Published online: 10-4-2016

A Framework for Problem-Based Learning: Teaching Mathematics with a Relational Problem-Based Pedagogy

Carmel Schettino

Independent Researcher, carmel.schettino@gmail.com

IJPBL is Published in Open Access Format through the Generous Support of the [Teaching Academy at Purdue University](#), the [School of Education at Indiana University](#), and the [Jeannine Rainbolt College of Education at the University of Oklahoma](#).

Recommended Citation

Schettino, C. (2016). A Framework for Problem-Based Learning: Teaching Mathematics with a Relational Problem-Based Pedagogy. *Interdisciplinary Journal of Problem-Based Learning*, 10(2).

Available at: <https://doi.org/10.7771/1541-5015.1602>

This document has been made available through Purdue e-Pubs, a service of the Purdue University Libraries. Please contact epubs@purdue.edu for additional information.

This is an Open Access journal. This means that it uses a funding model that does not charge readers or their institutions for access. Readers may freely read, download, copy, distribute, print, search, or link to the full texts of articles. This journal is covered under the [CC BY-NC-ND license](#).

THE INTERDISCIPLINARY JOURNAL OF PROBLEM-BASED LEARNING

ARTICLE

A Framework for Problem-Based Learning: Teaching Mathematics with a Relational Problem-Based Pedagogy

Carmel Schettino (Independent Researcher)

Abstract

One recommendation for encouraging young women and other underrepresented students in their mathematical studies is to find instructional methods, such as problem-based learning (PBL), that allow them to feel included in the learning process. Using a more relationally centered pedagogy along with more inclusive instructional methods may be a way to foster an interest in studying mathematics in the fields of science, technology, engineering, and mathematics. However, many mathematics teachers are at a loss in not only how to instruct with PBL methods but also how to create the environment that encourages optimal learning.

Keywords: PBL, secondary mathematics, gender equity, equity, relational pedagogy

During the past few decades, researchers have called for change in the way mathematics is taught in American schools to provide equity and accessibility for all (Leder, 2003), including changes specifically focused on underrepresented and underperforming students in our society due to gender, race/ethnicity, class, or socioeconomic status (SES) (McGraw, Lubienski, & Strutchens, 2006). Some studies found safety and equity in mathematics classes especially to be issues for underrepresented groups such as females and students of color or those with lower ability levels (Boaler, 2008; Kell-ermeier, 1996). For girls especially, it seems that the mathematics classroom environment has a great influence on their attitudes toward learning and is greatly affected by the relationships and beliefs that are forged in those classrooms.

Some gender theorists and educational researchers claim that the “level of interaction and exchanges” in social and interpersonal learning relations is “perhaps the least studied and most potentially informative area of research on gender equality” (Riegler-Crumb et al., 2012). It is time that

we looked at how young women view learning mathematics and the subject of mathematics in their secondary education and whether or not the method of learning plays a part in that experience. In my view, the instructional methods that are employed in mathematics classrooms should allow all students, regardless of gender, race/ethnicity, or SES, the safe, secure space to build those relationships and beliefs that would make their learning experience optimal. Therefore, it should be a goal of mathematics educators to find instructional approaches that satisfy the relational needs of a diverse group of learners and improve the experiences of those learners in mathematics classrooms. However, given the inequities that persist in the fields of science, technology, engineering, and mathematics (STEM) and the problems that exist in retaining women in STEM careers, it remains of crucial importance to examine girls’ learning and paths to STEM fields of work and study.

To that end, the purpose of this qualitative study was to explore the nature of adolescent females’ experiences

The research reported in this essay stems from the dissertation work of the author, who would like to thank her supporters Dr. Vick Kouba, Dr. Kristen Wilcox, and Dr. Carol Rodgers of the University at Albany–SUNY as well as the student and teacher participants without whom this work could not have occurred.

learning in a classroom utilizing a relational problem-based pedagogy. I sought to explore the question of how adolescent girls experience a mathematics classroom situated in a pedagogy of feminist relation and using an instructional approach that I called relational problem-based learning (RPBL). RPBL intends to foster a different type of learning environment, potentially positively impacting the feelings of adolescent females (and other underrepresented groups of students) about their potential success in the field of mathematics. I defined RPBL as an approach to curriculum and pedagogy whereby student learning and content material are (co-)constructed by students and teachers through mostly contextually based problems in a discussion-based classroom where student voice, experience, and prior knowledge are valued in a nonhierarchical environment utilizing a relational pedagogy (Schettino, 2013). To investigate how the use of RPBL related to young women's experiences of mathematics, I endeavored to address the following questions:

What is the nature of the relationship between girls' attitudes toward mathematics and their learning of mathematics during and after experiencing it in an RPBL environment? How do they describe their experiences?

Theoretical Framework

To situate this study, and hence my own framework for mathematics education, I must put forth the following two premises, as stated by Burton (2002):

- Learning in the mathematics classroom is social, not individual.
- Coming to know mathematics depends on active participation in the enterprises so valued in that community of mathematics practice that they are accepted within that community.

Within this view, mathematics knowledge is understood to be constructed within the classroom community in which it exists, and a learner "knows" mathematics based on the values that are prescribed within that community. For many, this is a very different view of mathematics learning and knowledge. For example, a traditional lecture-based mathematics classroom that many adults today presume as the typical mathematics classroom has been found to have teacher lecture or demonstration of methods followed by individual practice that take up 84% of classroom time (Boaler, 2008). This method of instruction implies a philosophy that values one version of the truth of knowledge (it stems from the instructor): that the learning of mathematics is mostly individual (since students learn from the instructor and then practice themselves), and listening to the teacher allows

students to learn the information they need to know. If a learner "knows" mathematics based on the values prescribed within such a learning environment, I put forth that in a traditional mathematics classroom, a learner comes to "know" mathematics in a very individual, superficial, rote way.

Further, and in contrast to the context described above, I situate mathematical learning, and learning in general, within the context of the greater relational approach to knowing—whereby "knowers are social beings-in-relation-to-others," and these relationships must be built on respect and care, not oppression and power (Thayer-Bacon, 2004). According to this view, education has a relational character, and it is precisely that relationship between the teacher and the student, and even possibly the student and his or her classmates, that affords the community the opportunity for the interaction in education (Biesta, 2004). The communication in these interactions between individuals is not about the transport of meaning but instead is about the participation in and co-construction of meaning between individuals and those members of the community in relationship to each other that, in turn, allows "education [to] exist only in and through the communicative interaction between the teacher and the learner" (Biesta, 2004, p. 21). In this relational world of knowing, learners improve their knowledge and further develop understanding by making greater connections—with material, concepts, and others (Thayer-Bacon, 2004). This is consistent with the definition of mathematical learning for understanding that has been widely encouraged and supported in the mathematics teaching community:

A mathematical idea or procedure or fact is *understood* if it is part of an internal network. . . . The degree of understanding is determined by the number and the strength of the connections. A mathematical idea, procedure or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections. . . . Understanding involves recognizing *relationships* between pieces of information. (Hiebert & Carpenter, 1992, p. 67)

The task, then, is to craft a pedagogical framework for mathematics instruction that facilitates construction of knowledge, creating strong connections between "existing networks"—both knowledge-based and relation-based. It should also incorporate the ideologies that enable as many students as possible the freedom to create those connections and relationships. My theoretical framework, which includes relational trust, relational authority, relational equity, and voice and agency, has at its roots what was historically known as feminist mathematics pedagogy, stemming from the gender difference movement of the 1990s (Becker, 1995; Boaler, 1997; Burton, 1995; Solar, 1995; Willis, 1996).

Relational Trust, Inclusion, and Active Participation

In the greater workings of a school, relationships are extremely important for success in communication, motivation, morale, and many other interpersonal beliefs in the community. Viewing trust through a relational lens can help support that success (Bryk & Schneider, 2003). However, in the microcosm of the classroom, this relational view of meaning making could also be seen in the collaborative learning experience between the members in a learning community, which inherently implies a level of trust between those members. Creating that connection in the classroom is not always easy and does not always come naturally for all individuals—both teachers and learners. However, it can be nurtured if an environment of trust is established based on relational ideals that are generally led by teacher beliefs and behaviors. I focus my definition of relational trust on the aspects that pertain most directly to classroom interactions between members of the learning community.

The first two facets of relational trust that stem from the teacher are somewhat intertwined. They link the teacher's ability to connect to the learners (and hence the learning community as a whole) and her ability to actualize the "genuine interest" she has in the students' own ideas (Raider-Roth, 2005). This "connectedness" can be interpreted as a willingness to question further, a sincere interest in the well-being of the student, or a mindfulness of the holistic nature of the individual. At one point in educational theory this concept of "connectedness" was specifically formalized to support women's and girls' ways of knowing and learning, specifically in mathematics education (Becker, 1995; Belenky, Clinchy, Goldberger, & Tarule, 1986). However, more recently opponents of gender difference theory in mathematics education promote an "unfixing" of the differences "to see mathematics as an opportunity to develop relations with others and re-make themselves" (Mendick, 2005b, p. 142). Mendick goes on to say that "By aligning separate-ness with masculinity and connected-ness with femininity, these approaches feed the oppositional binary patterning of our thinking and in the final analysis reiterate it" (p. 163). Supporters of this more humanizing approach to the multiplicities of student relationships with mathematics agree that rethinking gender differences in a larger framework would benefit both boys and girls. It may be possible to do this if mathematical learning is viewed in less of an oppositional way (male vs. female, objective vs. subjective, etc.) and in more of an interhuman relational way—appreciating all of the various needs of connection including being "authentic" and "feeling seen" by the other (Raider-Roth, 2005).

To allow for this more inclusive view of feminist mathematics pedagogy, we must consider the gendered nature

of the classroom while also valuing each student as a doer of mathematics—valuing students' intuition, risk taking, and exploration—while also finding ways of validating the knowledge with which they come to the problem-solving table (Anderson, 2005). This necessitates active participation in the pursuits within the context of the learning community. There is an accepted challenging of the norm that mathematics is cultured and objective and values certain ways of knowing above others. "Demystifying the construction of knowledge" by making the internal process of problem solving external and "valuing intuition and emotions as opposed to rationality and objectivity" are distinct ways to actively include multiple perspectives on a regular basis in the classroom (Solar, 1995).

To foster this type of active learning environment within this connected relation of trust, the teacher would also be able to sincerely express interest in listening to and following up on students' original ideas. In order for this expression to come through in the classroom, the teacher needs to attend to being "present"—as defined in terms of relational connections to self, students, pedagogy, and subject matter:

A key aspect of being present to students' experience means assuming a connected stance. In this stance students must have a sense that their teachers can see them and their learning, their strengths and their weaknesses. Not only do they see but they also accept what they see without judging it as good or bad. It is mutuality that strengthens the vision. . . . They [the students] know that they can extend themselves to the very edges of their learning, to the borders of their known world, because they know that someone will be there to meet them. . . . In short, a teacher who is "present" is a real learning partner. (Rodgers & Raider-Roth, 2006, pp. 278–279)

Allowing the teacher to be seen as a partner in collaboration builds trust in the classroom and also helps to redefine the vision of classroom authority and dissolves the traditional structure of hierarchy in relational and feminist ways. This helps to build an environment of safety and risk taking that empowers student agency and encourages student voice—both furthering the relationships that will enable learning to take place.

Relational Authority and Relational Equity

Considering that learning is a relational enterprise, one must also consider that traditional classrooms in the United States and mathematics classrooms especially are fraught with problems of equity. Authority is often described as something that one single person holds and possesses. Although many authors describe the concept of "sharing" authority, it

is difficult to get away from the concept of authority being held by one person who is the sole leader and wielder of the “influence over another” (Bingham, 2004, p. 26). Gadamer’s philosophy of authority is elaborated on here:

For authority to succeed in its aim of educating the student, the student must acknowledge that there is an important insight to be gained from the teacher. The student has an active role of authorizing the teacher by following the teacher’s pedagogical lead. To learn thus entails the authorization of the teacher by the student. (Bingham, 2004, p. 31)

This concept of relational authority is at the heart of a pedagogy of relation. If education happens relationally in the interactions between individuals in the community of learning, then there must be an acceptance that all members of the community have authorized the learning to take place. It is that respectful and reflexive interaction that allows for the opportunities to arise in order for learning to happen. Connected to this construct of authority is a similar view of equity. The term “relational equity” in regard to the classroom (Boaler, 2008) has been used to describe classroom relations between students, and I would extend that to teachers and students, where respect for others’ ideas is held as a priority, as is treating different viewpoints fairly. There is also a commitment to learning from others’ ideas, and this mutual respect and common commitment leads to positive intellectual relations (Boaler, 2008).

Voice and Agency

In theory, relational authority and equity in the classroom is a very idealistic notion, with the goal of fostering an environment that allows students to freely express ideas, grapple with learning tasks openly, and question not only authority but also knowledge in general. Those of us who strive for these ideals in our practice know the realities of the obstacles that encumber the development of student voice and agency in the learning process. We are all too aware of the hidden curriculum, the unspoken social prescriptions that govern the classroom, and the habits of learning that have been subconsciously taught for years through the traditional educational process. Especially for those students who consider themselves in underrepresented groups because of gender, race, ethnicity, sexual orientation, or other categorization, including opportunities for dialogue in the classroom by itself might not be enough:

Student voice . . . may not currently have the practical or theoretical tools . . . to explain, or to contend with, the multifarious ways in which power relations work within school . . . processes. As a consequence, it may find

itself implicated in reproducing, rather than unsettling or transforming, the hegemonic-normative practices it sought to contest. In addition, it may remain bound by the presumption that . . . such dialogue is itself a manifestation of a classed, gendered and “raced” form of cultural capital. (Taylor & Robinson, 2009, p. 169).

In other words, if not done in a deliberate and careful way, dialogue, even when attempting to be emancipatory, can simply perpetuate the hierarchy that already exists in the community of practice. Voices that were silenced can remain silenced, and those that have been heard will continue to be heard. One view of student voice work is geared toward action, participation, and change (Taylor & Robinson, 2009). These are worthy goals that need to be focused toward allowing the individual student to use that action, participation, and change to move toward his or her own agency in the learning process. Taylor and Robinson (2009) discuss the focus of postmodernist theory on reflexivity—transparent and open sharing of thoughts—and the production of knowledge in the context of student voice. It is important that the dialogue move individuals toward growth in their agency in the educational process. In addition, one must keep in mind the multiplicities of identities that students construct as they move through the process of belonging to a community of practice (Maher & Thompson Tetreault, 2001), which can make the formation of student voice even more complex. Therefore, any empowerment that is promoted in dialogue needs to also consider the awareness of the subtleties of the race/class differences in students’ identities. In the context of creating a relational learning environment, empowering student voice and agency is facilitated by creating a safe environment, further demonstrating the interdependence of the relational framework on each of its parts.

The characteristics described in models based on the tenets of postmodern feminist epistemology that resist dichotomous thinking and focus on subjective thought and multiple perspectives (Hesse-Biber & Leavy, 2007) that are included in this framework are quite different from those of traditional pedagogies in mathematics. Such pedagogies include process-driven and objective perspectives of mathematics that create environments that are “highly ritualized” and surrender student agency while students “watch the teacher demonstrate procedures and then practice the procedures—alone” (Boaler & Greeno, 2000, p. 177). Therefore, a feminist mathematics classroom should be situated in a theoretical framework that is consistent with goals that allow for a sincere environment where the interhuman connectedness of relational learning takes place. Figure 1 (next page) shows the intersections of the theories.

Figure 1. Theoretical framework structure.

Literature Review

The growing racial, cultural, and overall diversity of our student body in the United States has caused a surge of concern for the inequity in mathematics education for underrepresented groups such as African Americans, Hispanics, Latinos, and those of lower SES. Many researchers have stated that similar to females, these students are not served by the traditional ways that mathematics has been taught in many school systems (Ladson-Billings, 1995; Lubienski, 2007; Vithal, 2002). Researchers have studied the needs of students when controlling for race, ethnicity, and SES in mathematics classrooms and have found that valuing their cultural perspective and their need for political empowerment, encouraging reciprocity and responsibility, and promoting equity in experience are common values that help improve success for marginalized groups of students (Boaler, 2008; Frankenstein, 1983; Gutstein, 2007). Lower SES and racially diverse mathematics classes were also found to have great success with classrooms that exhibited “relational equity” (Boaler, 2008).

Since females can be considered a specific subcategory of all of these marginalized groups, it seems prudent to consider the intersections and comparisons of the literature in mathematics education. When looking at the research on gender equity in mathematics education, there is evidence that the “gender gap” in mathematical ability is closing but that there is still concern about performance, an interest gap at the secondary level, and a lack of females choosing to enter math- and science-related fields (Hanna, 2003; Hill, Corbett, & St. Rose, 2010; Lloyd, Walsh, & Sheni, 2005; Modi, Schoenberg, & Salmond, 2012; Mullis, Martin, & Foy, 2005). Much of the minimizing of the gender gap in the past two decades has been attributed to “female-friendly” teaching techniques that have been motivated by the realms of mathematics and gender research (Belenky, Clinchy, Goldberger, & Tarule, 1986; Boaler, 1997, 2002; Jacobs & Becker, 1997). Many educational philosophers and researchers integrated these ideas and connected them to feminist perspectives and epistemologies and argued against the “deficit model,” positing that perhaps it was a problem not with girls’ ability to learn mathematics but

instead with the way the teaching of mathematics was being delivered to girls, not matching with their learning styles in mathematics (Boaler, 2002). In discussions of feminist mathematics pedagogies, several authors have explored a means by which gender equity might occur in mathematics classes with different instructional approaches (Anderson, 2005; Burton, 1995), which were often consistent with Belenky et al.'s (1986) research on women's ways of connected knowing and learning. These characteristics included equity and power sharing, valuing prior knowledge and experience, cooperating and collaborating, valuing intuition and emotion, allowing room for authorship and ownership of the material, and making space for discussion-based learning that values all voices (Kellermeier, 1996; Mau & Leitze, 2001; Weiler, 2001).

Once the "deficit model" was dismissed, it became acceptable to view mathematics and its learners in a broader way. Research began to focus less on females as a broad category of mathematics learners and more on the differences between groups of females—African American, Hispanic, or white girls' attitudes toward learning mathematics, the mathematics classroom, or the subject of mathematics (Hoang, 2008; Lim, 2008a, 2008b). Feminist standpoint theory, which is rooted in the concept that all perspectives, and thus knowledge, are situated in the individual's personal life experience standpoint, informs research methods so that investigators place their participants at the center of the research process and consider the unique perspectives from which they come. Taking a lesson from standpoint theory, researchers became wary that for too long they had been generalizing about the issues surrounding gender equity in mathematics, making assumptions about all types of girls by looking through too unfocused a lens. Looking through the filter of culturally relevant and relational pedagogies, what seems clear is that most mathematics classes in the United States even today are still "fundamentally grounded in separate, procedural, individual and competitive work" that is often opposing young women's cultural and social inclinations (Lim, 2008b). Communication characteristics such as free verbal expression and talking aloud are often considered disruptive behavior in a typical mathematics classroom. The preferred learning and pedagogical characteristics of holistic and relational interdependence (Ladson-Billings, 1995) are generally substituted by distant, objective interactions. This poses problems for holding interest and maintaining positive attitudes for many young women, specifically young women of color. Lim (2008b) found that in general adolescent girls of color struggle with accepted norms in traditional mathematics classrooms to which their cultural and learning communication behavior norms do not conform. These struggles may even go as far as purposefully repressing natural behaviors such as

excited discussion and emotional relationships in order to fit the norms in these classrooms.

Because of this, many researchers, including Meece & Jones (1996) and Zohar (2006), have noticed the overlap between the constructivist teaching movement and feminist pedagogies. Both the National Council of Teachers of Mathematics and the U.S. National Research Council have prepared documents citing new standards and principles of mathematics learning that coincided with the values of feminist mathematics pedagogy (Donovan & Bransford, 2005; NCTM, 2000). In order to find ways in which teachers could better prepare students for these new outcomes, problem solving as an instructional outcome became the focus of a number of studies (Kurz & Batarelo, 2005; Lampert, 2001; Renkl, Atkinson, & Maier, 2002).

Relational Pedagogy and PBL

In comparing the literature on the desired outcomes for these pedagogical frameworks and PBL, it is interesting to note the intersections of the two. For example, group work, which is a foundational part of PBL, when done collaboratively and with respectful discussion would be supporting feminist mathematics pedagogy—valuing all voices and thereby creating a nonhierarchical group setting. In critical pedagogy, the concept of respect goes one step further and reaches toward reciprocity and responsibility for others' learning. In PBL, discourse in community is foundational for construction of learning—between teacher and students and between students and students—because in order for construction to be truly owned by the whole community, all voices must take part. This discourse also is foundational in both pedagogical practices because the methods used to exhibit the values of the theories need to ensure that all voices are heard, fairly and without bias. These intersections also resemble the theoretical framework of the feminist pedagogy of relation in which I am framing my study. Unfortunately, there is little to no literature on connecting the mathematics classroom and relational pedagogy. Database searches that include such key words as "pedagogy," "relational," "relation," "mathematics," and "instruction" only seem to turn up past studies that have interpreted culturally relevant pedagogy or critical pedagogy in a relational way (Cobb & Hodge, 2002).

It also seems that to optimize the PBL learning environment, the teacher must make the classroom environment as open and safe as possible when it comes to the potentially risky practices of conjecture and stating one's perspectives and opinions. From a feminist perspective, *belonging* and *becoming*, in terms of "learning in community," are key agents in an individual's practice in that community (Griffiths, 2008). In other words, how one enters that community of practice helps not only define who he or she is individually, but it also

defines the practice of that community. Using a pedagogy of relation and focusing on respectful learning sets the tone for individuals to be who they are and to support one another as a community of learners.

In Savery's (2006) overview of problem-based learning, 10 bullet points are listed that summarize the main tenets of the instructional approach, but none include the relational connection that I describe in my definition of RPBL wherein safety, trust, and student agency are of extreme importance in the learning process. The main difference between RPBL and other definitions of PBL (in mathematics classrooms or other disciplines) is the overarching awareness integrated into the pedagogy of the need for relational pedagogy in the framework of the classroom culture. Otherwise, the PBL classroom may simply perpetuate the same hierarchical authoritarian structures that have existed in traditional learning environments for decades.

Unlike a traditional classroom that might have practice problems that follow a lecture, PBL classrooms are places where communication skills, prior knowledge, metacognitive skills, lifelong learning skills, and content knowledge are practiced by focusing on problems prior to or, more often, in lieu of explicit instruction. RPBL classroom practice is based on student presentation of solution ideas that are partially complete or not necessarily known to be fully correct at times. The curriculum is an open-source problem set that is adapted and edited annually based on an integrated algebra and geometry college-preparatory syllabus (e.g. Schettino, 2015). However, the problems have different purposes, some of which are introducing new material, triggering prior knowledge, offering a different perspective on a new concept, setting up abstraction of a new or old concept, and, of course, practice (Schettino, 2011/2012).

Individual time to grapple with problems is an important part of the problem-solving process, so the teacher assigns approximately six to eight problems to read, reflect on, and possibly follow through with a complete solution on a nightly basis. It is not presumed that students will come to class with full and correct solutions. In class the next day, students share their thoughts from the night before in at-board presentations or in small group discussion, and then larger group discussion follows in order to draw conclusions, compare and critique others' ideas, and find connections between prior knowledge and potential new material through discussion. Class typically begins with students randomly assigned, volunteering, or pairing up to share their partially complete solutions or ideas on each problem. A whiteboard or digitally enhanced presentation is generally the beginning of the discussion of a problem, as the student becomes the leader of the discourse. Classmates can question the presenter directly about the methods, ideas, errors observed, connections to

other topics, or overarching themes. Many times, the leader of the discussion must hand off questions to other students, and the teacher then steps in to facilitate open dialogue and fair reciprocal discourse. After the students have agreed upon the goal of the problem being met or solution methods have been shared to their satisfaction, another student then becomes the leader of the discussion for the next problem. Summaries of theorems proven, conjectures made, and solution methods that might be connected to other problems are useful parts of the dialogue as well and are often done in the voice of the student or the teacher.

Other aspects of problem discussion and learning in the RPBL classroom might include working on student communication through feedback on students' presentation and questioning skills as well as metacognitive journaling to reflect on errors, thought processes, and others' perspectives (Schettino, 2014). Listening to each other and learning to take risks are skills that are encouraged throughout the class time together. Students utilize technology and other resources in the process of problem solving in order to become more independent and aware of the multitude of mathematical resources at their disposal.

Methods

This study took place in an all-girl's independent boarding and day school; approximately 60% of its students are boarding, and 26% are international. The sample of participants from the school is of course limited in that students at this selective private school are not fully representative of the general population, since this is a tuition- and admission-based school, and students are generally more academically motivated and may not reflect the diversity that would exist more widely in a public setting. However, with almost 18% students of color in the student body and 53% of the student body receiving some form of financial aid, the diversity of the school (race, ethnicity, SES) allowed for a diverse selection of the students in the study.

Teacher Participants

The mathematics department at the school had decided to change its geometry curriculum to a problem-based one three years prior to this study, the premise being that incorporating more discussion and deliberate problem solving would allow students to foster the 21st-century skills needed to develop independent and higher-order thinking (McCain, 2005). The three teachers of the course during the year in which the study was conducted were myself, Ms. Brown, and Ms. Johnson—all three of us were the original collaborators on the department's curricular RPBL project (Table 1, next page). Ms. Brown and Ms. Johnson had both been there for

Table 1. Participant teachers' information.

Teacher	Number of Sections Taught	Education	Years at Current School	Years of Teaching Experience	Years of Teaching RPBL
Ms. Brown	1	BA, math MAT, education	6	13	3
Ms. Johnson	2	BS, physics MS, physics and engineering	6	8	3
Ms. Schettino	3	BA, math MA, math	10	19	15

six years and had been teaching with RPBL for three years. Ms. Brown was a mathematics educator at midcareer and was the chair of the department at the time of the study, while Ms. Johnson was a younger teacher with a background in physics and was newer to the classroom. The classes that year varied in length from 50 minutes to 75 minutes (two of each class period length per week). The classes utilized inquiry activities that ranged from computer lab activities with dynamic geometry software to having students in groups at the board working on problems that motivate new ideas. After each activity, however, large group discussion always came back to summarizing conjectures and having the teacher facilitate a discussion that had students agree upon what was learned.

Student Participant Selection

In any given year, there were usually five or six sections of the course that over a period of four years had come to be taught with RPBL. It was titled "Integrated Algebra and Geometry: M210" and generally enrolled students from grades 9–11; each class had an average size of 13 students. It was important to have a range of students in the study who captured the diversity of the current students enrolled in the course. My hope was to recruit a maximum of approximately 8 students from the total number of girls ($n=46$) who were enrolled in M210 in that academic year. The recruitment of participants began with my short visits to each of the five M210 classes during which I read from a "Student Recruitment Script" in order to personally introduce them to the concept of the study.

Initially, 14 students expressed interest in becoming participants and returned an assent form, and at that time I e-mailed the "Parent Consent Form" to those parents. Once assent and consent were attained, I obtained the metacognitive writing journals from the RPBL class of those 14 students who had shown interest in becoming participants. My main goal was to be sure there was diversity among the final participants in

the study over a variety of categories. In Figure 2 (next page), I have attempted to outline the diversity of variables I hoped to achieve among the population of students taking M210. I read through these students' journals to ascertain whether their written communication would be helpful in telling the story of their experience by giving snapshots of their problem solving or explaining their processes in detail. Some students started the year out in a less articulate way and grew, which gave insight into their experiences, and others were skilled in this method of communication from the start of the year. Other students' journals did not give helpful insight into their experience in the classroom because they had not learned about writing mathematically or been able to use the journal as a tool to describe their problem solving usefully at that point in the year. At times I found it difficult to ascertain from the many varieties of writing styles at that point in the year which students might be the most suitable candidates for participation. However, I used the range of grades on the journal entries, student capability to articulate mathematical ideas and processes, and also their expressiveness in their writing as guidelines to help decide who would be interviewed. I do believe that in the end it was most important for me to include a variety of demographic information to be sure that all teachers were represented and to allow for a range of interest and ability.

I identified a set of eight students to participate in the interviews and obtained student assent and parental permission. I had the wonderful experience of conducting initial and final individual interviews with all eight young women who examined their experiences with this pedagogical approach. After completing all data collection, however, I had to narrow the eight participants down to five due to time constraints and data management. Although not always optimal, I found ways to balance the diversity in all seven categories as best I could. The five final participants can be seen in the diversity of their characteristics in Table 2 (next page).

Figure 2. Desired demographic for student participants.

The data that were collected over six months included student metacognitive journals, classroom observations, teacher interviews, and initial and final student interviews. (See Figure 3, next page, for a summary of all data collected and Appendix B for interview protocol.)

This collection of data allowed for triangulation through observation of the student’s work in the classroom, student metacognitive journals, teacher interviews, and student pre- and postinterviews, which provided each student’s perspective on the experience. The interviews allowed students to reflect on their change and growth, while the journals provided more consistent and longitudinal data.

Data Analysis

In keeping with the theoretical framework of education as a relational phenomenon, I used the Listening Guide

(Brown & Gilligan, 1991, 1992; Gilligan, Spencer, Weinberg, & Bertsch, 2003), a voice-centered relational approach to narrative data analysis. With this method, a researcher employs multiple readings, or “listennings,” of interview transcripts. In each reading a different participant perspective is identified and “listened for” (Doucet & Mauthner, 2008), because one’s discourse has multiple layers. The first reading is done while listening for plot—that is, the basic story of what the interviewee is telling. It also includes how the reader has responded to that story. During the second reading, the voice of the self should be listened for, and it is in this stage where phrases that are described in the first person (with the pronouns “I” and “we”) are contrasted with phrases described in the second person (with the pronoun “you”). These I-poems, as they are called, provide an alternative way of viewing the interview text in poetic form. In each consecutive reading thereafter, “contrapuntal voices” are read for. This reading brings out voices that seem to be in potential contradiction with each other. With this method, it is important for the researcher to respect the participants’ experiences without judgment and as she navigates the often coded, indirect language of girls and women (Beauboeuf, 2007). In Table 3 (next page), I describe the different readings and the questions I looked at while analyzing the participants’ narratives for coding.

During each reading of all interviews and journals, I utilized the coding software MaxQDA in order to consistently use codes for student pre- and postinterviews, teacher interviews, and journal entry texts. (A sample coding map is included in Appendix A for reader reference.) The coding helped to sort the themes that emerged from the I-poems as the listennings happened in each iteration. In answering the questions (in the third column of Table 3) during each reading, I highlighted segments of text as well as the personal pronouns that were used by the interviewee (I-you-we), which helped in structuring the poems as well as recognizing emerging themes.

Table 2. Student participation information.

Name	Leona	Isabelle	Kacey	Sarah	Alanna
Grade	10	9	10	9	9
Teacher	Schettino	Johnson	Schettino	Brown	Schettino
Race	White	Mixed	White	White	African-American
SES	Upper	Middle	Middle	Upper Middle	Lower
Ability	Low	Middle	Low	Middle	High
Interest	Low	Medium	High	Low	Low
Boarder/Day	Boarder	Boarder	Boarder	Day	Day

Figure 3. Summary of data collected.

Data analysis of the classroom observations on each participant included open coding prior to the use of the Listening Guide on the narrative data. This allowed for an overall general view of the stories of the girls' work in their classes—the similarities and differences in their behavior and interactions in the classroom setting and any consistencies that I might see in their mathematical learning.

Discussion and Findings

The five participants were a diverse group of young women who had much in common in terms of their overall

characteristics—adolescent girls in the 9th or 10th grade all participating in the same RPBL learning experience. However, they all had unique stories to tell.

Sarah

Sarah was an artistic freshman coming from a public school background where most of her mathematics classroom experience was described as traditional.

[T]he teacher would just stand at the board and she'd just like read off notes and how to do the problem, so you never actually got to figure them out with each other.

Table 3. Listening guide process.

Reading (Listening)	Theme	Questions
First	Plot/Reader Response	What is happening? What has occurred? What actions are described? What stories are told? What are my interpretations of the story?
Second	Voice of the Self (are there subvoices?)	Who is the actor? Can I engage with the speaker? Can I identify "I statements"? Are there multiple voices speaking?
Third/Fourth	Contrapuntal Listeners for attitudes in research question	Which voices seem to speak out about the experience in mathematics class? What are the juxtapositions of the experience? Where do they happen and how they relate to each other?

Figure 4. Sarah's pizza slice.

Although she had been grouped with honors students, she never had considered herself a “math person.” She commented that “even in, in elementary school I never liked math, it was always like my least favorite subject.” Sarah entered high school with a lack of confidence in mathematics, a feeling of frustration and disappointment in her ability, and a fear of being left behind and confused in math class. However, observing Sarah in the RPBL classroom, there was a different person learning. One example of this was when Ms. Brown had students work on a problem where they were finding the area of the cheese on a piece of pizza. Students did not have a formula for the area of a sector of a circle at this point in the course.

What is the area of the cheese on one piece of a 16” cheese pizza if it is cut into 12 slices?

The goal of this question is to lead students to the relationship between central angles and sector area as well as arc length in circles. Sarah and her classmates were at the board working on this problem, and after discussing what it meant to have a 16-inch pizza, they easily realized that if they found the area of the pizza, they could take a 12th of it to find the area of one slice. Quickly, Sarah thought of another question

and said “What if it only asked for the area of the crust?” and drew a diagram (Figure 4).

Suddenly the class was very interested, and Sarah went on to say that she wanted to subtract the Isosceles triangle’s area from the sector area. It was a great example of a moment when she was able to follow her curiosity and extend a problem into something that was more complex than the question asked.

Sarah described how much she valued her ability to go deeper into her own questions and the questions of the group in this classroom (“I think it, it like helps you remember how to do the problem more and you understand it rather than just knowing the steps”). The other aspect of the class that seemed to foster Sarah’s sense of inquiry came through in her voice every time she spoke about being “at the board” (“I think going to the board helps me more, like it’ll, it’ll help me like remember how to do the problems.”) In this classroom, student presentation of their ideas is a valued and focused part of the class discussion. In the following I-poem, I could hear Sarah’s voice of appreciation for what she learned from being “at the board.”

Although this poem (Figure 5) starts with an inclusive “we” voice, Sarah alternates between the “I” and “you” voices later, denoting more of a sharing between the first and second person. She wants to describe the experience from her perspective but also share the views of a general student in the class. In the first person, she is sharing her own experience of going up to the board and the mistakes that she has made herself. In the “you” voice, she is speaking as a student in the class and how, as a student, “you” actually learn from those mistakes, and the experience enables not only “you”

I	You	We
I think going up to the board I find interesting		we sit around the table
	you kind of have to learn on your own help you along the way you like do your homework	
I go up to the board I always find mistakes I did the night before I go up and do it		
	you’re up there explaining you get a better understanding	
I think I think I think about it		

Figure 5. Sarah's I-poem.

but the others in the class to learn as well. It is quite telling that she starts with “we sit around the table” and ends up with “I think about it,” which shows the connectedness in the learning between the whole group, the individual, and the material (we-I-it). This feeling of connectedness and a unified community is part of the learning environment that is definitely something that Sarah felt was missing in her prior mathematical experiences.

In class, Sarah worked with others, laughed, and communicated about mathematics while remaining positive about problem solving. Ms. Brown was optimistic about her attitude toward mathematics and was certain that she had positive feelings toward both mathematics and the class (“she has one of the best attitudes about math—she loves it,” “she came in here and suddenly just like looks forward to class every day,” “she talks about how much she loves math”). However, even in asking Sarah about her attitude toward math class, she responded with the tension between enjoyment and aversion from the past:

Well I, I don't know. I think I'm a better math student now and I think this class has made me, like, have a better understanding of math and that I can actually do problems and . . . I think, *I think it's helped me learn a lot better and I have a, like, better respect for math class [both laugh]* because before, even in, in elementary school *I never liked math, it was always like my least favorite subject.*

Even when Sarah was talking to me about how proud her parents are now about this change in her attitude, she became a bit modest and changed the subject to what she sees as good about enjoying mathematics now:

Sarah: Well I mean, I—I tell my parents that I like math class, and they think it's really great that I have a good teacher . . . and everything, like even when I bring home my journal entries, there's like pages and pages of how to do centroids and orthocenters and I was trying to explain it to my dad one day. *[pause]*

Ms. S.: Yeah. But they, they're impressed?

Sarah: Yes.

Ms. S.: That you had this change?

Sarah: They're definitely impressed *[both laugh]*.

Ms. S.: OK. *[pause]* That's great.

Sarah: Because I have like pages of how to, like in my

. . . my um, journal *I* have like color-coded. . . It's great to have, um, like not maybe a love for math, but if **you** understand it and **you** like math, I think it's better and **you** can use it in like everything else.

Sarah's switch to using “you” instead of “I” in the last statement indicates a disconnection from the idea of enjoying the mathematics as if she was not talking about herself anymore, but a student in general. There is a certain amount of pride that Sarah feels in her excellent work in her journal and also in her enjoyment of mathematics, but there's something stopping her from taking total ownership of this part of her identity. It is clear from research that the formation of an individual's identity in mathematics learning is a complex and subtle process (Lim, 2008a). Recent research points out that identity formation in mathematics for both boys and girls often stems from a culture that relies on gendered stereotypes and conceptions of a binary oppositional system of relationship with mathematics (Mendick, 2005a); you either get it or you don't, you're either fast or you're slow, you like math or you don't, and often these dichotomous views are linked to specific genders, although sometimes they are not, depending on the experiences that individuals have had. From the tension in Sarah's voices, it sounds as though she still struggles with her mathematical identity. Perhaps this course helped break down those clear distinctions of dichotomous mathematical identity and muddled the waters for her in order to allow her to gain a different perspective to enjoy mathematical activity a bit more.

I was encouraged by how Sarah found a place for herself and made a connection with this classroom and Ms. Brown. Sarah saw that mathematics could be seen with a different lens (“I try to solve problems in different ways”), and although she still struggles with the strength of her ability and being solid in her confidence, she is moving forward with this idea, which is certainly progress from where she was.

Leona

As a returning sophomore, Leona was a very confident, outgoing young woman who characterized herself as having somewhat midlevel ability in mathematics and relatively low interest in the subject. She loved theater and debate and so found herself attracted to humanities-based courses because they allowed her to utilize her strengths. However, in her final interview she summarized her thoughts about learning mathematics in the RPBL classroom as follows:

It's not the teacher sitting in front of the classroom being like, “Oh, do you remember when we did this? Well, this is like that.” . . . On my homework for example, using Pythagorean Theorem to find the length of the hypotenuse and then having to find a distance on

a coordinate plane, and relating the concept back and applying it to that. . . . It kind of gives me a satisfaction of being like, “Oh I’m smart enough to connect that point and understand that.”

Leona’s comments here describe her overarching feeling of this course giving her a larger sense of ownership of and control over her own learning. They also confirm the feeling and belief that she was “smart enough” to make the connections on her own or that she would be able to not need the teacher to tell her which way to do a problem.

In Leona’s interviews I heard a tension between her value and the strength of her independence and her interest in and desire for interdependence with others; it made me wonder about her feelings about relational learning. This is consistent with what is known about girls (Brown & Gilligan, 1992) but is not necessarily utilized or focused on in mathematics classes in the United States. Leona was very articulate about what it was about the relational aspect of this classroom that helped her learning. She said that she liked how it “kind of put you through another person’s mind, in a way.” She even extends herself to say that “for me, when I have a better relationship with a person, I want to listen to them more.” She tries to explain that wanting to listen to them more and wanting to learn from them are inextricably tied together, since “seeing the way another person thinks, [allows me to] develop a respect for them.” She follows that thought by saying that “I just think it opens up a lot of discussion . . . which promotes learning inevitably . . . and creating new ideas and things like that.” At one point in our initial interview, I asked Leona what she thought about how the open discussion allowed students to share their own ideas:

It’s nice because we all do things differently, like as different people, everyone has a different personality and everyone thinks differently and it’s really nice to see how I think or look at something versus how someone else like in my class looks at something and being like “wow, that could work, I could use that,” or “I could use my way, whichever feels most comfortable.” But it’s nice to have that option presented by not only the teacher, but the student too because, I think, in a way, it develops like a relationship with your class that you don’t really have because you’re talking to them and you’re learning how they think.

This might be something that Leona is used to in an English or history class but is actually very novel in a mathematics class, where she is used to there being “no other way to look at it” than the way the teacher showed you. This idea of bringing multiple perspectives on a problem to the discussion really worked for Leona, mostly because of the relational

aspect of learning. She had such a deep respect and appreciation for other people’s ideas that it was natural for her to learn this way. When asked for an anecdote from class, Leona gave an example from a class period that I remembered vividly. Here was the problem:

An airplane is flying at 36,000 feet directly above Lincoln, Nebraska. A little later the plane is flying at 28,000 feet directly above Des Moines, Iowa, which is 160 miles from Lincoln. Assuming a constant rate of descent, predict how far from Des Moines the airplane will be when it lands.

In class, another student had presented this problem by using slope as the rate of change (i.e., 8,000 feet/160 miles); she used 28,000 as a y-intercept and wrote the equation of the line. She had then graphed the line and found the x-intercept to find how far from Des Moines the plane would be when it landed. This made no sense to about half the class, who were thinking geometrically, including Leona. So, another student said that she just did it by “counting”—she started at 36,000 and went down by 8000 and tried to see how many times she needed to do that to get to the ground (i.e., $36,000/8,000 = 4.5$). So, she figured that she needed to go over to the right 4.5 times 160 miles, and that’s where the plane would land. That seemed to make more sense to a few more students, but then Leona got up and said, “Oh, so it’s like drawing a bunch of triangles with sides of 8,000 and 160 from 36,000 to the ground?” (Figure 6, next page).

It took a few minutes of discussion for her to show how what the other student said had inspired her geometric approach to this solution, but then a great connection was made between the other student’s algebraic approach and this one. The students realized that finding the x-intercept of the line was actually the same as finding the landing point the way Leona and the other student did. It was experiences and discussions such as these that allowed Leona to grow in her appreciation of the multiple ways in which students viewed different problems. She learned a great deal from seeing these different perspectives, and this only added to her learning experience. In our initial interview she made the statement that “I really like that you get that ‘why’ in a few different ways—from your teacher, from your friends, well, I consider them my friends.” And because of the relational aspect of the learning, she really did consider the majority of the class her friends even if they were not close friends outside of class.

One part of the relational learning that pleases Leona the most is the fact that there is interaction and connection between the students in the class. This interconnectedness and responsibility for each other seems to give her some satisfaction not only in her own learning but also in the learning process in the classroom as a whole:

Figure 6. Student's geometric problem solving method.

I feel accomplished that I get to . . . not influence, but in a way influence others and at the same time receive influence from others, because . . . then I feel accomplished like I've done something [that] not only affects myself as a learner, but others as well. And . . . it's just a good feeling that I could hope to make others understand, if I'm correct with what I'm saying. And even if I'm not, I mean, everyone learns from mistakes so to present myself and kind of put myself out there, too, in front of people, it's nice to have them accept what I'm saying, or choose not to. And so, I feel accomplished.

When I asked her to talk about how this course has possibly changed her as a mathematics learner or her identity as a mathematics learner, her narrative created the following I-poem (next page).

In this passage, it is striking that Leona began with the "you" voice, or the second person, distancing herself from the idea of growing up, getting older, maturing, and having power. She may see this as something that will happen in the future, perhaps when she is out of school—that is when you get to express yourself. She then takes the "I" voice, or the first-person narrative stance, where she says that she "likes to solve it this way," where you can distinctly hear her voice expressing her own opinion, something she said she didn't think would happen, or should happen, until you are older. She then moves into the third person, into the "We" voice,

speaking as the class as a whole or two classmates who disagree on their ideas in class coming to the conclusions that even if they had both used different methods that disagree, "both of us is right." This idea that there might be more than one "right" solution is actually the very essence of the freedom that Leona is looking forward to in the future. The idea that she can independently come to conclusions based on her own ideas is what is freeing, what has changed her identity and given her a voice (one she didn't have before in mathematics class). It is clear in the last stanza of the I-poem (Figure 7) that Leona is still conflicted between what she can and cannot do (by the alternating "could" and "could not" lines), but in the end she is clear that she was deeply affected by the methods utilized in this class.

Leona summarized her appreciation for the empowerment of her agency in her own learning of mathematics by commenting on how her experience in this course has changed her ability to speak in class:

It's changed my identity and given me kind of like a voice in math—whereas I didn't really have one before. It was a silent voice.

Leona's experience of having a "silent voice" in the mathematics classroom can be extended to many marginalized students in the United States today, where the "silencing" constitutes the process by which contradictory evidence,

I	You	We
	As you grow When you turn 18 You have the power You get to express yourself No matter what side you're on	
I feel like I could be on I like to solve it this way		We both get to express One of us is wrong If one of us is right Or even if both of us is right
Changed my identity Given me a voice I didn't really have one before I think I could always I guess I could go I needed		
I could ask I didn't really feel I could go I couldn't go I could ask how I could ask what I couldn't ask why before I think I mean I hope so I mean I mean I feel like I'm affected		We had each day

Figure 7. Leona’s I-poem.

ideologies and experiences find themselves buried, camouflaged and discredited” (Fine, 1987/2012). Whether she was actually silent by not talking at all or was silenced in this way where her ideas were buried or discredited by a learning environment that was not conducive or welcoming to them is really irrelevant—what was important is that this is how Leona felt. She spoke of not wanting to “go to listen to her talk to us,” which can be interpreted as students being “talked at” instead of having interaction with others. This type of oppression on the part of the teacher reduces the

students’ agency in learning in that it does not allow them to express their ideas or investigate their questions. Leona also described a form of self-silencing that came from knowing that the type of questions that were acceptable were ones that kept things moving along and were not creative or interesting (“questions were always a possibility. Teachers never denied us of that, like, *privilege*; I guess you could call it”). I was impressed with the depth of understanding of the subtleties of the classroom that she was able to share with me and how articulately she verbalized her thoughts.

I	We
I'm helping somebody	
I know	
I'm in a lot of situations	
I'm the one needing	
	We're all pretty much friends
	We had to do this problem
	We didn't know
I got it right	
	We needed to know
I think	
I don't remember	
I think	

Figure 8. Isabelle's I-poem.

Isabelle

Isabelle was another student who came from public middle school and was “moved up” from a “regular” track to an accelerated one. She had left that system with the feeling of being a bit “behind” the other students who had been together in the sixth grade. Isabelle was a rather mature, articulate freshman of mixed race who described herself as having midlevel mathematical ability and interest in the subject. However, her teacher, Ms. Johnson, noted that Isabelle lacked passion and interest in the classroom, but Ms. Johnson regularly counted on Isabelle as a strong contributor to class discussion. Although I observed her to be a valued member of the classroom community, in our discussions Isabelle would regularly admit to not seeing the value in doing the mathematics. Also, although she freely admitted that math historically had not been her favorite class, she does “like math” because she thinks “it’s really interesting when you can connect different ideas together.”

All of this begs the question, what would make a student who does not see the value in a subject or think they are particularly able enjoy studying it? What seems to have worked well for Isabelle in this situation was that she had an inherent sense of confidence in herself and what she was asked to do in this particular classroom setting. While reading for the contrapuntal voices of value and worthlessness in some of Isabelle’s narrative, I could hear a voice of doubt in her ability in mathematics. Although she is a confident young woman, she has had experiences that have led her to doubt her abilities in mathematics. Seeing herself as “average” in the accelerated class and having her teacher choose to place her in those classes later than the other students in her grade have led her to believe that she may not really belong and perhaps

may not be as able as the others. This shadow of doubt comes up when she talks about times when she is confused and how this classroom has helped her (“if I didn’t know something and I didn’t think it was right, I wouldn’t put it up on the board”) However, the voice of confidence can also be heard when she realizes how much she can accomplish on her own. For example, on individual assessments, it seems that although she may have had times when she doubted her abilities, it is also true that she had times when she saw problem solving as fun (“it’s more like a puzzle than a test”). She ended up feeling accomplished when she tried something on her own or with her classmates. This I-poem (Figure 8) shares her confidence in the mutuality of the relationship she has with her class (above).

In this segment, Isabelle spoke only in the “I” and “we” voices, indicating that she was totally inclusive in what she said. She moved back and forth narrating her feelings about what she did, knew, and needed for herself and what the class as a whole (including herself) did, knew, and needed. However, the processes for problem solving somewhat parallel each other, and she has a role that she played in both. I believe that her own confidence has helped play a part in her ability to see that she can be a more active participant in mathematics in this classroom and part of a community of problem solvers.

More than once in our conversations, Isabelle identified herself as a mathematics student who “really likes algebra” because of its procedural nature—traditional classrooms really worked for her in the past (“I like steps.”). However, she also stated that “if more math classes were taught like this I might like them a lot more.” However, she theorized that a “math person,” which Isabelle described with “button-up shirt, pants, tie, glasses, ruler, you know, really

straight-forward and stuff,” might not like an RPBL class because of the ambiguity in the lack of directness and the open-ended discussion that occurs.

As Isabelle started describing more attributes that seem to be adding up to her enjoying the class more, I tried to paint a picture of what it is that produced her enjoyment. The interesting thing is that it is not the mathematics she is enjoying but rather the class—the interaction between the people in the class—and should the class be solving some interesting problems that pertain to mathematics, that’s OK too. What Isabelle described enjoying about the class is the way in which she saw mathematics as no longer black and white, with only the teacher’s information as what counts. I asked her to describe for me what it’s like in class with Ms. Johnson:

Isabelle: Like it’s, if you have a question you can just ask it and then that can lead into like some conversation or she can ask a question and then kind of leaves it out there for us, the kids to answer it, so . . .

Ms. S: OK, and why do, why do you like that better?

Isabelle: Um, because it’s not so uptight and *[laughs]*, like it’s not like focused, “memorize all of this stuff.”

Ms. S: Hmm.

Isabelle: It’s more relaxed, and that helps me learn better I think.

Isabelle’s more traditional view of the mathematics classroom with its “uptight” and rigid nature reminds her of memorizing facts and formulas, and she stated that she responds better to a classroom that, in her eyes, is more “relaxed” and interactive, allowing her views and responses to matter. This is consistent with Maher’s (2001) view of the feminist classroom’s responsibility to “deliberately position students as academic authorities” in order to allow them the input for the feeling that their responses matter but also so they do not “dismiss their own emerging sense of themselves” (p. 92). Also, Isabelle’s feelings are consistent with what Keller (1985) once called “dynamic objectivity,” which she defined in terms of how we might be inclined to think about the idea of integrating student input with factual mathematical knowledge:

Dynamic objectivity is a form of knowledge that grants to the world around us its independent integrity but does so in a way that remains cognizant of, indeed relied on, our connectivity with that world. In this, dynamic objectivity is not unlike empathy, a form of knowledge of other persons that draw explicitly on the commonality of feelings and experience in order

to enrich ones’ understanding of another in his or her own right (Keller, 1985, p. 117).

We can consider this more flexible way of viewing knowledge as necessary for including students such as Isabelle who find the more rigid mathematics classroom not conducive to learning. She would rather remain connected to the material and the persons in the classroom with her in order to facilitate learning for herself. Isabelle truly enjoys the fact that students are the contributors to the knowledge and share in the presence of authority in the classroom. Because of the openness to the dynamic objectivity of the knowledge, the students (and she) are able to accept that their input is valuable. When I asked her why she thought the students felt so compelled to participate in the classroom, she had this to say:

Ms. S: Yeah, there’s almost a guarantee that people will. . . I wonder why? I wonder what guarantees that everyone will have something to say.

Isabelle: Well *[both laugh]* it’s probably just because geometry has like twen . . . like a lot of different ways to do certain problems so there’s a lot of variations in the way that people do them, so. . . That might be it, or it might just be that people feel comfortable in the situation they’re in to participate and it’s not like, “OK nobody ask questions so we can leave now.”

Ms. S: *[laughs]* Yeah. Ok. So there’s a certain amount of like motivation to want to talk about it?

Isabelle: Yeah.

Ms. S: Because it’s like interesting to hear what other people did? *[pause]* Um, yeah, I can’t figure that out.

Isabelle: I think everybody like shares the same curiosity level and like when somebody . . . like I know in our physics class he never tells us the answer to questions and it drives everybody crazy . . .

Ms. S: Huh . . .

Isabelle: And then we all start talking about it to try and figure out if like we can find out the answer ourselves so and the same thing happens in my math class so . . .

Ms. S: Yeah?

Isabelle: I think it’s just the motivation to find the right answer and like, because I know everybody in my class wants to understand.

Isabelle's newfound appreciation for both the dynamically subjective nature of mathematical learning and the connected community of learners of the RPBL classroom has influenced her learning experience greatly.

Alanna

Alanna was an African American high-ability ninth grader growing up in low-income circumstances with a single mother and moving from school to school. She often found herself unchallenged in many of the public schools she attended. When asked, she described herself as "lazy" and "distracting to others" in math class mostly because she didn't see any value in it. In reality, her ability was much higher than the care that her teachers could provide for her, and although she did well grade-wise, she never really enjoyed mathematics. Her past experiences in math class were isolated, passive, and lonely, since she would finish work early and her teachers would give her work to do on her own. There was no appreciation for the material, and it was an easy A. "It was just like *talking*," but there was no interaction or actual communication of concepts or ideas going on in the classroom.

Alanna described to me how she didn't understand the reasoning behind mathematics class. When I listened to the voice of the self, there was a clear sense of frustration, even sadness, when she spoke about this lack of understanding. The following I-poem (Figure 9, next page) came from a passage from my initial interview with Alanna when she and I were discussing her memories of her past mathematics classes in comparison to her experience so far in the RPBL classroom. She tried to summarize what those experiences meant to her.

What struck me most as meaningful about this I-poem is the initial use of the "you" voice to describe her experience of the lecture-practice method, which is very standard and assumes a set of objective factors. It would be natural for Alanna to disassociate herself from that process if she did not feel that it is the way she should be learning or that it did not work for her. She then speaks in the "we" voice as the students in the class are talking about "learning," "investigating," and "practicing" the things that are taught in class, but somehow it all sounds very passive and disassociated from herself in the first-person plural voice. She claims in frustration that she was "screwed" on the test since she never really fully constructed any knowledge or have any opportunity to do so. Once she gets to her "I" voice in this poem, she is extremely active in her frustration with the expectations of knowledge that she has never gained from the processes of the class. She's not even sure she can remember something that she was supposed to have learned at all. Most touching is the fact that "we just learned *words*," not concepts that they would go back to and have them actually make meaning in the context of something else once again. Alanna's voice

in this I-poem is clearly expressing her frustration with the lack of relationship she had with the material in her past class—it was what was missing for her and perhaps what would've answered the question of what the "point" was in being in the mathematics classroom.

Alanna had a difficult time putting into words that it was the relationships between the people that were integral to her engagement, but she was able to list the people and the interactions between the people that made the relationships important. Expressing herself in relation to the others in the classroom community allowed her to be more comfortable and find purpose in learning. Like many African American young women from urban culture, Alanna considers herself very loud and outspoken, and she may see her cultural and social personality in conflict with what is acceptable in the traditional mathematics classroom. Lim (2008b) said this of the internal conflict that young African American students grapple with in a traditional mathematics classroom: "Black students' communication style (e.g., free verbal expression and talking aloud) and learning preference (e.g., holistic, relational, and field-dependent) were rarely respected in the classroom space; rather they were considered disruptive behaviors or, at best, an attitude non-conducive to mathematics learning" (p. 92).

Alanna found that her personality and outspoken attitude were valued in the RPBL classroom because sharing her thoughts and creating relationships were encouraged. This actually worked in her favor. There has been evidence especially for students of color and low SES that a more cooperative learning environment and attempting to create processes that relate to their everyday life (such as authentic problem-solving scenarios) fostered deeper appreciation and higher achievement (Boaler, 2008; Lim 2008a). Alanna sums up her appreciation for this pedagogical style when she says:

'Cause that's like basically the essence of the class—just working together . . . incorporating what they say into what I say and just making something out of it.

It is just this relational aspect of the RPBL that Alanna seems most grateful for. In class, I would observe Alanna truly enjoying putting problems on the board and sharing her solutions with the class, but as the year went on, I watched as she learned to sit back and allow her classmates to present their solutions because she knew that they learned just as much from making their own mistakes at the board and not necessarily always watching her present. This was part of Alanna's realization in her growth, which there was much of throughout the year.

Kacey

Kacey was a new student who was repeating her sophomore year, so she was 17 years old. She came from a rural town

I	You	We
	they would teach you something you'd go home and practice you have to be able	
I think I've learned		we'll learn something we'll investigate something teaching us something we go home and practice it we didn't have midterms
I was pretty much screwed		we'd take a test
I'd forget it I remember stuff I take a test I feel I have, I have to retain I don't know yet I would have to know I want to do I'm not really sure I remember them		we'd just learn words we never went back

Figure 9. Alanna’s I-poem.

in a mid-Atlantic U.S. state and was very athletic—a star on the school’s track team. She was widely respected in the school community for her ability to speak her mind on all accounts—politics, school matters and her openly gay identity. Kacey’s past experiences with learning mathematics were back and forth from homeschooling and a large regional public school from 6th through 10th grade. The inconsistencies in her knowledge were tremendous, as was her ability to make connections that led to poor grades and a low feeling of self-efficacy. However, she had a tremendous curiosity for learning and a positive attitude.

Kacey consistently contended that she was not a “math” person, as her identity centered on sports and humanities, but she admitted that she developed in this course and came to value mathematics in a way that many self-identified weaker students in mathematics usually do not. Kacey saw how RPBL helped her to understand mathematics better in ways that a direct instruction classroom teacher just “telling you how to do something” did not. I asked her to elaborate on why she thought something that someone else tells her would leave her understanding less as opposed to something she figured out on her own.

I like to think about like *compared to me throwing the shot put without the technique*. Because like you could

just do it with just brute strength . . . and you could do it faster. But you would have to backtrack and go through the steps through the technique and learn it like down from . . . like start from the bottom, and build yourself back up and it goes slower, but *in the end you’ll like throw it so much farther*. Just like when you do geometry, . . . *I feel like once you understand the connection, you actually become smarter and you can make connections in other things as well*. And it just helps everything.

What Kacey is describing is a phenomenon that many educators have realized through experience and many researchers have confirmed through experimentation—that retention of knowledge and development of learning comes with experience and deliberate thought processes. The educational researcher and teacher Caleb Gattegno once said of learning that “We are retaining systems and do not need to stress memorization as much as most teachers do. We hold better in our minds what we meet with awareness” (1976, p. vii). Because RPBL stresses the process of problem solving and the collaborative relationships between those involved in the process, the learning is enhanced by making students aware of processes by their own realizations and discoveries. That awareness is often more meaningful or creates more sense making in terms of the mathematics for them in the

I	You
I think	
I've taken	
I can learn history	
I can get an A	
I have to study	
I still don't get As	
I love	
	The feeling you get
	You know
	You make a connection
	You know
	You were the one

Figure 10. Kacey’s I-poem.

long-run. As was stated by the National Research Council report on student learning in mathematics, “Metacognition and adaptive reasoning both describe the phenomenon of ongoing sense making, reflection, and explanation to oneself and others” (Donovan & Bransford, 2005, p. 218). Deeper understanding and more active participation definitely increased Kacey’s enjoyment of studying mathematics.

Kacey was clearly aware of her strengths and weaknesses based on her background. She was also aware of what she appreciated about the classroom and how she learned best. When being homeschooled, she could remember times when she craved interaction with other people (“All I want to do is talk to somebody or do a math problem. I would try to go upstairs and talk to my parents.” “I think everybody has a need to talk about it.”) When she was in a more traditional public school classroom, she was frustrated with the way students would silence themselves. When asked how Kacey viewed the traditional classroom now that she had experienced the RPBL classroom, she focused on her need for independence and agency in her own learning as well as a relationship of mutual respect with the teacher.

Like many weak mathematics students, Kacey saw her mathematical limitations as innate inadequacies in her ability as opposed to problems with her foundational preparation. (“I think there comes a time when you realize there’s like a block that some students put up against math and science and say, “Oh, I’m an English person and I still don’t like to say I’m an English person”). I could hear Kacey’s voice surrendering to her perceived lack of ability and how the external measures of the college process and grades judge her. However, I can still hear the voice of her appreciation for the satisfaction of finding a solution on her own and the value of problem solving and logical thinking. The ownership of the material and knowledge is hers and her learning community. In this I-poem (Figure

10), Kacey describes how her ability in the humanities is reciprocated with grades, but not so in math. There she accepts the wonderful feeling that she gets from mathematics, which is encouraged by the ownership she has found in the learning.

The Framework

After coding and comparing all of the collected data, themes emerged. It was important to question how the girls would explain their growth through the utilization of RPBL. I would posit that it is the combination of the pedagogy of relation and the PBL curriculum that fosters the outcomes they described in their stories. Figure 11 attempts to illustrate the relationship between the recurring themes in these five girls’ stories and how the RPBL classroom attributes support those themes. Each part of the results from data analysis described previously can relate to one of the four aspects of the RPBL classroom framework, but specific examples follow.

Because qualitative research allows for deep and rich views of the personal experiences of specific examples of participants, we cannot generalize to all students. However, when themes emerge from the analysis and perspectives of all participants, this does help guide a framework for aspects of the classroom or teacher choices that fostered the outcomes for the students. The themes of (1) ownership of knowledge, (2) justification—not prescription, (3) the connected curriculum, and 4) shared authority emerged from code maps of these five girls’ descriptions of their experiences of the RPBL classroom. Referring to dynamic objectivity once again, many of the participants referenced this more flexible way of viewing knowledge as helpful for including students who find the more rigid mathematics classroom less conducive to learning. This concept seemed to summarize all four aspects in many ways. They would rather remain connected to the material and the persons in the classroom in order to

Figure 11. A framework for relational PBL classroom.

facilitate learning. Because of the openness to the dynamic objectivity of the knowledge, the students are able to accept that their input is valuable. Isabelle mentioned the multiple solution methods and the different perspectives that each student brought to the discussion of each problem. When presented with a problem, and the solution is unknown. The teacher presumes a certain level of authority in the students, and the students take on a level of responsibility and curiosity in finding solutions and methods for those solutions.

All participants commented on how student ownership of the material allowed them to have more agency and that RPBL allowed this through metacognitive journaling, student presentation of partial solutions, and the deliberate discourse moves that the teacher-as-facilitator used to create the discourse-driven classroom. Sarah admitted that working with her peers and figuring something out “means more than just a teacher telling you how to do the problem.”

A classroom “lesson” focus and summarization that did not focus on prescribing methods was also a main theme. Leona commented on how seeing multiple perspectives on problems has opened her eyes to mathematics:

I could use my way, whichever feels most comfortable. But it’s nice to have that option presented by not only

the teacher, but the student too because, I think, in a way, it develops like a relationship with your class that you don’t really have because you’re talking to them and you’re learning how they think.

For many of the students, having a mathematics classroom that focused on curiosity and inquiry instead of processes changed the way they viewed mathematics as process driven, allowing them to take advantage of their creativity for the first time.

Using a scaffolded curriculum and connected problems, as opposed to traditional units that were compartmentalized and disconnected, made a huge difference for many students. Alanna described her appreciation for the connected curriculum:

The ability to connect other things . . . ’cause before they would teach you something and you’d go home and practice it. But in this class you have to like be able to bring back other information and then do the problem, so . . . I think I’ve learned that skill.

The awareness that mathematics is not a discipline made up of discrete topics but that they are all related showed many of the students that they are capable of making those connections themselves.

The shared authority was evident when many of the girls made reference to times when although no solution was clear, they started discussing their ideas, and the integration of the new ideas with their own helped move their thinking forward.

Kacey: You think you say, “Oh, I’m stumped, I don’t know what to do,” but then someone says something and someone else says something and maybe the group doesn’t get it as a whole but somehow what they said makes a connection in your head and you know how to do the problem.

Mathematics teachers must become more comfortable with sharing the mathematical authority in the classroom with students. Dissolving the traditional authoritarian hierarchy that generally exists in traditional mathematics classrooms can be a difficult task but is a very important part of the RPBL framework. It allows students the freedom of agency to find their voice and change their mind-set about learning mathematics.

Conclusions

Because of the positive nature of the experiences of these five girls in relation to their learning, it would be wise to follow up with further study on whether this framework is transferable to other classrooms and populations. Clearly, no intent of generalizability was implied from this qualitative study, only obtaining a rich description of student experiences relating to interest, engagement, enjoyment, empowerment, and agency. Further study may include populations of other underrepresented students and in coed environments. However, should further research find that RPBL is an effective means by which underrepresented students’ learning in mathematics can be improved, professional development will be needed for teachers in addition to curriculum work and support, all of which will need to be assessed for effectiveness and delivery.

In a study of two schools with different pedagogical methods, Boaler wrote that “The Amber Hill girls [at the traditional school] found that they were unable to improve their situation, not because they were disillusioned by their own inadequacies, but because they were powerless to change the pedagogical traditions of their institution” (1997, p. 302). In short, her advice was to “change the system, not the girls.” Still, 16 years later schools in the United States have not learned how best to teach our underrepresented students so that they feel empowered to learn in the ways that meet their needs. Personally, I have spent my career attempting to reach out not only to students but also to teachers who are interested in this type of change in the hope of making a difference in mathematics education. I have been encouraged by how many individual teachers are looking for a change in

their pedagogical approach to mathematics in order to have some semblance of equity, communication, and sense making actively occurring in the classroom.

At its lowest levels, what this study has done for me is confirmed my beliefs about how RPBL is valued in the experiences of young women studying mathematics. Their journeys, as told in their stories, touched me deeply and moved me as an educator. At the highest levels, my hope for this research is to inspire further study with problem-based learning and a movement in the education community to look for alternative and powerful ways in which all students can have experiences in the mathematical classroom that are valuable and meaningful to enrich their lives and affect their futures with enough depth to see some of the beauty in mathematics.

References

- Anderson, D. L. (2005). A portrait of a feminist mathematics classroom: What adolescent girls say about mathematics, themselves, and their experiences in a “unique” learning environment. *Feminist Teacher*, 15, 175–193.
- Beauboeuf, T. (2007). The listening guide: Using a voice-centered feminist tool to research “strength” among black women. Paper presented at the Annual Meeting of the American Sociological Association. Retrieved August 11, 2007, from http://citation.allacademic.com/meta/p178102_index.html
- Becker, J. R. (1995). Women’s ways of knowing in mathematics. In P. K. Rogers & G. Kaiser (Eds.), *Equity in mathematics education: Influences of feminism and culture* (pp. 163–174). Washington, DC: Falmer.
- Belenky, M., Clinchy, B., Goldberger, N., & Tarule, J. (1986). *Women’s ways of knowing: The development of self, voice, and mind*. New York: Basic Books.
- Biesta, G. (2004). Mind the gap. In C. Bingham & A. M. Sidorkin (Eds.), *No education without relation* (pp. 11–22). New York: Peter Lang.
- Bingham, C. (2004). Let’s treat authority relationally. In C. Bingham & A. M. Sidorkin (Eds.), *No education without relation* (pp. 23–38). New York: Peter Lang.
- Boaler, J. (1997). Reclaiming school mathematics: The girls fight back. *Gender and Education*, 9(3), 285–305.
- Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning*. Mahwah, NJ: Lawrence Erlbaum.
- Boaler, J. (2008). Promoting “relational equity” and high mathematics achievement through an innovative mixed-ability approach. *British Educational Research Journal*, 32(2), 167–194.
- Boaler, J., & Greeno, J. G. (2000). Identity, agency and knowing in mathematical worlds. In J. Boaler (Ed.), *Multiple*

- perspectives on mathematics teaching and learning* (Vol. 1, pp. 171–200). Westport, CT: Ablex Publishing.
- Brown, L. M., & Gilligan, C. (1991). Listening for voice in narratives of relationship. In M. B. Tappan & M. J. Packer (Eds.), *Narrative and storytelling: Implications for understanding moral development* (pp. 43–61). San Francisco: Jossey-Bass.
- Brown, L. M., & Gilligan, C. (1992). *Meeting at the crossroads: Women's psychology and girls' development*. Cambridge, MA: Harvard University Press.
- Bryk, A. S., & Schneider, B. (2003). Trust in schools: A core resource for school reform. *Educational Leadership*, 60(6), 40–44.
- Burton, L. (2002). Recognising commonalities and reconciling differences in mathematics education. *Educational Studies in Mathematics*, 50, 157–175.
- Cobb, P., & Hodge, L. L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. *Mathematical Thinking & Learning*, 4, 249–284.
- Donovan, M. S., & Bransford, J. D. (Eds.). (2005). *How students learn: Mathematics in the classroom*. Washington, DC: National Academies Press.
- Doucet, A., & Mauthner, N. S. (2008). What can be known and how? Narrated subjects and the listening guide. *Qualitative Research*, 8(3), 399–409.
- Fine, M. (1987/2012). Silencing in public schools. In C. Grant & T. Chapman (Eds.), *History of multicultural education* (Vol. 2). New York: Routledge.
- Frankenstein, M. (1983). Critical mathematics education: An application of Friere's epistemology. *Journal of Education*, 165, 315–339.
- Gattegno, C. (1976). *The common sense of teaching foreign languages*. New York: Educational Solutions.
- Gilligan, C., Spencer, R., Weinberg, M. K., & Bertsch, T. (2003). On the listening guide: A voice-centered relational method. In P. Camic, J. Rhodes & L. Yardley (Eds.), *Qualitative research in psychology: Expanding perspectives in methodology and design* (pp. 157–169). Washington, DC: American Psychological Association.
- Griffiths, M. (2008). A feminist perspective on communities of practice [electronic version]. Retrieved August 4, 2009, from http://orgs.man.ac.uk/projects/include/experiment/morwenna_griffiths.pdf.
- Gutstein, E. (2007). "And that's just how it starts": Teaching mathematics and developing student agency. *Teachers College Record*, 109(2), 420–448.
- Hanna, G. (2003). Reaching gender equity in mathematics education. *Education Forum*, 67, 204–214. <http://dx.doi.org/10.1080/00131720309335034>
- Hesse-Biber, S. N., & Leavy, P. L. (Eds.). (2007). *Feminist research practice: A primer*. Thousand Oaks, CA: Sage.
- Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65–97). Reston, VA: NCTM.
- Hill, C., Corbett, C., & St. Rose, A. (2010). *Why so few: Women in science, technology, engineering, and mathematics*. Washington, DC: American Association of University Women.
- Hoang, T. N. (2008). The effects of grade level, gender and ethnicity on attitude and learning environment in mathematics high school. *International Electronic Journal of Mathematics Education*, 3(1): 47–59.
- Jacobs, J. E., & Becker, J. R. (1997). Creating a gender-equitable multicultural classroom using feminist pedagogy. In *Yearbook (National Council of Teachers of Mathematics)* (pp. 107–114). Reston, VA: NCTM.
- Keller, E. F. (1985). *Reflections on gender and science*. New Haven, CT: Yale University Press.
- Kellermeier, J. (1996). Feminist pedagogy in teaching general education mathematics: Creating the riskable classroom. *Feminist Teacher*, 10(1), 8–12.
- Kurz, T., & Batarello, I. (2005). Using anchored instruction to promote mathematical growth and understanding. *Journal of Educational Technology Systems*, 33, 421–436.
- Ladson-Billings, G. (1995). But that's just good teaching: The case for culturally relevant pedagogy. *Theory into Practice*, 34, 159–165.
- Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.
- Leder, G. C. (2003). Mathematics and gender: Changing perspectives. In D. A. Grouws (Ed.), *Handbook on research in mathematics teaching and learning* (pp. 597–617). Reston, VA: NCTM Publications.
- Lim, J. H. (2008a). Adolescent girls' construction of moral discourses and appropriation of primary identity on a mathematics classroom. *ZDM—International Journal of Mathematics Education*, 40, 617–631.
- Lim, J. H. (2008b). Double jeopardy: The compounding effects of class and race in school mathematics. *Equity and Excellence in Education*, 4(1), 81–97.
- Lloyd, J. E. V., Walsh, J. Y., & Shen, M. (2005). Sex differences in performance attributions, self-efficacy, and achievement in mathematics: If I'm so smart, why don't I know it? *Canadian Journal of Education*, 28, 384–408.
- Lubienski, S. T. (2007). What we can do about achievement disparities? *Educational Leadership*, 65, 54–59.
- Maher, F. A., & Thompson Tetreault, M. K. (2001). *The feminist classroom*. New York: Rowman & Littlefield.
- Mau, S. T., & Leitz, A. R. (2001). Powerless gender or genderless power? The promise of constructivism for females in the mathematics classroom. In J. E. Jacobs, J. R. Becker,

- & G. Gilmer (Eds.), *Changing the faces of mathematics: Perspectives on gender* (pp. 37–41). Reston, VA: NCTM.
- McCain, T. (2005). *Teaching for tomorrow: Teaching content and problem-solving skills*. Thousand Oaks, CA: Corwin.
- McGraw, R., Lubienski, S. T., & Strutchens, M. (2006). A closer look at gender in NAEP mathematics achievement and affect data: Intersections with achievement, race/ethnicity, and socioeconomic status. *Journal for Research in Mathematics Education*, 37(2), 129–150.
- Meece, J., & Jones, G. (1996). Girls in mathematics and science: Constructivism as a feminist perspective. *High School Journal*, 79, 242–248.
- Mendick, H. (2005a). A beautiful myth? The gendering of being/doing “good at maths.” *Gender & Education*, 17, 203–219. <http://dx.doi.org/10.1080/0954025042000301465>
- Mendick, H. (2005b). Only connect: Troubling oppositions in gender and mathematics. *International Journal of Inclusive Education*, 9, 161–180.
- Modi, K., Schoenberg, J., & Salmond, K. (2012). *Generation STEM: What girls say about science, technology, engineering and math*. No. 978-0-88441-793-4. New York: Girl Scout Research Institute.
- Mullis, I. V. S., Martin, M. O., & Foy, P. (2005). Achievement by gender in the mathematics cognitive domains at the fourth and eighth grades. In *IEA TIMSS 2003 international report on achievement in mathematics cognitive domains*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.
- NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.
- Raider-Roth, M. (2005). *Trusting what you know: The high stakes of classroom relationships*. San Francisco: Jossey-Bass.
- Renkl, A., Atkinson, R. K., & Maier, U. H. (2002). From example study to problem solving: Smooth transitions help learning. *Journal of Experimental Education*, 70, 293–315. <http://dx.doi.org/10.1080/00220970209599510>
- Riegle-Crumb, C., King, B., Grodsky, E., and Muller, C. (2012). The more things change, the more they stay the same? Prior achievement fails to explain gender inequality in entry into STEM college majors over time. *American Educational Research Journal*, 49 (6), 1048–1073.
- Rodgers, C. R., & Raider-Roth, M. (2006). Presence in teaching. *Teachers and Teaching: Theory and Practice*, 12, 265–287. <http://dx.doi.org/10.1080/13450600500467548>
- Savery, J. R. (2006). Overview of problem-based learning: Definitions and distinctions. *Interdisciplinary Journal of Problem-Based Learning*, 1(1), 9–20.
- Schettino, C. (2011/2012). Teaching geometry through problem-based learning. *Mathematics Teacher*, 105(5), 346–351.
- Schettino, C. (2013). Dismantling the birdcage: Adolescent girls’ attitudes towards learning mathematics with a relational pedagogy in a problem-based environment. Doctoral dissertation. Retrieved from ProQuest Dissertations and Theses (Accession Order Number 11007).
- Schettino, C. (2014, August, 10). What does “making students metacognitive” mean?—Answering “why should someone learn?” Math [web log post]. Retrieved from <http://www.carmelschettino.com/wp/2014/08/10/what-does-making-students-metacognitive-mean/>
- Schettino, C. (2015). *Mathematics 202: Geometry*. Retrieved from <http://www.carmelschettino.com/wp/in-the-classroom/teaching/mat202-problem-book-2015-16-2/>
- Solar, C. (1995). An inclusive pedagogy in mathematics education. *Educational Studies in Mathematics*, 28(4), 311–333.
- Taylor, C., & Robinson, C. (2009). Student voice: Theorising power and participation. *Pedagogy, Culture and Society*, 17, 161–175. <http://dx.doi.org/10.1080/14681360902934392>
- Thayer-Bacon, B., J. (2004). Personal and social relations in education. In C. Bingham & A. M. Sidorkin (Eds.), *No education without relation*. New York: Peter Lang.
- Vithal, R. (2002). A pedagogy of conflict and dialogue for mathematics education from a critical perspective. *For the Learning of Mathematics*, 22, 29–41.
- Weiler, K. (2001). Rereading Paulo Friere. In K. Weiler (Ed.), *Feminist engagements: Reading, resisting and revisioning male theorists in education and cultural studies* (pp. 67–85). New York: Routledge.
- Willis, S. (1996). Gender justice and the mathematics curriculum: Four perspectives. In L. Parker, L. Rennie, & B. Fraser (Eds.), *Gender, science and mathematics: Shortening the shadow* (pp. 41–51). Dordrecht/Boston/London: Kluwer Academic Publishers.
- Zohar, A. (2006). Connected knowledge in science and mathematics education. *International Journal of Science Education*, 28(13), 1579–1599.

Appendix A: Sample Coding Map Preliminary for I-Poems

Appendix B: Schettino Student Interview Protocol

Inside the Class (Adapted) – Student Interview Protocol (Semi-Structured)

I appreciate your letting me interview you today. I have some questions I'd like to ask you related to your experiences in your math class. Would you mind if I recorded our interview? It will help me stay focused on our conversation, and it will ensure I have an accurate record of what we discussed.

Preliminary

If applicable, ask:

What is the name/title of this course?

What class period was this? Who is your teacher?

Experience in Learning

1. I'd like to know a bit more about your learning in this class.
2. How do you think this class is going for you?
3. *Tell me what goes on in the classroom that affects the quality of learning for you.* Can you give an example of a specific time when a classroom interaction affected your learning?
4. Can you tell me about a story about how this type of teaching method works with your learning?

5. Do you have any stories from your previous math class experiences and how they worked for your learning in mathematics?

Feelings toward Mathematics and Mathematics Class: Specific to Attitudes in the Study

1. What feelings come to mind when you think of your time in this mathematics class? *Can you think of a time when you felt this way?*
2. What feelings come to mind when you think of mathematics as a subject? What experiences or relationships in your life create those feelings for you?
3. If you had a magic wand that could change any one thing about the class without it adversely affecting you, what would you change? Why?
4. *Follow-up Questions:*
5. Are there any specific anecdotes that you can think of that specifically speak to your feelings toward the problem-based pedagogy in this course?
6. How do you see yourself as a learner of mathematics? What parts of your identity play a part in what you think of yourself in the problem-based learning classroom?

Is there any other experience that happened in math class that you would like to share with me? Thank you for your time. If I have need for additional clarification, how and when is the best time for me to contact you?