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Calculations of Pressure Pulsations in Pipelines In Case of Non-Sinusoidal Input Flows

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CALCULATIONS OF PRESSURE PULSATIONS IN PIPELINES IN CASE OF NONSINUSOIDAL INPUT FLOWS

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ABSTRACT

To calculate the pressure pulsations in pipelines in frequency domain, it is necessary to linearize the nonlinear damping function $|q|q$, under the condition that the flow q is periodic, but not sinusoidal. The authors have proposed an idea that two equivalent linearization coefficients should be determined in such cases. To verify the idea, the responses of pressure pulsation in a simple pipeline are calculated for several input flows, using three linearization schemes based on this idea. The results are all in good agreement with those by Runge-Kutta-Gill integration method.

NOMENCLATURE

q_{in} dimensionless input flow

q dimensionless flow in pipe

$\tilde{q}$ fluctuating component of q

Q_{in0} steady component of q_{in}

Q_{inj} amplitude of the jth component of q_{in}

ψ_{inj} phase angle of the jth component of q_{in}

Q_0 steady component of q

Q_j amplitude of the jth component of q

ψ_j phase angle of the jth component of q

p dimensionless pressure in pipe

p_0 steady component of p

p_j amplitude of the jth component of p

θ_j phase angle of the jth component of p

α constant included in nonlinear damping

K_0 equivalent linear coefficient of $|q|q$ for the steady component of q

K_1 equivalent linear coefficient of $|q|q$ for the fluctuating component of q

ω dimensionless angular frequency

t dimensionless time

In the following, all variables are described as if they had physical dimensions, but they are all treated as dimensionless variables.

INTRODUCTION

Pressure pulsations are frequently observed in pipelines connected to reciprocating compressors due to their periodic discharges or suctions, and thereby harmful vibrations are generated in the piping systems. For the avoidance of such vibrations, so-called pulsation studies are required in the design stage of chemical plants including reciprocating compressors.

Methods are developed by Abe, et al[1] and Sakai and Saeki[2] to calculate natural frequencies of pressure pulsations in complicated piping systems by transfer matrix method. Hayama, et al[3] have proposed a method to calculate resonant pressure amplitudes, in which nonlinear damping forces proportional to the square of fluid velocity are employed. Mohri and Hayama[4] have generalized this method to calculate frequency responses of pressure pulsations, in which nonlinear damping forces are linearized by Fourier development method, assuming that the flow q in pipe is sinusoidal.

In practice, however, input flows to pipelines, generated by reciprocating compressors, are certainly periodic in time, but far from sinusoidal. Then, the pressure pulsations caused by them are also periodic, but not sinusoidal. In order to calculate such pressure pulsations in frequency domain by transfer matrix method, the function $|q|q$ which represents the non-linear damping, common in fluid oscillations, should be linearized, in some way, under that condition.

When q is periodic, the function $|q|q$ is also periodic with the same period as q, and then developed into Fourier series. So, the linearization coefficients are easily determined for each frequency component contained. It is, however, found that in some cases some of them happen to be negative, which means that the linearization method by Fourier development can produce unreasonable negative damping.

In this paper linearization methods for $|q|q$, not producing negative damping, are presented. The author's idea is that, for the equivalent linearization of $|q|q$ to be done correctly under the present condition, two coefficients should be determined, one for the steady component of q and the other for the fluctuating one including all frequency components.

To verify the idea to be reasonable for the problem under consideration, three linearization schemes based on this idea are tried. In the first scheme, the flow q is approximated by the steady component and a single sinusoidal wave with the double amplitude equal to the difference between the maximum and the minimum value during one period, and Fourier development method [5] is applied. In the second, the derivative of the function $|q|q$, which is proportional to a damping coefficient, is replaced by its mean value over one period. In the third, the least square method is applied.

By using these linearization schemes, pressure pulsations caused by nonsinusoidal, periodic input flows are calculated for a simple pipeline. The results obtained by the three schemes are all in good agreement with those by RKG (Runge-Kutta-Gill) integration method, which shows the authors' calculation methods to be reasonable.

DIFFICULTIES ENCOUNTERED IN FOURIER DEVELOPMENT OF $|q|q$

When an input flow q_{in} is periodic in time with period $2\pi/\omega$, the flow q caused by it in pipe is also periodic with the same period as q_{in}, and then, it is expressed in Fourier series as Eq.(1),

$$q = Q_0 + \sum_{j=1}^{\infty} Q_j \cdot \sin(j\omega t + \psi_j) \quad \cdots\cdots\cdots\cdots\cdots\cdots (1)$$

As, in this case, the function $|q|q$ is a periodic function with the same period as q, it is also expressed in Fourier series as Eq.(2),

$$|q|q = K_{0F}Q_0 + \sum_{j=1}^{\infty} K_{jF}Q_j \cdot \sin(j\omega t+\psi_j) \quad \cdots\cdots\cdots(2)$$

where the coeffients K_{0F}, K_{jF} are expressed as

$$\left.\begin{aligned} K_{0F} &= \frac{\omega}{2\pi Q_0} \int_0^{2\pi/\omega} |q|q dt \\ K_{jF} &= \frac{\omega}{\pi Q_j} \int_0^{2\pi/\omega} |q|q \cdot \sin(j\omega t+\psi_j) dt \\ & (j=1,2,\cdots\cdots) \end{aligned}\right\} \cdots\cdots\cdots\cdots(3)$$

In case where Q_0 vanishes, K_{0F} is determined as a limiting value when Q_0 tends to zero.

Eq.(2) shows a linear relationship between the nonlinear damping function $|q|q$ and frequency components contained in q. It is, then, possible to calculate pressure pulsations in frequency domain for each frequency component required.

It happens, however, that some of the coefficients K_{jF} are negative for certain values of Q_0, Q_j, and ψ_j. Fig.1 shows an example of such situations, where K_{1F}, K_{2F} are plotted versus Q_2, as Q_0 being parameters, and letting $Q_1=1$, $\psi_1=0$, $\psi_2=\pi/2$. As seen from Fig.1, K_{1F} is positive, but K_{2F} becomes negative for smaller values of Q_2. By the way, K_{1F} has been positive for all other parameters employed for calculations.

Negative K_{jF} produces unreasonable negative damping for the jth component and makes it unstable. This fact shows that Fourier development method fails to give the correct linearization of the function $|q|q$ in case of nonsinusoidal flow q and is not applicable to the present problem.

EQUIVALENT LINEARIZATION OF $|q|q$, NOT PRODUCING NEGATIVE DAMPING

For the equivalent linearization of the function $|q|q$ under the condition that q is periodic in time, but not sinusoidal, it seems not to be correct to determine linearization coefficients for all components contained in q, as done in Fourier development method.

After considerations to solve the difficulties, the authors have come to the idea that, for the correct linearization of the function $|q|q$ to be done under the present condition, the flow q should be expressed as the sum of the steady component Q_0 and the fluctuating one $\tilde{q}$, and two coefficients, K_0 for Q_0 and K_1 for $\tilde{q}$, should be determined, in such a way as written in Eqs.(4) and (5),

$$q = Q_0 + \tilde{q} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (4)$$

$$|q|q = K_0 Q_0 + K_1 \tilde{q} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (5)$$

where the zero mean value is assumed for $\tilde{q}$.

Based on this idea, three schemes are introduced for the equivalent linearization of the function $|q|q$, not producing unreasonable negative damping.

Scheme 1 (Single Sinusoidal Approximation) In this scheme the fluctuating component $\tilde{q}$ in each pipe element is approximated by a single sinusoidal wave with double amplitude $2Q_{pp}$ equal to the difference between the maximum value of q and the minimum during one period, and then K_0 and K_1 are obtained by Fourier development method. The results are already published [5] and K_0 and K_1 of this scheme are plotted in Fig.2 by broken lines versus $\beta(=Q_0/Q_{pp})$.

Scheme 2 (Mean Derivative Approximation) It is well known that flow oscillations in pipe are simulated by electric lattice circuits. An example of a straight pipe with an input flow q_{in} at one end is shown in Fig.3, where the pipe is divided into N pipe elements with the same length [5]. m and c are equivalent inductance and capacitance of the pipe element, determined by its dimensions and fluid properties. $\alpha|q_i|$ represent nonlinear resistance in the ith pipe element, and $\alpha_{ex}|q_{ex}|$ is the exit nonlinear resistance determined by the boundary condition.

Now, regarding q_i as a current through the ith pipe element and p_i as a voltage across the ith capacitance, we have the following relations

$$\left.\begin{aligned} m\dot{q}_i &= p_{i-1} - p_i - \alpha|q_i|q_i, \quad (i=1,2,\cdots,N) \\ c\dot{p}_i &= q_i - q_{i+1}, \quad (i=1,2,\cdots,N-1) \\ c\dot{p}_0 &= 2(q_{in} - q_1), \quad c\dot{p}_N = 2(q_N - q_{ex}) \end{aligned}\right\} \quad (6)$$

where $\cdot$ denotes the derivative with respect to time.

Elimination of p_{i-1} and p_i from Eqs.(6) yields Eq.(7),

$$m\ddot{q}_i + 2\alpha|q_i|\dot{q}_i + 2q_i/c = q_{i-1}/c + q_{i+1}/c \quad \cdots\cdots (7)$$

It is seen that $2\alpha|q_i|$ represents damping coefficient for q_i. Though the absolute value $|q_i|$ is not constant, it is replaced, in the second approximation, by its mean value during one period. Then, the equivalent linear coefficient K_1 is determined for each pipe element as, dropping the suffix i,

$$K_1 = \frac{\omega}{\pi}\int_0^{2\pi/\omega} |q|dt \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(8)$$

K_0 or K_0Q_0 is determined from the steady component of $|q|q$, as seen from Eq.(5).

When the fluctuating component $\tilde{q}$ is sinusoidal, the integration in Eq.(8) is easily evaluated and K_1 is expressed as, putting $\beta = Q_0/Q_1$,

$$K_1/Q_1 = \begin{cases} (4/\pi)(\beta\sin^{-1}\beta + \sqrt{1-\beta^2}) & \text{for } 0 \leq \beta < 1 \\ 2\beta & \text{for } \beta \geq 1 \end{cases} \qquad (9)$$

The values of K_1/Q_1 are plotted versus $\beta = Q_0/Q_1$ by a solid line in Fig.2. K_0/Q_1 is the same as K_0/Q_{pp} in the first scheme.

<u>Scheme 3 (Least Square Method)</u> The right-hand side of Eq.(5) is, in general, not an exact expression for $|q|q$. So, there exists an error ε, as expressed in Eq.(10),

$$\varepsilon = |q|q - K_0Q_0 - K_1\tilde{q} \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(10)$$

In the third scheme the coeffients K_0 and K_1 are determined, so as to minimize the integration of square of the error over one period. This is satisfied by the following relations,

$$\int_0^{2\pi/\omega} \varepsilon\frac{\partial\varepsilon}{\partial K_0}dt = 0, \quad \int_0^{2\pi/\omega} \varepsilon\frac{\partial\varepsilon}{\partial K_1}dt = 0 \qquad \cdots\cdots\cdots(11)$$

Then, K_0 and K_1 in this scheme are expressed as,

$$\left.\begin{aligned} K_0 &= \frac{\omega}{2\pi Q_0}\int_0^{2\pi/\omega} |q|q\,dt \\ K_1 &= \int_0^{2\pi/\omega} |q|q\cdot\tilde{q}\,dt \Big/ \int_0^{2\pi/\omega} \tilde{q}^2 dt \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots(12)$$

K_0 in Eq.(12) is the same as K_{0F} in Eq.(3).

When flow q consists of the steady component Q_0 and a sinusoidal wave $Q_1\sin(\omega t+\psi_1)$, K_0 and K_1 in Eq.(12) coincide with the results in reference (5), which are shown in Fig.2 by broken lines in the form of K_0/Q_1 and K_1/Q_1.

CALCULATION PROCEDURES

Applying assumed input flows with higher harmonics up to the nth order, as expressed in Eq.(13), to one end of a simple pipe shown in Fig.3,

$$q_{in} = Q_{in0} + \sum_{j=1}^{n} Q_{inj} \cdot \sin(j\omega t + \psi_{inj}) \quad \cdots\cdots\cdots(13)$$

pressure pulsations are calculated under the boundary condition

$$p_N = \alpha_{ex}|q_{ex}|q_{ex} \quad \cdots\cdots\cdots(14)$$

The results are compared with those calculated by RKG integration method. The calculation procedures by means of the above linearization schemes are as follows.

Step 1 Write the input flow q_{in}, the flow q_i, and the pressure p_i in complex form as,

$$\left.\begin{aligned} q_{in} &= Q_{in0} + \sum_{j=1}^{n} Q_{inj} \cdot \exp\{\mathcal{J}(j\omega t + \psi_{inj})\} \\ q_i &= Q_{i0} + \sum_{j=1}^{n} Q_{ij} \cdot \exp\{\mathcal{J}(j\omega t + \psi_{ij})\} \\ p_i &= P_{i0} + \sum_{j=1}^{n} P_{ij} \cdot \exp\{\mathcal{J}(j\omega t + \psi_{ij})\} \end{aligned}\right\} \cdots\cdots\cdots(15)$$

where Q_{ij} and P_{ij} are complex. Linearize $|q_i|q_i$ and $|q_{ex}|q_{ex}$ as shown in Eq.(5),

$$\left.\begin{aligned} |q_i|q_i &= K_{i0}Q_{i0} + K_{i1}\sum_{j=1}^{n} Q_{ij} \cdot \exp\{\mathcal{J}(j\omega t + \psi_{ij})\} \\ |q_{ex}|q_{ex} &= K_{ex0}Q_{ex0} + K_{ex1}\sum_{j=1}^{n} Q_{exj} \cdot \exp\{\mathcal{J}(j\omega t + \psi_{exj})\} \end{aligned}\right\} \cdots\cdots\cdots(16)$$

Then, we have the following relations for steady components and jth harmonics, respectively.

$$\left.\begin{aligned} P_{i0} &= P_{i-10} - \alpha K_{i0}Q_{i0}, \quad (i=1,2,\cdots\cdots,N) \\ Q_{i0} &= Q_{in0}, \quad (i=1,2,\cdots,N), \quad Q_{ex0} = Q_{in0} \end{aligned}\right\} \cdots(17)$$

$$\left.\begin{aligned} P_{ij} &= P_{i-1j} - (\mathcal{J}j\omega m + \alpha K_{i1})Q_{ij}, \quad (i=1,2,\cdots,N) \\ Q_{1j} &= Q_{inj} - \mathcal{J}j\omega c P_{0j}/2 \\ Q_{i+1j} &= Q_{ij} - \mathcal{J}j\omega c P_{ij}, \quad (i=1,2,\cdots\cdots,N-1) \\ Q_{exj} &= Q_{Nj} - \mathcal{J}j\omega c P_{Nj}/2 \end{aligned}\right\} (18)$$

The boundary condition is also linearized as,

$$\left.\begin{aligned} P_{N0} &= \alpha_{ex} K_{ex0} Q_{ex0} \\ P_{Nj} &= \alpha_{ex} K_{ex1} Q_{exj} \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots\cdots (19)$$

<u>Step 2</u> Assume P_{00} and P_{0j}. Then, first of all, Q_{1j} are determined from the second equation of Eqs.(18), because Q_{inj} is known, and the response curve of q_1 in time domain is easily obtained, taking the absolute value and the phase angle of Q_{1j} into consideration and summing up to the nth order together with Q_{10}. So, the linearization coefficients K_{10} and K_{11} for the first pipe element are determined using one of the three linearization schemes described above, by which we get P_{10} from the first equation of Eqs.(17) and P_{1j} from the first one of Eqs.(18), respectively. The relations between (P_{1j}, Q_{1j}) and (P_{0j}, Q_{inj}) are expressed in matrix form as,

$$\begin{pmatrix} P_{1j} \\ Q_{1j} \end{pmatrix} = \begin{pmatrix} 1-(j\omega)^2 cm/2 + \vartheta j\omega\alpha K_{11} c/2 & -\alpha K_{11} - \vartheta j\omega m \\ -\vartheta j\omega c/2 & 1 \end{pmatrix} \begin{pmatrix} P_{0j} \\ Q_{inj} \end{pmatrix} \quad (20)$$

As P_{1j} and Q_{1j} are now known, Q_{2j} are known from the third equation of Eqs.(18) and the response curve of q_2 in time domain is obtained in the same way as stated above, from which the linearization coefficients K_{20} and K_{21} for the second pipe element are determined using one of the three linearization schemes. Then, P_{20} and P_{2j} are determined from the first equation of Eqs.(17) and the first one of Eqs.(18), respectively. This procedure continues to the Nth pipe element and the following relations are obtained.

$$\begin{pmatrix} P_{i+1j} \\ Q_{i+1j} \end{pmatrix} = \begin{pmatrix} 1-(j\omega)^2 cm + \vartheta j\omega\alpha K_{i1} c & -\alpha K_{i1} - \vartheta j\omega m \\ -\vartheta j\omega c & 1 \end{pmatrix} \begin{pmatrix} P_{ij} \\ Q_{ij} \end{pmatrix} \quad (21)$$

$$(i=1,2,\cdots\cdots,N-1)$$

Finally, Q_{exj} are obtained by P_{Nj} and Q_{Nj}. This is expressed in matrix form as

$$\begin{pmatrix} P_{Nj} \\ Q_{exj} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\vartheta j\omega c/2 & 1 \end{pmatrix} \begin{pmatrix} P_{Nj} \\ Q_{Nj} \end{pmatrix} \qquad \cdots\cdots\cdots\cdots\cdots (22)$$

<u>Step 3</u> From the matrix equations (20), (21), and (22), the following expression is obtained, multiplying successively,

$$\begin{pmatrix} P_{Nj} \\ Q_{exj} \end{pmatrix} = \begin{pmatrix} T_{11j} & T_{12j} \\ T_{21j} & T_{22j} \end{pmatrix} \begin{pmatrix} P_{0j} \\ Q_{inj} \end{pmatrix} \qquad \cdots\cdots\cdots\cdots\cdots (23)$$

The steady component P_{N0} is written as,

$$P_{N0} = P_{00} - (\sum_{i=1}^{N} \alpha K_{i0}) Q_{in0} \quad \cdots\cdots\cdots\cdots\cdots\cdots (24)$$

P_{N0}, P_{Nj} and Q_{exj} must satisfy the boundary conditions expressed in Eqs.(19). However, P_{N0}, P_{Nj}, and Q_{exj} just obtained by the assumed P_{00} and P_{0j}, in general, cannot satisfy the boundary conditions in Eqs.(19). Then, by equating P_{N0} in Eqs.(19) to that in Eq.(24), a renewed P_{00} is obtained as,

$$P_{00} = (\alpha_{ex} K_{ex0} + \sum_{i=1}^{N} \alpha K_{i0}) Q_{in0} \quad \cdots\cdots\cdots\cdots (25)$$

Renewed P_{0j} are obtained, by equating P_{Nj} in Eqs.(19) to those in Eqs.(23), as,

$$P_{0j} = - \frac{T_{12j} - \alpha_{ex} K_{ex1} T_{22j}}{T_{11j} - \alpha_{ex} K_{ex1} T_{21j}} Q_{inj} \quad \cdots\cdots\cdots (26)$$

Using these renewed P_{00} and P_{0j}, the calculation procedures after Step 2 are iterated until P_{00} and P_{0j} converge within 1% relative error, and K_{i0}, K_{i1}, Q_{ij}, and P_{ij} are fixed.

Then, the time history of the inlet pressure pulsation p_0 over one period is obtained, by summing up P_{00} and P_{0j} in the form

$$\left.\begin{aligned} p_0 &= P_{00} + \sum_{j=1}^{n} |P_{0j}| \sin(j\omega t + \psi_{inj} + \theta_{0j}) \\ \theta_{0j} &= \tan^{-1}\{\mathcal{I}m(P_{0j}) / \mathcal{R}e(P_{0j})\} \end{aligned}\right\} \quad \cdots\cdots\cdots (27)$$

RESULTS OF CALCULATIONS

The wave forms of inlet pressure pulsation p_0 over one period calculated by each linearization scheme and RKG method are shown in Figs.4, 5 and 6, where broken lines represent the results obtained by RKG method, and solid lines correspond to those by scheme 1, 2, and 3 from above, respectively. In the calculations the number of division N=10 is chosen. The values of equivalent inductance m and capacitance c are set equal to 1/N. The constant α=0.002 is chosen and the constant α_{ex} is chosen as α_{ex}=0.05 for positive q_{ex} and α_{ex}=0.075 for negative q_{ex}. Input flows used for calculations are listed in Tables 1, 2, and 3, together with the fundamental forcing angular frequency ω.

The steady components P_{00} and the amplitudes $|P_{0j}|$ of jth harmonics obtained by each calculation method are also tabulated in Tables 1, 2, and 3, respectively. Those values by RKG method are obtained by Fourier development of the calculated time history of pressure p_0. In the calculation by RKG method, the time interval is chosen as $1/(16 \times N)$ of one period of the highest harmonic contained in the input flow. K_{i0} and K_{i1} are calculated by Simpson's integration formula, using the time interval of 1/16 of the one period of the highest harmonic contained in the input flow.

It is found from the results shown in Figs.4, 5, and 6, and tables 1, 2, and 3 that the pressure pulsations calculated by the proposed equivalent linearization schemes for $|q|q$ are all in good agreement with those by RKG method. This shows that authors' idea for the equivalent linearization of $|q|q$ in case of nonsinusoidal flows and the calculation procedures are reasonable.

By the way, a personal computer is used for the above calculations. The computer times in calculations by the three linearization schemes, on an average, are reduced to 1/30~1/10 of those by RKG method.

CONCLUDING REMARKS

To calculate pressure pulsations in pipelines caused by nonsinusoidal, periodic input flows in frequency domain by transfer matrix method, it is necessary to linearize the nonlinear damping function $|q|q$. It is found that in some cases the linearization by Fourier development method produces negative coefficients for higher harmonics, leading to negative damping.

The idea is proposed that for the equivalent linearization of $|q|q$ in case of nonsinusoidal input flow, two coefficients should be determined, one for the steady component and the other for the fluctuating one. To verify this idea, three linearization schemes based on this idea are introduced and, applying input flows with several harmonics, pressure pulsations are calculated for a simple pipe.

As the results, it is found that the wave forms calculated by three schemes are all in good agreement with those by RKG method, which shows the authors' idea for the equivalent linearization of $|q|q$ in case of nonsinusoidal flow input and calculation procedures to be reasonable.

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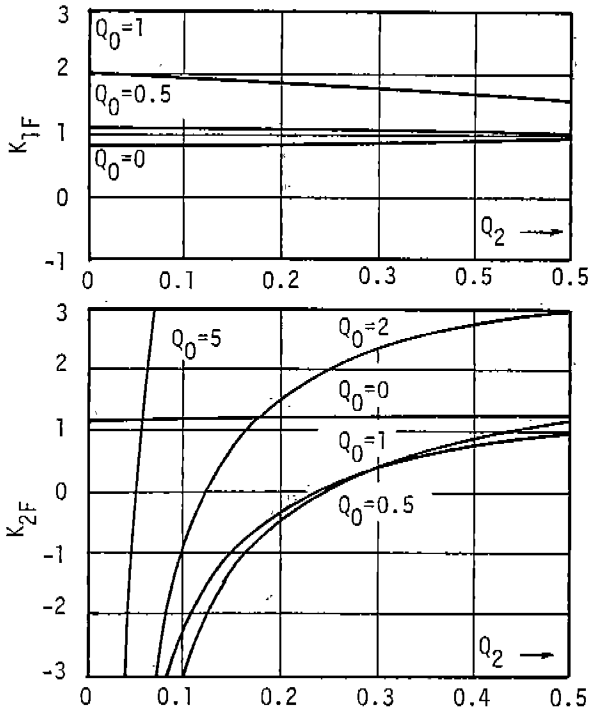

Fig.1 The coefficients K_{1F} and K_{2F} calculated by Eq.(3). $Q_1=1$, $\psi_1=0$, and $\psi_2=\pi/2$.

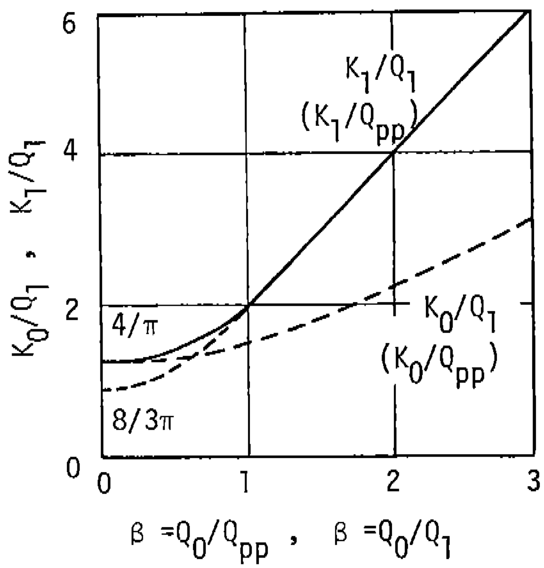

Fig.2 Equivalent linearization coefficients, in case where flow q consists of the steady component and a single sinusoidal wave.

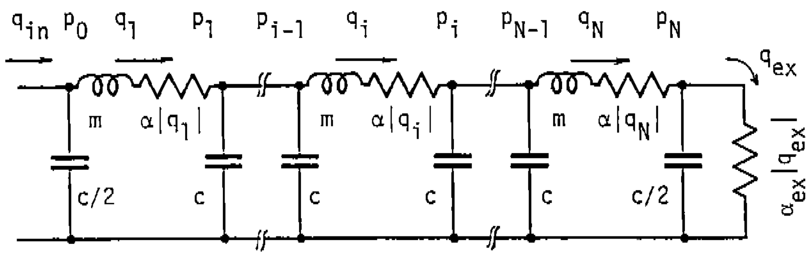

Fig.3 Equivalent lattice circuit of a straight pipe.

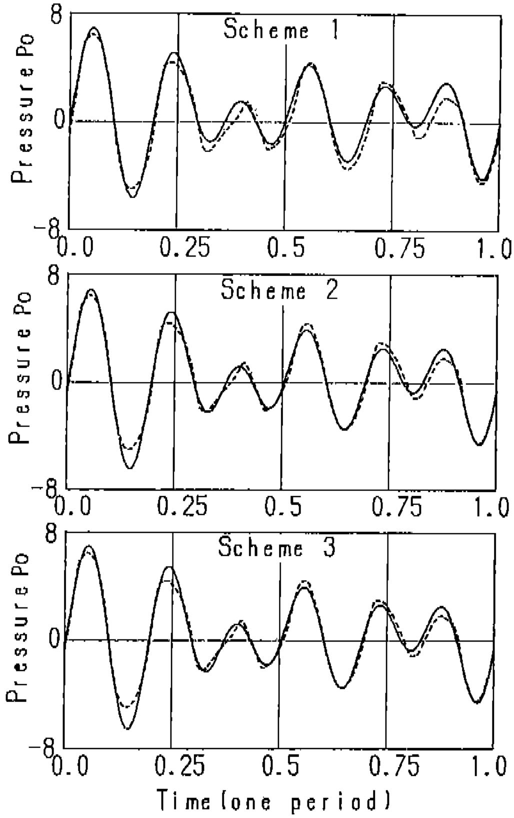

Fig.4 Calculated wave forms of the inlet pressure pulsation p_0 caused by q_{in} given in Table 1.

ω	Input Flow q_{in}			
0.314	$q_{in}= 1+\sin(4\omega t)+0.5\sin(5\omega t)+1.5\sin(6\omega t)$			
Responses of jth harmonics				
Method	P_{00}	$\|P_{04}\|$	$\|P_{05}\|$	$\|P_{06}\|$
RKG	0.25	2.14	1.13	3.12
Scheme 1	0.60	2.10	1.37	3.05
Scheme 2	0.25	2.20	1.50	3.19
Scheme 3	0.25	2.23	1.55	3.25

Table 1 Fundamental forcing angular frequency ω, input flow q_{in}, and the steady component P_{00} and the amplitudes of jth harmonic $|P_{0j}|$.

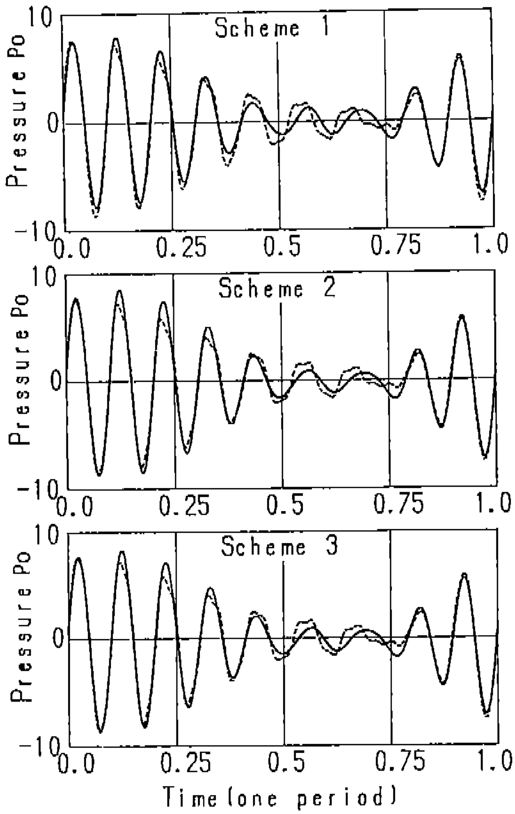

Fig.5 Calculated wave forms of the inlet pressure pulsation p_0 caused by q_{in} given in Table 2.

ω	Input Flow q_{in}			
0.47	q_{in} = 2sin(9ωt)+1.5sin(10ωt)+sin(11ωt)			
Responses of jth harmonics				
Method	P_{00}	$\lvert P_{09}\rvert$	$\lvert P_{010}\rvert$	$\lvert P_{011}\rvert$
RKG	-0.15	3.28	3.56	1.42
Scheme 1	0.0	3.15	3.33	1.44
Scheme 2	-0.18	3.34	3.85	1.51
Scheme 3	-0.17	3.29	3.70	1.50

Table 2 Fundamental forcing angular frequency ω, input flow q_{in}, and the steady component P_{00} and the amplitudes of jth harmonic $\lvert P_{0j}\rvert$.

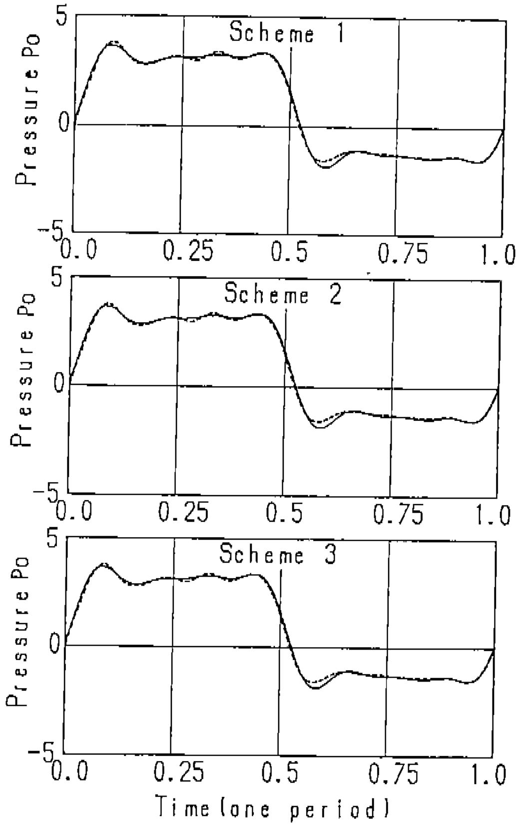

Fig.6 Calculated wave forms of the inlet pressure pulsation p_0 caused by q_{in} given in Table 3.

ω	Input Flow q_{in}				
1.57	$q_{in}=3+\sin(\omega t)+0.333\sin(3\omega t)+0.2\sin(5\omega t)+0.147\sin(7\omega t)$				
Responses of jth harmonics					
Method	P_{00}	$\lvert P_{01}\rvert$	$\lvert P_{03}\rvert$	$\lvert P_{05}\rvert$	$\lvert P_{07}\rvert$
RKG	0.90	2.80	0.95	0.52	0.22
Scheme 1	0.88	2.85	0.98	0.55	0.22
Scheme 2	0.90	2.85	0.98	0.55	0.22
Scheme 3	0.90	2.85	0.98	0.55	0.22

Table 3 Fundamental forcing angular frequency ω, input flow q_{in}, and the steady component P_{00} and the amplitudes of jth harmonic $\lvert P_{0j}\rvert$.