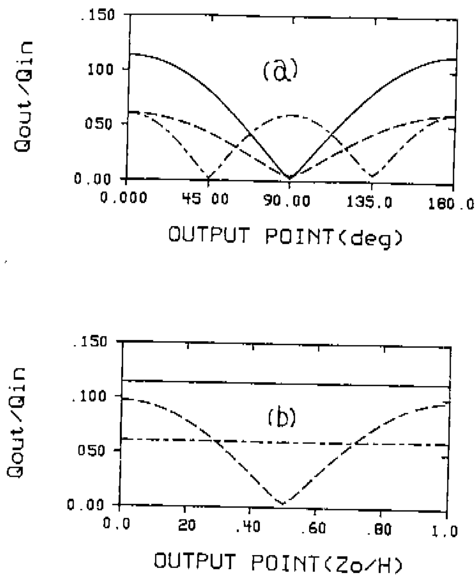

Figure 4. Change of transfer functions due to the output port location for the first resonance, ———, the second resonance, -----, and the third, —·— (a) circumferential variation for Z_o/H = 5/7, (b) axial variation for $\theta = 180^{o}$.

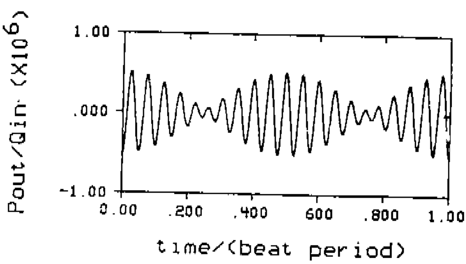

Figure 5. Beating phenomenon of the output pressure