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An Eulerian framework for modeling and simulating the nonlinear dynamics of soft hyperelastic materials

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ABSTRACT

In this discussion, we present a novel framework for modeling and simulating the nonlinear elastodynamics of soft hyperelastic materials such as rubbers, elastomers, and biological tissues. The proposed framework differs fundamentally from the standard paradigm in computational mechanics in concept, structure, and methodology. A key feature of our approach is the use of a fixed mesh in tandem with a novel shock-capturing scheme from computational fluid dynamics designed specifically for first-order hyperbolic systems. Our use of a fixed mesh demands an Eulerian formulation of the mathematical model. The mathematical model we employ bridges the Eulerian conservation laws of mass, linear momentum, and angular momentum with a hyperelastic (i.e., strain-energy-based) constitutive equation. The recently developed reference map technique is used to retain “memory” of the reference configuration, a necessary feature for computing the deformation and strain (relative to the reference configuration) at each node in the fixed computational mesh. This departs from the customary practice of specifying an evolution equation for the deformation gradient that must satisfy a set of compatibility conditions, or jettisoning hyperelasticity altogether in favor of a hypoelastic (i.e., rate-type) constitutive equation that violates the second law of thermodynamics. For our numerical scheme, we employ the space–time Conservation Element and Solution Element (CESE) method. The CESE method differs fundamentally from modern shock-capturing finite-volume schemes in that (i) a fixed space–time mesh is employed, (ii) flux is conserved in both space and time, (iii) no complex reconstruction techniques are needed, and (iv) no computationally expensive Riemann solver is required to compute numerical fluxes at cell boundaries. As demonstrated through the numerical solution of several benchmark problems, the CESE method accurately and efficiently captures the evolution and interaction of nonlinear traveling waves, such as shocks and rarefactions, without excessive numerical diffusion or spurious oscillations.