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The Harmonic Impact of Rectifiers Serviced by Scott and LeBlanc Connected Transformers

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The Harmonic Impact of Rectifiers Serviced by Scott and LeBlanc Connected Transformers

William P. Butler

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School of Electrical Engineering
Purdue University
West Lafayette, Indiana 47907
THE HARMONIC IMPACT OF RECTIFIERS

SERVICED BY SCOTT AND LEBLANC

CONNECTED TRANSFORMERS

A Thesis
Submitted to the Faculty

of

Purdue University

by

William P. Butler

In Partial Fulfillment of the
Requirements for the Degree

of

Master of Science in Electrical Engineering

May 1987
this is dedicated
to our Creator
ACKNOWLEDGMENTS

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NOMENCLATURE

- A zero-peak amplitude in phase A
- A,B,L circuit incidence matrices
- a 1 /120°
- a² 1 /240°
- aₙ,bₙ Fourier coefficients in rectangular form
- δV ripple voltage
- Hₙ magnitude of the nth harmonic
- Hz Hertz
- h harmonic order
- I current
- I̅ identity matrix
- I(t) time-varying current
- I(2ϕ) two-phase current
- I(3ϕ) three-phase current
- I₊ positive sequence current
- I₋ negative sequence current
- I₀ zero sequence current
- Im imaginary part of the (complex) argument
- j square root of -1 (√-1)
- kV kilovolt
- kV*A kilo-voltampere
- M symmetrical component transformation
- N transformer turn
- xi -

- NSS non-sinusoidal scheme
- $\omega_o$ power frequency (radians/second)
- $P$ active power, Watts
- $p$ pulse number
- $\phi$ power phase angle, power factor angle
- $\phi_n$ phase angle of the nth harmonic
- $Q$ reactive voltamperes, vars
- $R-C$ resistor-capacitor
- $R-L$ resistor-inductor
- $Re$ real part of the (complex) argument
- $SS$ sinusoidal scheme
- $T$ period of a wave
- $THD$ total harmonic distortion
- $TIF$ telephone influence factor
- $u$ unbalance factor
- $V$ voltage
- $V-I$ voltage-current
- $w_n$ telephone influence factor weight for harmonic n
- $x(t)$ arbitrary waveform
ABSTRACT

Butler, William P. M.S.E.E., Purdue University. May 1987. THE HARMONIC IMPACT OF RECTIFIERS SERVICED BY SCOTT AND LEBLANC CONNECTED TRANSFORMERS. Major Professor: G. Thomas Heydt.

Conventional three-phase, six-pulse rectifiers produce harmonic currents which propagate into the ac power system from which they are supplied. A two-phase, four-pulse rectifier can be operated such that there are no ac harmonic currents generated. In this connection, although there is no harmonic impact, the three-phase load current is imbalanced. The operation of a line-commutated two-phase, four-pulse rectifier served by the Scott and LeBlanc three-phase to two-phase transformer connections is simulated using a digital computer program in this thesis. Several different methods of rectification for a two-phase bridge are simulated and their current waveforms analyzed for harmonic content and balance. Also, the total harmonic distortion and the telephone influence factors of alternative connections and configurations are calculated. Finally, general comparisons with three-phase, six-pulse rectifiers served by conventional transformer connections are presented.
CHAPTER 1
INTRODUCTION

1.1 Introduction, Motivation, and Objectives

Electric power engineering has its origins in direct current applications. However, the development of the induction motor and the transformer in the late 1800's made ac power economically more feasible than dc. Alternating current transmission and distribution of electric energy soon afterwards become the accepted practice. Yet, there are still numerous applications of dc power that include, among others, high voltage dc (HVDC) transmission systems, dc drive systems, electrolysis and smelting applications, pulse width modulated controllers, and electronic applications requiring dc bus voltages for solid state and vacuum tube devices. Most, if not all, commercial power systems in the United States and Europe are of based on alternating current transmission and distribution. Thus the conversion, or rectification, of alternating current into direct current is of considerable importance and relevance, particularly in industrial applications. Also, the process of inversion, or conversion of dc to alternating current, is nearly identical to rectification except for the direction of flow of energy. Inverter applications include HVDC systems, alternate energy sources, and isolation of power supplies.

The rectification process can be facilitated in several different ways. Regardless of the method of rectification, the nonlinear nature of the conversion process causes harmonic currents to propagate into the ac supply system unless appropriate precautions are taken. The effects of waveform distortion can have audible effects (such as telephone interference) or worse, destructive effects (such as the over-stressing of power factor correction capacitors). Other undesirable effects include high ac motor losses and heating, timing signal interference (in digital computers), protective relay malfunction, inaccurate metering of power, and over-voltage conditions on transmission system components. The topic of the harmonic impact of rectification systems is therefore an important one. In this thesis, the focus
is on rectifiers although identical techniques and conclusions are applicable for inverters.

The principal goal of this research is to study the feasibility, harmonic impact, and the impact in general of the use of converter transformers which convert three-phase ac power to two-phase ac power before rectification or inversion. The obvious advantage to converting to two-phase first is a one-third reduction in solid state parts and control devices needed. It is true that the cost of solid state devices used in converters has dropped dramatically in recent years, but there is a consideration of savings nonetheless. Also, there is the prospect of lower conversion losses. Another advantage is that the initial investment may be reduced without generating unacceptable levels of harmonic currents.

1.2 The Scott and LeBlanc Connections and the Graetz Bridge

The method of rectification assumed is a static converter employing a thyristor, or a gate controlled solid state diode, as a switching device. The principal parts of a rectifier are the transformer connections, the thyristors, and the filters. Because of the rectification system under consideration, the focus will be upon the transformer connections used.

The rectification scheme considered is a two-phase, four-pulse version of the three-phase, six-pulse Graetz Bridge [1]. The Graetz bridge is shown in Figure 1.1 and the two-phase, four-pulse bridge in Figure 1.2. The two-phase power delivered to the rectifier can be derived from a three-phase supply in either of two not-so-common transformer connections: the Scott connection [2,3,4] and the LeBlanc connection [5,6]. Figure 1.3 shows a diagram of the Scott connection and the voltage and current relationships, and similarly in Figure 1.4 for the LeBlanc connection. The Scott connection is a variation of a wye-wye connection, and the LeBlanc that of a wye-delta connection.

Modern HVDC conversion systems take advantage of the 30° phase shift in wye-delta transformers to eliminate some harmonics [7]. This is accomplished by supplying a wye-delta transformer and a wye-wye transformer in parallel with each transformer supplying it's own Graetz bridge. The output of the Graetz bridges are then connected in series to form a twelve-pulse converter. On the three-phase side, the phase shift introduced by the wye-delta transformer will cause some harmonic currents to cancel with the unshifted harmonic currents of the wye-wye transformer. A diagram of this arrangement can be found in Figure 1.5. A similar
Figure 1.1. Three-phase Graetz bridge.
Figure 1.2. Two-phase, four-pulse bridge.
Figure 1.3. Scott transformer connection and its voltage and current relationships.
Figure 1.4. LeBlanc transformer connection and its voltage and current relationships.
Figure 1.5. Twelve-pulse converter using parallel wye-wye and wye-delta transformers.
arrangement can be made with the Scott and LeBlanc connections, a diagram of which can be found in Figure 1.6. This arrangement, using the Scott and LeBlanc connections, and its impact on the ac supply system is the focus of this thesis.

1.3 Literature Summary

This section presents some of the relevant information necessary to understand the nature of power system harmonics and the operation of the Scott and LeBlanc connections. Due to the fact that these connections first appeared over 80 years ago, the full literature on the topic is considerable. The intent of the section is not to be complete in its survey of these topics, but rather to set forth some of the 'fundamentals' in a concise manner.

*Power System Harmonics*

The issue of harmonics has been seriously addressed since Dallenbach and Gerecke wrote their paper concerning the effects of harmonic currents in 1925 [8]. More recently this topic has been the subject of at least three books [9,10] and a host of papers. The more specific issue of HVDC converter harmonics has also been well documented in recent years as well [11,12,13].

The ideal $p$-phase two way converter [see Figure 1.7] will have phase currents that are rectangular pulses of width $2\pi/p$ times the system frequency [9]. A Fourier series analysis of this waveform shows that the current pulses can be expressed as [14]

$$I(t) = \frac{4}{\pi} \left\{ \sin \frac{a}{2} \cos \omega t + \frac{1}{3} \sin \frac{3a}{2} \cos 3\omega t + \frac{1}{5} \sin \frac{5a}{2} \cos 5\omega t + \ldots \right\}$$  \hspace{1cm} (1.1)

where $a=2\pi/p$. The amplitudes of the harmonic currents are seen to decrease as $1/h$ by inspection of (1.1). The order of the harmonics present (3rd,5th,7th,etc) is a function of the number of the phases used for conversion and can be expressed as

$$h = pk \pm 1$$  \hspace{1cm} (1.2)

where $p$ is the pulse number and $k = 1,2,3\ldots$ [13].
Figure 1.6. Eight-pulse converter using parallel Scott and LeBlanc transformers.
Figure 1.7. P-phase two-way ideal converter.
The Scott Connection

The Scott connection was named for Charles F. Scott, 1903 President of the American Institute of Electrical Engineers, forerunner of the Institute of Electrical and Electronic Engineers. In reference [26], Blackwell gives turns ratios and dielectric insulation stress for various three-phase connections. Scott discussed this paper and described the two-phase to three-phase connection which we now know as the Scott connection. The Scott connection is used primarily for getting a two-phase transformation from three-phase [2,3,4]. It requires two transformers, one being the main transformer and the other being the teaser transformer. The main transformer is connected line-to-line and the teaser from the remaining line to the midpoint of the main transformer [see Figure 1.3]. The teaser has 86.6% of the number of turns as the main, and also has a neutral tap at 66.6% on the three-phase side. The Scott connection does not change the power factor from two-phase side to three-phase side.

The LeBlanc Connection

The LeBlanc connection is named for Charles LeBlanc, a French engineer who was an early experimenter in a range of subjects from electolysis to transformers. The LeBlanc connection is another scheme that can be used for three-phase to two-phase conversion [5,6]. The LeBlanc connection may, in fact, predate the Scott connection [26]. The LeBlanc connection is not a well known method of phase conversion (as the lack of literature in comparison to the Scott connection indicates). The primary of a LeBlanc connection is connected in three-phase delta, and the secondary in quadrature [see Figure 1.4], thus three transformers are required. Two of the transformers require taps at 36.6% on the two-phase side, and the third transformer a conventional arrangement. The LeBlanc connection and the Scott connection have identical two-phase voltage and current relationships [cf. Figure 1.3 and Figure 1.4]. These connections can be used in parallel operation quite readily with proper attention given to percent impedance, kVA requirements, etc.

1.4 Organization of Thesis

The remainder of this thesis is organized into three additional chapters. In Chapter 2, the framework for analyzing the voltage and current relationships between the two-phase side and the three-phase side for the Scott and LeBlanc connections is developed. Next, the method of
symmetrical components is applied to the three-phase system in terms of the two-phase variables. The chapter concludes with a discussion of methods to compute voltages and currents in a two-phase rectifier. Chapter 3 presents the computed voltages and currents for several different schemes of Scott and LeBlanc connections. The total harmonic distortion (THD) and telephone influence factor (TIF) are also presented. The computed THD and TIF of the Scott and LeBlanc connections are compared with that of rectifiers with alternative transformer connections. Chapter 4 presents conclusions and recommendations for further research. The Appendices contain the methods of calculating the harmonic series, THD, and TIF for the connection schemes under consideration, and also data too numerous to be placed in the body of this thesis.
CHAPTER 2
THE CALCULATION OF VOLTAGES AND CURRENTS IN A TWO-PHASE BRIDGE RECTIFIER

In this chapter, the calculation of voltages and currents in a two-phase rectifier is described. The conversion system is represented as an ideal diode with a resistive load. The transformers supplying power to the diodes are represented as ideal voltage sources.

2.1 The Definition of Voltage and Current Relationships in a Scott Connection

To determine the voltage and current relationships between the three-phase primary side of a Scott connection and the two-phase secondary side, the transformer connection shown in Figure 2.1 will be used as a basis. The three-phase side of the transformer is assumed to be the primary side and the two-phase side the secondary. The three-phase voltages sequence will be labeled ABC, consistent with common practice [15], and the two-phase side labeled 1 and 2. Also shown in Figure 2.1 are the turns and the dots used for determining voltages. The dot convention used specified states that a current entering a dot on one coil produces a positive voltage at the dotted terminal of the other coil [16].

At this point the voltage expressions are written as follows

\[ V_1 = \frac{N_5}{N_1} V_{an} \quad \text{or} \quad \frac{-N_5}{N_2} V_x \]  

(2.1)

where,

\[ V_x = \frac{N_4}{N_3 + N_4} V_{bn} + \frac{N_3}{N_3 + N_4} V_{cn} \]  

(2.2)

let

\[ N_{34} = N_3 + N_4 \]

thus
Figure 2.1. Scott transformer connection showing windings, dots, and currents.
\[ V_1 = \frac{N_5}{N_1} V_{an} = -\frac{N_4 N_5}{N_2 N_{34}} V_{bn} - \frac{N_3 N_5}{N_2 N_{34}} V_{cn} \quad (2.3) \]

Similarly

\[ V_2 = -\frac{N_3}{N_{34}} V_{bc} \quad \text{or} \quad -\frac{N_4}{N_{34}} V_{bc} \frac{N_6}{N_4} \quad (2.4) \]

hence

\[ V_2 = -\frac{N_6}{N_{34}} V_{bc} \quad (2.5) \]

The current relationships are expressed similarly by referring to Figure 2.1 by writing equations expressing the conservation of magnetomotive force (MMF) around a closed loop. This is equivalent to insuring that the Ampere-turns, or the current in a coil times the number of turns in the coil, must sum to zero. The Ampere-turns for transformer #1 are

\[ N_1 I_a - N_2 I_x - N_5 I_1 = 0 \quad (2.6) \]

where

\[ I_x = I_b + I_c . \]

Thus

\[ N_1 I_a - N_2 I_b - N_2 I_c - N_5 I_1 = 0 \quad (2.7) \]

Similarly for transformer #2,

\[ N_3 I_b - N_4 I_c + N_6 I_2 = 0 . \quad (2.8) \]

The currents \( I_1 \) and \( I_2 \) are expressed as functions of the three-phase currents as follows

\[ N_1 I_a - N_2 I_b - N_2 I_c = N_5 I_1 \quad (2.9) \]

\[ N_3 I_b - N_4 I_c = -N_6 I_2 . \quad (2.10) \]

Equations (2.9) and (2.10) are augmented with

\[ I_a + I_b + I_c = 0 \quad (2.11) \]

to form a three-by-three system of equations in matrix notation as shown in (2.12),
\[
\begin{bmatrix}
N_1 & -N_2 & -N_2 \\
0 & N_3 & -N_4 \\
1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix} =
\begin{bmatrix}
N_5 & 0 \\
0 & -N_6 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}.
\quad (2.12)
\]

Let,
\[
A =
\begin{bmatrix}
N_1 & -N_2 & -N_2 \\
0 & N_3 & -N_4 \\
1 & 1 & 1
\end{bmatrix}
\quad (2.13)
\]

\[
B =
\begin{bmatrix}
N_5 & 0 \\
0 & -N_6 \\
0 & 0
\end{bmatrix}
\quad (2.14)
\]

Therefore
\[
A I(3\phi) = B I(2\phi).
\quad (2.15)
\]

Solving for the three-phase currents yields
\[
I(3\phi) = A^{-1}B I(2\phi),
\quad (2.16)
\]

where
\[
A^{-1} =
\begin{bmatrix}
\frac{1}{N_{12}} & 0 & \frac{N_2}{N_{12}} \\
-\frac{N_4}{N_{12}N_{34}} & \frac{1}{N_{34}} & \frac{N_1 N_4}{N_{12}N_{34}} \\
-\frac{N_3}{N_{12}N_{34}} & \frac{1}{N_{34}} & \frac{N_1 N_3}{N_{12}N_{34}}
\end{bmatrix}
\quad (2.17)
\]

Note: \(N_{12} = N_1 + N_2\) and \(N_{34} = N_3 + N_4\). Substituting (2.17) into (2.16) results in \(I(3\phi)\),
In summary, the two-phase secondary voltages for a Scott connection, \( V_1 \) and \( V_2 \), are functions of the three-phase primary voltages as in equations (2.3) and (2.5) respectively. The equations expressing the conservation of MMF (or equivalently the zero sum of Ampere-turns) in the transformers used in the Scott connection are stated in (2.7) and (2.8). When augmented with the definition of balanced three-phase currents (2.11), these equations are solved to yield an expression for the three-phase currents as a function of the two-phase currents (2.18).

2.2 The Definition of Voltage and Current Relationships in a LeBlanc Connection

The voltage and current relationships between the primary and secondary of a LeBlanc connection can be determined in a manner exactly analogous to that which was done in Section 2.1 for the Scott connection. The transformer arrangement to be used for deriving the various circuit relationships can be found in Figure 2.2.

The voltage expressions for a LeBlanc connection are written as follows

\[
V_1 = V_{aa} + V_{bb} \tag{2.19}
\]

where

\[
V_{aa} = -\frac{N_4}{N_3} V_{ca} \tag{2.20}
\]

\[
V_{bb} = \frac{N_5}{N_1} V_{ab} \tag{2.21}
\]

Thus:

\[
I(3\phi) = \begin{bmatrix}
\frac{N_5}{N_{12}} & 0 \\
-\frac{N_4N_5}{N_{12}N_{34}} & -\frac{N_6}{N_{34}} \\
-\frac{N_3N_5}{N_{12}N_{34}} & \frac{N_5}{N_{34}}
\end{bmatrix} \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}. \tag{2.18}
\]
Figure 2.2. LeBlanc transformer connection showing windings, dots, and currents.
Similarly

\[ V_2 = V_{cc} + V_{dd} + V_{ee}, \]  

where

\[ V_{cc} = -\frac{N_6}{N_2} V_{bc}, \]  

\[ V_{dd} = \frac{N_7}{N_1} V_{ab}, \]  

\[ V_{ee} = \frac{N_8}{N_3} V_{ca}. \]

Hence

\[ V_2 = \frac{N_7}{N_1} V_{ab} - \frac{N_6}{N_2} V_{bc} + \frac{N_8}{N_3} V_{ca}. \]

The current relationships are written from Figure 2.2 by observing the zero sum of Ampere-turns,

\[ N_1 I_{ab} - N_5 I_1 - N_7 I_2 = 0 \]  

(2.28)

\[ N_2 I_{bc} + N_6 I_2 = 0 \]  

(2.29)

\[ N_3 I_{ca} + N_4 I_1 - N_8 I_2 = 0. \]  

(2.30)

Equations (2.28) - (2.30) are rewritten in matrix form as

\[
\begin{bmatrix}
N_1 & 0 & 0 \\
0 & N_2 & 0 \\
0 & 0 & N_3 \\
\end{bmatrix}
\begin{bmatrix}
I_{ab} \\
I_{bc} \\
I_{ca} \\
\end{bmatrix}
= 
\begin{bmatrix}
N_5 & N_7 \\
0 & -N_6 \\
-N_4 & N_8 \\
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\end{bmatrix}.
\]

Equation (2.31) relates the two-phase secondary currents to the three-phase currents of the delta connected primary. However, for the purpose of analyzing the impact on the three-phase side of the LeBlanc connection, the actual line currents are desired rather than the currents in the delta.
The line currents are found by summing the currents into a node. At node 1 in Figure 2.2, for instance, the currents entering the node are $I_a$, $I_{ca}$, and $-I_{ab}$. From Kirchhoff's current law, it is known that these currents must sum to zero. Relationships between the line currents and the currents in the delta connected transformers can be formed as,

\[
I_a + I_{ca} - I_{ab} = 0
\]

\[
I_b + I_{ab} - I_{bc} = 0
\]

\[
I_c + I_{bc} - I_{ca} = 0.
\]

These are expressed in matrix form as

\[
\begin{bmatrix}
-1 & 0 & 1 \\
1 & -1 & 0 \\
0 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
I_{ab} \\
I_{bc} \\
I_{ca}
\end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]  

(2.35)

The line currents are easily solved for by linear algebraic techniques. Let

\[
A = \begin{bmatrix}
N_1 & 0 & 0 \\
0 & N_2 & 0 \\
0 & 0 & N_3
\end{bmatrix}
\]

(2.36)

\[
B = \begin{bmatrix}
-1 & 0 & 1 \\
1 & -1 & 0 \\
0 & 1 & -1
\end{bmatrix}
\]

(2.37)

\[
C = \begin{bmatrix}
N_5 & N_7 \\
0 & -N_6 \\
-N_4 & N_8
\end{bmatrix}
\]

(2.38)

Then
A \begin{bmatrix} A & B \\ B & \bar{I} \end{bmatrix} \begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \\ I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{2.39}

where \( \bar{I} \) is the identity matrix. Then

\[ A \begin{bmatrix} (3\phi)_{\text{delta}} \end{bmatrix} = C \begin{bmatrix} (2\phi) \end{bmatrix} \tag{2.40} \]

\[ B \begin{bmatrix} (3\phi)_{\text{delta}} \end{bmatrix} + \begin{bmatrix} (3\phi)_{\text{line}} \end{bmatrix} = 0 \tag{2.41} \]

Equations (2.40) are solved for the three-phase delta currents,

\[ (3\phi)_{\text{delta}} = A^{-1} C \begin{bmatrix} (3\phi) \end{bmatrix} \tag{2.42} \]

and substituted into (2.41) to solve for the three-phase line currents,

\[ (3\phi)_{\text{line}} = -B A^{-1} C \begin{bmatrix} (2\phi) \end{bmatrix}. \tag{2.43} \]

The calculation of \(-B A^{-1} C\) yields

\[ \begin{bmatrix} \frac{N_5 + N_4}{N_1 N_3} & \frac{N_7 - N_8}{N_1 N_3} \\ -\frac{N_5}{N_1} & -\frac{N_7 - N_6}{N_1 N_2} \\ -\frac{N_4}{N_3} & \frac{N_8 + N_6}{N_3 N_2} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}. \tag{2.44} \]

In summary, the two-phase voltages for a LeBlanc connection are given by (2.22) and (2.27). The equations relating the three-phase line currents to the two-phase currents, in matrix form, are given by (2.44).

### 2.3 Use of the Symmetrical Component Transformation

The method of symmetrical components is used to analyze a potentially unbalanced three-phase system in terms of balanced phasors. This method is applied to the three-phase system supplying a two-phase rectifier.
by relating the two-phase currents to the three-phase currents. There are several different methods of transformations from known broadly as symmetrical components. The method used to transform phase variables a, b, c, into positive, negative, and zero sequence, +, −, and 0 respectively, as shown in (2.45) is,

\[
\begin{bmatrix}
W_+ \\
W_- \\
W_0
\end{bmatrix} = M
\begin{bmatrix}
W_a \\
W_b \\
W_c
\end{bmatrix},
\]

(2.45)

where W can be voltage (V) or current (I). The linear transformation, M [17], is defined below as

\[
M = \frac{1}{\sqrt{3}} \begin{bmatrix}
1 & a & a^2 \\
1 & a^2 & a \\
1 & 1 & 1
\end{bmatrix}
\]

(2.46)

where \(a = 1/120^\circ\) and \(a^2 = 1/240^\circ\)

The scaling factor of \(1/\sqrt{3}\) causes M to be unitary and its rows and columns to form an orthonormal set [18].

2.3.1 The Scott Connection

The three-phase currents in a Scott connection have been defined to be a function of the two-phase currents, in matrix form, as given by (2.18). Let the matrix relating the two-phase currents to the three-phase currents be "L" (2.18), then

\[
I(3\phi) = L I(2\phi).
\]

(2.47)

Substituting (2.47) into (2.45) yields

\[
I = M L I(2\phi).
\]

(2.48)

Evaluation of the product \(M L\) left in polar form yields
Converting to complex rectangular notation results in

\[
\begin{bmatrix}
\frac{\sqrt{3} N_5 N_{34} + j N_5 (N_3 - N_4)}{2N_{12} N_{34}} & j2 N_6 N_{12} \\
\frac{\sqrt{3} N_5 N_{34} - j N_5 (N_4 - N_3)}{2N_{12} N_{34}} & -j2 N_6 N_{12} \\
0 & 0
\end{bmatrix}
\]

(2.50)

The symmetrical components, then, of the three-phase currents as a function of the two-phase currents are obtained by substituting (2.50) into (2.48),

\[I_+= \left\{ \frac{\sqrt{3} N_5 N_{34} + j N_5 (N_3 - N_4)}{2N_{12} N_{34}} \right\} I_1 + \left\{ j \frac{N_6}{N_{34}} \right\} I_2 \quad (2.51)\]

\[I_- = \left\{ \frac{\sqrt{3} N_5 N_{34} - j N_5 (N_3 - N_4)}{2N_{12} N_{34}} \right\} I_1 - \left\{ j \frac{N_6}{N_{34}} \right\} I_2 \quad (2.52)\]

\[I_o = 0 \quad (2.53)\]

2.3.2 The LeBlanc Connection

As in the case of the voltage and current relationships, the LeBlanc symmetrical component relationships is derived in a manner completely analogous to that which was done for the Scott. Let the matrix relating the two-phase currents to the three-phase currents be "L" (2.44). Thus the
symmetrical components is obtained by substituting $L$ into (2.48). The result of the evaluation of the product $M^L$ will simply be presented,

\[
I_+ = \left\{ \frac{\sqrt{3}}{2N_1N_3} (N_3N_5 + N_1N_4) + j \frac{1}{2N_1N_3} (N_3N_5 - N_1N_4) \right\} I_1
\]
\[
+ \left\{ \frac{\sqrt{3}}{2N_1N_2N_3} (N_2N_3N_7 - N_1N_2N_8) + \frac{j}{2N_1N_2N_3} (N_2N_3N_7 + N_1N_2N_8 + 2N_1N_3N_6) \right\} I_2
\]
(2.54)

\[
I_- = \left\{ \frac{\sqrt{3}}{2N_1N_3} (N_3N_5 + N_1N_4) - j \frac{1}{2N_1N_3} (N_3N_5 - N_1N_4) \right\} I_1
\]
\[
+ \left\{ \frac{\sqrt{3}}{2N_1N_2N_3} (N_2N_3N_7 - N_1N_2N_8) - \frac{j}{2N_1N_2N_3} (N_2N_3N_7 + N_1N_2N_8 + 2N_1N_3N_6) \right\} I_2
\]
(2.55)

\[
I_0 = 0.
\]
(2.56)

2.4 Voltage Constraints For a Two-Phase Bridge

There are several conditions that are requirements for proper rectifier action in a two-phase bridge. A condition that insures proper balance between the two-phases is that the two-phase voltages be orthogonal. This is depicted geometrically in Figures 1.3 and 1.4. Stated mathematically for a two-phase set of voltages, $V_1$ and $V_2$, the product of the real parts plus the product of the imaginary parts, or the inner product, must equal zero. In equation form, then

\[
\text{Re}(V_1) \text{Re}(V_2) + \text{Im}(V_1) \text{Im}(V_2) = 0
\]
(2.57)

The voltages $V_1$ and $V_2$ for a Scott connection are given in (2.3) and (2.5), respectively. Substituting into (2.57) yields
\[ \text{Re}(\frac{N_5}{N_1} V_{an}) \text{Re}(\frac{-N_6}{N_{34}} V_{bc}) + \text{Im}(\frac{N_5}{N_1} V_{an}) \text{Im}(\frac{-N_6}{N_{34}} V_{bc}) = 0 \] (2.58)

or

\[ \text{Re}(\frac{-N_4N_5}{N_2N_{34}} V_{bn} - \frac{N_3N_5}{N_2N_{34}} V_{cn}) \text{Re}(\frac{-N_6}{N_{34}} V_{bc}) + \text{Im}(\frac{-N_4N_5}{N_2N_{34}} V_{bn} - \frac{N_3N_5}{N_2N_{34}} V_{cn}) \text{Im}(\frac{-N_6}{N_{34}} V_{c}) = 0. \] (2.59)

Similarly the two-phase voltages for a LeBlanc connection are given in (2.22) and (2.27). Substituting into (2.57) yields

\[ \text{Re}(\frac{N_5}{N_1} V_{ab} - \frac{N_4}{N_3} V_{ca}) \text{Re}(\frac{N_7}{N_1} V_{ab} - \frac{N_6}{N_2} V_{bc} + \frac{N_8}{N_3} V_{ca}) + \text{Im}(\frac{N_5}{N_1} V_{ab} - \frac{N_4}{N_3} V_{ca}) \text{Im}(\frac{N_7}{N_1} V_{ab} - \frac{N_6}{N_2} V_{bc} + \frac{N_8}{N_3} V_{ca}) = 0. \] (2.60)

A second constraint is that the voltages are "balanced". This condition is that the two-phase voltages are equal in amplitude, chosen here to be 1.0 per-unit. The equations expressing this condition for a Scott connection are

\[ \frac{N_5}{N_1} V_{an} = 1.0 \] (2.61)

\[ \left| \frac{N_4N_5}{N_2N_{34}} V_{bn} - \frac{N_3N_5}{N_2N_{34}} V_{cn} \right| = 1.0 \] (2.62)

\[ \left| \frac{N_6}{N_{34}} V_{bc} \right| = 1.0 . \] (2.63)

Similarly for a LeBlanc connection

\[ \frac{N_5}{N_1} V_{ab} - \frac{N_4}{N_3} V_{ca} = 1.0 \] (2.64)

\[ \left| \frac{N_7}{N_1} V_{ab} - \frac{N_6}{N_2} V_{bc} + \frac{N_8}{N_3} V_{ca} \right| = 1.0 . \] (2.65)
2.5 Current Constraints For a Two-Phase Bridge

A solid state diode conducts current whenever it is positive forward biased. Applied to dc rectification, this means that during a given time interval, some diodes will be positive forward biased to conduct current while other diodes will be off. For example, the two-phase bridge in Figure 2.3 with voltage $V_1 = V_m \cos(t)$ and $V_2 = V_m \sin(t)$, has diode 1 forward biased during the interval $0 < t < \pi/4$, and it conducts current to the load during this interval. Also during the interval diode 4 conducts the return current from the load. During the interval $\pi/4 < t < 5\pi/4$ diode 3 conducts current to the load while diode 1 is off, and diode 2 conducts the return current while diode 4 is off. This is a periodic process that repeats with a period of $\pi$.

A simple analysis of the conduction process reveals that the current in the two-phases are opposite, i.e. when one phase is conducting the other must be conducting the return current. Stated mathematically,

$$I_1 = -I_2 .$$

(2.66)

2.6 The Computation of Voltages and Currents in a Two-Phase Bridge

There are numerous ways to calculate voltages and currents in a dc bridge. One approach is consider a dc bridge as a source of harmonic current on the ac side and a source of harmonic voltage on the dc side [9]. It is convenient to use a digital computer to compute the ac-side harmonics. The applied voltage is taken to be 1.0 per-unit magnitude for convenience. Also for convenience, the transformation is taken to be such that 1.0 per-unit Volts occurs at the secondary terminals. The two-phase voltages are then calculated according to (2.3)-(2.5) and (2.22)-(2.27) for the Scott and LeBlanc connections respectively. Once the load current, $I_{dc}$ and subsequently $I_1$ and $I_2$, have been determined then the three phase currents can be calculated. The three-phase currents for the Scott connection can be calculated from (2.18), and those for the LeBlanc connection from (2.44). The symmetrical component currents can then be calculated as well. Those for the Scott from (2.51)-(2.53), and those for the LeBlanc from (2.54)-(2.56).
Figure 2.3. Conduction sequence and resultant voltages and currents for a two-phase bridge.
CHAPTER 3
COMPUTATION RESULTS AND COMPARISON WITH ALTERNATIVE RECTIFIER CONFIGURATIONS

In this chapter the computed currents, voltages, and harmonic currents of rectifiers serviced by various schemes of Scott and LeBlanc transformer connections are presented. This is done for the case of resistive loads on the dc-side and for a parallel R-C load on the dc-side. The harmonic currents for comparable alternative rectification methods are presented as well.

3.1 The Calculation of AC Supply Currents

It is convenient to use a digital computer to calculate the ac-side current waveforms for the Scott and LeBlanc connected converter transformers. In this section, the parameters and assumptions made in these computations are stated. The detailed computation of currents in a rectifier in an actual power system would require much information about the balance of the supply voltages, what the fault capacity of the power system is, what the distributed inductance of the transformer-rectifier connection is, the circuit details of the dc-side, diode characteristics, firing angle and commutation characteristics. Several assumptions were made to reduce the complexity of the calculations:

1) infinite bus with balanced ac voltages
2) ideal transformers
3) ideal diodes.

The effect of considering the ac source to be an infinite bus with balanced voltages is primarily neglecting harmonic current impact on the source voltage. As mentioned in Section 2.6, rectifiers are typically considered sources of harmonic currents on the ac side and harmonic voltages on the dc side, so this assumption has a well-founded basis. Another effect is ignoring the rectifier impact on the frequency behavior of the source. The
system frequency is a function of the load configuration, generation configuration, tie-line considerations, and generation control [19]. Although it is possible to model these, it is beyond the scope of this thesis to consider these effects.

Transformer modeling is a subject of considerable controversy. The actual device is a complex, distributed parameter device which is frequency dependent [20]. In actual practice, most power engineering applications entail the simplified, lumped parameter, tee model [21]. For purposes of calculations in this thesis, a simple series R-L equivalent is used.

Diodes are also complex due to their nonlinear V-I characteristics [22]. for our purposes, however, the diodes are considered ideal switches at power frequency since the forward voltage drop and forward holding current during conduction are minimal [23].

3.2 Connection Schemes

There are at least two different methods, or schemes, of paralleling a Scott connection with a LeBlanc connection. The usual method of connection is with the voltages and currents of the two transformer connections to be in phase on the two-phase side [see Figure 1.3 and Figure 1.4]. This scheme of operation results in identical ac side performance for either connection, as well as identical dc side performance. As will be shown shortly, this scheme results in sinusoidal ac side currents. Thus, this method of parallel operation will be called the sinusoidal scheme (SS). Because the Scott and LeBlanc connections have identical operating characteristics in the SS, they will not be considered separately. They will, instead, be analyzed in parallel operation keeping in mind that the output, harmonic contribution, etc. of only one unit is one-half that of the group total.

Another method of parallel operation is to introduce a 180° phase reversal in either transformer connection of $V_1$ or $V_2$. This scheme results in non-sinusoidal currents. This method of parallel operation will be called the non-sinusoidal scheme (NSS). With the either case the rectified output of the parallel bridge can be either filtered or unfiltered. Both cases for both schemes will be observed and their behavior analyzed.

3.3 Sinusoidal Scheme Without DC Filtering

A study of the SS of parallel operation [see Figure 1.6 for reference] without filtering on the dc side with the load being a 1.0Ω resistor was
performed. A complete set of graphs showing the two-phase voltages and currents, and the three-phase currents for the three-phase system are shown in Figures 3.1 to 3.10. The three-phase voltages were omitted because they were stated in Section 2.6 as being a balanced set of 1.0 per-unit magnitude.

The difference between the two-phase voltages shown in Figures 3.1 and 3.2 is rectified to produce the dc voltages which are shown in Figures 3.3 and 3.4. The sum of the rectified voltages from the LeBlanc and Scott serviced converters is then the rectified voltage delivered to the load which is shown in Figure 3.5. The two-phase currents are shown in Figures 3.6 and 3.7. The corresponding three-phase currents are shown in Figures 3.8 and 3.9. The total three-phase currents are shown in Figure 3.10. The three-phase currents are all sinusoidal, thus no harmonic currents exist. The rms voltage at the load is 2.0 V. The ripple voltage can be defined as

\[ \delta V = 0.5 (V_{\text{max}} - V_{\text{min}}) \]  

In such case, \( \delta V \) is 1.414 V.

### 3.3.1 Unbalance in the Unfiltered Sinusoidal Scheme

The three-phase currents are not distorted with harmonic currents for the unfiltered SS case. However, inspection of Figure 3.10 reveals that there are serious problems with the balance of the currents. The A-phase current has a peak amplitude of 3.77, B-phase 1.38, and C-phase 5.15. The A- and B-phase currents are in phase at \(-45^\circ\) and the C-phase currents is \(180^\circ\) put of phase with A and B at \(135^\circ\).

Unbalance in three-phase systems can be analyzed in terms of balanced phasors by the method of symmetrical components as mentioned in Section 2.3. This method applied to the Scott and LeBlanc connections using (2.51)-(2.56) reveals that the positive and negative sequences are of the same magnitude and \(90^\circ\) out of phase. Figures depicting the positive and negative sequence magnitudes and relative phase angles for the LeBlanc connection, Scott connection, and the parallel combination (or total three-phase) are shown in Figures 3.11 to 3.13.

A standard of three-phase system unbalance, \( u \), defined by Wagner and Evans [24] is
The unbalance factor can range from 0 for no negative sequence, to infinity for no positive sequence, and it can have any complex value in between. The most desirable case is \( u = 0 \). As applied to the LeBlanc, Scott, and parallel combination, \( u = 0 + j1.0 \) for each case. A magnitude of 1.0 for \( u \) reaffirms that the positive and negative sequence amplitudes are equal. This could be undesirable operation in an actual power system. Under low power and/or low load applications, this condition could be acceptable. Under high power conditions, this imbalance may not be tolerable.

### 3.3.2 Power Factor Correction of the Unfiltered Sinusoidal Scheme

The three-phase currents, again referring to Figure 3.10, were found to be not only unbalanced in magnitude, but in phase as well. In the sinusoidal steady state the phase unbalance can be overcome with power factor correction. The A-phase power factor is 0.707 lagging \( (\phi = -45^\circ) \), the B-phase power factor is 0.259 leading \( (\phi = 75^\circ) \), and the C-phase current power factor is 0.966 leading \( (\phi = 15^\circ) \). The amount of \( Q \) needed to inject into the A-phase to bring it up to \( pf = 0.966 \) lagging is approximately 1.95 var. The amount of \( Q \) needed to be consumed in the B-phase to bring it down to a \( pf = 0.966 \) lagging is approximately 1.43 var. Finally, the amount of \( Q \) needed to be consumed to bring the C-phase down to \( pf = 0.966 \) lagging is approximately 2.66 var.

### 3.4 Sinusoidal Scheme With DC Filtering

The load voltage for no filtering [see Figure 3.5] is not "dc". It has a time-varying nature characterized by \( |V_m \sin \omega t| \). Up to this point, the only type of load considered has been a resistance. In addition, the reactances of the transformers have been neglected because they are assumed to simply supply a phase shift to the ac system. If it is desired to filter the voltage supplied to the load, the transformer reactances must be taken into account. A diagram of an eight-pulse bridge with output filtering is in Figure 3.14.

For the diagram shown in Figure 3.14 representative values of \( R, L, \) and \( C \) were chosen. They are

\[
R = 1.0 \ \Omega
\]
The resultant voltage waveform at the load and the three-phase current waveform are shown in Figures 3.15 and 3.16, respectively. The dc voltage is nearly perfectly flat with very little ripple. The ac current waveforms are square waves with period $2\pi$. Shown in Figure 3.17 is the harmonic amplitude and spectrum of the three-phase currents.

3.4.1 Harmonic Analysis of the Filtered Sinusoidal Scheme

The filtered SS, like its unfiltered counterpart, has unbalanced three-phase currents. However, the harmonic content in each phase will be identical except for magnitude because the waveforms in each phase are the same, except for amplitude. The total harmonic distortion (THD) for this waveform is 48.59% and the telephone influence factor (TIF) is 794. Appendix A contains the method by which these were calculated. Appendix B contains the data created by the calculations.

3.4.2 Harmonic Sequence of the Filtered Sinusoidal Scheme

The harmonic currents can be analyzed by the method of symmetrical components using Equation 2.46 as the transformation. The method of calculating the harmonic sequences is contained in Appendix C, and the data calculated is contained in Appendix D. The positive and negative sequence are both present in all harmonics. In addition, the positive and negative sequences have the same magnitude at all harmonics. However, there is no zero sequence present. The relative magnitudes of the harmonic sequences are the same as that of the harmonics themselves. The phase angle of the positive sequence harmonics starts at 360° (0°) and rotates +90° at every odd harmonic. The phase angle of the negative sequence harmonics is always 90° lagging.

3.5 Unfiltered and Filtered Non-sinusoidal Scheme

By introducing a phase shift of 180° in $V_2$ in the Scott connection, a more desirable dc side waveform was obtained for the unfiltered case, as shown in Figure 3.18. The rms voltage is approximately 1.83 V, which is lower than that for the SS (2.0 V). The ripple as defined in Equation 3.1 is 0.292 V for the NSS which is significantly lower than the ripple voltage for
the SS case (1.414 V). The voltage at the load for the filtered NSS case, shown in Figure 3.19, is of the same form as that for the SS case. The three-phase currents for the unfiltered case are shown in Figures 3.20 to 3.22. They are considerably worse in unbalance and harmonic content than any other rectification scheme. The three-phase currents for the filtered case are shown in Figures 3.23 to 3.25. They, too, are considerably worse in unbalance and harmonic content than any other rectification scheme. Because of these facts, the NSS will not be analyzed or compared with any other rectification schemes; they have been presented for completeness of the topic.

3.6 Alternative Rectification Methods

The two-phase, four-pulse method of rectification employing Scott and LeBlanc connected transformers to derive two-phase power from three-phase is in a league by itself. There are no other schemes of rectification using the same method (i.e. two-phase, four-pulse) with which to compare. There are other methods of rectification which are similar. The most similar method is three-phase, six-pulse bridge, or in other words, the Graetz bridge. Alternative forms of this scheme of rectification are derived by changing the transformer connections supplying the rectifier. The conventional connection schemes used are the delta-delta and wye-delta connections. The work done by Pinneo [23] in this area will be used as a basis for comparison.

3.7 Discussion of Comparison with Alternative Rectification Methods

Total harmonic distortion is a measure of harmonic magnitudes compared with the fundamental frequency signal. With respect to power systems, the THD is a measure of distortion compared to the 60 Hz current. There are no harmonic distortion standards in the United States, however limits have been proposed [25]. For the 2.4 kV to 69 kV voltage class a limit of 5.0% THD has been proposed. For dedicated systems a limit of 8.0% THD has been proposed.

The effect a power system has on communication circuits is not uniform over the audio frequency range. Therefore, a weighted average of effect power frequency harmonics have on communication circuits was developed, and that is the telephone influence factor. The TIF weights frequencies in the audible hearing range much more heavily than those outside that range. The TIF is a dimensionless number the measures the cumulative effect power system harmonics have on communication circuits.
For conventional rectifier circuits the harmonic sequences are

positive \quad 1,4,7,...,3n+1
negative \quad 2,5,8,...,3n-1
zero \quad 3,6,9,...,3n

Conventional rectifiers typically have only positive and negative sequence harmonics present.

The THD and TIF for the connections mentioned in Section 3.9 are summarized in Table 3.1. The corresponding harmonic information is contained in Appendix E [23].

Table 3.1. THD and TIF for delta-delta and wye-delta connections.

<table>
<thead>
<tr>
<th></th>
<th>Delta-Delta</th>
<th>Wye-Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>THD%</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>TIF</td>
<td>247</td>
<td>267</td>
</tr>
</tbody>
</table>

3.8 Comparison of THD and TIF with Alternative Connections

The THD and TIF for the unfiltered SS are, basically, zero because there are no harmonic currents generated. The filtered SS, as mentioned in Section 3.7, has a THD of 46.6% and a TIF of 794. The THD and TIF of the unfiltered SS, the delta-delta connected Graetz bridge, and the wye-delta Graetz bridge are summarized in Figure 3.26.

The filtered SS case exhibits a much higher harmonic current distortion than either of the other two cases. This is attributable to the square wave nature of the filtered SS case. The telephone influence factor is also much worse than the other two cases. This is attributable to the harmonic spectrum generated by the filtered SS.

3.9 Comparison of Harmonics with Alternative Connections.

The harmonic spectra produced by the four-pulse and six-pulse bridges are quite different. The harmonic order produced is given by Equation 1.2, repeated here for reference
where \( p \) is the pulse number and \( k = 1,2,3,\ldots,n \). The harmonic spectrum of a four-pulse rectifier is, then, \( 3,5,7,9,\ldots \) whereas the spectrum of a six-pulse rectifier is \( 5,7,11,13,17,19,\ldots \).

The harmonic magnitudes decrease as \( 1/n \) for both the four-pulse and six-pulse methods. However, the noncharacteristic harmonics for conventional six-pulse rectifiers, i.e. \( 3,9,15,\ldots \) are present in the four-pulse rectifier. The third harmonic alone is of great concern because of its potentially damaging effects to nearby induction machines. A diagram of the relative harmonic magnitude for the filtered SS, delta-delta connected Graetz bridge, and wye-delta connected Graetz bridge is also in Figure 3.26.

The sequence of the harmonics in the four-pulse rectifier are also different than that for conventional rectifiers. The harmonic sequence for the four-pulse circuit is

- positive: \( 1,3,5,\ldots \)
- negative: \( 1,3,5,\ldots \)
- zero: \( 0 \)

This is not a surprise in light of the fact that the positive and negative sequences were found to have the same magnitude for the fundamental frequency in Section 3.4 for the unfiltered SS.

### 3.10 Summary

A two-phase, four-pulse rectifier can be connected in two different schemes. One scheme without any filtering on the dc side results in unbalanced sinusoidal three-phase currents that have no harmonic impact on the ac supply. The other scheme results in unbalanced non-sinusoidal three-phase currents that have a considerably worse harmonic impact on the ac supply than conventional rectifier systems. The first scheme can have filtering on the dc side to produce a better dc voltage waveform. Its harmonic impact on the ac supply is worse than conventional rectifiers. It produces noncharacteristic harmonics that adversely affect the THD and TIF. The harmonic sequence currents contain equal positive and negative currents for all harmonics.
Figure 3.1. LeBlanc two-phase voltages.
Figure 3.2. Scott two-phase voltages.
Figure 3.3. Rectified voltage of a LeBlanc serviced converter.
Figure 3.4. Rectified voltage of a Scott serviced converter.
Figure 3.5. Rectified voltage delivered to the load.
Figure 3.6. LeBlanc two-phase currents.
Figure 3.7. Scott two-phase currents.
Figure 3.8. LeBlanc three-phase currents.
Figure 3.9. Scott three-phase currents.
Figure 3.10. Total three-phase currents.
Figure 3.11. LeBlanc connection sequence currents.
Figure 3.12. Scott connection sequence currents.
Figure 3.13. Parallel combination (total three-phase) sequence currents.
Figure 3.14. Parallel four-pulse rectifiers with filtered output.
Figure 3.15. Rectifier load voltage with filtering.
Figure 3.16. Three-phase currents with filtering.
Figure 3.17. Harmonic amplitudes versus frequency.
Figure 3.18. Load voltage with reversed two-phase sequence (NSS configuration).
Figure 3.19. Filtered load voltage with reversed two-phase sequence (NSS configuration).
Figure 3.20. Phase A current, reversed two-phase sequence (NSS configuration) with no filtering.
Figure 3.21. Phase B current, reversed two-phase sequence (NSS configuration) with no filtering.
Figure 3.22. Phase C current, reversed two-phase sequence (NSS configuration) with no filtering.
Figure 3.23. Phase A current, reversed two-phase sequence (NSS configuration) with filtering.
Figure 3.24. Phase B current, reversed two-phase sequence (NSS configuration) with filtering.
Figure 3.25. Phase C current, reversed two-phase sequence (NSS configuration) with filtering.
Figure 3.26. Comparison of THD and TIF.
CHAPTER 4
CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

A four-pulse rectifier serviced by a Scott or LeBlanc connection has considerably different operating characteristics than a conventional Graetz bridge serviced by delta-delta or wye-delta transformer connections. In addition, a four-pulse rectifier generates harmonic currents that are considerably different than those generated by six-pulse rectifier. Some conclusions which follow from these observations are:

- Two four-pulse bridges operated in parallel serviced by Scott and LeBlanc transformer connections can have two schemes of operation. One produces pulsating dc power with sinusoidal ac currents, and the other produces more constant dc power with non-sinusoidal ac currents.
- A four-pulse rectifier will draw power from each phase of the three-phase supply. However, the power and current is unbalanced in phase and magnitude.
- A four-pulse rectifier serviced by a Scott or LeBlanc transformer connection will produce non-characteristic harmonics. The current harmonics exist at every odd harmonic.
- The magnitudes of the positive and negative sequences are equal at each harmonic frequency.
- The total harmonic distortion is higher for a four-pulse rectifier than for a conventional rectifier. The THD in each phase is the same.
- The telephone influence factor calculated was significantly higher for the Scott and LeBlanc serviced rectifiers than for conventional rectifiers.
- The presence of all odd harmonics in the Scott and LeBlanc serviced rectifiers results in a higher spectrum of frequencies. This causes the increase in the TIF.
The Scott and LeBlanc connections may be applicable for servicing four-pulse and six-pulse rectifiers under certain conditions. At lower current levels where phase unbalance does not significantly effect power systems, these connections may be appealing. Further, these nonstandard connections may find application in cases which do not have stringent ripple requirements. Their use in emergency contingencies where a six-pulse rectifier has lost a phase is feasible under low power conditions.

4.2 Recommendations

The noncharacteristic harmonics produced by four-pulse rectifiers serviced by Scott and LeBlanc transformer connections would certainly require harmonic filters to minimize their effect on the power grid. The cost of filtering versus transformer costs should be investigated to determine the power level at which four-pulse rectifiers are economic.

Of additional interest is the prospect of using pulse modulation (e.g., pulse width modulation, pulse position modulation) to shift the spectrum of the ac side harmonics to higher frequencies. This would reduce the filter requirements. However, there would be incurred a higher cost of electronic components to implement the modulation. This tradeoff and application of four-pulse bridges should be studied.

The possibility remains of supplying multiple sets of parallel LeBlanc and Scott connected rectifiers. This may produce equal current magnitudes in the three-phase supply: this should be investigated.
LIST OF REFERENCES
LIST OF REFERENCES


APPENDICES
Appendix A - Harmonic Magnitude and Phase, Total Harmonic Distortion, and Telephone Influence Factor Calculations

The Fourier series representation of a periodic time function is

\[ x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n \omega_0 t + b_n \sin n\omega_0 t \]  

(A.1)

where \( a_0 \) is the average value, \( \omega_0 \) is the fundamental frequency of \( x(t) \), and the coefficients \( a_n \) and \( b_n \) are

\[ a_n = \frac{2}{T} \int_{t}^{t+T} x(t) \cos n\omega_0 t \, dt \]  

(A.2)

and

\[ b_n = \frac{2}{T} \int_{t}^{t+T} x(t) \sin n\omega_0 t \, dt \]  

(A.3)

For the current waveforms in Figure 3.10, \( x(t) \) is piece-wise continuous signal that can be broken into regions of constant amplitude for integrating to get \( a_n \) and \( b_n \). If the 0 to peak amplitude of the A-phase is, say, \( A \) then the the integrals for \( \omega_0 = 1.0 \text{ rad/s} \) and \( T = 2\pi \) are

\[ a_n = \frac{2}{2\pi} \int_{\pi/4}^{3\pi/4} A \cos nt \, dt + \frac{2}{2\pi} \int_{3\pi/4}^{7\pi/4} -A \cos nt \, dt \]  

(A.4)

and

\[ b_n = \frac{2}{2\pi} \int_{\pi/4}^{3\pi/4} A \sin nt \, dt + \frac{2}{2\pi} \int_{3\pi/4}^{7\pi/4} -A \sin nt \, dt \]  

(A.5)

Upon evaluation, then, \( a_n \) and \( b_n \) are
The current waveforms in Figure 3.20 have an average value of zero by inspection. Therefore the first term in (A.1) can be dropped and \( a_n \) and \( b_n \) are given by (A.6-7), respectively.

The magnitude of the \( n \)th harmonic is

\[
H_n = \sqrt{a_n^2 + b_n^2}
\]

and the phase angle is

\[
\phi_n = \arctan\left(\frac{b_n}{a_n}\right)
\]

The total harmonic distortion is defined as

\[
\text{THD} = \frac{\left[\sum_{n=2}^{\infty} H_n^2\right]^{1/2}}{H_1}
\]

The telephone influence factor is defined as

\[
\text{TIF} = \frac{\left[\sum_{n=2}^{\infty} w_n^2 H_n^2\right]^{1/2}}{\left[\sum_{n=1}^{\infty} H_n^2\right]^{1/2}}
\]

where \( w_n \) is a weighting factor associated with the \( n \)th harmonic. The following is a listing of a program used to calculate the Fourier series components, magnitudes and phase angles, THD, and TIF.

```plaintext
c----------
c amp = vector of current magnitudes
```
can = vector of a coefficients
bn = vector of b coefficients
mag = vector of harmonic magnitudes
nend = number of coefficients required
phase = vector of phase angles
thd = thd
tif = tif

open(1,file='coeffA')
rewind 1
open(2,file='coeffB')
rewind 2
open(3,file='coeffC')
rewind 3
open(4,file='weights')
rewind 4
open(5,file='phaseA')
rewind 5
open(6,file='phaseB')
rewind 6
open(7,file='phaseC')
rewind 7

amp(1) = 3.77
amp(2) = 1.38
amp(3) = -5.15
nend = 30

do 200 m = 1,3

  if(m.eq.1) write(1,10)
  if(m.eq.2) write(2,11)
  if(m.eq.3) write(3,12)

10 format('Phase A',/,'n',5x,'An',10x,'Bn',10x,'Mag',10x,'Phase')
11 format('Phase B',/,'n',5x,'An',10x,'Bn',10x,'Mag',10x,'Phase')
12 format('Phase C',/,'n',5x,'An',10x,'Bn',10x,'Mag',10x,'Phase')

do 100 n = 1,nend
call coeffs(n,amp(m),an(n),bn(n),mag(n),phase(n))

if(m.eq.1) write(1,20) n,an(n),bn(n),mag(n),phase(n)
if(m.eq.2) write(2,20) n,an(n),bn(n),mag(n),phase(n)
if(m.eq.3) write(3,20) n,an(n),bn(n),mag(n),phase(n)

20 format (i4,3x,f8.4,4x,f8.4,4x,f8.4,4x,f8.2)

if(m.eq.1) write(5,25) mag(n),phase(n)
if(m.eq.2) write(6,25) mag(n),phase(n)
if(m.eq.3) write(7,25) mag(n),phase(n)

25 format(2fl4.7)

100 continue

call cthd(nend,mag,thd)
if (m.gt.1) rewind 4
call ctif(nend,mag,tif)

if (m.eq.1) write(1,110) thd
if (m.eq.2) write(2,110) thd
if (m.eq.3) write(3,110) thd
if (m.eq.1) write(1,120) tif
if (m.eq.2) write(2,120) tif
if (m.eq.3) write(3,120) tif

110 format(/,'Total Harmonic Distortion = ',f8.5)
120 format(/,'Telephone Influence Factor = ',f8.3,/)

200 continue

end

subroutine catan(x,y,ang)
c-----------------
c subroutine catan calculates the
c angle in degrees of any rectangular
c coordinates in the any of the four
c quadrants
real ang, arg, c, epsilon, pi, r, q, x, y

epsilon = 1.0e-5
pi = 3.141592653589793
r = abs(x)
q = abs(y)
arg = q/r
c = 0.0

if (r.lt.epsilon.and.q.lt.epsilon) go to 100
if (r.lt.epsilon.and.y.gt.0.0) c = 90.0
if (r.lt.epsilon.and.y.lt.0.0) c = 270.0
if (x.gt.0.0.and.y.gt.0.0) c = (180.0/pi)*atan(arg)
if (x.lt.0.0.and.y.gt.0.0) c = 180.0-(180.0/pi)*atan(arg)
if (x.lt.0.0.and.y.lt.0.0) c = 180.0+(180.0/pi)*atan(arg)
if (x.gt.0.0.and.y.lt.0.0) c = 360.0-(180.0/pi)*atan(arg)
100 ang = c
return
end

subroutine coeffs(n,amp,a,b,mag,phase)

integer n
real a,amp,b,mag,phase,pi

pi = 3.141592653589793

a = 2*sin(0.75*pi*n)-sin(-0.25*pi*n)-sin(1.75*pi*n)
a = amp*(a/(n*pi))

b = 2*cos(0.75*pi*n)-cos(-0.25*pi*n)-cos(1.75*pi*n)
b = amp*(b/(n*pi))
mag = sqrt(a\*a+b\*b)
call catan(a,b,phase)

return
dern

subroutine cthd(n,x,thd)
c-------------------
c subroutine cthd calculates
c the THD for a waveform
c-------------------
  integer i,n
  real x,thd
  dimension x(30)

  thd = 0.0

  do 10 i = 2,n
    thd = thd + x(i)*x(i)
  10 continue

  thd = sqrt(thd)/x(1)

  return
dern

subroutine ctif(n,x,tif)
c---------------
c subroutine ctif calculates
c the TIF for a waveform
c---------------
  integer i,n
  real x,tif,weight,top,bot
  dimension x(30),weight(30)

  do 10 i = 1,n
    read(4,5) weight(i)
  5 format(f7.1)
10 continue

    top = 0.0
    do 20 i = 2,n
        top = top + weight(i)*weight(i)*x(i)*x(i)
    20 continue
    top = sqrt(top)

    bot = 0.0
    do 30 i = 1,n
        bot = bot + x(i)*x(i)
    30 continue
    bot = sqrt(bot)

    tif = top/bot

    return
end
### Table B1 Phase A harmonics.

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Total Harmonic Distortion = 0.46588
Telephone Influence Factor = 794.114
Table B2 Phase B harmonics.

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Total Harmonic Distortion = 0.46588
Telephone Influence Factor = 794.113
Table B3 Phase C harmonics.

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Total Harmonic Distortion = 0.46588
Telephone Influence Factor = 794.113
Appendix C - Calculation of Positive and Negative Sequence Components

The method of symmetrical components, used to study unbalanced electric power systems in balanced phasors, is a linear transformation. Symmetrical component currents will be defined as the transformation

\[
\begin{bmatrix}
I_+ \\
I_- \\
I_0
\end{bmatrix} = M
\begin{bmatrix}
I_{an} \\
I_{bn} \\
I_{cn}
\end{bmatrix}
\]  \hspace{1cm} (C.1)

where

\[
M = \frac{1}{\sqrt{3}}
\begin{bmatrix}
1 & a & a^2 \\
1 & a^2 & a \\
1 & 1 & 1
\end{bmatrix}
\]  \hspace{1cm} (C.2)

and

\[
a = 1/120^\circ \quad a^2 = 1/240^\circ
\]

The harmonic sequences can be calculated by adding the appropriate phase shift to a line current then perform the complex arithmetic to get the complex rectangular form of the sequence. The polar complex form can be easily computed once the rectangular form is known. The following is a listing of a computer program to find the harmonic sequence magnitudes. Note: the zero sequence is not computed in this program because the systems under consideration in this research contained no zero sequence.
integer n
---------
c n = the number of harmonics the sequences are to be
c computed for.
---------
real cneg, posre, posim, negre, negim, posmag, posang, negmag, negang
type real amag, aph, bmag, bph, cmag, cph, bpos, bneg, cpos, k, rad3

c The files "phaseA", "phaseB", and "phaseC" need to have the magnitude
c and angle (in degrees) of the first n harmonics of the A-phase, B-phase
c and C-phase, respectively, in 2f14.7 format. The file "output" will
c contain the generated output.

open(1, file='phaseA')
rewind 1
open(2, file='phaseB')
rewind 2
open(3, file='phaseC')
rewind 3
open(4, file='output')
rewind 4

k = 3.1414926535/180.0
rad3 = 1.7302050808

write (4,1)
1 format(' N Pos(mag) Pos(ang) Neg(mag) Neg(ang) ')
do 100 n = 1,30

read(1,10) amag, aph
read(2,10) bmag, bph
read(3,10) cmag, cph
10 format(2f14.7)

aph = k*aph
bpos = k*(bph + 120.0)
\[ b_{\text{neg}} = k \cdot (b_{\text{ph}} + 240.0) \]
\[ c_{\text{pos}} = k \cdot (c_{\text{ph}} + 240.0) \]
\[ c_{\text{neg}} = k \cdot (c_{\text{ph}} + 120.0) \]

\[ \text{posre} = \text{amag} \cdot \cos(\text{aph}) + \text{bmag} \cdot \cos(\text{bpos}) + \text{cmag} \cdot \cos(\text{cpos}) \]
\[ \text{posim} = \text{amag} \cdot \sin(\text{aph}) + \text{bmag} \cdot \sin(\text{bpos}) + \text{cmag} \cdot \sin(\text{cpos}) \]
\[ \text{negre} = \text{amag} \cdot \cos(\text{aph}) + \text{bmag} \cdot \cos(\text{bneg}) + \text{cmag} \cdot \cos(\text{cneg}) \]
\[ \text{negim} = \text{amag} \cdot \sin(\text{aph}) + \text{bmag} \cdot \sin(\text{bneg}) + \text{cmag} \cdot \sin(\text{cneg}) \]

\[ \text{posmag} = \sqrt{\text{posre} \cdot \text{posre} + \text{posim} \cdot \text{posim}} \]
\[ \text{call catan(posre, posim, posang)} \]

\[ \text{negmag} = \sqrt{\text{negre} \cdot \text{negre} + \text{negim} \cdot \text{negim}} \]
\[ \text{call catan(negre, negim, negang)} \]

\[ \text{write(4,20) n, posmag, posang, negmag, negang} \]
\[ 20 \text{ format(i4,3x,f7.4,3x,f7.2,3x,f7.4,3x,f7.2)} \]

100 continue

end
### Appendix D - Calculated Harmonic Sequence Data

Table D1  Harmonic sequence data.

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Appendix E - Harmonic Sequence Data for Alternative Rectification Methods

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Table E2 Wye-delta connected Graetz bridge.

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