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ABSTRACT

The single screw compressor is proving to be much advantages and broad prospects as the twin compressor. It's not subjected to any radial or axial thrust. Also from weight, size, noise level,reliability etc. many points are promising.

This paper presents a systematic analysis of the close angle, the area of the gaterotor interventing in the groove, its centroid and the volumetric flow rate. A series of formulas have been derived and the principles governing the selection of the design parameters are discussed

NOMENCLATURE

The surface through the mainscrew and perpendicular with the gatero-tors axis is called main plane, most dimensions can be expressed in the main plane (fig.1).

r_1, r_2 — radious of mainrotor and gaterotor

θ_1 — rotation angle of mainrotor

θ_2 — rotation angle of gaterotor

Z_1 — teeth number of mainrotor

Z_2 — teeth number of gaterotor

i — transmission ratio

A — distance between screw and gaterotor axis

a — distance between gaterotor axis and column surface of mainscrew

l — axis length at discharge side

l' — axis length at suction side

$\Delta l = l - l'$

r_1' — radious of teeth at the bottom of entrance

α — engaging angle at discharge side

α' — engaging angle at suction side

α'' — close angle

$\zeta = \alpha'/\alpha$

b — teeth width of gaterotor

δ — half angle of teeth width

e — axis width of gaterotor at teeth top

E— rejected area at suction side

$\xi = \varepsilon / 2r_2$

γ— division angle of gaterotor

F— interventing area of gaterotor into mainrotor

$\bar{x},\bar{y}$— coordinates of centroid F

$\bar{r}_1$— distance between centroid and mainrotor axis

v— groove volume at any angle

v_o— maximum groove volume

V— theorectical volumetric flow rate

n— r.p.m.

k— polytropic exponent

p, p_o— pressure at discharge and suction side

G_2— weight of gaterotor and its related axis, bearing etc.

G_1— weight of main screw and its related axis, bearing etc.

G— weight of single screw compressor

Close angle

During the working process, the gaterotors action similarly to the piston of a reciprocating compressor. Only when the gaterotor entered the entrance of mainrotor and formed a sealed volume, it start to compress (fig.2).

The front and back side of gaterotor teeth sequantly intersect with the main line of conic surface at suction side. Given out the equation seperately and substitute x = a, solved close angle α''.

Normal equation at the front and back side of gaterotor teeth

$$X\cos(\theta_2+\frac{\pi}{2})+Y\sin(\theta_2+\frac{\pi}{2})\pm\frac{b}{2}=0 \qquad (1)$$

or

$$-X\sin\theta_2+Y\cos\theta_2\pm\frac{b}{2}=0 \qquad (2)$$

Negative indicates front side, positive indicates back side

Main line of conic surface passes through point P. Its equation can be expressed as:

$$y=-l'-tg\beta(x-\sqrt{r_2^2-l'^2}) \qquad (3)$$

Combine (2) and (3), then

$$X\sin\theta_2+(l'+tg\beta(X-\sqrt{r_2^2-l'^2}))\cos\theta_2=\pm\frac{b}{2} \qquad (4)$$

The solution of equation (4) is

$$\alpha''=tg^{-1}\frac{l'+tg\beta(\alpha-\sqrt{r_2^2-l'^2})}{a}-\sin^{-1}\frac{b}{2\sqrt{\alpha^2+[l'+tg\beta(\alpha-\sqrt{r_2^2-l'^2})]^2}} \qquad (5)$$

There are three conditions:

1. $\beta > \alpha' - \delta$ (fig.2a)

When back side of teeth passed through point P, it starts to compress.

2. $\beta = \alpha' - \delta$ (fig.2b)

The back side of teeth at the moment of $\theta_2 = -\alpha' + \delta$, coincide with the main line of conic surface, so $\alpha'' = \alpha' - \delta$.

3. $\beta < \alpha' - \delta$ (fig.2c)

The entering of back side teeth into the conic surface starts from point P, until extending to the top side, then completely closed. At situation 1 and 3, α'' can be calculated with eq.(5).

AREA OF THE GATEROTOR INTERVENTING IN THE SCREW GROOVE AND ITS CENTROID

From the area interventing by gaterotor and the position of centroid, we may calculate volumetric flow rate. Also it's necessary for strength and power calculation.

According to fig.3

$$y = \sqrt{r_2 - x^2}$$

$$h = y - a\sec\theta_2 + x\,tg\,\theta_2$$

Interventing area

$$F = \int_{-b/2}^{b/2} h\,dx = -\frac{b}{2}\sqrt{r_2^2 - \frac{b^2}{4}} + r_2^2\delta - ab\sec\theta_2$$

After integration

$$F\bar{x} = \frac{b^3}{12} tg\,\theta_2$$

$$F\bar{y} = \frac{b}{2}\left[r_2^2 - \sec^2\theta_2 \left(a^2 + \frac{b^2}{12}\right)\right]$$

THEORECTICAL VOLUMETRIC FLOW RATE

After reaching the close angle, the gaterotor reduce the confined volume and the pressure in the groove increased.

The sealed groove can be devided into two parts(fig.4):

1. v_1 starts from the sealed angle " until the gaterotor aparture from screw groove, i.e. from to .

2. v_2 tail of groove, from to .

$$v_o = v_1 + v_2$$

$$v_1 = i\Big\{A\Big(r_2^2\delta + \frac{b}{2}\sqrt{r_2^2 - \frac{b^2}{4}}\Big)(\alpha - \delta + \theta_2) - \Big(\frac{r_1^2 b}{2} - \frac{b^3}{12}\Big) \cdot \left[\sin(\alpha - \delta) - \sin\theta_2\right] - \frac{1}{4}\left[(A + r_1)ab + \frac{b^3}{12}\right] \cdot \left[\ln\frac{1 + \sin(\alpha - \delta)}{1 - \sin(\alpha - \delta)} - \ln\frac{1 + \sin\theta_2}{1 - \sin\theta_2}\right]\Big\} \tag{6}$$

$$v_2 = \frac{i(\alpha + \delta - \theta_2)^3}{12\delta^2}\left[r_2^2\delta + \frac{b}{2}\sqrt{r_2^2 - \frac{b^2}{4}} - ab\cdot\sec(\alpha - \delta)\right] \cdot \Big\{r_1 - \frac{1}{8\delta}\left[r_2\cos(\alpha - 2\delta) - \alpha\right](\alpha + \delta - \theta_2)\Big\} \tag{7}$$

$$V = 2Z_1 v_o n \tag{8}$$

$$P = P_o\left(\frac{v_o}{v}\right)^k \tag{9}$$

SELECTION OF PARAMETERS

1. β . β related to the close angle α'', the larger the β , the smaller the α'', then the swept volume is smaller too.

2. ξ . The smaller the ξ , the greater the b, also the swept volume will be greater. But for the reason of diminishing leakage between the control volume and avoiding the stress overconcentrated, ξ has to keep in a certain range

3. $\Delta 1$, ζ. The greater the $\Delta 1$, the greater the E, but the smaller the α'' and v_o. So $\Delta 1$ should select smaller value in case of E satisfied.

4. If we take r as the proportional constant of other dimensions, G and V can be considered proportional to the cubic of r.
Approximately, specific weight

$$g = \frac{G_1+G_2}{V} = \frac{c_1 r_1^2 r_2 + 2c_2 r_2^3}{k_1 r_1 r_2^2} \qquad (16)$$

Here c_1, c_2, k_1 are constants, greater than zero
When

$$\frac{dg}{4r} = 0,$$

Then

$$r = \sqrt{\frac{2c_2}{c1}}$$

and

$$\frac{d^2 g}{dr^2} = \frac{4c}{k_1}\left(\frac{c_1}{2c_1}\right) > 0 \qquad (17)$$

There exists a r value can make g minimized

5. $\frac{A}{d_1}$

$$v_o = \eta r_1^{\,3} \qquad (18)$$

and

$$\frac{A}{d_1} = \frac{1}{2}\left(1+\frac{a}{r_2}\right)$$

η is function of $Z_1, Z_2, \xi, \zeta, \beta$ and $\frac{a}{r_2}$. In between, $\frac{a}{r_2}$ is the most important.

From fig.5, we can find the optimum value of $\frac{a}{r_2}$. Consider of the strength, general $\frac{A}{d_1}$ = o.55~o.6o.

Table 1 is the value of . From eq. (14), (18) and table 1, may estimate V quickly.

CONCLUSION

To design the single screw compressor, it is necessary to calculate the theorectical performance, estimate volumetric flow rate and decide the geometric shape.

Some basic equations are derived and optimization of geometric size are discussed. It'll be helpful to predict the performances and computer added design.

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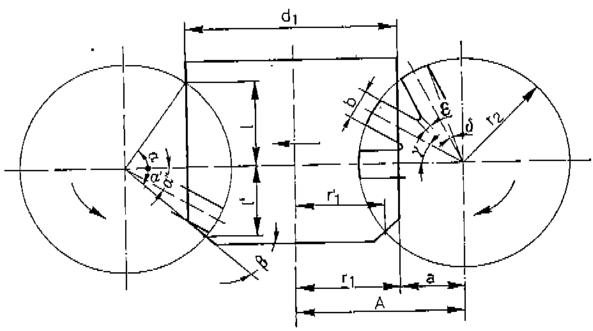
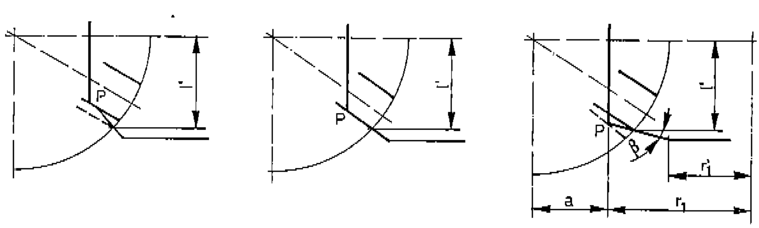


Figure 1

TABLE 1

a \ ζ	0.65	0.66	0.67	0.68	0.69	0.70	0.71	0.72	0.73	0.74	0.75
					$\xi=0.010$						
0.55	0.2211	0.2230	0.2250	0.2269	0.2289	0.2308	0.2327	0.2346	0.2365	0.2384	0.2403
0.56	0.2175	0.2194	0.2214	0.2233	0.2253	0.2272	0.2290	0.2309	0.2328	0.2347	0.2365
0.57	0.2138	0.2157	0.2176	0.2195	0.2214	0.2233	0.2252	0.2270	0.2289	0.2307	0.2325
0.58	0.2099	0.2118	0.2136	0.2155	0.2174	0.2192	0.2211	0.2229	0.2247	0.2265	0.2283
0.59	0.2057	0.2076	0.2094	0.2113	0.2131	0.2149	0.2167	0.2185	0.2203	0.2221	0.2239
0.60	0.2014	0.2032	0.2051	0.2069	0.2081	0.2104	0.2122	0.2140	0.2137	0.2175	0.2192
					$\xi=0.015$						
0.55	0.2150	0.2169	0.2188	0.2206	0.2225	0.2243	0.2262	0.2280	0.2298	0.2316	0.2334
0.56	0.2117	0.2136	0.2155	0.2173	0.2191	0.2210	0.2228	0.2246	0.2264	0.2282	0.2299
0.57	0.2082	0.2101	0.2119	0.2137	0.2155	0.2173	0.2191	0.2209	0.2227	0.2244	0.2262
0.58	0.2045	0.2064	0.2082	0.2100	0.2110	0.2135	0.2153	0.2170	0.2188	0.2205	0.2222
0.59	0.2006	0.2024	0.2042	0.2060	0.2077	0.2095	0.2112	0.2129	0.2147	0.2164	0.2181
0.60	0.1965	0.1983	0.2000	0.2018	0.2035	0.2052	0.2069	0.2086	0.2103	0.2120	0.2137
					$\xi=0.020$						
0.55	0.2089	0.2107	0.2125	0.2143	0.2160	0.2178	0.2196	0.2213	0.2231	0.2248	0.2265
0.56	0.2058	0.2076	0.2094	0.2112	0.2129	0.2147	0.2164	0.2181	0.2199	0.2216	0.2233
0.57	0.2026	0.2043	0.2061	0.2070	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198
0.58	0.1991	0.2009	0.2026	0.2043	0.2060	0.2077	0.2094	0.2111	0.2128	0.2144	0.2161
0.59	0.1955	0.1972	0.1989	0.2006	0.2023	0.2039	0.2056	0.2073	0.2089	0.2106	0.2122
0.60	0.1916	0.1933	0.1950	0.1966	0.1983	0.2000	0.2016	0.2032	0.2048	0.2065	0.2081
					$\xi=0.025$						
0.55	0.2026	0.2044	0.2061	0.2078	0.2095	0.2112	0.2129	0.2145	0.2162	0.2178	0.2195
0.56	0.1998	0.2016	0.2033	0.2050	0.2066	0.2083	0.2100	0.2116	0.2133	0.2149	0.2165
0.57	0.1968	0.1985	0.2002	0.2019	0.2036	0.2052	0.2069	0.2085	0.2101	0.2117	0.2133
0.58	0.1936	0.1953	0.1970	0.1986	0.2002	0.2019	0.2035	0.2051	0.2067	0.2083	0.2099
0.59	0.1902	0.1918	0.1935	0.1951	0.1967	0.1983	0.1999	0.2015	0.2031	0.2047	0.2062
0.60	0.1866	0.1882	0.1898	0.1914	0.1930	0.1946	0.1962	0.1977	0.1993	0.2008	0.2024



(a) (b) (c)

Figure 2

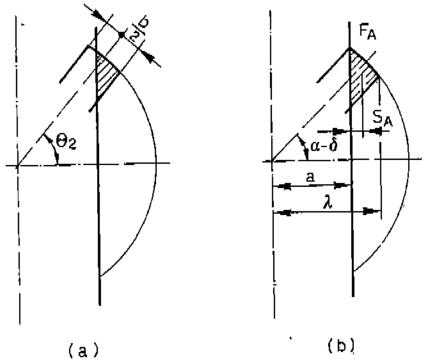


Figure 3

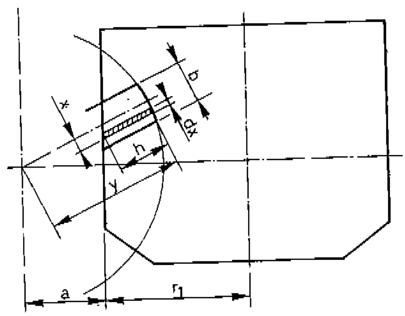


Figure 4

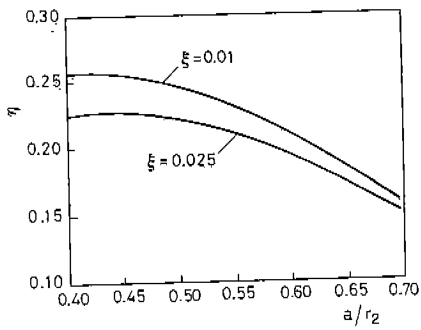


Figure 5