1986

Finite Element Modeling of Compressor Discharge Tubes

M. R. Seidel

R. J. Bernhard
bernhard@purdue.edu

FINITE ELEMENT MODELING OF COMPRESSOR DISCHARGE TUBES

Manfred R. Seidel*[1] and Robert J. Bernhard[2]

[1]Dipl. Ing., Plass-str. 58, 4800 Bielefeld 1, West Germany
[2]Assistant Professor, School of Mechanical Engineering, Purdue University, West Lafayette, Indiana 47907

Abstract

Discharge tubes in hermetically sealed refrigeration compressors are often a source of fatigue failure which is not serviceable and necessitates the replacement of the compressor. The finite element method can be used to model the discharge tube to help prevent such failures and to reduce costs by optimizing the tube. This investigation considers techniques for modeling discharge tubes with finite elements and predicting the dynamic motion and dynamic stresses in the tube during compressor operation. The relative merits of direct versus modal time integration techniques are studied specifically to evaluate which technique is most accurate and efficient.

Introduction

In the past the design of discharge tubes was based mainly on the experience of the designer and tedious, time consuming testing of prototype designs. Due to the steadily increasing power of micro and personal computers, complete modeling of the discharge tube motion, and of the resulting stresses, can now be efficiently done with a number of readily available finite element programs. As a consequence, valuable time and expense is saved by avoiding the experimental

* Investigations conducted as a research assistant at the Ray W. Herrick Laboratories, School of Mechanical Engineering, Purdue University

trial and error design process. This paper discusses various aspects of modeling the dynamic motion and stress distribution in discharge tubes using finite elements.

The geometry of a typical discharge tube problem is shown schematically in Figure 1. Modeling the dynamic behavior of the discharge tube requires; a finite element model of the discharge tube, information about the sources that excite the discharge tube, and algorithms for efficient response calculations and computation of resulting stresses. Each of these aspects of the model will be discussed in detail.

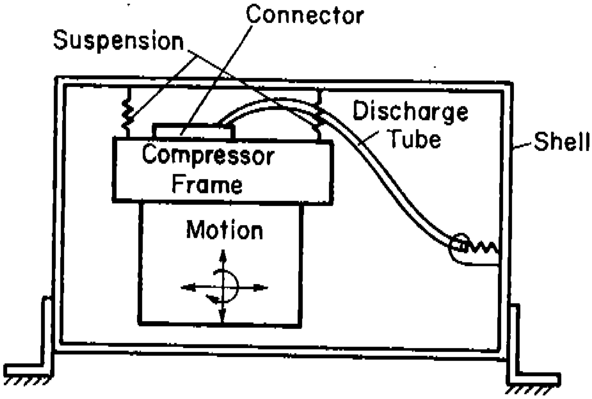

Figure 1. Schematic of a Hermetic Compressor and Discharge Tube

The Finite Element Model Of The Tube

The finite element method is well suited to model the discharge tube. The finite element model yields a familiar system of equations:

$$[M]\{\ddot{x}\} + [K]\{x\} = \{f\} \qquad (1)$$

where [M] is the mass matrix, [K] is the stiffness matrix, {x} is the displacement vector representing the displacements of the tube at the nodal node points and $\{\ddot{x}\}$ is the nodal acceleration vector. For a tube with three dimensional geometry, each node of the tube has six degrees of freedom (i.e. three translational and three rotational degrees of freedom). When refrigerant flow induced forces are neglected and the discharge tube is attached only at its ends, the force vector {f} has nonzero entries only at the nodes at the end of the tube. External forces are induced only by the compressor motion and by the reaction forces at the shell. However, these external forces are unknown. On the other hand, the enforced displacements at the discharge tube ends can be measured, estimated or calculated from the compressor motion and imposed as boundary conditions on the model. The implementation of this time-variant boundary condition will be discussed later.

When no branches exist in the tube, as is the usual case, each finite element is connected only to the next element. The resulting stiffness and mass matrices are banded, symmetric and highly sparse. With finite element programs which take advantage of the sparse and banded characteristic of the model, large amounts of computer memory and time can be saved. It is possible to efficiently run such models on personal computers.

Derivation of the finite element method for straight beam elements is well documented and will not be discussed here [1-4]. Discharge tubes usually have predominantly curved geometries and curved finite elements have been formulated. Such elements are discussed and compared with straight beam elements by Sabir and Ashwell [5] and Thomas and Wilson [6]. It was found that straight elements can successfully be used for curved geometries. The main advantage of straight elements is their general availability in most commercial codes and their convenient data input format.

To obtain good results with straight elements for curved sections, the geometry input must be adjusted such that the combined length of all elements equals the total length of the real tube. When a 180 degree bend is modeled with three elements with nodes on the centerline of the actual geometry, the length of the model will be 5% less than the length of the actual geometry. The result is less mass and higher stiffness in the model than for the actual geometry. As a result, the predicted natural frequencies will be high. By moving the nodes outside the centerline as shown in Figure 2, a much improved prediction can be made. In tests, a typical discharge tube with two short, straight sections covering only approximately 20% of the total tube length, was modeled with straight elements. The 180 degree bends of the tube were modeled with three elements. The difference between the measured and computed fundamental frequency was less than 1%.

Figure 2. Modeling of Curved Beams with Straight Finite Elements

When straight beam elements are used to model beams of circular cross section only the coordinates of the two end nodes need be input since the orientation of the principal bending axis is arbitrary. Many codes, however, require orientation data which makes the geometry input inconvenient. When an existing commercial code does not have axisymmetric or circular element, a short preprocessor program might be written which generates the orientation information automatically.

When material properties of the tubing material are not accurately known they may be measured. For the current investigations, Young's modulus was measured by freely supporting a piece of tubing material from a long, flexible string attached to one end of the tube or by rigidly clamping one tube end. The fundamental bending frequency was measured, and EI was computed using the equation:

$$EI = \left(\frac{\omega l^2}{\beta}\right)^2 \rho A \tag{2}$$

where: ω = first natural frequency

β = 22.37 for the free-free case
3.516 for the clamped-free case

l = length of tube

ρA = mass per length of the tubing material

Due to compressor motion, a time-variant boundary condition is applied at the compressor end of the discharge tube. The formulation of the boundary conditions for the motion of the compressor body is discussed later in the section on response calculations. However, when the tube is connected to a flexible shell or to a flexible discharge muffler, a model of the flexible coupling between the known boundary motions and the end of the tube must be included. For a flexible shell, stiffness effects are generally more important than inertial effects since the shell natural frequencies are much higher than the tube natural frequencies. Furthermore, in most cases only the rotational and translational stiffness effects shown in Figure 3 need be modeled.

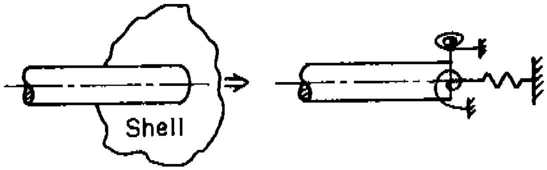

Figure 3. Schematic Model of the Shell/Discharge Tube Interface Stiffness

The in-plane stiffness of the shell can generally be assumed to be infinite. The rotational boundary stiffnesses can be measured by attaching a short straight piece of tube to the shell and measuring the fundamental frequency of the system. Since the fundamental frequency of the tube can be made much lower than the natural frequencies of the shell, the measurements can be performed in the stiffness controlled region of the shell, such that the boundary stiffness can be extracted. The system can be modeled as spring-hinged beam as shown in Figure 4 and the spring constant K can be calculated as discussed by Blevins [7] or Chun [8] using:

$$k = \frac{EI}{l} \lambda \frac{(\cos \lambda \cosh \lambda)(\tan \lambda - \tanh \lambda)}{(\cos \lambda \cosh \lambda + 1)} \qquad (3)$$

where

$$\lambda = \sqrt{2\pi f l^2 \sqrt{\frac{\rho A}{EI}}}$$

and f is the measured frequency of the fundamental mode.

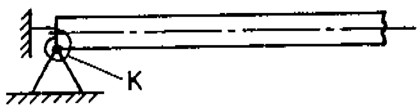

Figure 4. Model of the Rotational Stiffness of the Shell/Discharge Tube Interface

The axial boundary stiffness can be determined similarly by exciting the short straight tube in a longitudinal direction. However, it is difficult to excite purely longitudinal vibrations of the beam. Also, frequencies of the longitudinal modes of a clamped-free tube might lie in the same frequency region as the shell frequencies such that measurements are not conducted in the stiffness controlled region. If a reasonable measurement can be made, the formula to extract the axial stiffness for the model shown in Figure 5 are

$$k = \tan(\lambda) \frac{EA\lambda}{l} \tag{4}$$

where

$$\lambda = 2 \pi f l \sqrt{\frac{\rho A}{E}}$$

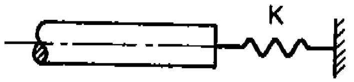

Figure 5. Model of the Axial Stiffness of the Shell/ Discharge Tube Interface

For the discharge muffler connection no general guidelines about which boundary effects will be important can be presumed. Shell and plate finite elements could be used to model the muffler in order to treat the muffler and the discharge tube as one problem. Also impedance measurements of the muffler could be conducted to extract stiffness and inertial boundary effects. However, difficulties in measuring the rotational impedances will limit such an approach. On the other hand, stiffness boundary effects can be extracted experimentally using static testing procedures.

Excitation Of The Discharge Tube

Discharge tube vibrations result from four excitations:

a) Transient vibrations during start-up and shut-down of the compressor

b) Vibrations induced by the motion of the compressor frame during steady state operation

c) Random vibrations during the shipment of the compressor

d) Steady state excitation by pulsating fluids in the tube and random excitation by refrigerant flow noise and airborne lubricant droplets

Obtaining reliable, quantitative information for each excitation source is difficult. During transient start-up and shut-down the discharge tube is excited by the compressor frame motion. Measurement of the motion of the compressor frame is quite complicated. Seidel uses six discrete accelerometer measurements on the compressor frame and a computer program to calculate the six acceleration degrees of freedom at the point where the discharge tube is attached to the frame [1]. When displacement motions are needed the acceleration signals must be integrated twice. Low frequency noise in the acceleration signals, when integrated twice, causes problematic high noise levels in the displacement signal. The noise may be filtered provided the desired motion is at a significantly higher frequency than the noise. For compressors in transient dynamic conditions, the motion of the compressor is dominated by response at the compressor natural frequencies. Thus the highpass filter frequency must be significantly lower than the suspension natural frequencies. The techniques for measuring steady state motion of the discharge tube attachment point are essentially the same as those for transient analysis except that filtering of the low frequency noise and double integration of the acceleration signals can be done in the frequency domain with an FFT analyzer. Alternatively, the steady state or transient compressor frame motion can be calculated as discussed by Gupta [9].

Random vibrations during shipment of the compressor are a complex problem which requires a statistical description of the motion of all six degrees of freedom of the compressor shell during shipment. When it can be assumed that the discharge tube motion during shipment is mainly a static problem, (i.e. that inertial forces of the tube are negligible), the design procedure outlined by Andersen can be used [10]. Andersen used the largest possible deflections of the frame within the compressor shell as a static input. The static stresses for the assumed maximum deflections were computed and the tube was then modified such that the stresses for the worst input were reduced. However, dynamic forces are not accounted for by this method and will be significant for high shock loads.

For small displacements, the interaction of the tube motion and the forces due to pulsating fluids can be uncoupled [11]. The excitation of the pulsating fluids in the tube can be modeled as a time varying distributed load on the tube. For high deflections of the tube, however, the fluid pulsations and the tube vibrations interact and the excitation and motion must be coupled.

Algorithms For Efficient Response Calculations

Due to the widely varying character of the excitation sources, (i.e. steady state, transient and random) different approaches are used for response calculation. Except for the flow induced vibrations, all excitations are imposed as time-variant boundary conditions.

The input boundary motions of the discharge tube end can be imposed by several techniques. The first is to add a very large inertia of mass M to the degree of freedom where acceleration is imposed and excite the system with a force F = M * A, where A is the known input acceleration. For predicting transient motion, a significant drawback of this method is the inherent numerical integration error which causes unstable results when direct integration schemes are used. For steady state response this method can be employed successfully.

An alternative method uses a spring with large stiffness, K, attached to the known degree of freedom. The model is driven with a force F = K * X, where X is the known input displacement. This method is stable but gives poor results when modal truncation procedures are used.

A third method for imposing the time-variant boundary conditions uses partitioned mass and stiffness matrices

$$\begin{bmatrix} [M11] & [M12] \\ [M21] & [M22] \end{bmatrix} \begin{Bmatrix} \ddot{x}1 \\ \ddot{x}2 \end{Bmatrix} + \begin{bmatrix} [K11] & [K12] \\ [K21] & [K22] \end{bmatrix} \begin{Bmatrix} x1 \\ x2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} \quad (5)$$

where {x1} are the unknown interior displacements and {x2} are the known boundary input displacements. Assuming no external forces (fluid forces neglected) acting on the internal nodes the vector {f1} is zero. The known degrees of freedom multiplied with their corresponding partitions of the matrices are brought to the right hand side of the equation. The first row of the partitioned matrix equation can be rewritten as:

$$[M11]\,\{\ddot{x}1\} + [K11]\,\{x1\} = -[M12]\,\{\ddot{x}2\} - [K12]\,\{x2\} \quad (6)$$

where $(-[M12]\{\ddot{x}2\} - [K12]\{x2\})$ is a force vector generated by the input displacements. The term $[M12]\{\ddot{x}2\}$ can usually be neglected when the last element of the tube is short because the associated inertial forces will be small. The system can now be solved for {x1}, and the stresses calculated from the tube deflections. This method is stable. The one drawback of the method is that, in this form modal truncation cannot be used to reduce the size of the problem. As shown in Figure 6, the natural modes computed from matrices [M11] and [K11] have very small deflections {x1e} at the node adjacent to the node where the displacements are imposed. Since the displacements will be large at these nodes, a large set, if not the complete set, of modes will be required to model the discharge tube motion. Thus, an accurate model reduction through modal truncation procedures is not likely. An example of the prediction of response using modal truncation is illustrated in Figure 7. In this example the modes of a cantilever beam are extracted using the M11 and K11 matrices. The model is excited at very low frequency (essentially a static displacement excitation). As Figure 7 indicates, a large set of modes must be used before the prediction is valid.

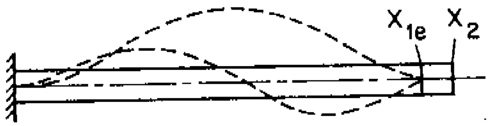

Figure 6. The First Two Bending Modes of a Clamped-Clamped Beam

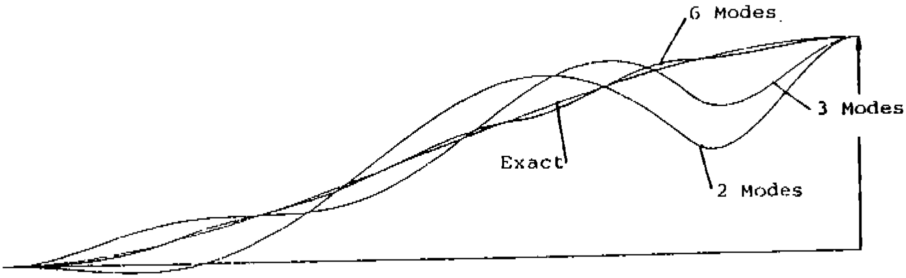

Figure 7. Predicted Low Frequency Response of a Fixed/Force Beam Using Modal Methods and the Modes of a Clamped-Clamped Beam

A similar modified modal method which allows for modal truncation was developed. The first 10-30 modes of the complete discharge tube model of Eq. 5, with a free end condition at the forced node are computed and assembled in a reduced, nonsquare modal matrix, $[U]^t = [[U1],[U2]]^t$, where [U2] corresponds to the input degrees of freedom. A reduced set of these eigenvectors [U] are still similar to the response, since they have large deflections at the forced end. The transformation

$$(x1) = [U1]\ (\eta) \tag{7}$$

is applied to Eq. 6 and the equation is premultiplied by $[U1]^T$ such that

$$[U1]^t[M11][U1](\ddot{\eta}) + [U1]^t[K11][U1](\eta) = -[U1]^t[K12](x2) \tag{8}$$

or

$$[\overline{M}](\ddot{\eta}) + [\overline{K}](\eta) = -[F](x2) \tag{9}$$

Equation 9 is an accurate reduced approximation of the response. The matrices $[\overline{M}]$ and $[\overline{K}]$ are not diagonal since the [U1] transformation matrix is not derived from the matrix equation of Eq. 6. However, the new system can be diagonalized with a second "standard" modal transformation which requires little effort due to the reduced size of the transformed problem. Thus, the method divides modal truncation and modal diagonalization into two transformations. Although two transformations are needed, the savings are still considerable. Application of the modified modal transformation for the low frequency deflection problem is shown in Figure 8. As shown, a truncation to two modes gives a very good approximation of the solution.

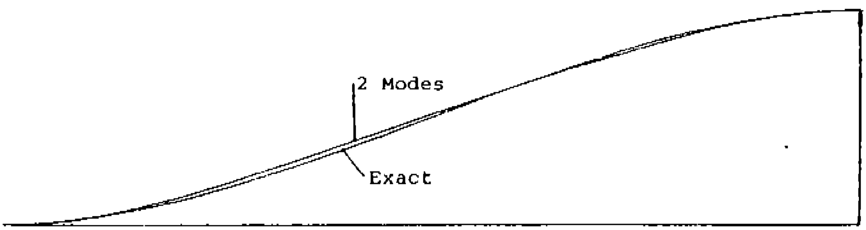

Figure 8. Predicted Low Frequency Response of a Fixed/Forced Beam Using Modal Methods and the Modes of a Clamped-Free Beam

Whether modal transformations and truncation should be used depends on the type of excitation. For steady state response, modal transformation is advantageous in most circumstances. However, it is inefficient to calculate eigenfrequencies of the clamped-clamped tube. As illustrated, a complete set of modes will be required to simulate the behavior of the tube. Using "free end" eigenvectors may not be much better, since the eigenvectors have zero moments and forces at the free end and cannot model force and moment reactions introduced by the compressor frame. The real response lies somewhere between these two limiting cases.

For transient response calculations, modal transformation is not necessarily advantageous. If a FEM code can exploit the banded and symmetric character of the matrices, tremendous savings in memory space can be accomplished. Furthermore, with the use of an active column equation solver [3], the time integration in the physical coordinate domain is faster than in the modal domain, where the back-transformation consumes considerable time.

Conclusions

The details of dynamic modeling of compressor discharge tubes using the finite element method has been discussed. In particular, techniques for measurement and application of tube end conditions are developed. The application of the tube excitation by the compressor is formulated. Several modal reduction procedures are also studied. It is the conclusion of these studies the modal reduction procedures must be done carefully to avoid error. However, if used appropriately, the finite element method is an excellent tool to be used for designing discharge tubes.

Acknowledgment

This investigation was supported by Zanussi Electtromeccanica S.p.A. The authors wish to express their appreciation to Mr. Greco, Mr. Pol, and Mr. Giusto for their assistance and advice.

References

[1] Seidel, M. R. , "Transient Vibrations of Compressor Discharge Tubes", Master's Theses, Purdue University, West Lafayette, IN, May 1985.

[2] Meirovitch, L. , "Elements Of Vibration Analysis", McGraw-Hill, 1975.

[3] Zienkiewicz, O. C. , "The Finite Element Method", 3rd Edition, McGraw-Hill, 1977.

[4] Bathe, K.J. , "Numerical Methods In Finite Element Analysis", Prentice-Hall, 1976.

[5] Sabir, A. B. and Ashwell, D. G. , "A Comparison Of Curved Beam Finite Elements When Used In Vibration Problems", Journal of Sound and Vibration, 18(4), pp. 555-563, 1971.

[6] Thomas, D. L. and Wilson, R. R. , "The Use Of Straight Beam Finite Elements For Analysis OF Curved Beams", Journal of Sound and Vibration, 26, pp. 155-158, 1973.

[7] Blevins, R. D. , "Formulas For Natural Frequency And Mode Shape", Van Nostrand Reinhold Company, 1979.

[8] Chun, K. R. , "Free Vibration Of A Beam With One End Spring Hinged And The Other Free", Journal of Applied Mechanics, pp. 1154-1155, December 1972.

[9] Gupta, N. , "Analysis Of The Transient Motion Of A Compressor", Master's Theses, Purdue University, May 1984. Confidential until May 1986

[10] Andersen, G. E. , "Compressor Discharge Line Stress Analysis Utilizing Finite Element Methods", Proceedings of the 1980 Purdue Compressor Technology Conference, pp. 64-68, Purdue, 1980.

[11] Kohli, A. K. and Nakra, B.C. , "Vibration Analysis Of Straight And Curved Tubes Conveying Fluid By Means Of Straight Beam Finite Elements", Journal of Sound and Vibration, 93(2), pp. 307-311, 1984.