Harmonic Impact of Rectifiers Served by Unbalanced Three-Phase Sources

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The Harmonic Impact of Rectifiers Served by Unbalanced Three-Phase Sources

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This is dedicated to my family.
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NOMENCLATURE

\( a \) \quad 1 \angle \frac{2\pi}{3}

\( a_h \) \quad \text{amplitude of cosine term for } h^{th} \text{ harmonic}

\( AC \) \quad \text{alternating current}

\( ASA \) \quad \text{American Standards Association}

\( b_h \) \quad \text{amplitude of sine term for } h^{th} \text{ harmonic}

\( DC \) \quad \text{direct current}

\( F \) \quad \text{Farad}

\( f_0 \) \quad \text{frequency of AC power system}

\( g_i \) \quad \text{state variable}

\( g_{i_0} \) \quad \text{fixed operating point of state variable } g_i

\( h \) \quad \text{order of harmonics}

\( H \) \quad \text{Henry}

\( H_n \) \quad \text{magnitude of the } n^{th} \text{ harmonic component}

\( Hz \) \quad \text{Hertz}

\( HVDC \) \quad \text{high voltage DC}

\( I_{DC} \) \quad \text{average DC load current of the rectifier (pu)}

\( I_n \) \quad \text{rms magnitude of the } n^{th} \text{ harmonic (pu)}
$I_1$  rms amplitude of the fundamental current (pu)

$j$  complex operator equal to $\sqrt{-1}$

$kW$  kilowatt

$L_{DC}$  inductance of DC circuit in pu or Henries

$L_{l,ac}$  AC transformer leakage inductance

$MVA$  megavoltampere

$MVAR$  megavoltampere reactive

$MW$  megawatt

$p$  pulse number or differential operator $\frac{d}{dt}$ where applicable

$P$  operational active power

$pu$  per unit

$rms$  root-mean-square

$R_{ac}$  AC transformer resistance

$R_{DC}$  resistance in the DC circuit

$S$  complex voltamperes

$SCR$  silicon controlled rectifier

$T$  period of wave

$THD$  total harmonic distortion

$TIF$  telephone influence factor

$t_n$  time at discrete interval $n$

$T_{SC}$  complex symmetrical component transformation

$u$  overlap angle
$UF$ unbalance factor

$V$ volt

$V_{an}$ AC phase to neutral voltage (e.g. phase A-to-neutral)

$v_{sc}^+$ positive sequence voltage

$v_{sc}^-$ negative sequence voltage

$X_{l,ac}$ AC transformer leakage reactance

$W_n$ telephone influence weighting factor for harmonic n

$x(t)$ periodic voltage or current waveform which is a function of time

$X_1(\omega)$ Fourier transform of $x_1(t)$

$\alpha_h$ complex amplitude of the $h^{th}$ harmonic

$\mu$ commutation angle

$\alpha$ delay or firing angle

$\rho$ tolerance (e.g. $1\times 10^{-6}$)

$\omega_0$ power frequency in radians per second

$\epsilon_i$ magnitude unbalance in phase i AC line-to-neutral voltage

$\theta_i$ phase unbalance in phase i AC line-to-neutral voltage

$\delta_{ij}, \delta_{ij}$ perturbation of commutation angle when phase i commutates to phase j in the upper and lower parts of the bridge respectively

$\phi_{ij}, \gamma_{ij}$ actual commutation angles in the upper and lower parts of a Graetz bridge respectively

$\Delta t$ time step

* denotes convolution

$\delta(\cdot)$ delta or impulse function
$Y-\Delta$  wye-delta transformer connection

$Y-Y$  wye-wye transformer connection

$(\cdot)^t$  denotes transpose of vector or matrix

$\Omega$  Ohm

$\infty$  infinity
ABSTRACT

Olejniczak Kraig J. M.S.E.E. Purdue University. August 1988. THE HARMONIC IMPACT OF RECTIFIERS SERVED BY UNBALANCED THREE-PHASE SOURCES. Major Professor: G. Thomas Heydt.

A converter is a rectifier or inverter which is intended to transfer electrical energy between AC and DC busses. A common industrial converter usually employs the familiar Graetz bridge configuration and usually has a rating in the kilowatt through lower megawatt range. The operation of such a device is nominally in the balanced three-phase mode in which the phase currents are nonsinusoidal. The Fourier series components of these currents, or harmonics, have been studied extensively, but relatively little has been done in the unbalanced operating mode. The principal goal of this research is to examine how unbalance in magnitude and phase of the AC supply alters the frequency spectrum of a line-commutated power converter. The topics considered in this thesis are for cases of small unbalance, infinite inductance in the DC circuit \((L_{DC}=\infty)\), and pure resistance in the DC circuit \((L_{DC}=0)\). Also, symmetrical component analysis is made, and reasons for the presence of uncharacteristic harmonics are studied.
CHAPTER I

INTRODUCTION

1.1 Motivation and research goals

This thesis concerns harmonic signals in electric power systems. Alternating current systems are designed to operate at a "power frequency", $f_0$, which may be impacted by harmonic signals due to the presence of transmission and distribution system components which distort the sinusoidal waveform. Also, nonlinear loads appearing in the network may cause harmonic currents to be injected into the supply bus. The term harmonic refers to a sinusoidal component of a periodic wave or quantity having a frequency that is an integer multiple of the power frequency. A voltage or current $f(t)$ may be resolved into a Fourier series,

$$f(t) = \frac{a_0}{2} + \sum_{h=1}^{\infty} \left[ a_h \cos(h\omega_0 t) + b_h \sin(h\omega_0 t) \right]$$

if the Dirichlet conditions hold [13]. Usually these conditions are satisfied for signals of practical interest. Note that

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T}$$

where $T$ is the period of the waveform. When $f(t)$ is a three-phase voltage or current, the terms $a_h$, $b_h$ are three-phase triplets. This thesis focuses on the study of three-phase converters (i.e., rectifiers or inverters) which are unbalanced in the three-phase sense. Power semiconductors used in this application are effectively converters or converters and modulators. Thus, these devices introduce current and voltage harmonics into the supply system and load circuit. Due to the various operating regimes of the network, the existence and propagation of these harmonics can cause serious problems of interference with communication systems, computers, protective relays, and many other devices. Harmonics have become a more prevalent problem since the development of the thyristor or silicon controlled rectifier (SCR) in the early 1960's [20]. These devices have been used in an increasing fashion in power conditioning devices
including equipment at the megawatt power level for applications in HVDC systems.

Because solid-state power converters have expanded applications in heavy industry, there is an increased concern by the electric power industry to maintain and preserve harmonic limits in the power system. The harmonic currents produced by industrial users result in harmonic voltages due to the interaction with the network impedance. Because of resonance and standing-wave effects, harmonic voltage sources are not necessarily located near the point on the power network where the maximum magnitude appears. Identification procedures by a state estimation technique could be used to identify violating consumers [15]. Power converters act as highly nonlinear loads which contaminate the AC network with harmonic currents of order $np \pm 1$, $n=1,2,3,\ldots$ for a $p$-pulse converter. The problem of minimizing and attenuating these harmonic currents is of great interest and motivation to the power engineer.

Virtually all transmission and most distribution circuits are three-phase systems. Most power converters above $2$ kW are three-phase devices. The principal goal of this research is to examine how unbalance in magnitude and phase of the AC supply alters the frequency spectrum of a line-commutated power converter. While much has been written on balanced operation of converters, less has been presented on the effects of unbalanced supply voltages to the harmonics injected at the AC bus. The topics considered here are for cases of small unbalance, infinite inductance in the DC circuit ($L_{DC}=\infty$), and pure resistance in the DC circuit ($L_{DC}=0$). Also, symmetrical component analysis is made, and reasons for the presence of uncharacteristic harmonics are studied.

The principal motivation of this work is the value and need to model the "real world" operating environment of power converters. It is desired to explain the deviations between ideally calculated and actually observed harmonic levels in the vicinity of power converters.

1.2 Literature summary

This section outlines specific issues of power system harmonics as it pertains to a six-pulse converter. As mentioned previously, many sources can be found discussing the analysis and operation of power converters. In the following sections, discussion will focus upon background information, harmonic sources, unbalanced conditions, effect of harmonics on power systems, harmonic attenuation, and modelling strategy for a six-pulse line-commutated converter.
1.2.1 Harmonics produced by p-pulse converters

Converters are used to convert power from an AC to a DC network. These large devices can be used to form a link between the AC supply and DC transmission circuits. At the distribution level, converters are used to interface the AC network to alternative energy sources and a wide variety of DC loads. Due to the nonlinear nature of these devices, harmonics are generated in the AC supply. For a converter operating under ideally balanced conditions only the harmonics

\[ pn \pm 1, \ n=0,1,2,3, \ldots \]  \hspace{1cm} (1.2-1)

on the AC side and

\[ pn, \ n=0,1,2,3, \ldots \]  \hspace{1cm} (1.2-2)

on the DC side are considered for a p-pulse converter. These harmonics are termed characteristic and can be expected when observed by a digital spectrum analyzer at the respective AC or DC bus. Figure 1.1 depicts the standard configuration for a six-pulse converter. The three-phase AC network is isolated from the DC circuit by the means of a transformer connected \( \Delta-Y \). Because of this specific connection, the current waveform on the secondary side of the transformer contains harmonics of \( 6n \pm 1, \ n=1,2,3, \ldots \) with currents of order \( 6n+1 \) and \( 6n-1 \) being of positive and negative sequence respectively \([2,3]\). In the ideal case, the harmonic root-mean-square (rms) magnitude of the \( n^{th} \) harmonic is

\[ I_n = \frac{I_1}{n} \]  \hspace{1cm} (1.2-3)

where \( I_1 \) is the rms amplitude of the fundamental current. Also, no triplen (order \( 3n, \ n=1,2,3, \ldots \)) harmonics are present. This \( \frac{1}{n} \) variation does not consider commutation, unbalance, diode mismatch, transformer mismatch, or other phenomena. Arrillaga \([2]\), Kimbark \([3]\), and Stratford \([4]\) discuss some of these realities and their effect on the AC-side current spectrum.

To examine higher pulse converters (i.e. 12, 24, 36,...), many aspects of analysis become more complex. Higher pulse order is termed phase multiplication. The main reason for increased complexity originates from various transformer connections which use phase displacement to take advantage of reduced DC ripple and cancellation of certain harmonics in the supply currents \([8]\). For a 12-pulse converter \( (p=12) \), we see from (1.2-1) that the lowest occurring characteristic harmonic is the \( 11^{th} \). This is true for the most common twelve-pulse connections shown in Figure 1.2. This connection has a common
Figure 1.1 Three-phase six-pulse bridge converter.
primary but has its AC supply voltages mutually displaced by \( \frac{\pi}{6} \). Another interesting result of ideal twelve-pulse converters is the fact that harmonic currents of \( 6k \pm 1 \) (\( k \) odd), circulate between the two converter transformers but do not penetrate the AC network [2]. In practice however, these converters will produce fifth and seventh current harmonics due to slight unbalances or mismatches in the converter transformer reactances.

Twenty-four and 36-pulse converters are used in large electrolytic plants. For a 24-pulse converter, a \( \frac{\pi}{12} \) phase displacement between converter supply currents is needed. These values cannot be provided by simple transformer connections so the transformers must be modified with phase-adjusting windings incorporated on the supply side or autotransformers with phase shifting windings connected in series [8]. In effect, a zig-zagged transformer winding is used to shift the phase voltages of the AC supply bus by \( \frac{2\pi}{p} \) radians for a \( p \)-pulse converter. High pulse numbers above 36 are rarely used, not only for reasons of transformer economy, but also due to the level of distortion found in the supply voltage waveforms which can affect voltage crossings and commutation. However, pulse orders of 104 have been used in aluminum smelting which requires DC power levels above 300 MW.

1.2.2 Harmonic sources in power systems

Harmonic sources can be classified into three main categories:

(i.) high-power (100 kW or above)
(ii.) medium-power (10 - 100 kW)
(iii.) low-power (below 10 kW) converters.

Reference [1] contains a wide variety of harmonic sources classified in these categories as either old sources, such as power transformers and mercury-arc rectifiers, or new sources which employ power semiconductor devices and switching for their operation. For example, industrial loads such as electric arc furnaces and electro-chemical plants produce highly nonlinear voltage/current characteristics which introduce harmonics and other signals during scrap melting and electrolytic phases. These large current fluctuations are irregular and cannot be precisely calculated or predicted. In fact, some of the phenomena present in these applications are stochastic in nature and supply currents have a component which is a stochastic process. In this case, the spectrum of the supply current, \( I(\omega) \), is a continuous function of \( \omega \) rather than the familiar "bar chart" which is characteristic of spectra of periodic signals. Strictly speaking, these stochastic phenomena are not harmonics since it is impossible to resolve a
Figure 1.2 Twelve-pulse converter in series using Y-Y and Y-Δ transformers.
nonperiodic signal into a Fourier series. The practical importance of arc processes can not be overstated since these devices can often produce considerable difficulty due to supply voltage dips and fluctuating currents through the supply system impedance.

Medium and low-power converters are overlapping categories. Medium size converters are used in converter fed DC drives, compressors, press welding machines, piston pumps, and other electromechanical devices for which variable power consumption is characteristic. Low-power converters are used in television and audio receivers, battery chargers, and multi-speed electric motor appliances. Fluorescent lighting loads are another large source of harmonics. At some metropolitan substations, over 50% of the demand is due to fluorescent lighting. This type of lighting is highly nonlinear and produces odd-ordered harmonic currents with the third being dominant [2].

1.2.3 Unbalanced conditions in power systems

Conditions of perfectly balanced three-phase systems are seldom achieved. Because of non-ideal conditions in conversion equipment and controls, and because of unbalance in the three-phase bus, uncharacteristic harmonics can be produced in the AC supply current of converters.

To conveniently assess unbalanced conditions, a method was developed by C.L. Fortescue to analyze power system voltages and currents under unbalanced fault conditions. The method is to perform a complex transformation, $T_{SC}$, on the three-phase currents and voltages in order to decouple the three state variables. The method is known as symmetrical components. This method of symmetrical components is also used in the analysis of: parameters of synchronous machines and transformers, transmission lines and cables, and harmonic studies. The method of symmetrical components is analogous in some respects to the resolution of a periodic function into its fundamental and higher harmonics by Fourier series expansion. By this method, a set of unbalanced voltages or currents may be resolved into a system of balanced sets equal in number to the number of phases involved [26]. Symmetrical components have been used for many years in classical power system applications by Clarke [25], and Wagner and Evans [26]. This approach is used in the remainder of this thesis to evaluate how unbalance in the three-phase supply affects the current spectrum.

Load unbalance. Three-phase networks may operate unbalanced due to loads. The presence of single-phase loads, usually in the 5 kW class and lower, are often placed on the three-phase bus. It is common to distribute these single-phase loads between the three phases either by phase-to-ground or phase-to-phase connections. However, if the number of loads is small, or some
individual loads are operated at a high power level, it is not unusual for phase powers to differ by 10% between phases. Proper balancing of single-phase loads among the three phases on both branch circuits and feeders is necessary to keep the load unbalance and the corresponding phase voltage unbalance within reasonable limits. In general, single-phase loads should not be connected to three-phase circuits supplying equipment sensitive to phase voltage unbalance. A separate circuit should be used to supply this equipment [30].

Even individual three-phase loads can contribute to bus unbalance. Rotating loads may not have phase windings exactly matched. Nonrotating three-phase loads can often be unbalanced because these devices are usually a collection of three-phase resistive circuits. Resistive heating, for example, in an industrial application may be served directly from a 440 V bus. Mismatched transformer connections and dissimilarities in the several heating circuits can cause high levels of unbalance.

Transmission and distribution component unbalance. Components in the transmission and distribution network may be mismatched in terms of their impedance among the three phases due to unbalanced component parameters. A few examples of these are: unbalanced leakage reactance of the converter transformer windings [6, 10], unequal rating of reactors [10], and unbalanced line impedances due to improperly transposed or untransposed lines. According to [17], phase current magnitudes can vary by as much as 14.5% due to improper line transposition. Higher power devices are matched within 0.5%; for example, high-power transmission transformers are matched in phase voltage rating to within 0.5% by ASA standard [19]. Another component problem which generates harmonics is the capacitance of the transformer windings which are distributed along the winding to ground. There may also be a significant amount of stray capacitance between winding pairs for some transformer constructions. The rapid changes of voltage impressed across these stray capacitances distort the voltage waveforms on the converter AC bus to ground as shown in [9].

Control unbalance. There are basically four methods to control a six-pulse converter: constant phase angle control, equidistant firing control, modulated phase angle control, and integral cycle control [2]. In the ideal case of converter operation, negligible inductance on the AC side allows for instantaneous commutation. However, this never happens in reality and a finite length of time, called the commutation angle, elapses while the transfer of current from one conducting valve to another takes place. In cases where a firing pulse is used, unbalanced firing angles among phases and inconsistent voltage zero-crossings will have an effect on the harmonic magnitudes. The commutation angle, \( \mu \), and firing (delay) angle, \( \alpha \), directly affect the characteristic harmonic magnitudes [5].
Supply unbalance. Supply unbalance is generally regarded as unequal magnitudes of phase voltages and unbalanced phase relationships. In addition, the supply bus may have a distorted voltage waveform - this is a phenomenon apart from unbalance. The AC system voltages of a three-phase system are never perfectly balanced because of continually changing loads and unequal impedances in the three phases of the converter transformer. Yao and Sharaf [14], and Arrillaga [2] have reported that unbalance existing in the AC network impedance and supply voltages generate uncharacteristic odd harmonics. Deviations from the perfectly balanced supply can be caused by presence of negative sequence fundamental frequency in the commutating voltage, harmonic voltage distortion of positive or negative sequence, and unbalance in the commutation reactances [2]. The ratio of the magnitudes of the negative sequence to positive sequence voltages at the power frequency is called the unbalance factor (UF). Negative sequence voltage (i.e., large UF) causes second harmonic currents to flow in the DC system producing a positive sequence third harmonic current on the AC side which will not be blocked by delta windings [11]. As mentioned in [8], supply unbalance and voltage variations sometimes cause variation in lighting intensity. In fact, at a flicker frequency of 7 Hz with incandescent lighting the effect of voltage variations of 0.5% or less is perceptible to the human eye. This phenomenon is not a harmonic effect.

A significant problem encountered with unbalanced supply voltages relates to the fact that zeroes are not equally spaced in time and the thyristors are not fired at equal intervals [3]. In line-commutated converters the AC supply is used as a commutating voltage. In other words, the line voltage is used to provide the negative bias across a diode or thyristor to turn it off. When the rectifier firing angle is zero or when diode rectifiers are used the thyristors commutate when the peak phase amplitude of one phase becomes the largest of the three.

1.2.4 Effect of harmonics on power systems due to unbalanced conditions

Harmonics propagating throughout the network create havoc with many customers connected to the system. Customers and electric utilities can experience [1-4,7,8],

(i.) capacitor bank failure from dielectric breakdown,
(ii.) losses in induction, synchronous, and other AC machines,
(iii.) increased dielectric stress of insulated cables resulting from harmonic overvoltages on the system,
(iv.) inductive interference with telecommunication systems,
(v.) error in kilowatt-hour meter readings,
(vi.) unstable operation of firing circuits based on zero voltage crossing
detecting or latching,
(vii.) signal interference and relay malfunction particularly in solid-state and
microprocessor-controlled systems,
(viii.) interference with large motor controllers and power plant excitation
systems,
(ix.) increased losses in supply components, and
(x.) interference with digital systems which derive timing signals from the
AC bus.

Other major effects of voltage and current harmonics within the power system
is the amplification of harmonic levels resulting from series and parallel reso­
nances. In Figure 1.3 two types of resonances are depicted pictorially. The
resonant frequency of a circuit occurs when the impedance of the circuit is
purely real. For an LC circuit, the resonant frequency $f_r$ is [3,4,7,8]

$$f_r = \frac{1}{2\pi \sqrt{L_{sc} C}} = \frac{f_1}{X_c} = f_1 \left( \frac{MVA_{sc}}{CMVAR} \right)^{\frac{1}{2}}$$  

where

$L_{sc}$ = inductance of power system, H
$C$ = capacitance of capacitor bank, F
$f_1$ = fundamental frequency
$X_c$ = reactance of capacitor bank in per unit or $\Omega$
$X_f$ = reactance of power system in per unit or $\Omega$
$MVA_{sc}$ = short circuit capacity of system in MVA
$CMVAR$ = capacitor value in MVAR.

In cases of parallel resonance the impedance is high to the flow of current at the
frequency of resonance. Therefore the voltage passes through a maximum. On
the other hand, a series resonant circuit is low impedance to the flow of current
at the frequency of resonance and therefore a high current exists. Due to these
conditions (overvoltages and excessive currents), overheating of generators and
capacitor banks can occur. It is estimated that a $3\frac{1}{2}$% voltage unbalance can
result in a 25% increase in motor temperature and shortening of the motor
Figure 1.3 Electric power system resonant configurations:
(a) parallel resonant circuit; (b) series resonant circuit.
insulation life by over 50% [16]. This in turn can cause a reduction of motor efficiency and other difficulties.

1.2.5 Harmonic attenuation

There are basically three principal means of attenuating the harmonic output of converters. This can be accomplished by increasing the pulse number (phase multiplication), installing filters, or using means to cancel the harmonics [1,3,4,7]. In each case undesirable components of current, whether phase components of the fundamental or higher harmonic components, must be either suppressed in the supply lines or diverted from the supply by alternative current paths [7]. System reconfiguration is sometimes necessary to achieve this goal [18].

Phase multiplication is not a popular solution for harmonic attenuation because of economic factors and levels of distortion in the three-phase supply. Harmonic currents are reduced or eliminated through transformer connections that reduce the zero sequence harmonics and act as two-way filters protecting the load and system [1]. As stated before, the twelve-pulse converter is the usual upper limit in this regard.

Filters can be designed to provide reactive power and control the flow of harmonic currents produced by the nonlinear characteristics of power converters [2,3,10,11,13,19,20-24]. Filters serve two main purposes: attenuating AC harmonics and supplying reactive power at the fundamental frequency to the load. At a bus where an inverter or rectifier is located, filters are nearly always used on the AC side. Shunt filters [4,13] are used to provide low impedance paths for harmonic currents to ground. In typical medium power level applications, the filter cost can be 30% of the total project cost. At high power levels, the filter cost is a smaller percentage of the total project cost. As stated in Stratford [4], the expense of designing and installing reactors to tune power factor capacitors is small compared to the expense caused by problems when power factor capacitors are applied with converters.

Cancelling harmonics by various switching schemes is another popular method for harmonic control [2,3,20]. In [16], pulse-width-modulation (PWM) switching scheme for optimum current distortion (PSOCD) and PWM switching scheme for specific harmonic elimination (PSSHE) is discussed in detail. With PWM techniques, it is possible to improve power factors and reduce the harmonic currents generated into the power supply. The optimum switching pattern is found by varying the width of the pulses until the distortion factor of the input line current is a minimum. In most cases, three, five, or seven pulses per half cycle is used. With three pulses per half cycle, the fifth and seventh
harmonic components can be eliminated [16].

1.2.6 Modelling strategy for a six-pulse line-commutated converter

The six-pulse line-commutated converter can be modelled in varying degrees of complexity. The traditional techniques of analyzing power converters are based on the frequency domain approach whereby balanced voltages, purely inductive sources, equal firing angles and infinite inductance on the DC side are assumed [3]. These assumptions infer that the converter acts as a source of harmonic voltage on the DC side and of harmonic current on the AC side [2, 8]. If the AC supply voltage is sinusoidal at the source, the voltage at the transformer input terminals of a loaded rectifier will inevitably be distorted. When a converter is modelled very simply, the idealized mode of operation implies that it is not possible for three thyristors to conduct simultaneously in transitional periods. Under such an analysis, only the thyristor which has the highest voltage applied in the forward direction conducts. This is an uncontrolled line-commutated converter. The idealized approach as a first stage in considering converter problems affords clarity and gives good insight into operating modes and characteristics. A more detailed degree of modelling includes the commutation inductances on the AC side and several thyristors carrying current simultaneously. For some applications, it is desirable to model the converter in considerable detail; in such cases, all details of the converter circuit are included (i.e. thyristor characteristics, suppression, temperature, etc.) [8].

Some important problems in modelling occur when non-ideal commutation, interaction between converters, nonsinusoidal supply voltage, mutual network effects, and unbalanced loads are considered. Reference [13] discusses in considerable detail about methods and concerns of modelling an inverter. The data required to model a converter include: converter transformer resistance and reactance at the power frequency, inductance and resistance of the DC circuit, operational active power, $P$, and complex voltamperes, $S$ of the inverter, filter designs, and converter transformer tap settings. Additional modelling is needed in order to accommodate nonsinusoidal bus voltages, obtain harmonics remote from the converter site, and to calculate unknown bridge firing angles accurately. References [9] and [18] discuss concerns involved with converter stray capacitance and transmission line model accuracy on the computation of harmonic resonance parameters respectively. This additional modelling mentioned above, is important for valid results of a harmonic analysis.
1.3 Organization of this thesis

The remainder of this thesis is organized into four additional chapters. In Chapter II, a discussion of commutation angles is presented for a six-pulse converter. These are the angles in the sinusoidal bus voltage waveform at which the positive and negative poles of the DC circuit are switched or commutated between phases. In that chapter, the analysis and development of approximate commutation deviation angles are introduced as well as assumptions for all analyses. Also, harmonic analysis is discussed with phase variable and symmetrical component representations. Chapter III presents results and comparisons between all methods discussed for the analysis of the balanced operating condition. Among the factors which determine the AC current waveform is the DC circuit inductance. In this chapter, theoretical and detailed simulation cases are examined for infinite and zero inductance respectively. Chapter IV follows the same development as Chapter III except the unbalanced operating condition is considered. The last chapter, Chapter V, discusses the author's conclusions and recommendations for further research.
CHAPTER II

ANALYTICAL DEVELOPMENT OF COMMUTATION ANGLE DEVIATIONS AND FOURIER ANALYSIS OF A SIX-PULSE CONVERTER

2.1 Introduction

A Graetz bridge converter is a device which is used to convert energy from an AC to a DC bus and vice-versa. The two most familiar configurations are the six- and twelve-pulse converters. For a six-pulse converter operating in the balanced, line-commutated mode with no delay angle, when the supply voltage passes through \( \frac{\pi n}{3} \) radians, \( n = 0,1,2,3,... \), the positive and negative poles of the DC bus are switched or commutated between the phases of the AC bus. The cited angles are called commutation angles. As the mode of operation is changed, the commutation angles, in general, will also change. In this chapter, approximations for calculating these commutation angles of a six-pulse rectifier are described. The formulas, utilizing the method of symmetrical components, will then be used to perform a Fourier analysis of a rectifier in the balanced or unbalanced operating mode. A Fourier series expansion will then be presented for the case of infinite inductance in the DC circuit with stated assumptions. From this expansion, a relationship is derived relating the Fourier series of the infinite inductance case \( (L_{DC}=\infty) \) to one of zero inductance \( (L_{DC}=0) \). The latter case corresponds to a purely resistive circuit on the DC side of the converter. Lastly, a state space detailed rectifier simulation is presented based on Kron's method of tensor analysis [27-29].

2.2 Development of approximate commutation angle deviation equations

In the following analyses, several simplifying assumptions are made about the converter fed from balanced and unbalanced supply voltages of Figure 2.1:

(i.) The balanced or unbalanced sources are connected to an infinite bus.
Balanced and unbalanced voltages applied to a six-pulse converter.

Balanced Applied Voltages

Unbalanced Applied Voltages
(ii.) The commutating inductance of the AC network is negligible.

(iii.) The converter is operating with no phase delay (delay angle, $\alpha$, is zero).

(iv.) The DC current, $I_{DC}$, is constant.

(v.) In the conduction mode the voltage drop across the diode (by virtue of 2,3, and 4) is negligibly small.

For these assumptions, Figure 2.2 displays the three balanced phase to neutral voltages and the respective phase currents in the case of infinite inductance in the DC circuit. As expected, the commutation angles are equidistant at $\frac{2\pi}{p}$ radians for the ideally balanced case. These six commutation angles are the exact angle in which one phase in the upper (lower) part of the bridge becomes more positive (negative) than any other phase.

In the case of an unbalanced three-phase supply, Figure 2.3 depicts how magnitude and phase unbalance will change the angles of commutation of a six-pulse converter from that of the balanced case shown in Figure 2.2. It is convenient to represent this phase and magnitude unbalance of the three phase to neutral voltages as a vector

$$v(t) = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} \cos(t) \\ (1+\epsilon_b) \cos(t - \frac{2\pi}{3} - \theta_b) \\ (1+\epsilon_c) \cos(t + \frac{2\pi}{3} - \theta_c) \end{bmatrix}$$

where $\epsilon_i$ and $\theta_i$ denote the per unit magnitude and phase unbalance in phase $i$ respectively. Observing Figure 2.3, the commutation angles have deviated from the balanced case,

$$\frac{\pi}{3} i, \quad i = 1, ..., 6$$

to that of

$$\frac{\pi}{3} i + \delta_{ij} \text{ or } \frac{\pi}{3} i + \bar{\delta}_{ij}, \quad i = 1, ..., 6$$

where $\delta_{ij}$ and $\bar{\delta}_{ij}$ represent perturbations of commutation angles when phase $i$ commutates to phase $j$ in the upper and lower parts of the bridge respectively. The bar notation used simply denotes correspondence with the negative arm of the Graetz bridge - this should not be confused with other meanings of the bar notation which is used by other authors. The actual angles of commutation due to the unbalance of the three-phase supply can be expressed as
Figure 2.2  Phase currents from a balanced source for the case $L_{DC} = \infty$. 
Figure 2.3  Phase currents from an unbalanced source for the case $L_{DC}=\infty$. 
\[
\begin{align*}
\phi_{ab} &= \frac{\pi}{3} + \delta_{ab} \\
\phi_{bc} &= \pi + \delta_{bc} \\
\phi_{ca} &= \frac{5\pi}{3} + \delta_{ca}
\end{align*}
\]
\[
\begin{align*}
\gamma_{ca} &= \frac{2\pi}{3} + \delta_{ca} \\
\gamma_{ab} &= \frac{4\pi}{3} + \delta_{ab} \\
\gamma_{bc} &= 2\pi + \delta_{bc}
\end{align*}
\]  
(2.2-2)

where \(\phi_{ij}\) and \(\gamma_{ij}\) are the actual angles of commutation in the upper and lower part of the bridge respectively. The use of the bar notation in (2.2-2) simply denotes those deviations which correspond to commutation in the lower portion of the bridge.

In order to find how the magnitude and phase unbalance of the AC voltage supply affect the commutation angle deviations, the six commutation times result in six equations by setting the appropriate phase voltages equal to each other (see a-e in Figure 2.3) at their point of intersection,

\[
\cos(\delta_{ab} + \frac{\pi}{3}) = (1 + \epsilon_b) \cos(\delta_{ab} + \frac{\pi}{3} - \frac{2\pi}{3} - \theta_b)  
\]

(2.2-3)

\[
(1 + \epsilon_b) \cos(\delta_{bc} + \pi - \frac{2\pi}{3} - \theta_b) = (1 + \epsilon_c) \cos(\delta_{bc} + \pi - \frac{4\pi}{3} - \theta_c) 
\]

(2.2-4)

\[
(1 + \epsilon_c) \cos(\delta_{ca} + \frac{5\pi}{3} - \frac{4\pi}{3} - \theta_c) = \cos(\delta_{ca} + \frac{5\pi}{3}) 
\]

(2.2-5)

\[
\cos(\delta_{ca} + \frac{2\pi}{3}) = (1 + \epsilon_c) \cos(\delta_{ca} + \frac{2\pi}{3} - \frac{4\pi}{3} - \theta_c) 
\]

(2.2-6)

\[
\cos(\delta_{ab} + \frac{4\pi}{3}) = (1 + \epsilon_b) \cos(\delta_{ab} + \frac{4\pi}{3} - \frac{2\pi}{3} - \theta_b) 
\]

(2.2-7)

\[
(1 + \epsilon_b) \cos(\delta_{bc} - \frac{2\pi}{3} - \theta_b) = (1 + \epsilon_c) \cos(\delta_{bc} - \frac{4\pi}{3} - \theta_c) 
\]

(2.2-8)

Knowing the magnitude and phase unbalance of the supply voltages allows for the exact solution of these six transcendental equations (2.2-3) - (2.2-8) for the commutation angle deviations \(\delta_{ij}\) and \(\delta_{ij}\) respectively. This can be accomplished by a search technique where the search begins sufficiently far away to insure proper convergence to the correct commutation angle. By subtracting the right-hand side from the left of (2.2-3) - (2.2-8), an expression set to zero is obtained. If each respective equation, evaluated for some value of \(\delta_{ij}\) or \(\delta_{ij}\), has a change in sign from \(t_n\) to \(t_n + \Delta t\), then clearly the "exact solution" (to some tolerance, \(\rho\)) has been overstepped. At this point, the step size, \(\Delta t\), can be decreased accordingly and the search continues in the opposite direction until the "exact solution" is obtained for a desired tolerance, \(\rho\). It should be quite
clear that this method is computationally burdensome and a relatively quick and accurate approximation would be useful.

Developing a set of approximate commutation angle deviation equations demand the linearization of (2.2-3) - (2.2-8). One method is to employ the familiar Taylor series expansion to a perturbed variable \( g_i \), about its fixed operating value, \( g_{i0} \), as

\[
h(g_i) = h(g_{i0}) + h'(g_{i0}) \Delta g_i + \frac{h''(g_{i0})}{2!} \Delta g_i^2 + \ldots \]

(2.2-9)

where

\[
g_i = g_{i0} + \Delta g_i. \quad (2.2-10)
\]

If small perturbations from the fixed point are experienced, all terms higher than the first-order term may be neglected. Now, \( h(g_i) \) can be approximated by

\[
h(g_i) \approx h(g_{i0}) + h'(g_{i0}) \Delta g_i. \quad (2.2-11)
\]

The small displacement characteristics of the system are given by the first-order term of Taylor's series as

\[
\Delta h(g_i) = h(g_i) - h(g_{i0}) = h'(g_{i0}) \Delta g_i. \quad (2.2-12)
\]

An equivalent method of linearizing nonlinear equations is to write all variables in the form of (2.2-10). After all multiplications are performed, then the following approximations are made:

\[
\cos \theta_i = \cos \delta_{ij} = \cos \overline{\delta}_{ij} = 1 \quad (2.2-13)
\]

\[
\sin \theta_i = \theta_i, \quad \sin \delta_{ij} = \delta_{ij}, \quad \sin \overline{\delta}_{ij} = \overline{\delta}_{ij}, \quad (2.2-14)
\]

\[
\delta_{ij} \theta_i = \overline{\delta}_{ij} \theta_i = 0. \quad (2.2-15)
\]

In this study, the latter method was chosen since (2.2-1) was in the form of (2.2-10). By expanding (2.2-3) - (2.2-8), and simplifying with the approximations of (2.2-13) - (2.2-15), the resulting approximate commutation angle deviations are

\[
\delta_{ab} = \overline{\delta}_{ab} = \frac{\sqrt{3} \theta_b + \sqrt{3} \theta_b - \varepsilon_b}{\sqrt{3}(\varepsilon_b + 2)} \quad (2.2-16)
\]
\[ \delta_{ca} = \delta_{ca} = \frac{\sqrt{3} \theta_e + \sqrt{3} \epsilon e \theta_e + \epsilon_e}{\sqrt{3}(\epsilon_e + 2)} \quad (2.2-17) \]

\[ \delta_{bc} = \delta_{bc} = \frac{\epsilon_b(1 + \sqrt{3} \theta_b) + \epsilon_e(-1 + \sqrt{3} \theta_e) + \sqrt{3}(\theta_b + \theta_e)}{\sqrt{3}(\epsilon_b + \epsilon_e + 2)} \quad (2.2-18) \]

### 2.3 Implementing approximate commutation angle deviation formulas in Fourier analysis

The nonsinusoidal phase currents, \( i(t) \), shown in Figure 2.3 are resolved into a one-sided Fourier series, where

\[ i(t) = \frac{\alpha_0}{2} + \sum_{h=1}^{\infty} \alpha_h e^{j\omega_0 t} \quad (2.3-1) \]

\[ \alpha_0 = \frac{1}{T} \int_T i(t) \, dt \quad (2.3-2) \]

\[ \alpha_h = \frac{1}{T} \int_T i(t) e^{-j\omega_0 t} \, dt \quad (2.3-3) \]

The term, \( \frac{\alpha_0}{2} \), is the DC component, \( \alpha_h \) are the complex Fourier coefficients, \( \omega_0 \) is the power frequency, and \( T \) is the period of the power frequency. When \( i(t) \) is a vector containing three components, \( \alpha_0 \) and \( \alpha_h \) in (2.3-1) - (2.3-3) are used to represent the harmonic amplitude of a three-phase current. Note that the one-sided series in (2.3-1) is commonly used in engineering work and this form is readily derived from the two-sided series [12]. The harmonic content of the waveforms shown in Figure 2.3 is readily found from the Fourier integral. As a reminder, note that the phase currents in Figure 2.3 are for the case of infinite inductance in the DC circuit with the assumptions stated in the beginning of Section 2.2. The harmonic coefficients of the phase currents in Figure 2.3 are
\[
\alpha_h = \frac{1}{j\pi h} \begin{pmatrix} 1 & 1 & 1 \\
0 & 1 & 1 \\
0 & 0 & 1 \end{pmatrix} \left( \cos h_{\phi_{ab}} - \cos h_{\phi_{bc}} + j \sin h_{\phi_{ab}} - j \sin h_{\phi_{bc}} \right) + \begin{pmatrix} 0 \\
1 \\
-1 \end{pmatrix} \left( \cos h_{\phi_{bc}} - \cos h_{\gamma_{bc}} + j \sin h_{\phi_{bc}} - j \sin h_{\gamma_{bc}} \right) + \begin{pmatrix} 0 \\
-1 \\
1 \end{pmatrix} \left( \cos h_{\phi_{ca}} - \cos h_{\gamma_{ca}} + j \sin h_{\phi_{ca}} - j \sin h_{\gamma_{ca}} \right)
\]

(2.3-5)

where the \( \phi \) and \( \gamma \) angles are the actual commutation angles defined in (2.2-4).

This expression, \( \alpha_h \), is the vector of complex Fourier coefficients of the current spectrum of the vector \( v(t) \).

### 2.4 Use of symmetrical components in Fourier analysis

In the case of balanced operation of three-phase circuits, only positive sequence voltages and currents exist from a positive sequence excitation. However, if unbalances occur due to unbalanced system operation, negative and zero sequence components may exist. To handle this situation, a complex transformation \( T_{SC} \) is used to form three decoupled circuits representing positive, negative, and zero sequence respectively. This symmetrical component transformation is

\[
T_{SC} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & a & a^2 \\
1 & a^2 & a \\
1 & 1 & 1 \end{pmatrix}
\]

where the sequence ordering of \( T_{SC} \) is +, -, 0, and \( a \) is defined as \( 1 \angle \frac{2\pi}{3} \).

Writing (2.2-1) in phasor notation and applying \( T_{SC} \)

\[
v_{SC} = \frac{1}{\sqrt{3}} \begin{pmatrix} 3 + \epsilon_b (1+j\theta_b) + \epsilon_c (1+j\theta_c) + j(\theta_b + \theta_c) \\
1 + a^2 (1+j\theta_b)(1+\epsilon_b) + a(1+j\theta_c)(1+\epsilon_c) \\
1 + a^2 (1+j\theta_b)(1+\epsilon_b) + a(1+j\theta_c)(1+\epsilon_c) \end{pmatrix}
\]

(2.4-1)

The ratio of the magnitudes of \( v_{SC} \) to \( v_{SC}^* \) is sometimes called the unbalance factor. The symmetrical component sequence of the current spectrum in (2.3-5) can be found by applying transformation \( T_{SC} \). This results in an expression identical to (2.3-5) with the following substitution of vector coefficients,
phase variable → symmetrical component

\[
(1 -1 0)^t \rightarrow (1-a 1-a^2 0)^t \\
(0 1 -1)^t \rightarrow (a-a^2 a^2-a 0)^t \\
(-1 0 1)^t \rightarrow (a^2-1 a-1 0)^t.
\]

Note that there are neither zero sequence components nor even harmonics present.

### 2.5 Relationship between the Fourier series expansion of the \( L_{DC} = \infty \) and \( L_{DC} = 0 \) cases

Since the beginning of Chapter II, all discussion and analysis centered around the case of infinite inductance in the DC circuit. In order to conjecture about all effects of harmonics, the other extreme of zero inductance \((L_{DC}=0)\) must be examined. It is possible to obtain a relationship between the Fourier expansions of the \( L_{DC}=\infty \) and \( L_{DC}=0 \) cases. Referring to Figure 2.4, \( x_4(t) \) is the phase \( A \) current for the case of \( L_{DC}=0 \). In this case, the current in a purely resistive load is a sinusoid matched in phase with the source, but having the waveform of the phase \( A \) current for \( L_{DC}=\infty \). The waveform \( x_3(t) \) is a modulating cosine wave \((\cos \omega_0 t)\) with \( \omega_0 \) equal to unity. By multiplying the waveforms \( x_1(t), x_2(t), \) and \( x_3(t) \), the resultant waveform is that of \( x_4(t) \),

\[
x_1(t) = \sum_{m=0}^{\infty} a_m \cos(m\omega_0 t) \tag{2.5-1}
\]

\[
x_2(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega_0 t) \tag{2.5-2}
\]

\[
x_3(t) = \cos(\omega_0 t) \tag{2.5-3}
\]

\[
x_4(t) = x_1(t) \cdot x_2(t) \cdot x_3(t) = \sum_{\ell=0}^{\infty} \beta_{\ell} \cos(\ell\omega_0 t). \tag{2.5-4}
\]

Because all waves exhibit even symmetry, all \( b_i \) terms of the Fourier expansion are zero and no DC component exists. In the time domain, \( x_4(t) \) is formed by multiplying \( x_1(t), x_2(t), \) and \( x_3(t) \). Working in the frequency domain, the Fourier transforms of (2.5-1) - (2.5-3) are
Figure 2.4 Relationship between the $L_{DC} = \infty$ and $L_{DC} = 0$ current waveforms.

$$x_4(t) = x_1(t) \cdot x_2(t) \cdot x_3(t)$$
\[ X_1(\omega) = \sum_{m=0}^{\infty} a_m \pi[\delta(\omega-m\omega_0) + \delta(\omega+m\omega_0)] \quad (2.5-5) \]
\[ X_2(\omega) = \sum_{n=0}^{\infty} a_n \pi[\delta(\omega-n\omega_0) + \delta(\omega+n\omega_0)] \quad (2.5-6) \]
\[ X_3(\omega) = \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)] \quad (2.5-7) \]
\[ X_4(\omega) = \sum_{\ell=0}^{\infty} \beta_\ell \pi[\delta(\omega-\ell\omega_0) + \delta(\omega+\ell\omega_0)]. \quad (2.5-8) \]

In all cases \( \omega_0 \) is unity. The Fourier transform corresponding to multiplication in the time domain is the complex convolution in the frequency domain,

\[ X_{12}(\omega) = X_1(\omega) \ast X_2(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1(\xi) X_2(\omega-\xi) d\xi \]
\[ X_3(\omega) \ast X_{12}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_3(\xi) X_{12}(\omega-\xi) d\xi. \]

The "sifting property" of the delta function is used as follows

\[ F(q) = \int_{-\infty}^{\infty} F(\xi) \delta(\xi-q) d\xi. \quad (2.5-9) \]

The resulting expression for \( X_4(\omega) \) is

\[ X_4(\omega) = \frac{\pi}{4} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} a_m a_n \pi[\delta(\omega-1-m-n) + \delta(\omega-1-m+n) + \delta(\omega+1-m-n) + \delta(\omega+1+m-n) + \delta(\omega+1-m+n) + \delta(\omega+1+m+n)]. \quad (2.5-10) \]

Equating (2.5-10) and (2.5-8), the transform of the \( L_{DC} = 0 \) case in terms of the transform of \( L_{DC} = \infty \) is
\[
\frac{1}{4} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \alpha m \alpha n [\delta(\omega-1-m-n) + \delta(\omega-1-m+n) \\
+ \delta(\omega-1+m-n) + \delta(\omega-1+m+n) + \delta(\omega+1-m-n) \\
+ \delta(\omega+1-m+n) + \delta(\omega+1+m-n) + \delta(\omega+1+m+n)]
\]

\[
= \sum_{\ell=0}^{\infty} \beta_{\ell} \delta(\omega-\ell) + \delta(\omega+\ell).
\]

The expression (2.5-11) yields impulse functions which are located in the interval from \(-\infty\) to 0 and 0 to \(+\infty\). Equating terms of like arguments on both sides of (2.5-11) allows for the computation of \(\beta_i\) as a function of the \(\alpha_m, \alpha_n\) terms,

\[
\beta_0 = 0
\]

\[
\beta_1 = \frac{3}{4} a_1^2 + \frac{1}{2} (a_5^2 + a_7 a_7 + a_7^2) + \frac{1}{2} (a_{11}^2 + a_{13} a_{13} + a_{13}^2) + ...
\]

\[
\beta_2 = 0
\]

\[
\beta_3 = \frac{1}{4} a_1^2 + \frac{1}{2} (a_1 a_5 + a_5 a_7 + a_7 a_{11} + a_{11} a_{13} + ...) 
\]

\[
\beta_4 = 0
\]

\[
\beta_5 = a_1 a_5 + \frac{1}{2} (a_1 a_7 + a_5 a_{11} + a_7 a_{11} + a_{11} a_{13} + ...)
\]

\[
\beta_5 = 0
\]

and so forth.

2.6 A time-varying state space model of a six-pulse converter

The familiar first order linear differential equation

\[
pX = AX + BU
\]

(2.6-1)

(where \(p\) denotes \(\frac{d}{dt}\) and \(X\) and \(U\) denote the system states and inputs respectively) describes either time independent or time-varying systems. For a six-pulse converter operating with an overlap angle less than 60° \((u < 60°)\), the
number of states (i.e., dimension of \( X \)) depends on whether the converter is in one of the six conduction or six commutation modes.

In the analysis of three-phase thyristor bridge circuits, it is most convenient and computationally efficient to use an average-value model for functional representation of the converter where harmonics are neglected. At times unusual fault conditions, commutation failure, and harmonic analysis demand a detailed representation for the simulation of the converter. To cope efficiently with complex commutation conditions where the source impedance is significant, a method based on Kron's tensor analysis is used [32]. This approach develops a systematic method of assembling and solving the network differential equations automatically.

For a six-pulse converter connected in \( Y - \Delta \) and served from a three-phase supply, circuit mesh analysis dictates that six meshes (branches - nodes + 1) are necessary to completely describe the network topology shown in Figure 2.5. If the nonplanar graph of the converter contains only voltage sources and impedances, then the operation of the circuit is completely described for all diodes conducting by the equation

\[
V_m = Z_m I_m
\]  

(2.6-2)

where \( V_m \) denotes the mesh primary line-to-line voltages referred to the transformer secondary and \( I_m \) is the independent link currents. Due to changes in circuit topology from the varying conduction modes, nonconducting diodes can be eliminated from (2.6-2) by a matrix transformation \( C_n \). This matrix relates the independent link currents of the original network, \( I_m \), to the reduced set of independent currents with nonconducting diodes eliminated, \( I_n \). These currents are related by \( C_n \) as follows,

\[
I_m = C_n I_n
\]  

(2.6-3)

Substituting (2.6-3) into (2.6-2)

\[
V_m = Z_m C_n I_n
\]  

(2.6-4)

and then premultiplying (2.6-4) by \( C_n^{\prime} \),

\[
C_n^{\prime} V_m = C_n^{\prime} Z_m C_n I_n
\]

Now, describe the reduced network with nonconducting diodes eliminated as

\[
V_n = Z_n I_n
\]  

(2.6-5)

where \( V_n = C_n^{\prime} V_m \) and \( Z_n = C_n^{\prime} Z_m C_n \). Rewriting the complex matrix \( Z_n \) in differential form, (2.6-5) can be expressed as
Figure 2.5  Circuit and mesh diagram for a six-pulse rectifier served by a $Y - \Delta$ connection.
Rearranging,

\[ V_n = (R_n + pL_n)I_n. \]

is now in standard state space form required for a numerical integrator. The matrices \( R_n \) and \( L_n \) are the resistance and inductance matrices corresponding to existing links of the reduced set. Equation (2.6-6) is used at each time step to solve for the reduced independent currents, \( I_n \), from which the mesh currents \( I_m \) are found using (2.6-3). Clearly, the diode and transformer secondary currents are algebraic functions of the mesh currents \( I_m \). The applications of this method in more detail can be found in [27-29].

2.7 Summary

In this chapter, all of the mathematical analysis in this research project has been presented. The six-pulse converter is considered exclusively although the techniques shown may readily be extended to the twelve-pulse case. The development of approximate commutation angle deviation formulas using small-angle approximations is presented. These results are then extended to the implementation of Fourier analysis. A mathematical relation has been derived relating the Fourier expansions for the \( L_{DC}=\infty \) and \( L_{DC}=0 \) cases. Lastly, a state space time-varying model for detailed rectifier representation based on Kron's tensor analysis is presented in order to verify harmonic phenomena in the AC supply currents.
CHAPTER III
COMPARISON OF RESULTS FOR THE BALANCED OPERATING CONDITION

3.1 Introduction

In this chapter, the balanced operating condition is examined for the cases of infinite and zero inductance in the DC circuit. Although the emphasis of this research lies in the examination of supply unbalance, much can be observed about the effects of other system parameters on the Fourier spectrum of the AC supply currents. Because the three-phase source is balanced, changes in the Fourier spectrum cannot be attributed to commutation deviations or other results of supply unbalance. This chapter will examine the effects of varying the DC load and AC transformer leakage reactance under balanced operating conditions. Referring to Figure 3.1, the six-pulse converter is fed by a balanced three-phase source whose phase angles are mutually displaced by 120°. This balanced triplet, with commutation angles equally spaced in time, will be used for all cases presented in this chapter. The DC resistance and inductance, $R_{DC}$ and $L_{DC}$, will be varied from zero to one and zero to 377 pu, respectively throughout this chapter. The AC network is modelled as an AC equivalent resistance and reactance representing the equivalent impedance looking back towards the source from the rectifier terminals. Since the AC equivalent impedance is almost entirely due to the converter transformer, the parameters $R_{ac}$ and $X_{\ell,ac}$ will be used from this point forward, to represent the converter transformer resistance and leakage reactance, as well as the AC equivalent resistance and reactance.

In Section 3.2, the balanced case results for infinite inductance in the DC circuit are presented. In order to simulate the infinite inductance case, a value of $L_{DC}$ was selected to be sufficiently large relative to the AC converter transformer leakage reactance, $L_{\ell,ac}$. Typical impedances appearing in the AC network have an approximate R/X (resistance to reactance) ratio of one-tenth (e.g., $R_{ac} + jX_{\ell,ac} = 0.01 + j0.10$ per unit) at the power frequency. When a purely resistive load in the DC circuit is of interest, $L_{DC}$ is simply set to zero.
Balanced voltages applied to a six-pulse converter.

\[
\begin{align*}
V_{an} &= \cos(t) \\
V_{bn} &= \cos(t - 2\pi/3) \\
V_{cn} &= \cos(t + 2\pi/3)
\end{align*}
\]
Once the DC parameters are established, it is useful to examine the current spectrum of the AC supply to see how the complex magnitude of each harmonic, \( \alpha_h \), varies as a function of the AC transformer leakage reactance, \( X_{\ell,ac} \). Since the reactive part of the AC impedance is significantly larger in magnitude than the real part, it is assumed that the resistance \( R_{ac} \) has little influence on the magnitude and phase of the complex Fourier coefficients. A Fourier analysis program (Appendix A), utilizing 4096 point trapezoidal integration, is used to calculate the Fourier expansion for the three AC phase currents in each case. Total harmonic distortion (THD) and telephone influence factor (TIF) are compared for various values of \( X_{\ell,ac} \) to determine their effect on the spectrum of the unfiltered AC supply currents.

Section 3.3 discusses the balanced case results with zero inductance in the DC circuit. As in Section 3.2, the DC resistance and converter transformer leakage reactance is varied to observe changes in the Fourier spectrum, THD, and TIF of the supply currents. Lastly, detailed rectifier simulations are presented for constant DC resistance (1 pu) and intermediate DC inductances which lie between the two extremes, \( L_{DC} = 0 \) and \( L_{DC} = \infty \). These cases will be useful in conjecturing whether all harmonic magnitudes in other cases are bound by these two extremes.

### 3.2 Balanced case results for infinite inductance in the DC circuit

The balanced operating case with infinite inductance in the DC circuit has the theoretical current waveforms shown in Figure 3.2. Referring to the simplifying assumptions stated in Section 2.2, these nonsinusoidal phase currents result from instantaneous commutation due to the negligibly small commutating inductance in the AC network (by virtue of 2 and 3). In addition, since the balanced three-phase source is connected to an infinite bus, the supply line-to-neutral voltages will be of constant magnitude and frequency. With the constraints mentioned above and infinite inductance in the DC circuit, the resulting DC current is constant with a superimposed 60 Hz ripple. Figure 3.3 displays the three AC phase currents obtained from a detailed rectifier simulation. In this specific case, the theoretical case was simulated using very small values of transformer resistance and leakage reactance to satisfy the negligible commutating inductance constraint. This was accomplished by forcing the AC resistance and reactance, \( R_{ac} \) and \( X_{\ell,ac} \), to a \( R/X \) ratio of one. The last necessary requirement was to set the DC resistance to zero in order to maintain a purely inductive load. To compare the theoretic and detailed simulation case of Figure 3.3, the Fourier spectra of both cases are presented in Figure 3.4. This comparison strongly supports the validity of this simulation method. Observing
Figure 3.2 Theoretical phase currents for the $L_{DC} = \infty$ case.
Figure 3.3 AC phase currents obtained from a detailed rectifier simulation: $R_{ac} = 0.01 \text{ pu}$, $X'_{ac} = 0.01 \text{ pu}$, $R_{DC} = 0 \text{ pu}$, and $L_{DC} = 377 \text{ pu}$. 
the phase currents of Figure 3.3, the exponential rise and decay during commu-
tication intervals are nearly vertical representing the instantaneous switching of
the thyristors. When comparing the AC phase currents of Figure 3.3 to the
theoretical case of Figure 3.2, a distinct difference is noted and must be
clarified. The AC line currents have the same wave shape as Figure 3.2 only if
the transformer connection is $Y-Y$ or $\Delta-\Delta$, and that of Figure 3.3 if the
transformer connection is $\Delta-Y$ or $Y-\Delta$. The detailed rectifier simulation used
here is designed for a $Y-\Delta$ transformer connection while the theoretical case is
chosen for a $Y-Y$ connection to take advantage of wave symmetry during
Fourier analysis. In any case, the former's primary current waveform is the
latter's secondary waveform and vice-versa.

To examine the effect of the DC load on the AC currents, Figures 3.5 and
3.6 display detailed simulation results for zero and one per unit DC resistances
respectively. In both cases, $R_{ac}$ and $X_{ac}$ are set to a $R/X$ ratio of one-tenth.
Pictorially, it is clear that the AC current magnitudes are dependent upon the
DC load resistance and inductance. To compare these cases quantitatively, the
Fourier spectra were evaluated for the theoretical case and those cases
represented by Figures 3.5 and 3.6. Table 3.1 summarizes the real and ima-
ginary components of $a_h$, the complex Fourier coefficients, in the phase A sup-
ply currents for the three cases mentioned above. From Table 3.1, it seems rea-
sonable to conjecture that the harmonic magnitudes for all detailed simulations
between $R_{DC} = 0$ and $R_{DC} = 1$ pu would be bounded by these two extreme
cases. Of course cases involving resonance conditions and other system
anomalies would be excluded. The THD and TIF of Figures 3.5 and 3.6 also
add insight into the effects of changing DC loads. As the DC resistance
increased from zero to one per unit, the THD and TIF increased by 1.32% and
4.04%, respectively. The THD is a measure of all harmonic frequencies present
in a signal in relation to its fundamental component. The THD is defined as

$$THD = \left[ \sum_{h=2}^{\infty} \frac{H_h^2}{H_1^2} \right]^{1/2}$$

where $H_1$ is the fundamental component of the periodic signal and $H_h$ denotes
the $h^{th}$ harmonic component. The TIF is a measure of the influence that the
power system harmonics have on the audible frequencies used in telecommuni-
cations. The TIF is defined as
Figure 3.4 Fourier spectra of theoretical values compared to theoretical detailed rectifier simulation: $R_{ac} = 0.01$ pu, $X_{ac} = 0.01$ pu, $R_{dc} = 0$ pu, and $L_{dc} = 377$ pu.
Figure 3.5 AC phase currents obtained from a detailed rectifier simulation: $R_{ac} = 0.01$ pu, $X_{ac} = 0.10$ pu, $R_{dc} = 0$ pu, and $L_{dc} = 377$ pu.
Figure 3.6 AC phase currents and DC current obtained from a detailed rectifier simulation: $R_{ac} = 0.01$ pu, $X_{\ell,ac} = 0.10$ pu, $R_{DC} = 1$ pu, and $L_{DC} = 377$ pu.
### Table 3.1 The real and imaginary parts of $\alpha_h$ for cases of $L_{DC} = \infty$.

<table>
<thead>
<tr>
<th>$h$</th>
<th>CASE 1</th>
<th>CASE 2</th>
<th>CASE 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_{ac} = 0.01 , \text{pu}$</td>
<td>$R_{dc} = 0 , \text{pu}$</td>
<td>$X_{\ell,ac} = 0.1 , \text{pu}$</td>
</tr>
<tr>
<td></td>
<td>$a_h$</td>
<td>$b_h$</td>
<td>$a_h$</td>
</tr>
<tr>
<td>1</td>
<td>1.164</td>
<td>-j 0.113</td>
<td>1.087</td>
</tr>
<tr>
<td>5</td>
<td>0.184</td>
<td>-j 0.100</td>
<td>0.177</td>
</tr>
<tr>
<td>7</td>
<td>-0.108</td>
<td>+j 0.087</td>
<td>-0.107</td>
</tr>
<tr>
<td>11</td>
<td>-0.027</td>
<td>+j 0.057</td>
<td>-0.032</td>
</tr>
<tr>
<td>13</td>
<td>0.004</td>
<td>-j 0.044</td>
<td>0.010</td>
</tr>
<tr>
<td>17</td>
<td>-0.014</td>
<td>-j 0.016</td>
<td>-0.010</td>
</tr>
<tr>
<td>19</td>
<td>0.012</td>
<td>+j 0.008</td>
<td>0.010</td>
</tr>
<tr>
<td>23</td>
<td>0.007</td>
<td>-j 0.007</td>
<td>0.008</td>
</tr>
<tr>
<td>25</td>
<td>-0.007</td>
<td>+j 0.009</td>
<td>-0.008</td>
</tr>
</tbody>
</table>

(1) Case 1 refers to the phase A current of Figure 3.5
(2) Case 2 refers to the phase A current of Figure 3.6
(3) Case 3 refers to the phase A current of Figure 3.2
where $W_h$ is the weighting factor of the $h^{th}$ harmonic and $H_h$ is as defined above [27]. These two measures will be used below to portray the relative impact of varying transformer leakage reactance in the AC network and their effect on the current spectrum. Figure 3.7 displays the complex magnitude, $|\alpha_h|$, versus the AC converter transformer leakage reactance. To isolate and quantify the effect of $X_{t,ac}$ on the harmonic amplitudes, the DC resistance was set to 1 pu and $L_{DC}$ remained at infinity. The AC system R/X ratio was varied from 1 to 0.2 and the magnitudes of the fifth, seventh, and eleventh harmonics were plotted. The AC transformer reactance clearly attenuates the harmonic magnitudes as its value increases. To gain a more analytic insight, again the THD and TIF are used. In Table 3.2, the THD and TIF are tabulated for all three phases as $X_{t,ac}$ was increased from 0.01 to 0.5 pu. From Table 3.2, the THD and TIF decreases as $X_{t,ac}$ is increased. Intuitively, this decreasing trend of THD and TIF values are to be expected as $X_{t,ac}$ is increased. One justification for this behavior is that as the commutating inductance in the AC network increases, the AC currents become more sinusoidal due to increased damping. As a result, the supply currents contain less high frequency components. The numerators of (3.2-1) and (3.2-2) also support the trend of Table 3.2. Because high frequency components are attenuated, the contribution to the numerator of (3.2-1) decreases. As $X_{t,ac}$ becomes larger, more damping occurs, and the numerator of (3.2-2) decreases rapidly.

3.3 Balanced case results for zero inductance in the DC circuit

The balanced operating case with zero inductance in the DC circuit has the theoretical current waveforms shown in Figure 3.8. As mentioned in Section 3.2, the simplifying assumptions of Chapter II are still valid with one exception: since the DC inductance is no longer infinite, the DC current will not be constant for the $L_{DC} = 0$ case. When the phase currents of Figure 3.8 are compared to those of Figure 3.2, the currents for the $L_{DC} = 0$ case have rounded crests instead of the flat tops of the $L_{DC} = \infty$ case. During the conducting portions of the AC phase currents for the $L_{DC} = 0$ case, the current in a purely resistive load is sinusoidally matched in phase with the source. Figure 3.9 displays the three AC phase currents and DC current obtained from a detailed rectifier simulation. For this case, the AC parameters, $R_{ac}$ and $X_{\ell,ac}$ were
Figure 3.7 Graph of the complex magnitude $|\alpha_h|$ versus AC transformer leakage reactance $X_{l,ac}$ for phase A with $R_{DC} = 1.0$ pu and $L_{DC} = 377$ pu.
Table 3.2  THD and TIF values for varied transformer leakage reactance, $X_{\ell,ac}$ ($L_{DC} = \infty$).

<table>
<thead>
<tr>
<th>CONVERTER TRANSFORMER IMPEDANCE</th>
<th>THD (%)</th>
<th>TIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ac} + jX_{\ell,ac}$</td>
<td>$\phi A$</td>
<td>$\phi B$</td>
</tr>
<tr>
<td>0.01 + j 0.01</td>
<td>28.1</td>
<td>27.7</td>
</tr>
<tr>
<td>0.01 + j 0.10</td>
<td>23.0</td>
<td>22.6</td>
</tr>
<tr>
<td>0.01 + j 0.125</td>
<td>22.1</td>
<td>21.6</td>
</tr>
<tr>
<td>0.01 + j 0.25</td>
<td>18.6</td>
<td>18.0</td>
</tr>
<tr>
<td>0.01 + j 0.375</td>
<td>16.1</td>
<td>15.6</td>
</tr>
<tr>
<td>0.01 + j 0.50</td>
<td>13.9</td>
<td>13.6</td>
</tr>
</tbody>
</table>
selected to have a R/X ratio of one-tenth. In addition, the DC resistance was set to one per unit. In Figure 3.9, the three phase currents appear more sinusoidal than any other waveforms presented in Section 3.2. The DC current still contains a 60 Hz component but lacks the constancy of the $L_{DC} = \infty$ case. Because the DC current is not constant, the average value of the DC current has decreased by approximately 10%.

To examine the effect of the DC load on the AC currents, Figure 3.10 displays a detailed simulation where the DC inductance is increased to 10 pu. Comparing Figures 3.9 and 3.10, features of the AC and DC currents have changed quite drastically in response to the 10 pu increase of DC inductance. With this change the AC currents are becoming more nonsinusoidal in nature and the DC current is already becoming more constant. The average value of the DC current has increased by nearly 3%. To compare these cases quantitatively, the Fourier spectra were evaluated for the theoretical case and for those cases represented by Figures 3.9 and 3.10. Table 3.3 summarizes the real and imaginary parts of $\alpha_k$, the complex Fourier coefficient, in the phase A supply currents for the three cases mentioned above. Pertaining to Table 3.3, the theoretical spectrum displays harmonics of the order $3k$ ($k$ odd), which do not appear in the detailed rectifier simulation. Due to the transformer connection used in this simulation ($Y-A$), the triplen harmonics are absorbed in the delta winding of the secondary of the transformer and will not penetrate into the AC network.

In evaluating the effect of $X_{\ell, ac}$ on the harmonic magnitudes, the DC resistance was set to 1 pu and $L_{DC}$ remained at zero. The AC system R/X ratio was varied from 1 to 0.2 and the THD and TIF are tabulated for all three phases as $X_{\ell, ac}$ was varied. Like that of the $L_{DC} = \infty$ case, as the transformer reactance is increased, the THD and TIF decrease accordingly. In order to conjecture about all cases lying between the two extreme cases $L_{DC} = \infty$ and $L_{DC} = 0$, an intermediate case was simulated for a DC inductance of 100 pu. Table 3.4 contains the complex magnitudes for the cases of $L_{DC}$ taking the value of zero, ten, 100, and 377 pu. The standard R/X ratio of one-tenth was used on the AC side and the DC resistance was maintained at 1 pu. From this table, it seems that no conjecture can be made for this case. It is interesting to note that the harmonic magnitudes are found in a very close proximity of each other for the four respective cases.
Figure 3.8  Theoretical phase currents for the $L_{DC} = 0$ case.
Figure 3.9  AC phase currents and DC current obtained from a detailed rectifier simulation: $R_{ac} = 0.01$ pu, $X_{L_{ac}} = 0.10$ pu, $R_{DC} = 1$ pu, and $L_{DC} = 0$ pu.
Figure 3.10 AC phase currents and DC current obtained from a detailed rectifier simulation: $R_{ac} = 0.01$ pu, $X_{L_{ac}} = 0.10$ pu, $R_{DC} = 1$ pu, and $L_{DC} = 10$ pu.
Table 3.3 THD and TIF values for varied transformer leakage reactance, $X_{\ell,ac}$ ($L_{DC} = 0$).

<table>
<thead>
<tr>
<th>CONVERTER TRANSFORMER IMPEDANCE</th>
<th>THD (%)</th>
<th>TIF</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ac} + jX_{\ell,ac}$</td>
<td>$\phi A$</td>
<td>$\phi B$</td>
</tr>
<tr>
<td>0.01 + j 0.01</td>
<td>28.5</td>
<td>28.1</td>
</tr>
<tr>
<td>0.01 + j 0.10</td>
<td>24.9</td>
<td>24.5</td>
</tr>
<tr>
<td>0.01 + j 0.125</td>
<td>24.2</td>
<td>23.8</td>
</tr>
<tr>
<td>0.01 + j 0.25</td>
<td>21.5</td>
<td>20.6</td>
</tr>
<tr>
<td>0.01 + j 0.375</td>
<td>18.9</td>
<td>18.1</td>
</tr>
<tr>
<td>0.01 + j 0.50</td>
<td>16.4</td>
<td>15.9</td>
</tr>
</tbody>
</table>
Table 3.4  Complex magnitude $|\alpha_h|$ for 0, 10, 100, and 377 pu DC inductance.

<table>
<thead>
<tr>
<th>h</th>
<th>$L_{DC} = 0$ pu</th>
<th>$L_{DC} = 10$ pu</th>
<th>$L_{DC} = 100$ pu</th>
<th>$L_{DC} = 377$ pu</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.014</td>
<td>1.014</td>
<td>1.075</td>
<td>1.092</td>
</tr>
<tr>
<td>5</td>
<td>0.224</td>
<td>0.185</td>
<td>0.195</td>
<td>0.198</td>
</tr>
<tr>
<td>7</td>
<td>0.087</td>
<td>0.123</td>
<td>0.129</td>
<td>0.130</td>
</tr>
<tr>
<td>11</td>
<td>0.063</td>
<td>0.059</td>
<td>0.060</td>
<td>0.061</td>
</tr>
<tr>
<td>13</td>
<td>0.033</td>
<td>0.043</td>
<td>0.044</td>
<td>0.044</td>
</tr>
<tr>
<td>17</td>
<td>0.023</td>
<td>0.021</td>
<td>0.021</td>
<td>0.021</td>
</tr>
<tr>
<td>19</td>
<td>0.011</td>
<td>0.014</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td>23</td>
<td>0.009</td>
<td>0.008</td>
<td>0.008</td>
<td>0.009</td>
</tr>
<tr>
<td>25</td>
<td>0.006</td>
<td>0.010</td>
<td>0.010</td>
<td>0.010</td>
</tr>
</tbody>
</table>
3.4 Summary

In Chapter III, the balanced operating condition was examined for cases of infinite and zero inductance in the DC circuit. Section 3.2, focusing on the infinite inductance case, presents important results supporting the validity of the detailed rectifier simulation. In Figure 3.4 the Fourier spectra are presented for the theoretical case and that of a detailed rectifier simulation representing the theoretical case. This comparison enables other simulation results to be evaluated with a high degree of confidence. To see how changes in the DC load impedance affect the supply current spectra, the DC resistance was varied for the \( L_{DC} = \infty \) case. The following observations were made:

(i.) as the DC resistance increases the magnitude of the AC phase currents decrease,

(ii.) a conjecture is made that as the DC resistance is increased from 0 to 1 pu, the harmonic magnitudes of intermediate cases lie between these two extremes, and

(iii.) as the DC resistance is increased, the THD and TIF increases also.

The AC transformer leakage reactance was also varied to investigate its effect on the supply current spectra. In this case, the AC \( R/X \) ratio was decreased from unity to 0.02; this corresponds to increasing \( X_{\epsilon, ac} \) from 0.01 pu to 0.5 pu while maintaining \( R_{ac} \) constant at 0.01 pu. The following observations were made for this case:

(i.) the harmonic magnitudes, \( |\alpha_h| \), are attenuated for low order harmonics and to a lesser degree for harmonics greater than the 13th,

(ii.) as \( X_{\epsilon, ac} \) is increased, the THD and TIF decrease in all phases, and

(iii.) as \( X_{\epsilon, ac} \) is increased, the AC supply currents become more sinusoidal.

In Section 3.3 the zero inductance case was examined. For the case of zero DC circuit inductance \( (L_{DC} = 0) \), the following observations are made:

(i.) the AC phase currents for the \( L_{DC} = 0 \) case are much more sinusoidal compared to the \( L_{DC} = \infty \) case,

(ii.) for the \( L_{DC} = 0 \) case, the DC current lacks the constancy of the \( L_{DC} = \infty \) case,

(iii.) the average value of the DC current is decreased by 10% compared to the \( L_{DC} = \infty \) case,
(iv.) as \( L_{DC} \) is increased, the AC currents become more non-sinusoidal,
(v.) as the DC inductance is increased, the DC current becomes more constant and its average value increases,
(vi.) in the theoretical case, triplen harmonics \((3k, k \text{ odd})\) are present
(vii.) \( Y-\Delta \) transformer connection, no triplen harmonics are observed in the current spectrum by the detailed simulation (absorbed by \( \Delta \) winding), and
(viii.) as the transformer leakage reactance is decreased from an \( R/X \) ratio of 1 to 0.02, the THD and TIF levels decrease in all phases with phase B being almost exclusively the smallest.

Since the supply voltages were balanced for all cases observed, system parameters could be perturbed to evaluate their impact on the AC supply spectra. This information implies that the AC and DC parameters have an impact on the spectra of the AC supply currents. Understanding how these relationships interact enable the effect of supply voltage magnitude and phase unbalance on the AC current spectrum to be isolated.
CHAPTER IV
COMPARISON OF RESULTS FOR THE UNBALANCED OPERATING CONDITION

4.1 Introduction

In this chapter, the unbalanced operating condition is examined for cases of infinite and zero inductance in the DC circuit. Unlike Chapter III where system parameters were varied, Chapter IV focuses on supply voltage magnitude and phase unbalance to determine what effect this unbalance has on the AC current spectrum. It is still important that the observations of Chapter III be kept in mind since it is probable that these considerations also apply to some degree in the unbalanced case. To verify the approximate commutation angle deviation formulas of Chapter II (2.2-16)-(2.2-18), 625 cases were calculated for magnitudes ranging from -0.2 to 0.2 pu and phase angle deviations ranging from -0.2 to 0.2 radians. Each case was evaluated for the following information: actual and approximate commutation angle deviations in radians, percent error of actual and approximate commutation angle deviations, actual and approximate commutation angles in degrees, and the percent error of actual versus approximate commutation angles. The word "approximate" is used throughout this chapter in reference to any information calculated using equations (2.2-16)-(2.2-18).

In Section 4.2 and 4.3, four cases will be presented for the infinite and zero inductance case respectively. These representative cases are selected from the 625 cases by their unbalance factor (UF). The UF, as stated in Chapter II, can be expressed with the aid of (2.4-1) as

\[
UF = \frac{v_{sc}^-}{v_{sc}^+} = \frac{1+a(1+j\theta_b)(1+\epsilon_b)+a^2(1+j\theta_c)(1+\epsilon_c)}{3+\epsilon_b(1+j\theta_b)+\epsilon_c(1+j\theta_c)+j(\theta_b+\theta_c)}.
\]

For the 625 cases examined, the UF had a range from zero to 18%. The number of cases examined prohibit full description here, but four representative cases are presented for unbalance factors of 5, 10, 15, and 18%. These cases are
used to validate the accuracy of (2.2-16)-(2.2-18) and formulate observations about the effect of supply voltage unbalance on the AC current spectrum. In addition to the theoretical $L_{DC} = \infty$ and $L_{DC} = 0$ cases, results obtained from a detailed rectifier simulation are used for comparison. In view of the results of Chapter III, the AC R/X ratio is fixed at one-tenth and $R_{DC}$ is constant at 1 pu.

4.2 Unbalanced case results for infinite inductance in the DC circuit

Figure 4.1 displays unbalanced supply voltages applied to a six-pulse converter. Due to the supply voltage unbalance, the commutation angles will deviate from the theoretical angles as displayed in Figure 4.2. Chapter II described a method of iteratively searching for the "exact" solution of the six transcendental commutation equations (2.2-3)-(2.2-8). An alternative to this method is to linearize the six commutation equations about their ideal operating points. By the use of Taylor series expansion, six algebraic equations are developed which are functions of the supply voltage magnitude and phase unbalance. Equations (2.2-16)-(2.2-18) is this alternative solution and Tables 4.1-4.8 display the accuracy of these equations.

4.3 Unbalanced case results for zero inductance in the DC circuit

For the case of zero inductance in the DC circuit, the results of Tables 4.1, 4.3, 4.5, and 4.7 remain valid. The only difference resulting from forcing $L_{DC}$ to zero is a change in shape during the current waveform conducting interval. As discussed previously in Chapter II, the Fourier coefficients of the $L_{DC} = \infty$ case is related to the $L_{DC} = 0$ case by the relation formulated in Chapter II. Because these current waveforms vary in shape, the Fourier coefficients for the $L_{DC} = 0$ case must be evaluated for the four cases to observe the effects of supply unbalance on the current spectra. These cases are presented in Tables 4.9-4.12.

4.4 Summary

The specific goal in Chapter IV was to demonstrate the validity of the approximate commutation angle deviations developed by the linearization of equations (2.2-3)-(2.2-8). To demonstrate the accuracy of these approximate formulas, four representative cases were presented here for varying degrees of unbalance. In Section 4.2, the results of Cases I-IV are presented for infinite inductance in the DC circuit. The observations for this condition are:
(i.) the commutation angle deviation equations (2.2-16)-(2.2-18) are surprisingly accurate; typically less than 15% error,

(ii.) the detailed rectifier simulation current spectra do not include odd triplen harmonics due to the delta winding on the transformer secondary,

(iii.) the detailed rectifier simulation current spectra varied little to none for Cases I-IV,

(iv.) this lack of variation can be attributed to a high value of DC inductance and a small supply unbalance (maximum UF = 18%),

(v.) for a UF of 5, 10, 15, and 18%, the amplitude of the 3rd harmonic was 1.6, 2.9, 2.4, and 2.4% of the phase A fundamental, and

(vi.) the 9th and 15th harmonics had approximately the same ratios to the fundamental (±0.05%) as the 3rd above.

In Section 4.3, the results of Cases I-IV are presented for the case of zero inductance in the DC circuit. Observations for this case are:

(i.) the commutation angles and their deviations are the same as the \( L_{DC} = \infty \) case,

(ii.) the magnitude of the 3rd, 9th, and 15th harmonics varied for different UF factors,

(iii.) the magnitudes of the 3rd, 9th, and 15th harmonics observed for this case were approximately 17%, 3.3%, and 2.3% of the fundamental current in phase A.

Since the system parameters were constant for the cases observed, all preceding results can be attributed to supply unbalance. In [14], a graph of uncharacteristic harmonics versus unbalance factor is shown for an AC side impedance of 0.015 + j0.15 pu per phase and a 1.0 H smoothing reactor in the DC circuit. The results from an extensive digital simulation agree reasonably well with the results contained here. A slight attenuation of the 3rd harmonic in [14] is noticed compared to the theoretical results presented in this chapter. It should be noted that for a DC inductance of 1.0 H, the detailed simulation results presented here and in [14] are quite close to the theoretical results for infinite inductance in the DC circuit.
Unbalanced Applied Voltages

\[ V_{an} = \cos(t) \]
\[ V_{bn} = (1 + \varepsilon_b) \cos(t - 2\pi/3 + \theta_b) \]
\[ V_{cn} = (1 + \varepsilon_c) \cos(t + 2\pi/3 + \theta_c) \]

Figure 4.1 Unbalanced supply voltages applied to a six-pulse converter.
Figure 4.2 Commutation angle deviations caused by unbalanced source voltages applied to a six-pulse converter ($L_{DC} = \infty$ case).
Table 4.1 Comparison of exact versus approximate solutions for Case I.

| CASE I. $\epsilon_b = 0.0$ pu $\theta_b = -0.1$ rad $\theta_c = -0.2$ rad $\epsilon_c = 0.1$ pu $\theta_c = -0.2$ rad $\text{UF} = 5\%$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | 1               | 2               | 3               | 4               | 5               | 6               |
| $\bar{\delta}_{bc}$ | -0.180755      | -0.179874       | 0.49            | -10.36          | -10.31          | 0.49            |
| $\delta_{ab}$    | -0.050000       | -0.050000       | 0               | 57.14           | 57.14           | 0               |
| $\delta_{ca}$    | -0.078533       | -0.077269       | 1.61            | 115.50          | 115.57          | -0.06           |
| $\delta_{bc}$    | -0.180755       | -0.179874       | 0.49            | 169.64          | 169.69          | -0.03           |
| $\bar{\delta}_{ab}$ | -0.050000       | -0.050000       | 0               | 237.14          | 237.14          | 0               |
| $\delta_{ca}$    | -0.078533       | -0.077269       | 1.61            | 295.50          | 295.57          | -0.02           |

1 actual commutation angle deviations in radians
2 approximate commutation angle deviations in radians
3 percent error, $\Delta\%$, defined as $\frac{1-2}{1} \times 100$
4 actual commutation angles in degrees
5 approximate commutation angles in degrees
6 percent error, $\Delta\%$, defined as $\frac{4-5}{4} \times 100$
Table 4.2  Comparison of Fourier spectra for Case I.

<table>
<thead>
<tr>
<th>h</th>
<th>Detailed Simulation</th>
<th>Theoretical</th>
<th>Approximate Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_h$</td>
<td>$b_h$</td>
<td>$a_h$</td>
</tr>
<tr>
<td>1</td>
<td>1.087</td>
<td>+j 0.105</td>
<td>1.109</td>
</tr>
<tr>
<td>3</td>
<td>-0.017</td>
<td>+j 0.003</td>
<td>-0.016</td>
</tr>
<tr>
<td>5</td>
<td>0.177</td>
<td>-j 0.088</td>
<td>-0.200</td>
</tr>
<tr>
<td>7</td>
<td>-0.107</td>
<td>+j 0.074</td>
<td>0.149</td>
</tr>
<tr>
<td>9</td>
<td>-0.015</td>
<td>+j 0.010</td>
<td>-0.014</td>
</tr>
<tr>
<td>11</td>
<td>-0.032</td>
<td>+j 0.052</td>
<td>-0.068</td>
</tr>
<tr>
<td>13</td>
<td>0.010</td>
<td>-j 0.043</td>
<td>0.061</td>
</tr>
<tr>
<td>15</td>
<td>-0.010</td>
<td>+j 0.014</td>
<td>-0.010</td>
</tr>
<tr>
<td>17</td>
<td>-0.010</td>
<td>-j 0.019</td>
<td>-0.025</td>
</tr>
<tr>
<td>19</td>
<td>0.010</td>
<td>+j 0.009</td>
<td>0.022</td>
</tr>
<tr>
<td>21</td>
<td>-0.004</td>
<td>+j 0.017</td>
<td>-0.004</td>
</tr>
<tr>
<td>23</td>
<td>0.008</td>
<td>-j 0.004</td>
<td>-0.003</td>
</tr>
<tr>
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<td>-0.008</td>
<td>+j 0.006</td>
<td>-0.002</td>
</tr>
<tr>
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<td>0.003</td>
<td>+j 0.017</td>
<td>0.002</td>
</tr>
</tbody>
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Table 4.3 Comparison of exact versus approximate solutions for Case II.

<table>
<thead>
<tr>
<th>CASE II.</th>
<th>$\epsilon_b = 0.2,\text{pu}$</th>
<th>$\epsilon_c = 0.2,\text{pu}$</th>
<th>$\theta_b = -0.1,\text{rad}$</th>
<th>$\theta_c = -0.2,\text{rad}$</th>
<th>UF = 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>$\delta_{bc}$</td>
<td>-0.150000</td>
<td>-0.150000</td>
<td>0</td>
<td>-8.59</td>
<td>-8.59</td>
</tr>
<tr>
<td>$\delta_{ab}$</td>
<td>-0.108665</td>
<td>-0.107032</td>
<td>1.5</td>
<td>53.77</td>
<td>53.87</td>
</tr>
<tr>
<td>$\delta_{ca}$</td>
<td>-0.059033</td>
<td>-0.056605</td>
<td>4.11</td>
<td>116.62</td>
<td>116.76</td>
</tr>
<tr>
<td>$\delta_{bc}$</td>
<td>-0.150000</td>
<td>-0.150000</td>
<td>0</td>
<td>-8.59</td>
<td>-8.59</td>
</tr>
<tr>
<td>$\delta_{ab}$</td>
<td>-0.108665</td>
<td>-0.107032</td>
<td>1.5</td>
<td>233.77</td>
<td>233.87</td>
</tr>
<tr>
<td>$\delta_{ca}$</td>
<td>-0.059033</td>
<td>-0.056605</td>
<td>4.11</td>
<td>296.62</td>
<td>296.76</td>
</tr>
</tbody>
</table>

1 actual commutation angle deviations in radians
2 approximate commutation angle deviations in radians
3 percent error, $\Delta\%$, defined as $\frac{1-2}{1} \times 100$
4 actual commutation angles in degrees
5 approximate commutation angles in degrees
6 percent error, $\Delta\%$, defined as $\frac{4-5}{4} \times 100$
Table 4.4 Comparison of Fourier spectra for Case II.

<table>
<thead>
<tr>
<th>h</th>
<th>Detailed Simulation</th>
<th>Theoretical</th>
<th>Approximate Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_h$</td>
<td>$b_h$</td>
<td>$a_h$</td>
</tr>
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<td>1.087</td>
<td>-j 0.105</td>
<td>1.083</td>
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<tr>
<td>3</td>
<td>0.03</td>
<td></td>
<td>0.0078</td>
</tr>
<tr>
<td>5</td>
<td>0.177</td>
<td>-j 0.088</td>
<td>-0.214</td>
</tr>
<tr>
<td>7</td>
<td>-0.107</td>
<td>+j 0.074</td>
<td>0.116</td>
</tr>
<tr>
<td>9</td>
<td>0.022</td>
<td>-j 0.021</td>
<td>0.024</td>
</tr>
<tr>
<td>11</td>
<td>-0.032</td>
<td>+j 0.052</td>
<td>-0.067</td>
</tr>
<tr>
<td>13</td>
<td>0.0097</td>
<td>-j 0.043</td>
<td>0.030</td>
</tr>
<tr>
<td>15</td>
<td>0.0092</td>
<td>-j 0.029</td>
<td>0.010</td>
</tr>
<tr>
<td>17</td>
<td>-0.0097</td>
<td>-j 0.019</td>
<td>-0.010</td>
</tr>
<tr>
<td>19</td>
<td>0.010</td>
<td>+j 0.0087</td>
<td>-0.0011</td>
</tr>
<tr>
<td>21</td>
<td>-0.0059</td>
<td>-j 0.029</td>
<td>-0.0045</td>
</tr>
<tr>
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<td>0.0078</td>
<td>-j 0.0039</td>
<td>0.019</td>
</tr>
<tr>
<td>25</td>
<td>-0.0085</td>
<td>+j 0.0059</td>
<td>-0.011</td>
</tr>
<tr>
<td>27</td>
<td>-0.019</td>
<td>-j 0.022</td>
<td>-0.018</td>
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</tbody>
</table>
Table 4.5 Comparison of exact versus approximate solutions for Case III.

<table>
<thead>
<tr>
<th>CASE III.</th>
<th>$\epsilon_b = -0.2$ pu</th>
<th>$\epsilon_e = -0.1$ pu</th>
<th>$\theta_b = -0.1$ rad</th>
<th>$\theta_e = 0.2$ rad</th>
<th>UF = 15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{bc}$</td>
<td>0.026945</td>
<td>0.024862</td>
<td>7.73</td>
<td>1.54</td>
<td>1.42</td>
</tr>
<tr>
<td>$\delta_{ab}$</td>
<td>0.021661</td>
<td>0.019706</td>
<td>9.03</td>
<td>61.24</td>
<td>61.13</td>
</tr>
<tr>
<td>$\delta_{ca}$</td>
<td>0.062157</td>
<td>0.064350</td>
<td>-3.53</td>
<td>123.56</td>
<td>123.69</td>
</tr>
<tr>
<td>$\overline{\delta}_{bc}$</td>
<td>0.026945</td>
<td>0.024862</td>
<td>7.73</td>
<td>181.54</td>
<td>181.42</td>
</tr>
<tr>
<td>$\overline{\delta}_{ab}$</td>
<td>0.021661</td>
<td>0.019706</td>
<td>9.03</td>
<td>241.24</td>
<td>241.13</td>
</tr>
<tr>
<td>$\delta_{ca}$</td>
<td>0.062156</td>
<td>0.064350</td>
<td>-3.53</td>
<td>303.56</td>
<td>303.69</td>
</tr>
</tbody>
</table>

1. actual commutation angle deviations in radians
2. approximate commutation angle deviations in radians
3. percent error, $\Delta\%$, defined as $\frac{1-2}{1} \times 100$
4. actual commutation angles in degrees
5. approximate commutation angles in degrees
6. percent error, $\Delta\%$, defined as $\frac{4-5}{4} \times 100$
Table 4.6  Comparison of Fourier spectra for Case III.

<table>
<thead>
<tr>
<th>h</th>
<th>Detailed Simulation</th>
<th>Theoretical</th>
<th>Approximate Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_h$</td>
<td>$b_h$</td>
<td>$a_h$</td>
</tr>
<tr>
<td>1</td>
<td>1.087</td>
<td>-j 0.105</td>
<td>1.088</td>
</tr>
<tr>
<td>3</td>
<td>0.025</td>
<td>+j 0.0032</td>
<td>0.028</td>
</tr>
<tr>
<td>5</td>
<td>-j 0.088</td>
<td>-0.227</td>
<td>-j 0.048</td>
</tr>
<tr>
<td>7</td>
<td>+j 0.074</td>
<td>0.137</td>
<td>+j 0.041</td>
</tr>
<tr>
<td>9</td>
<td>0.024</td>
<td>+j 0.0095</td>
<td>0.026</td>
</tr>
<tr>
<td>11</td>
<td>+j 0.052</td>
<td>-0.099</td>
<td>-j 0.049</td>
</tr>
<tr>
<td>13</td>
<td>-j 0.043</td>
<td>0.059</td>
<td>+j 0.036</td>
</tr>
<tr>
<td>15</td>
<td>0.020</td>
<td>+j 0.015</td>
<td>0.022</td>
</tr>
<tr>
<td>17</td>
<td>-j 0.019</td>
<td>-0.056</td>
<td>-j 0.048</td>
</tr>
<tr>
<td>19</td>
<td>+j 0.0087</td>
<td>0.029</td>
<td>+j 0.029</td>
</tr>
<tr>
<td>21</td>
<td>0.018</td>
<td>+j 0.019</td>
<td>0.017</td>
</tr>
<tr>
<td>23</td>
<td>-j 0.0039</td>
<td>-0.031</td>
<td>-j 0.045</td>
</tr>
<tr>
<td>25</td>
<td>+j 0.0059</td>
<td>0.013</td>
<td>+j 0.023</td>
</tr>
<tr>
<td>27</td>
<td>0.010</td>
<td>+j 0.022</td>
<td>0.011</td>
</tr>
</tbody>
</table>
Table 4.7 Comparison of exact versus approximate solutions for Case IV.

<table>
<thead>
<tr>
<th>CASE IV.</th>
<th>$\epsilon_b = -0.2$ pu</th>
<th>$\epsilon_c = -0.2$ pu</th>
<th>$\text{UF} = 18%$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta_b = -0.2$ rad</td>
<td>$\theta_c = 0.2$ rad</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta_{bc}$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.020241</td>
<td>-0.024739</td>
<td>-22.22</td>
<td>58.84</td>
<td>58.58</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>0.020241</td>
<td>0.024739</td>
<td>-22.22</td>
<td>121.16</td>
<td>121.42</td>
<td>-0.21</td>
<td></td>
</tr>
<tr>
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<td>180.0</td>
<td>180.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>-0.020241</td>
<td>-0.024739</td>
<td>-22.22</td>
<td>238.84</td>
<td>238.58</td>
<td>0.11</td>
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</tr>
<tr>
<td>0.020241</td>
<td>0.024739</td>
<td>-22.22</td>
<td>301.16</td>
<td>301.42</td>
<td>-0.09</td>
<td></td>
</tr>
</tbody>
</table>

1 actual commutation angle deviations in radians

2 approximate commutation angle deviations in radians

3 percent error, $\Delta\%$, defined as $\frac{1 - 2}{1} \times 100$

4 actual commutation angles in degrees

5 approximate commutation angles in degrees

6 percent error, $\Delta\%$, defined as $\frac{4 - 5}{4} \times 100$
Table 4.8 Comparison of Fourier spectra for Case IV.

<table>
<thead>
<tr>
<th>h</th>
<th>( a_h )</th>
<th>( b_h )</th>
<th>( a_h )</th>
<th>( b_h )</th>
<th>( a_h )</th>
<th>( b_h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.087</td>
<td>-j 0.105</td>
<td>1.089</td>
<td>-j 0.0</td>
<td>1.086</td>
<td>+j 0.0</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>0.026</td>
<td>-j 0.0</td>
<td>0.032</td>
<td>+j 0.0</td>
</tr>
<tr>
<td>5</td>
<td>0.177</td>
<td>-j 0.088</td>
<td>-0.232</td>
<td>+j 0.0</td>
<td>-0.235</td>
<td>+j 0.0</td>
</tr>
<tr>
<td>7</td>
<td>-0.107</td>
<td>+j 0.074</td>
<td>0.143</td>
<td>-j 0.0</td>
<td>0.139</td>
<td>+j 0.0</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td>0.026</td>
<td>-j 0.0</td>
<td>0.032</td>
<td>+j 0.0</td>
</tr>
<tr>
<td>11</td>
<td>-0.032</td>
<td>+j 0.052</td>
<td>-0.111</td>
<td>+j 0.0</td>
<td>-0.112</td>
<td>+j 0.0</td>
</tr>
<tr>
<td>13</td>
<td>0.0097</td>
<td>-j 0.043</td>
<td>0.069</td>
<td>+j 0.0</td>
<td>0.064</td>
<td>+j 0.0</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td>0.026</td>
<td>+j 0.0</td>
<td>0.031</td>
<td>+j 0.0</td>
</tr>
<tr>
<td>17</td>
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<td>-j 0.019</td>
<td>-0.074</td>
<td>+j 0.0</td>
<td>-0.075</td>
<td>+j 0.0</td>
</tr>
<tr>
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<td>+j 0.0087</td>
<td>0.041</td>
<td>+j 0.0</td>
<td>0.036</td>
<td>+j 0.0</td>
</tr>
<tr>
<td>21</td>
<td></td>
<td></td>
<td>0.025</td>
<td>+j 0.0</td>
<td>0.031</td>
<td>+j 0.0</td>
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<td>-0.055</td>
<td>+j 0.0</td>
<td>-0.055</td>
<td>+j 0.0</td>
</tr>
<tr>
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<td>+j 0.0059</td>
<td>0.026</td>
<td>+j 0.0</td>
<td>0.020</td>
<td>+j 0.0</td>
</tr>
<tr>
<td>27</td>
<td></td>
<td></td>
<td>0.025</td>
<td>+j 0.0</td>
<td>0.030</td>
<td>+j 0.0</td>
</tr>
</tbody>
</table>

**CASE IV.**

- \( \epsilon_h = -0.2 \) pu
- \( \epsilon_c = -0.2 \) pu
- \( R_{ac} = 0.01 \) pu
- \( R_{DC} = 1.0 \) pu
- \( UF = 18\% \)
- \( \theta_h = -0.2 \) rad
- \( \theta_c = 0.2 \) rad
- \( X_{\epsilon,ac} = 0.10 \) pu
- \( L_{DC} = \infty \)
Table 4.9  Comparison of exact and approximate Fourier coefficients for Case I ($L_{DC} = 0$).

<table>
<thead>
<tr>
<th>h</th>
<th>Theoretical</th>
<th>Approximate Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_h$</td>
<td>$b_h$</td>
</tr>
<tr>
<td>1</td>
<td>0.944</td>
<td>-j 0.035</td>
</tr>
<tr>
<td>3</td>
<td>0.131</td>
<td>+j 0.001</td>
</tr>
<tr>
<td>5</td>
<td>-0.129</td>
<td>+j 0.033</td>
</tr>
<tr>
<td>7</td>
<td>0.064</td>
<td>-j 0.035</td>
</tr>
<tr>
<td>9</td>
<td>0.008</td>
<td>+j 0.004</td>
</tr>
<tr>
<td>11</td>
<td>-0.041</td>
<td>+j 0.029</td>
</tr>
<tr>
<td>13</td>
<td>0.027</td>
<td>-j 0.033</td>
</tr>
<tr>
<td>15</td>
<td>0.002</td>
<td>+j 0.006</td>
</tr>
<tr>
<td>17</td>
<td>-0.016</td>
<td>+j 0.024</td>
</tr>
<tr>
<td>19</td>
<td>0.009</td>
<td>-j 0.029</td>
</tr>
<tr>
<td>21</td>
<td>0.002</td>
<td>+j 0.007</td>
</tr>
<tr>
<td>23</td>
<td>-0.004</td>
<td>+j 0.019</td>
</tr>
<tr>
<td>25</td>
<td>-0.001</td>
<td>-j 0.024</td>
</tr>
<tr>
<td>27</td>
<td>0.003</td>
<td>+j 0.006</td>
</tr>
</tbody>
</table>
Table 4.10 Comparison of exact and approximate Fourier coefficients for Case II ($L_{DC} = 0$).

<table>
<thead>
<tr>
<th>h</th>
<th>Theoretical</th>
<th>Approximate Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_h$</td>
<td>$b_h$</td>
</tr>
<tr>
<td>1</td>
<td>0.930</td>
<td>-0.047</td>
</tr>
<tr>
<td>3</td>
<td>0.157</td>
<td>-0.005</td>
</tr>
<tr>
<td>5</td>
<td>-0.136</td>
<td>+j 0.050</td>
</tr>
<tr>
<td>7</td>
<td>0.045</td>
<td>-j 0.039</td>
</tr>
<tr>
<td>9</td>
<td>0.028</td>
<td>-j 0.013</td>
</tr>
<tr>
<td>11</td>
<td>-0.039</td>
<td>+j 0.047</td>
</tr>
<tr>
<td>13</td>
<td>0.009</td>
<td>-j 0.028</td>
</tr>
<tr>
<td>15</td>
<td>0.012</td>
<td>-j 0.018</td>
</tr>
<tr>
<td>17</td>
<td>-0.006</td>
<td>+j 0.039</td>
</tr>
<tr>
<td>19</td>
<td>-0.004</td>
<td>-j 0.016</td>
</tr>
<tr>
<td>21</td>
<td>0.000</td>
<td>-j 0.018</td>
</tr>
<tr>
<td>23</td>
<td>0.010</td>
<td>+j 0.027</td>
</tr>
<tr>
<td>25</td>
<td>-0.008</td>
<td>-j 0.006</td>
</tr>
<tr>
<td>27</td>
<td>-0.008</td>
<td>-j 0.014</td>
</tr>
</tbody>
</table>

CASE II. $\epsilon_b = 0.2$ pu $\epsilon_c = 0.2$ pu
UF = 10% $\theta_b = -0.1$ rad $\theta_c = -0.2$ rad
Table 4.11 Comparison of exact and approximate Fourier coefficients for Case III \((L_{DC} = 0)\).

<table>
<thead>
<tr>
<th>(h)</th>
<th>Theoretical</th>
<th>Approximate Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a_h)</td>
<td>(b_h)</td>
</tr>
<tr>
<td>1</td>
<td>0.935</td>
<td>+j 0.024</td>
</tr>
<tr>
<td>3</td>
<td>0.152</td>
<td>+j 0.002</td>
</tr>
<tr>
<td>5</td>
<td>-0.142</td>
<td>-j 0.025</td>
</tr>
<tr>
<td>7</td>
<td>0.058</td>
<td>+j 0.021</td>
</tr>
<tr>
<td>9</td>
<td>0.027</td>
<td>+j 0.005</td>
</tr>
<tr>
<td>11</td>
<td>-0.055</td>
<td>-j 0.025</td>
</tr>
<tr>
<td>13</td>
<td>0.025</td>
<td>+j 0.018</td>
</tr>
<tr>
<td>15</td>
<td>0.016</td>
<td>+j 0.008</td>
</tr>
<tr>
<td>17</td>
<td>-0.030</td>
<td>-j 0.025</td>
</tr>
<tr>
<td>19</td>
<td>0.012</td>
<td>+j 0.015</td>
</tr>
<tr>
<td>21</td>
<td>0.011</td>
<td>+j 0.010</td>
</tr>
<tr>
<td>23</td>
<td>-0.016</td>
<td>-j 0.023</td>
</tr>
<tr>
<td>25</td>
<td>0.005</td>
<td>+j 0.011</td>
</tr>
<tr>
<td>27</td>
<td>0.007</td>
<td>+j 0.012</td>
</tr>
</tbody>
</table>

**CASE III.**

- \(\epsilon_b = -0.2\) pu
- \(\epsilon_c = -0.1\) pu
- \(\theta_b = -0.1\) rad
- \(\theta_c = 0.2\) rad
- \(U_F = 15\%\)
Table 4.12 Comparison of exact and approximate Fourier coefficients for Case IV (LDC = 0).

<table>
<thead>
<tr>
<th>h</th>
<th>CASE IV. $\epsilon_b = -0.2$ pu</th>
<th>$\epsilon_e = -0.2$ pu</th>
<th>Theoretical</th>
<th>Approximate Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$UF = 18%$ $\theta_b = -0.2$ rad</td>
<td>$\theta_e = 0.2$ rad</td>
<td>$a_h$</td>
<td>$b_h$</td>
</tr>
<tr>
<td>1</td>
<td>0.935</td>
<td>-j 0.000</td>
<td>0.934</td>
<td>-j 0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.151</td>
<td>-j 0.000</td>
<td>0.154</td>
<td>-j 0.000</td>
</tr>
<tr>
<td>5</td>
<td>-0.144</td>
<td>+j 0.000</td>
<td>-0.145</td>
<td>+j 0.000</td>
</tr>
<tr>
<td>7</td>
<td>0.061</td>
<td>-j 0.000</td>
<td>0.059</td>
<td>-j 0.000</td>
</tr>
<tr>
<td>9</td>
<td>0.027</td>
<td>-j 0.000</td>
<td>0.030</td>
<td>-j 0.000</td>
</tr>
<tr>
<td>11</td>
<td>-0.060</td>
<td>+j 0.000</td>
<td>-0.061</td>
<td>+j 0.000</td>
</tr>
<tr>
<td>13</td>
<td>0.031</td>
<td>-j 0.000</td>
<td>0.029</td>
<td>-j 0.000</td>
</tr>
<tr>
<td>15</td>
<td>0.018</td>
<td>-j 0.000</td>
<td>0.021</td>
<td>-j 0.000</td>
</tr>
<tr>
<td>17</td>
<td>-0.039</td>
<td>+j 0.000</td>
<td>-0.039</td>
<td>+j 0.000</td>
</tr>
<tr>
<td>19</td>
<td>0.019</td>
<td>-j 0.000</td>
<td>0.016</td>
<td>-j 0.000</td>
</tr>
<tr>
<td>21</td>
<td>0.015</td>
<td>-j 0.000</td>
<td>0.018</td>
<td>-j 0.000</td>
</tr>
<tr>
<td>23</td>
<td>-0.029</td>
<td>+j 0.000</td>
<td>-0.029</td>
<td>+j 0.000</td>
</tr>
<tr>
<td>25</td>
<td>0.012</td>
<td>-j 0.000</td>
<td>0.009</td>
<td>-j 0.000</td>
</tr>
<tr>
<td>27</td>
<td>0.014</td>
<td>-j 0.000</td>
<td>0.017</td>
<td>-j 0.000</td>
</tr>
</tbody>
</table>
CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

In this thesis, much has been discussed in regard to supply unbalance and its effects on the power system. In Chapter I, background information was presented to set the stage for analytical developments contained in Chapter II. By solving the nonlinear commutation equations in Chapter II iteratively for a 6-pulse rectifier, an appreciation for a less computationally burdensome method is attained. The approximate commutation angle deviations due to supply unbalance are presented as algebraic equations which can be computed quickly and quite accurately for magnitude unbalances ranging from -0.2 to 0.2 pu and phase unbalance from -0.2 to 0.2 radians. In Chapter III, the AC supply voltages are held constant as the AC and DC parameters are varied to isolate their effect on the supply current spectrum. The conclusions drawn from this chapter are as follows:

\[ L_{DC} = \infty \text{ case - converter parameters unbalanced} \]

- (i.) as the DC resistance increases, the magnitude of the AC phase currents decrease,
- (ii.) a conjecture is made that as the DC resistance is increased from 0 to 1 pu, the harmonic magnitudes of intermediate cases lie between these two extremes,
- (iii.) as the DC resistance is increased, the THD and TIF increase also,
- (iv.) the harmonic current magnitudes, \( |\alpha_h| \), are attenuated for low order harmonics and to a lesser degree for harmonics greater than the 13\textsuperscript{th} when the AC transformer leakage reactance is increased from 0.01 pu to 0.5 pu,
- (v.) as \( X_{\ell, ac} \) is increased, the THD and TIF decrease in all phases, and
- (vi.) as \( X_{\ell, ac} \) is increased, the AC supply currents become more sinusoidal.
$L_{DC} = 0$ case - converter parameters unbalanced

(i.) the AC phase currents are much more sinusoidal compared to the $L_{DC} = \infty$ case,

(ii.) the DC current lacks the constancy of the $L_{DC} = \infty$ case,

(iii.) the average value of the DC current is decreased by 10% compared to the $L_{DC} = \infty$ case,

(iv.) as $L_{DC}$ is increased, the AC currents become more nonsinusoidal,

(v.) as $L_{DC}$ is increased, the DC current becomes more constant and its average value increases,

(vi.) in the theoretical case, odd triplen harmonics are present,

(vii.) due to the detailed rectifier simulation Y-$\Delta$ transformer connection, no triplen harmonics are observed in the current spectrum as calculated by the detailed simulation (i.e., they are blocked by the delta connected winding), and

(viii.) as the transformer leakage reactance is decreased from an R/X ratio of 1 to 0.02, the THD and TIF levels decrease in all phases with phase B being almost exclusively the smallest.

These conclusions mentioned above are a result of converter unbalance and not due to supply unbalance. In Chapter IV, the system parameters are now held constant and the supply voltages are varied to observe the Fourier spectra of the supply currents. It is important to keep in mind the results of Chapter III when conjectures are made for supply and system unbalance interactions and their effect on the supply currents. The conclusions drawn from Chapter IV are as follows:

$L_{DC} = \infty$ case - AC bus unbalanced

(i.) the commutation angle deviation equations (2.2-16)-(2.2-18) are surprisingly accurate; typically less than 15% error,

(ii.) the detailed rectifier simulation current spectra do not include odd triplen harmonics due to small unbalance and the delta winding on the transformer secondary,

(iii.) the detailed rectifier simulation current spectra varied little to none for Cases I-IV,

(iv.) this lack of variation can be most likely attributed to a high value of DC inductance and a small supply unbalance (maximum UF = 18%).
(v.) for a UF of 5, 10, 15, and 18%, the amplitude of the 3rd harmonic current was 1.6, 2.9, 2.4, and 2.4% of the fundamental in phase A, and
(vi.) the 9th and 15th harmonics had approximately the same ratios to the fundamental (±0.05%) for the various UF levels as the 3rd harmonic cited in (v.).

$L_{DC} = 0$ case - AC bus unbalanced

(i.) the commutation angles and their deviations are the same as the $L_{DC} = \infty$ case,
(ii.) the magnitude of the 3rd, 9th, and 15th harmonics varied for different UF factors,
(iii.) the largest magnitudes for Cases I-IV appeared at a UF = 10%, and
(iv.) the magnitudes of the 3rd, 9th, and 15th harmonic currents observed for this case were approximately 17%, 3.3%, and 2.3% of the fundamental current in phase A.

5.2 Recommendations

The simulation and analysis of a three phase converter is a surprisingly complex exercise. Field data of harmonic content of AC circuit currents often deviate from levels calculated using ideal models. These discrepancies are most pronounced for uncharacteristic harmonics which, according to idealized models, should not occur. In this thesis, a number of explanations and conclusions were developed for converters which are operated in the unbalanced mode. However, a number of idealizations were made in modelling. It is recommended that some of these idealizations be dropped and more detailed models used. These recommendations include the following:

(i.) examine the effect of nonzero forward voltage drop in the converter diodes,
(ii.) examine nonlinearities in the diode V-I characteristics including mismatched diodes,
(iii.) study the effects of stray capacitance in the unbalanced operating mode (note: this phenomenon has been studied in the balanced operating mode),
(iv.) determine the effects of converter transformer unbalance (such as unequal leakage reactance among the phases),
(vi.) detailed modelling of the converter transformer magnetization during unbalanced operation.
LIST OF REFERENCES
LIST OF REFERENCES


Appendix A

The following program was used to calculate the exact and approximate commutation deviation angles, exact and approximate commutation angles, and the Fourier spectra for the AC phase currents supplying a six-pulse line commutated rectifier. This program uses a search technique, as described in Chapter II, to solve for the commutation angles under varying degrees of supply unbalance. A 4096-point trapezoidal integrator is then used to calculate the Fourier expansions of the AC phase currents with infinite inductance in the DC circuit.

In order to solve for the Fourier coefficients with zero inductance in the DC circuit (purely resistive load), the function \( fcn \) can be modified as shown below.

```fortran
function fcn(t,iph,gbc,pab,.gca,pbc,gab,pca,gbcp2pi)
  f = cos(t)
  if (t.ge.gbc.and.t.lt.pab) int = 1
  if (t.ge.pab.and.t.lt.gca) int = 2
  if (t.ge.gca.and.t.lt.pbc) int = 3
  if (t.ge.pbc.and.t.lt.gab) int = 4
  if (t.ge.gab.and.t.lt.pca) int = 5
  if (t.ge.pca.and.t.lt.gbcp2pi) int = 6
  if (int.eq.0) print*, 'ERROR! INT = 0'
  if (iph.eq.1) go to 1
  if (iph.eq.2) go to 2
  if (iph.eq.3) go to 3
  1 fen = 0.
    if (int.eq.1.or.int.eq.6) fen = 1.* f * 1
    if (int.eq.3.or.int.eq.4) fen = -1.* f * (-1.)
  return
  2 fen = 0.
    if (int.eq.2.or.int.eq.3) fen = 1.* f * 1
    if (int.eq.5.or.int.eq.6) fen = -1.* f * (-1.)
  return
  3 fen = 0.
    if (int.eq.1.or.int.eq.2) fen = -1.* f * (-1.)
    if (int.eq.4.or.int.eq.5) fen = 1.* f * 1
  return
end
```
program urf

real a(3,27), b(3,27), ao(3), ph(3,27), c(3,27)
integer flag, flaga, step, temp
logical lease

open(unit=8, file='urf.out')
rewind 8

C **********************************************
C Initialize any needed values.
C **************************************

pi = 4.*atan(1.)
twopi = 2. * pi
fract = 0.05
dt = 0.01
toler = 1.e-06
tol = 1.e-04
step = 4000
period = twopi
nharm = 27
do 11 i = 1,3
do 11 j = 1,nharm
a(i,j) = 0.0
b(i,j) = 0.0
c(i,j) = 0.0
ph(i,j) = 0.0
11 continue
ao(1) = 0.0
ao(2) = 0.0
ao(3) = 0.0
print 'enter the value of eb in pu: "$'
read(5,*)eb
print 'enter the value of ec in pu: "$'
read(5,*)ec
print 'enter the value of tb in radians: "$'
read(5,*)tb
print 'enter the value of tc in radians: "$'
read(5,*)tc
\[ tadv = dt \]

C ******************************************************************************
C End of initialization section.
C ******************************************************************************

\texttt{write(8,*)}'
\texttt{write(8,*)}'PURDUE UNBALANCED RECTIFIER STUDY-PURELY RESISTIVE CASE 1'
\texttt{write(8,100)}
\texttt{100 format(80('_'))}
\texttt{write(8,101)eb,ec,tb,tc}
\texttt{101 format(\"GLOBAL VALUES OF EB, EC, TB, AND TC ARE: ',/,'EB = ',
 1f8.5,/,10x,'EC = ',f8.5,/,10x,'TB = ',f8.5,/,10x,'TC = ',f8.5)\"

\texttt{pab = pi/3.}
\texttt{pbc = pi}
\texttt{pca = 5.* pi/3.}
\texttt{gab = 4.* pi/3.}
\texttt{gbc = 0.}
\texttt{gca = 2.* pi/3.}
\texttt{dab = -.3}
\texttt{dbc = -.3}
\texttt{dca = -.3}
\texttt{dbca = -.3}
\texttt{dbab = -.3}
\texttt{dbbc = -.3}

1 \texttt{s = f1(dab+pab,eb,tb)}
\texttt{if (abs(s).lt.toler) go to 10}
\texttt{if (s.eq.abs(s)) then}
\texttt{flag = 1}
\texttt{else}
\texttt{flag = -1}
\texttt{end if}
\texttt{sa = f1(dab+pab+tadv,eb,tb)}
\texttt{if (sa.eq.abs(sa)) then}
\texttt{flaga = 1}
\texttt{else}
\texttt{flaga = -1}
end if
    temp = flag*flaga
    if (temp.gt.0.) dab = dab + tadv
    if (temp.gt.0.) go to 1
    dab = dab + tadv
    tadv = -tadv * fract
    go to 1
10 tadv = dt

2 s = f2(dbc+pbc,eb,tb,ec,tc)
    if (abs(s).lt.toler) go to 20
    if (s.eq.abs(s)) then
        flag = 1
    else
        flag = -1
    end if
    sa = f2(dbc+pbc+tadv,eb,tb,ec,tc)
    if (sa.eq.abs(sa)) then
        flaga = 1
    else
        flaga = -1
    end if
    temp = flag*flaga
    if (temp.gt.0.) dbc = dbc + tadv
    if (temp.gt.0.) go to 2
    dbc = dbc + tadv
    tadv = -tadv * fract
    go to 2
20 tadv = dt

3 s = f3(dca+pca,ec,tc)
    if (abs(s).lt.toler) go to 30
    if (s.eq.abs(s)) then
        flag = 1
    else
        flag = -1
    end if
    sa = f3(dca+pca+tadv,ec,tc)
    if (sa.eq.abs(sa)) then
        flaga = 1
else
flaga = -1
end if

temp = flag*flaga
if (temp.gt.0.) dca = dca + tadv
if (temp.gt.0.) go to 3
dca = dca + tadv
tadv = -tadv * fract
go to 3
30 tadv = dt

4  s = f4(gca+dbca,ec,tc)
if (abs(s).lt.toler) go to 40
if (s.eq.abs(s)) then
  flag = 1
else
  flag = -1
end if
sa = f4(gca-fdbca+tadv,ec,tc)
if (sa.eq.abs(sa)) then
  flaga = 1
else
  flaga = -1
end if
temp = flag*flaga
if (temp.gt.0.) dbca = dbca + tadv
if (temp.gt.0.) go to 4
dbca = dbca + tadv
tadv = -tadv * fract
go to 4
40 tadv = dt

5  s = f5(gab+dbab,eb,tb)
if (abs(s).lt.toler) go to 50
if (s.eq.abs(s)) then
  flag = 1
else
  flag = -1
end if
sa = f5(gab+dbab+tadv,eb,tb)
if (sa.eq.abs(sa)) then
  flaga = 1
else
  flaga = -1
end if

temp = flag*flaga
if (temp.gt.0.) dbab = dbab + tadv
if (temp.gt.0.) go to 5
  dbab = dbab + tadv
  tadv = -tadv * fract
  go to 5
50  tadv = dt

6  s = f6(gbc+dbbc,eb,tc,tc,eb)
if (abs(s).lt.toler) go to 60
if (s.eq.abs(s)) then
  flag = 1
else
  flag = -1
end if

sa = f6(gbc+dbbc+tadv,eb,tc,tc)
if (sa.eq.abs(sa)) then
  flaga = 1
else
  flaga = -1
end if

temp = flag*flaga
if (temp.gt.0.) dbbc = dbbc + tadv
if (temp.gt.0.) go to 6
  dbbc = dbbc + tadv
  tadv = -tadv * fract
  go to 6
60  tadv = dt

70  s3 = sqrt(3.)
  rtd = 180./pi
  dabo = (s3*tb+s3*eb*tb-eb)/(s3*eb+2.*s3)
  dcao = (ec+s3*tc*ec+s3*tc)/(s3*ec+2.*s3)
  dbco = ((eb*1+(s3*tb))+(ec*1+(s3*tc))+2+(s3*(tb+tc)))/s3*(eb+ec+2)
dbabo = dabo
dbeao = dcao
dbbco = dbco
rtddab = ((pab + dab) * rtd)
rtddbc = ((pbc + dbc) * rtd)
rtddeca = ((pca + dca) * rtd)
rtddbca = ((gca + dbca) * rtd)
rtddbab = ((gab + dbab) * rtd)
rtddbbbe = ((gbe + dbbe) * rtd)
rtddabo = ((pab + dabo) * rtd)
rtddbco = ((pbc + dbco) * rtd)
rtddcao = ((pca + dcao) * rtd)
rtddbcao = ((gca + dbcao) * rtd)
rtddbab = ((gab + dbabo) * rtd)
rtdddbe = ((gbe + dbbe) * rtd)
if (abs(dab-dabo).gt.ep) then
  e1 = ((dab - dabo) / dab) * 100.
else
  e1 = 0.0
end if
if (abs(dbc-dbco).gt.ep) then
  e2 = ((dbc - dbco) / dbc) * 100.
else
  e2 = 0.0
end if
if (abs(dca-dcao).gt.ep) then
  e3 = ((dca - dcao) / dca) * 100.
else
  e3 = 0.0
end if
if (abs(dbca-dbcao).gt.ep) then
  e4 = ((dbca - dbcao) / dbca) * 100.
else
  e4 = 0.0
end if
if (abs(dbab-dbabo).gt.ep) then
  e5 = ((dbab - dbabo) / dbab) * 100.
else
  e5 = 0.0
end if
if (abs(dbbc-dbbco).gt.ep) then
    e6 = ((dbbc - dbbco) / dbbc ) * 100.
else
    e6 = 0.0
end if
if (abs(rtddab-rtddabo).gt.ep) then
    e11 = ((rtddab - rtddabo) / rtddab ) * 100.
else
    e11 = 0.0
end if
if (abs(rtddbc-rtddbco).gt.ep) then
    e22 = ((rtddbc - rtddbco) / rtddbc ) * 100.
else
    e22 = 0.0
end if
if (abs(rtddca-rtddcao).gt.ep) then
    e33 = ((rtddca - rtddcao) / rtddca ) * 100.
else
    e33 = 0.0
end if
if (abs(rtddbab-rtddbabo).gt.ep) then
    e55 = ((rtddbab - rtddbabo) / rtddbab ) * 100.
else
    e55 = 0.0
end if
if (abs(rtddbbc-rtddbbco).gt.ep) then
    e66 = ((rtddbbc - rtddbbco) / rtddbbc ) * 100.
else
    e66 = 0.0
end if
write(8,102)
102 format('ANGLES OF COMMUTATION OF A SIX-PULSE CONVERTER')
write(8,*)' ' write(8,102)
155 format(16x,'RADIANS',25x,'DEGREES')
   write(8,103)
103 format(9x,'I',9x,'II',9x,'III',7x,'IV',9x,'V',9x,'VI')
   write(8,*)'
   write(8,104) dab,dabo,el,rtddab,rtddabo,e11,dbc,dbc0,e21,
1         rtddbc,rtddbc0,e22,dc,dc0,e3,rtddca,rtddca0,e33,
2         dbca,dbca0,e4,rtddca,rtddca0,e44,dbab,dbabo,e5,
3         rtddbab,rtddbab0,e55,dbc,dbc0,e6,rtdddbc,rtdddbc0,e66
104 format('dab ',2f10.6,4f10.2,'/','dbc ',2f10.6,4f10.2,'/','dca ',
12f10.6,4f10.2,'/','dbca',2f10.6,4f10.2,'/','dbab',2f10.6,4f10.2,
2,'/','dbbc',2f10.6,4f10.2)

C *******************************************************
C Begin of the Fourier Analysis.
C *******************************************************

write(8,105)
105 format(/,'FOURIER ANALYSIS - - EXACT ANGLES')
   dbbc = 0.
   gbc = dbbc
   pab = dab + (pi/3.)
   gca = dbca + (2. * pi/3.)
   pbc = dbc + pi
   gab = dbab + (4. * pi/3.)
   pca = dca + (5. * pi/3.)
   gbcp2pi = gbc + twopi
   case = .false.
9   call fourier(1,nharm,twopi,step,ao,a,b,c,ph,gbc,pab,gca,pbc,
1                 1   gab,pca,gbcp2pi)
   call fourier(2,nharm,twopi,step,ao,a,b,c,ph,gbc,pab,gca,pbc,
1                 1   gab,pca,gbcp2pi)
   call fourier(3,nharm,twopi,step,ao,a,b,c,ph,gbc,pab,gca,pbc,
1                 1   gab,pca,gbcp2pi)
   write(8,106)
106 format(/,'FOURIER COEFFICIENTS')
   do 7 i = 1,3
   write(8,107)
107 format(/,'N',10x,'A[l]',15x,'B[l]',14x,'C[l]',14x,'PH[l]')
   write(8,108)0,ao(i)
   do 8 j = 1,nharm
write(8,110)j,a(i,j),b(i,j),c(i,j),ph(i,j)
108 format(1x,i3,f18.8)
110 format(1x,i3,4f18.8)
8 continue
7 continue
  if (lcase) go to 999
  write(8,109)
109 format(/,/,/'FOURIER ANALYSIS--APPROXIMATE ANGLES')
  gbc = dbbc0
  pab = dabo + (pi/3.)
  gca = dbcao + (2.*pi/3.)
  pbc = dbco + pi
  gab = dbabo + (4.*pi/3.)
  pca = dcao + (5.*pi/3.)
  gbcp2pi = gbc + twopi
  lcase = .true.
  go to 9
999 stop
end

C **************************************************************
C Functions used to calculate the commutation points.
C **************************************************************

function f1(a,eb,tb)
  pi = 4.* atan(1.)
  elhs = cos(a)
  erhs = (eb+1.)*cos(a-(2.*pi/3.)-tb)
  f1 = elhs - erhs
return
end

function f2(a,eb,tb,ec,tc)
  pi = 4.* atan(1.)
  elhs = (eb+1.)*cos(a-(2.*pi/3.)-tb)
  erhs = (ec+1.)*cos(a-(4.*pi/3.)-tc)
  f2 = elhs - erhs
return
end
function f3(a,ec,tc)
pi = 4.* atan(1.)
elhs = (ec+1.)*cos(a-(4.*pi/3.)-tc)
erhs = cos(a)
f3 = elhs - erhs
return
end

function f4(a,ec,tc)
pi = 4.* atan(1.)
elhs = cos(a)
erhs = (ec+1.)*cos(a-(4.*pi/3.)-tc)
f4 = elhs - erhs
return
end

function f5(a,eb,tb)
pi = 4. * atan(1.)
elhs = cos(a)
erhs = (eb+1.)*cos(a-(2.*pi/3.)-tb)
f5 = elhs - erhs
return
end

function f6(a,eb,tc,ec,tc)
pi = 4.* atan(1.)
elhs = (eb+1.)*cos(a-(2.*pi/3.)-tb)
erhs = (ec+1.)*cos(a-(4.*pi/3.)-tc)
f6 = elhs - erhs
return
end

C *************************************************
C Integration subroutine.
C *************************************************

subroutine trap(ta,tb,n,area,iph,gbc,pab,gba,pc,gbp,
1     gab, pca, gbcp2pi)
  t := ta
  tn := n
\[ dt = (tb - ta) / tn \]

C Initialize the value of the integral.

\[ area = f(ta,iph,gbc,pab,gca,pbc,gab,pca,gbcp2pi) / 2. \]

C Compute the n-1 terms in the summation

\[
\begin{align*}
&\text{do } 8 \text{ i = 2,n} \\
&\quad t = t + dt \\
&\quad area = area + f(t,iph,gbc,pab,gca,pbc,gab,pca,gbcp2pi) \\
&8 \quad \text{continue}
\end{align*}
\]

C Calculate at the endpoint.

\[
\begin{align*}
&\quad area = (area + f(tb,iph,gbc,pab,gca,pbc,gab,pca,gbcp2pi) / 2.) \\
&1 \quad * dt \\
&\quad \text{return} \\
&\quad \text{end}
\end{align*}
\]

C Fourier coefficients subroutine.

C

subroutine fourier(iph,nharm,period,step,ao,a,b,c,ph,
1 gbc,pab,gca,pbc,gab,pca,gbcp2pi)
common omega,h,k
real a(3,27),b(3,27),c(3,27),ph(3,27),ao(3)
integer step
pi = 4. * atan(1.)
twopi = 2. * pi
omega = twopi / period

C Compute the dc component a[0].

\[
\begin{align*}
&\quad h = 0. \\
&\quad k = 1 \\
&\quad \text{call trap}(0.,period,step,ao(iph),iph,gbc,pab,gca,pbc,
1 gab,pca,gbcp2pi) \\
&\quad ao(iph) = ao(iph) / period \\
&\quad h = 1.
\end{align*}
\]
Ih = 1
9 if (lh.gt.nharm) return

C Compute the hth coefficient for the cosine terms.

k = 1
call trap(0.,period,step,a(iph,lh),iph,gbc,pab,gca,pbc,
1     gab,pca,gbcp2pi)
a(iph,lh) = a(iph,lh) * 2.0 / period
k = 2

C Compute the hth coefficient for the sine terms.

call trap(0.,period,step,b(iph,lh),iph,gbc,pab,gca,pbc,
1     gab,pca,gbcp2pi)
b(iph,lh) = b(iph,lh) * 2.0 / period
c(iph,lh) = sqrt(a(iph,lh)**2 + b(iph,lh)**2)
ph(iph,lh) = atan2(b(iph,lh),a(iph,lh))
h = h + 1.
lh = lh + 1
go to 9
end

C ********************************************
C Function for evaluating integrand.
C ********************************************

function f(t,iph,gbc,pab,gca,pbc,gab,pca,gbcp2pi)
common omega,h,k
if (k.gt.1) go to 4
f = cos(omega * h * t) * fcn(t,iph,gbc,pab,gca,pbc,gab,
1     pca,gbcp2pi)
return
4  f = sin(omega * h * t) * fcn(t,iph,gbc,pab,gca,pbc,gab,
1     pca,gbcp2pi)
return
end

C ********************************************
C Evaluates waveform at some time t.
C  **********************************************************************

function fcn(t,iph,gbc,pab,gca,pbc,gab,pca,gbcp2pi)
  f = cos(t)
  if (t.ge.gbc.and.t.lt.pab) int = 1
  if (t.ge.pab.and.t.lt.gca) int = 2
  if (t.ge.gca.and.t.lt.pbc) int = 3
  if (t.ge.pbc.and.t.lt.gab) int = 4
  if (t.ge.gab.and.t.lt.pca) int = 5
  if (t.ge.pca.and.t.lt.gbc2pi) int = 6
  if (int.eq.0) print*, 'ERROR! INT = 0'
  if (iph.eq.1) go to 1
  if (iph.eq.2) go to 2
  if (iph.eq.3) go to 3
  1  fcn = 0.
      if (int.eq.1.or.int.eq.6) fcn = 1.
      if (int.eq.3.or.int.eq.4) fcn = -1.
      return
  2  fcn = 0.
      if (int.eq.2.or.int.eq.3) fcn = 1.
      if (int.eq.5.or.int.eq.6) fcn = -1.
      return
  3  fcn = 0.
      if (int.eq.1.or.int.eq.2) fcn = -1.
      if (int.eq.4.or.int.eq.5) fcn = 1.
      return
end
Appendix B

This program was used to simulate the operation of a six-pulse rectifier supplied by a wye-delta transformer connection.

```fortran
program simulate

C ***********************************************************************************************************************
C This program is used to simulate the operation of a six-pulse rectifier supplied from a Y-DELTA transformer connection.
C ***********************************************************************************************************************
real vn(6), vn(6), vdiode(6), im(6), in(6), idiode(6),
1 iphase(3), delin(6), ln(6,6), llninv(6,6),
2 cn(6,6), enrans(6,6), tmp2(6), tmp3(6), acur(7000),
3 bcur(7000), ccur(7000), apha(30), aphb(30), bpha(30),
4 bphb(30), cpha(30), cphb(30), afour(6000), bfour(6000),
5 cfour(6000), tranrab, tranrbc, tranrac, dcr, dcl,
6 tranlab, tranlbc, tranlac, epsilonb, epsilone, thetab, thetae,
7 idccur(7000)
integer red(6)
complex zm(6,6), zn(6,6), tmp1(6,6)
logical link(6), x(6), change, plot

C ***********************************************************************************************************************
C Open all data files and initialize all variables.
C ***********************************************************************************************************************
open(unit=1,file='aphase.out')
open(unit=2,file='bphase.out')
open(unit=3,file='cphase.out')
open(unit=8,file='harm.out')
open(unit=15,file='idc.out')
rewind 1
rewind 2
rewind 3
rewind 8
rewind 15
print '("enter phase B magnitude imbalance in pu: ",epsilonsb)
read(5,*),epsiionsb
print '("enter phase C magnitude imbalance in pu: ",epsilone)
```

read(5,*)epsilonc
print('"enter phase B phase imbalance in radians: "$')
read(5,*)thetab
print('"enter phase C phase imbalance in radians: "$')
read(5,*)thetac
print('"enter xformer resistance for a-b delta circuit: "$')
read(5,6)tranrab
print('"enter xformer inductance for a-b delta circuit: "$')
read(5,6)tranlabc
print('"enter xformer resistance for b-c delta circuit: "$')
read(5,6)tranrbc
print('"enter xformer inductance for b-c delta circuit: "$')
read(5,6)tranlbc
print('"enter xformer resistance for a-c delta circuit: "$')
read(5,6)tranrac
print('"enter xformer inductance for a-c delta circuit: "$')
read(5,6)tranlac
print('"enter resistance for the DC circuit: "$')
read(5,6)dcr
print('"enter inductance for the DC circuit: "$')
read(5,6)dcl
6 format(fl0.5)
call initial(x,link,zm,vm,t,tend,step,period,tfore,in,nd,
1 nharm,trlr,plot,tplot,tranrab,tranlab,tranrbc,
2 tranlbc,tranrac,tranlac,epsilonb,epsilonc,
3 thetab,thetac,dcr,dcl)
ncount = 0
nfour = 0
if (nd.eq.0) stop 77

C *******************************************************
C Calculate the incidence matrix Cn.
C *******************************************************
10 call formcn(x,link,nl,cn,red)

C *******************************************************
C Produce Cn transpose.
C *******************************************************
do 20 i = 1,6
do 20 j = 1,nl
20  cntrans(i,i) = cn(i,j)

C **************************************************
C Generate the Z bus matrix Zm.
C **************************************************

do 30 i = 1,6
do 30 j = 1,6
  tmpl(i,j) = (0.,0.)
30  zm(i,j) = (0.,0.)
do 40 i = 1,nl
do 40 j = 1,6
do 40 k = 1,6

C **************************************************
C Tmpl is the product of Cn transpose * Zm.
C **************************************************

40  tmpl(i,j) = tmpl(i,j) + cntrans(i,k)*zm(k,j)
do 50 i = 1,nl
do 50 j = 1,nl
do 50 k = 1,6

C **************************************************
C Zn is the product Cn transpose * Zm * Cn.
C **************************************************

50  zn(i,j) = zn(i,j) + tmpl(i,k)*cn(k,j)

C **************************************************
C Generate the real and imaginary matrices of Zn.
C **************************************************

do 60 i = 1,nl
do 60 j = 1,nl
  rn(i,j) = real(zn(i,j))
  ln(i,j) = aimag(zn(i,j))
C Generate Ln invert.

if (nl.ne.1) go to 70
lninv(1,1) = 1./ln(1,1)
go to 80
70 call linrg(nl,ln,6,lninv,6)

C Calculate the new Vn vector.

80 do 90 i = 1,nl
   90 vn(i) = 0.0
   do 100 i = 1,nl
   do 100 j = 1,6
   100 vn(i) = vn(i) + cntrans(i,j)*vm(j)

C Calculate Im vector.

if (.not.change) go to 130

C If change = .false. the state hasn't changed, so integrate state variables.

do 110 i = 1,6
110 in(i) = 0.0
   do 120 i = 1,6
   120 if (link(i)) in(i) = im(i)
130 continue

C Integrate the differential equation pIn.
do 140 i = 1, nl
140   tmp2(i) = 0.0
   do 150 i = 1, nl
   do 150 j = 1, nl

C ********************************************
C Tmp2 is now the product of Rn * In.
C ********************************************

150   tmp2(i) = tmp2(i) + rn(i,j)*in(red(j))

C ********************************************
C Tmp3 contains the term Vn - (Rn * In).
C ********************************************

   do 160 i = 1, nl
160   tmp3(i) = vn(i) - tmp2(i)
   do 170 i = 1, nl
170   delin(i) = 0.0
   do 180 i = 1, nl
   do 180 j = 1, nl

C ********************************************
C Delin contains the term (L inverse * (Vn - (Rn * In))).
C ********************************************

180   delin(i) = delin(i) + lninv(i,j)*tmp3(j)

C ********************************************
C Update the In vector.
C ********************************************

   do 190 i = 1, 6
190   if(.not.link(i)) in(i) = 0.0
   do 200 i = 1, nl
200   in(red(i)) = in(red(i)) + delin(i)*step
   do 210 i = 1, 6
210   im(i) = 0.0
   do 220 i = 1, 6
   do 220 j = 1, nl
220 \ im(i) = \ im(i) + cn(i,j)*in(red(j))

C ***************************************************************
C Calculate the phase and diode currents.
C ***************************************************************

call current(im,iphase,idiode)

C ***************************************************************
C Calculate the diode voltages.
C ***************************************************************

call diodev(cn,nd,nl,zm,vm,im,delin,x,link,vdiode,t)

C ***************************************************************
C Save data for plots and Fourier analysis.
C ***************************************************************

if (.not.plot) go to 230
if (t.lt.tplot) go to 240
ncount = ncount + 1

C ***************************************************************
C Arrays for plotting the shapes of AC currents.
C ***************************************************************

acur(ncount) = iphase(1)
bcur(ncount) = iphase(2)
ccur(ncount) = iphase(3)
idccur(ncount) = im(5)

230 if (t.lt.tplot.or.t.ge.tfore) go to 240
nfour = nfour + 1

C ***************************************************************
C Arrays for saving points for Fourier analysis.
C ***************************************************************

afour(nfour) = iphase(1)
bfour(nfour) = iphase(2)
cfour(nfour) = iphase(3)
240 continue

C  **********************************************************************************
C Check for a change in state configuration.
C **********************************************************************************

call state(vdiode,idiode,x,change,tlr,nd)

C  **********************************************************************************
C Increment time and voltages.
C **********************************************************************************

t = t + step
call voltage(vm,t)

C  **********************************************************************************
C End of study?
C **********************************************************************************

if (t.gt.tend) go to 250

C  **********************************************************************************
C Check for system change.
C **********************************************************************************

if (change) write(8,245)x(1),x(2),x(3),x(4),x(5),x(6),t
245  format(6l3,3x,1f10.8)
if (change) go to 10
go to 80

C  **********************************************************************************
C Calculate the harmonic content.
C **********************************************************************************

250 write(8,251)epsilonb,thetab,epsilonc,thetac,tranrab,
251  format(/,/*, phase B magnitude imbalance (pu) is: ',f5.3,/, 
1' phase B phase imbalance (rad) is: ',f5.3,/, 
2' phase C magnitude imbalance (pu) is: ',f5.3,/, 
3' phase C phase imbalance (rad) is: ',f5.3,/,
4'transformer impedance Z [ab] (pu) is: ',f10.8,'+j',
5f10.8,/
6'transformer impedance Z [bc] (pu) is: ',f10.8,'+j',
7f10.8,/
8'transformer impedance Z [ac] (pu) is: ',f10.8,'+j',
9f10.8,/

write(8,260)
260 format(' ',15x,'PHASE A HARMONICS')
call fourier(afour,nfour,nharm,step,period,apha,aphb)
write(8,270)
270 format(' ',15x,'PHASE B HARMONICS')
call fourier(bfour,nfour,nharm,step,period,bpha,bphb)
write(8,280)
280 format(' ',15x,'PHASE C HARMONICS')
call fourier(cfour,nfour,nharm,step,period,cpha,cphb)

C Store data for plotting purposes.

if (.not.plot) go to 999
  do 291 i = 1,ncount
    write(1,292)acur(i)
    write(2,292)bcur(i)
    write(3,292)ccur(i)
    write(15,292)idccur(i)
292 format(1f15.5)
291 continue
999 print*, 'number of plotting points = ',ncount
   print*, 'number of points for fourier analysis = ',nfour
   stop 999

end

C Subroutine to initialize variables.

subroutine initial(x,link,zm,vm,tstart,tend,step,period,
  1 tfor,in,nd,nharm,tol,plot,tplot,tranrab,
  2 tranlab,tranrbc,tranlbc,tranracs,tranlac,
epsilonb, epsilone, thetab, thetac, dcr, dcl

logical x(6), link(6), plot
real vm(6), in(6), dcr, dcl, tranlab, tranrac,
1 tranlbc, tranlac, tranrab, tranrbc
complex zm(6,6), z1, z2, z3, z4, z5, z6, zab, zbc, zac, zload

pi = 4. * atan(1.)
omega = 120. * pi
dcl = dcl / omega
tranlab = tranlab / omega
tranlbc = tranlbc / omega
tranlac = tranlac / omega
plot = .true.
link(5) = .true.
link(6) = .true.
link(1) = .false.
link(2) = .false.
link(3) = .false.
link(4) = .false.
x(1) = .true.
x(6) = .true.
x(2) = .false.
x(3) = .false.
x(4) = .false.
x(5) = .false.

nd = 2
do 2 i = 1, 6

2
in(i) = 0.0
in(5) = 1.0
tstart = 0.0004
pts = 3000.
period = 1./60.
step = period / pts
nharm = 27
tplot = tstart + (period * 4.0)
tfor = tplot + period
tend = tplot + (period * 2.1)
tol = 0.0
z1 = (0., 0.)
z2 = (0., 0.)
z3 = (0., 0.)
z4 = (0.,0.)
z5 = (0.,0.)
z6 = (0.,0.)
zab = cmplx(tranrab,tranlab)
zbc = cmplx(tranrbc,tranlbc)
zac = cmplx(tranrac,tranlac)
zload = cmplx(dcr,dcl)
zm(l,1) = zab + zbc + z1 + z3
zm(l,2) = zbc + z3
zm(l,3) = -zab - zbc
zm(l,4) = -zbc
zm(l,5) = -z3
zm(l,6) = zab + zbc
zm(2,1) = zbc + z2 + z3
zm(2,2) = zbc + z2 + z3
zm(2,3) = -zbc
zm(2,4) = -zbc
zm(2,5) = -z3
zm(2,6) = zbc
zm(3,1) = zab + zbc + z4 + z6
zm(3,2) = zbc + z6
zm(3,3) = -z6
zm(3,4) = -zab - zbc
zm(3,5) = -zbc
zm(3,6) = zbc + z5 + z6
zm(4,1) = -z6
zm(4,2) = -zbc
zm(4,3) = -z5
zm(4,4) = -zbc
zm(4,5) = -z6
zm(4,6) = -zbc
zm(5,1) = z3 + z6 + zload
zm(5,2) = 0.0
zm(6,2) = zab + zbc + zac

do 3 i = 1,6
do 3 j = 1,6
3   zm(j,i) = zm(i,j)
call voltage(vm,tstart,epsilonb,epsilonc,thetab,thetac)
return
end

C **************************************************
C Subroutine voltage determines the mesh voltages.
C **************************************************
real tv(6,3), vm(6), v(3)
do 1 i = 1, 6
do 2 j = 1, 3
tv(i, j) = 0.0
2 continue
1 continue
sr3 = sqrt(3.)
tv(6, 1) = sr3
tv(6, 2) = sr3
tv(6, 3) = sr3
tv(2, 2) = sr3
tv(3, 3) = sr3
tv(4, 2) = -sr3
tv(1, 3) = -sr3
v(1) = volt(time, 1, epsilonb, epsilonc, thetab, thetac)
v(2) = volt(time, 2, epsilonb, epsilonc, thetab, thetac)
v(3) = volt(time, 3, epsilonb, epsilonc, thetab, thetac)
do 3 i = 1, 3
v(i) = v(i) / sr3
3 continue
do 4 i = 1, 6
vm(i) = 0.0
4 continue
do 5 i = 1, 6
do 5 j = 1, 3
vm(i) = vm(i) + tv(i, j) * v(j)
return
end

C ******************************************************************************
C Function atan3 which evaluates the inverse tan function.
C ******************************************************************************

function atan3(a, b)
ab = abs(a)
pidiv2 = 2. * atan(1.)
if (ab.gt.0.1e-5) atan3 = atan2(a, b)
if (ab.gt.0.1e-5) return
if (b.ge.0.) atan3 = pidiv2
if (b.lt.0.) atan3 = -pidiv2
C Function volt evaluates the phase voltages at time t.

function volt(time,i,epsilonb,epsilonc,thetab,thetac)
pi = 4.*atan(1.)
omega = 120.* pi
phaseb = (-2.*pi)/3.
phases = (2.*pi)/3.
amp = 1.0
if (i.eq.1) volt = amp*cos(omega * time)
if (i.eq.2) volt = (amp + epsilonb)*cos(omega * time + 1  phaseb + thetab)
if (i.eq.3) volt = (amp + epsilonc)*cos(omega * time + 1  phases + thetac)
return
end

C Subroutine formcn generates the incidence matrix Cn.

subroutine formcn(x,link,n,cn,temp)
logical x(6), link(6)
integer temp(6)
real cn(6,6)
do 1 i = 1,6
  link(i) = .false.
  if (x(1).and.x(3))      link(1) = .true.
  if (x(1).and.x(2).or.x(2).and.x(3))   link(2) = .true.
  if (x(4).and.x(6))     link(3) = .true.
  if (x(4).and.x(5).or.x(5).and.x(6))   link(4) = .true.
  if (x(1).or.x(2).or.x(3)) link(5) = .true.
  link(6) = .true.
do 2 i = 1,6
  temp(i) = 0
do 2 j = 1,6
2 \quad cn(i,j) = 0.0

C *******************************************************************************
C Check for the number of links that are zero.
C *******************************************************************************

n == 0
do 3 i = 1,6
  if (link(i)) n = n+1
  if (n.ne.0) go to 4
stop 111

C *******************************************************************************
C Check to see if diode 3 is a branch.
C*******************************************************************************
C If x(3) is conducting then x(3) is a branch.
C If x(3) is NOT conducting then
C  if x(1) is conducting and x(2) is conducting then x(1) is a
C  branch i[a]=Id-i[b], cn[1,2]=-1., and cn[1,5]=1.
C  else if x(1) is conducting alone, i[a]=Id and cn[1,5]=1.
C  If x(3) is not conducting and x(1) is not conducting then
C  x(2) better be conducting otherwise and ERROR has occurred.
C  If x(2) is conducting then cn[2,5]=1. and i[b]=Id.
C  *******************************************************************************

4  if (x(3)) go to 6
   if (x(1)) go to 5
   if (.not.x(2)) stop 112
   cn(2,5) = 1.
   go to 6
5  if (x(2)) cn(1,2) = -1.
     cn(1,5) = 1.

C *******************************************************************************
C Check to see if diode 6 is a branch.
C*******************************************************************************
C If x(6) is conducting then x(6) is a branch.
C If x(6) is NOT conducting then
C  if x(4) is conducting and x(5) is conducting then x(4) is a
C  branch i[c]=Id-i[d], cn[3,4]=-1., and cn[3,5]=1.
C  else if x(4) is conducting alone, i[c]=Id and cn[3,5]=1.
C  If x(6) is not conducting and x(4) is not conducting then


If \( x(5) \) is conducting then \( cn[4,5] = 1.0 \) and \( i[d] = I_d \).  

6   if \( x(6) \) go to 8
   if \( x(4) \) go to 7
   if \( \neg x(5) \) stop 113
   \( cn(4,5) = 1.0 \)
   go to 8
7   if \( x(5) \) \( cn(3,4) = -1 \).
   \( cn(3,5) = 1 \).

C Reduce the incidence matrix \( C_n \).  

8   \( n = 0 \)
   do 9 \( i = 1,6 \)
   if \( \neg \text{link}(i) \) go to 9
   \( temp(n+1) = i \)
   \( cn(i,i) = 1 \).
   \( n = n+1 \)
9   continue
   do 10 \( i = 1,6 \)
   do 10 \( j = 1,n \)
10   \( cn(i,j) = cn(i,\text{temp}(j)) \)
end

C Subroutine \texttt{current} calculates all phase and diode currents.  

subroutine \texttt{current}(im,ip,id)
   real \( im(6), ip(3), id(6), t(3,6), dt(6,6) \)
   do 1 \( i = 1,3 \)
   do 1 \( j = 1,6 \)
   \( t(i,j) = 0.0 \)
1   continue
C Fill the T matrix.
C *****************************************************************
sr3 = sqrt(3.)
t(1,1) = sr3
t(1,6) = sr3
t(2,1) = sr3
t(2,2) = sr3
t(2,6) = sr3
t(3,6) = sr3
t(1,3) = -sr3
t(2,3) = -sr3
t(2,4) = -sr3
do 2 i = 1,3
2
   ip(i) = 0.0
C *****************************************************************
C Calculate the primary phase currents.
C *****************************************************************
do 3 i = 1,3
do 3 j = 1,6
3
   ip(i) = ip(i) + t(i,j)*im(j)
do 4 i = 1,6
do 4 j = 1,6
4
   dt(i,j) = 0.0
C *****************************************************************
C Calculate the diode currents.
C *****************************************************************
do 5 i = 1,6
5
   id(i) = 0.0
C *****************************************************************
C Fill d transpose.
C *****************************************************************
dt(1,1) = 1.
dt(2,2) = 1.

```
dt(3,5) = 1.
dt(4,3) = 1.
dt(5,4) = 1.
dt(6,5) = 1.
dt(3,1) = -1.
dt(3,2) = -1.
dt(6,3) = -1.
dt(6,4) = -1.

do 6 i = 1,6
  do 6 j = 1,6
    id(i) = id(i) + dt(i,j)*im(j)
  end
return
end

C Subroutine diodev solves for the diode voltages at time t.
C

subroutine diodev(cn,nd,nl,zm,vm,im,delin,x,link,vdiode,

  real rm(6,6),vm(6),im(6),delin(6),vdiode(6),
  vxn(6),vvn(6),d(6,6),dinv(6,6),tmp1(6,6),tmp2(6),
  tmp3(6),cn(6,6),vx(6)

integer nrow(6),ncol(6)
complex zm(6,6)
logical x(6),link(6)

C Form Rn and Ln matrices.
C

do 1 i = 1,6
  do 1 j = 1,6
    rm(i,j) = real(zm(i,j))
    im(i,j) = aimag(zm(i,j))
  end
  do 2 i = 1,6
    tmp2(i) = 0.0
    tmp3(i) = 0.0
    vxn(i) = 0.0
    vx(i) = 0.0
end
```
do 2 j = 1,6
2     tmp1(i,j) = 0.0
do 3 i = 1,6
    vdiode(i) = 0.0
3     vvn(i) = 0.0

C ************
C Calculate vx vector.
C ************

   do 4 i = 1,6
   do 4 j = 1,nl
   do 4 k = 1,6
4     tmp1(i,j) = tmp1(i,j) + lm(i,k)*cn(k,j)
   do 5 i = 1,6
   do 5 j = 1,nl
5     tmp2(i) = tmp2(i) + tmpl(i,j)*delin(j)
   do 6 i = 1,6
   do 6 j = 1,6
6     tmp3(i) = tmp3(i) + rm(i,j)*im(j)
   do 7 i = 1,6
7     vx(i) = vm(i) - tmp3(i) - tmp2(i)

C ************
C Fill d matrix.
C ************

   do 8 i = 1,6
   do 8 j = 1,6
8     d(i,j) = 0.0
   d(1,1) = 1.
   d(2,2) = 1.
   d(3,4) = 1.
   d(4,5) = 1.
   d(5,3) = 1.
   d(5,6) = 1.
   d(1,3) = -1.
   d(2,3) = -1.
   d(3,6) = -1.
   d(4,6) = -1.
nr = 0
nc = 0

do 9 i = 1,6
  if (link(i)) go to 9
  nrows(nr + 1) = i
  nr = nr + 1
9 continue

do 10 i = 1,6
  if (x(i)) go to 10
  ncol(nc + 1) = i
  nc = nc + 1
10 continue

if (nc.ne.nr) stop 50

do 11 i = 1,nc
do 11 j = 1,nr
11 d(j,i) = d(nrows(j),ncol(i))

C *****************************************************************
G Calculate vxn vector.
Q  ^****************************************************************

do 12 i = 1,nr
12 vxn(i) = vx(nrows(i))

C ************************************************************************
C Calculate vvn vector.
C ************************************************************************

do 14 i = 1,nr
do 14 j = 1,nr
14 vvn(i) = vvn(i) + dinv(i,j)*vxn(j)
do 15 i = 1,nc
15 vdiode(ncol(i)) = vvn(i)
return
C Subroutine state.
C **********************************

subroutine state(vv,id,x,change,tlr,nd)
real vv(6),id(6)
logical x(6),temp(6),change
do I i = 1,6
  temp(i) = .false.
change = .false.
do 2 i = 1,6
2   if (vv(i).gt.0) temp(i) = .true.
do 4 i = 1,6
   if (x(i).and..not.temp(i)) go to 3
go to 4
3   if (id(i).gt.tlr) temp(i) = .true.
4   continue
   if (x(i).and.temp(i)) go to 5
   if (.not.x(i).and..not.temp(i)) go to 5
   x(i) = temp(i)
   change = .true.
5   continue
nd = 0
do 6 i = 1,6
6   if (x(i)) nd = nd+1
return
end

C Subroutine fourier will analyze the current spectrum.
C ******************************************************

subroutine fourier(f,ncnt,nharm,h,period,fca,fcb)
real f(ncnt),fca(30),fcb(30),fcmag(30),fcp(30),w(27)
double precision reals,aimags,period,time,omega,sum1,
1sqr,thd,sum2,tif
w(1) = 0.5
w(2) = 10.
w(3) = 30.
w(4) = 105.
w(5) = 225.
w(6) = 400.
w(7) = 750.
w(8) = 950.
w(9) = 1320.
w(10) = 1790.
w(11) = 2260.
w(12) = 2760.
w(13) = 3360.
w(14) = 3830.
w(15) = 4350.
w(16) = 4690.
w(17) = 5100.
w(18) = 5400.
w(19) = 5630.
w(20) = 5860.
w(21) = 6050.
w(22) = 6230.
w(23) = 6370.
w(24) = 6650.
w(25) = 6680.
w(26) = 6790.
w(27) = 6970.

omega = 120. * 4. * atan(1.)
do 2 i = 1,nharm
  j = 1
  time = 0.0
  reals = 0.0
  aimags = 0.0
  1  reals = reals + f(j)*cos(i*omega*time)*h
  aimags = aimags - f(j)*sin(i*omega*time)*h
  time = time + h
  j = j + 1
  if (j.lt.ncnt) go to 1
  fca(i) = reals * 2./period
  fcb(i) = aimags * 2./period
  fcmag(i) = sqrt(fca(i)*fca(i) + fcb(i)*fcb(i))
2 \( f_{cph}(i) = \frac{\arctan3(f_{cb}(i),f_{ca}(i)) \times 180}{4 \cdot \arctan(1)} \)
\[ \text{thd} = 0.0 \]
\[ \text{sum1} = 0.0 \]
\[ \text{sum2} = 0.0 \]
\[ \text{tif} = 0.0 \]
do 3 \( i = 1, \text{nharm} \)
\[ \text{sum1} = \text{sum1} + (f_{cma}(i) \times f_{cma}(i)) \]
\[ \text{sqr} = f_{cma}(1) \times f_{cma}(1) \]
\[ \text{thd} = \sqrt{\text{sum1}/\text{sqr}} - 1 \]
do 4 \( i = 1, \text{nharm} \)
\[ \text{sum2} = \text{sum2} + (w(i) \times w(i) \times f_{cma}(i) \times f_{cma}(i)) \]
\[ \text{tif} = \sqrt{\text{sum2}}/\sqrt{\text{sum1}} \]
write(8,5)
\[ \text{format}(rieving,49('')) \]
write(8,6)
\[ \text{format}('', 'harmonic', 2x, 'A[i]', 6x, 'B[i]', 7x, 'mag', 8x, 'ang') \]
do 7 \( i = 1, \text{nharm} \)
write(8,8)i, fca(i), fcb(i), fcmag(i), fph(i)
\[ \text{format}(rieving,2x, 3f10.5, 5x, f10.5) \]
write(8,')
write(8,')' 'total harmonic distortion ==', thd
write(8,')' 'telephone influence factor ==', tif
write(8,')' 'return
end
VITA
VITA

Kraig J. Olejniczak was born in Green Bay, Wisconsin on February 6, 1965. He received the B.S.E.E. degree from Valparaiso University, Valparaiso, Indiana on May 17, 1987. Since August of 1987, Mr. Olejniczak has been a research assistant with the Purdue Electric Power Center, Purdue University, West Lafayette, Indiana. In May of 1988, he accepted a Technical Assistant position in the Technical Assistance Program (TAP) which is administered by the Purdue University Schools of Engineering and is supported by the Indiana Economic Development Council. The TAP program utilizes the technical resources of Purdue University to assist Indiana businesses and industries. Presently he is serving on the Organizing Committee for the Third International Conference on Harmonics in Power Systems and assisting with the North American Power Symposium which will be held in September of 1988.

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