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Z. Zicheng

J. F. Hamilton

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Zicheng Zhou[1] and James F. Hamilton[2]

[1]Power Machinery Department, Xi'an Jiaotong University, Xi'an, Shaanxi Province, CHINA, [2]Ray W. Herrick Laboratories, Purdue University, West Lafayette, IN 47907, U.S.A.

ABSTRACT

A simulation model for multicylinder reciprocating refrigerating compressors has been developed. In this model, the following concepts have been considered: real gas properties; heat transfer between the gas and the cylinder wall during the working process; heat and mass transfer between the suction gas and the gas in the clearance volume; heat transfer between the gas and plenum wall; gas leakage through the clearance between piston ring and the cylinder wall; and pressure variations in the suction and discharge plenums.

Several efficiencies have been discussed which can be used to evaluate the compressor performance and to optimize the compressor parameters.

NOMENCLATURE

A - area M^2

C - specific heat J/Kg

c - valve stiffness Kg/M

d - cylinder bore M

e - internal energy J/Kg

E - excergy J/Kg

F - heat transfer area or force M^2 or N

h - heat transfer coefficient $J/(M^2 \cdot {}^oC)$

H - valve Lift M

k - ratio of specific heats

L - conecting rod length M

M - mass Kg

N - the number of valve plates

p - pressure Bar or N/M^2

Q - heat flow J

R - gas constant J/Kg^oC

r - crank radius M

S - stroke M

T - absolute temperature K

t - temperature ^oC or time S

U - the prepressing length of spring M

V - volume M^3

v - specific volum M^3/Kg

W - work J

Z - compressibility of gas refrigerant

Nu- Nusselt number

Pr- Prandlt number

Re- Reynolds number

RPM- the speed of the compressor revoluton per minite

α - coefficient of flow through the valve

ω - angule velocity of crank shaft Deg/S

ρ - density $Kg/(M^3)$

ε - pressure ratio

φ - the crank angle

λ - the ratio between the crank radio and conecting rod length

Δ - difference

ξ - coefficient of impulsive force

SUBSCRIPTS

b - brake

c - cylinder

d - discharge

e - evaporator

k - condeser
s - suction
v - valve
o - clearance
w - cylinder wall
in - flow into the cylinder
out- flow out of the cylinder
pl - plenum
ex - exergy

INTRODUCTION

Computer simulation and modeling of the reciprocating refrigeration compressor has been developing repidly in recent years and has become a useful tool for analyzing and improving compressor performance. Different models for predicting the performance of the compressor have been developed which differ in the completeness of the modeling of the physical processes.

The modeling of a multicylinder refrigerating reciprocating compressor is described in this paper. The interface between the cylinders must the considered in any multicylinder compressor modeling. Since the gas pressure and the temperature in the cyliner will be effected by the pressure and temperature in the suction and discharge plenums.

Computer simulation and modeling of reciprocating compressors is a combination of mathmatical modeling and numerical analysis. It is important to use mathmatical equations which will represent all physical phenomenona occuring in the compressor. Unfortunately, there are some that are difficult to include in our mathmatical model, such as: the heat transfer between the gas and the valve passage and the lubricating oil dissolved into and disasociated from the refrigerant. However, it is also possible to neglect some insignificant physical phenomenona in order to simplify the program and save computation time.

The numerical simulation method depends upon the model. A fourth order Runge-Kutta method is commonly used to solve the differential equations. This method is used in this program.

THE INTERFACE BETWEEN THE CYLINDERS

There are two kinds of multicylinder reciprocating compressor schemes used in refrigeration systems: the single stage and the multiple stage. In the former case, usually in the one stage refrigeration system, all cylinders are the same bore. In the latter case, usually in the two stage or three stage system, the bores could be different from each other, and have intercoolers connected between the different stage cylinders.

A typical system that was choosen for simulated is the vertical type, two cylinder, single stage, ring valve, reciprocating compressor (Fig.1) having a suction plenum around the cylinder and a discharge plenum above the cylinder head. The crank angle between the two cylinders is 180°.

If the cylinder bore and the stroke of two cylinders are the same, and the angle between two crank is γ, then the pressure (P_c), temperature (t_c), and the volume (V_c) of the first cylinder at the crank angle φ equal the P_c, t_c, V_c, of second cylinder at the crank angle $\varphi + \gamma$ deg. (in this scheme, γ equals 180°). Therefore,

$$P(2)|_{\varphi} = P(1)|_{\varphi+\gamma} = P(1)|_{\varphi+180^\circ}$$
$$T(2)|_{\varphi} = T(1)|_{\varphi+\gamma} = T(1)|_{\varphi+180^\circ}$$
$$V(2)|_{\varphi} = V(1)|_{\varphi+\gamma} = V(1)|_{\varphi+180^\circ}$$

It can be seen from above equations that all gas properties in the two cylinders will be the same if we compare the gas parameters P(1), V(1), T(1) at the crank angle φ with the gas parameters P(2), V(2), T(2) at the crank angle φ + 180° respectively. This method is called the characteristic cylinder method [1,2].

If the cylinder bore and the stroke of two cylinders are not the same, the relationship will be different from above.

In order to use the same mathmatical equations in one program, it was necessary take the different cylinders as an array in the two cylinder, single stage case:

Crank angle	$\varphi(2) = \varphi(1) + \gamma$	(1)
Cylinder bore	$D(2) = D(1)$	(2)
Stroke	$S(2) = S(1)$	(3)

Speed $\qquad rpm(2) = rpm(1)$ (4)

This method is more general than the characteristic cylinder method.

MATHMATICAL MODEL

1. Cylinder Model

1.1 The energy conservation equation

The first law of thermodynamics applied to the cylinder control volume (Fig.2) is

$$\dot{Q}(i) = M(i)C_V(i)\dot{T}(i) + \dot{W}(i) \tag{5}$$

using a compressibility coefficient z,

$$\dot{T}(i) = \frac{\dot{Q}(i)}{M(i)\,C_v(i)} - \frac{z(i)\,R\,T(i)}{C_v(i)\,V(i)}\,\dot{V}(i) \tag{6}$$

or

$$\frac{dT(i)}{d\varphi} = \frac{dQ(i)}{M(i)\,C_v(i)\,d\varphi} - \frac{z(i)\,R\,T(i)}{C_v(i)\,V(i)}\,\frac{dV(i)}{d\varphi} \tag{7}$$

1.2 The cylinder volume equation is

$$V(i) = \frac{\pi}{4} D^2(i)\,S(i)\,\{V_0 + \frac{1}{2}[1 - \cos(\varphi(i))$$

$$+ \frac{\lambda(i)}{8}(1 - \cos(2\varphi(i))]\} \tag{8}$$

The crank angle φ is measured from T.D.C. of the first cylinder.

1.3 Real gas equation

The properties p,v,t,h,s of real gas refrigerants can be determined by eigher the Martin-Hou equations [3] or the Rumbusch equations [4,5]. They can also be found from other equations[6,7].

After selecting the P,V,T relationship for a given refrigerant, the compressibility coefficient Z can be found [8].

$$z(i) = \frac{p(i)\,v(i)}{R\,T(i)} \tag{9}$$

1.4 Heat transfer equation

The equation of heat transfer between the gas and the cylinder wall is

$$\dot{Q}(i) = h(i)F(i)\;T_W(i) - T(i) \tag{10}$$

The coefficient of heat transfer can be calculated by the correlation

$$N_u = AR_e^{B} P_r^{C} \tag{11}$$

The constants, A,B,C, of eq.(11) can be found in literature [9,10].

The temperature distribution on the wall depends on the pressure ratio (ε), the relative distance ($\overline{X}$), and the suction plenum temperature ($t_{s.pl}$). There are correlations which can be used to calculated the temperature distribution on the cylinder wall and cylinder head [10].

1.5 Suction gas and cylinder gas mixture

In the suction process, the temperature variation of the gas is caused by two actions; one is the heat transfer between the gas and the cylinder wall, (equation (10)) and the other is the mixing of the suction gas and the cylinder gas.

The mixed gas in the cylinder can be calculated by using the heat and mass balance equations.

$$\begin{aligned} T(i)^{(2)} = [&M(i)^{(1)} C_p(i)^{(1)} T(i)^{(1)} + (M(i)^{(2)} C_p(i)^{(2)} \\ &- M(i)^{(1)} C_p(i)^{(1)}) T_{s.pl}^{(1)}] / (M(i)^{(2)} C_p(i)^{(2)}) \end{aligned} \tag{12}$$

where (1) - refers to the gas state in cylinder(i) before mixture.

(2) - refers to the gas state in cylinder(i) after mixture.

1.6 The equations of valve action

The equations for calculating the suction and discharge processes of the compressor depends the construction type and motion of the valves. In this ring type valve modeling, one degree of freedom is considered.

1.6.1 Mass flow through the valves

Assuming the gas flow as an adiabatic reversible process and using the fluid dynamic equations, gives:

Suction valve

$$\begin{aligned} \frac{dM_{sv}(i)}{d\varphi} = &\frac{1}{\omega(i)} A_{sv}(i) p_{s.pl} \sqrt{\frac{2K}{(K-1) R T_{s.pl}}} \\ &\times \sqrt{\left(\frac{p(i)}{p_{s.pl}}\right)^{2/K} - \left(\frac{p(i)}{p_{s.pl}}\right)^{(K+1)/K}} \end{aligned} \tag{13}$$

Discharge valve

$$\frac{dM_{d.v}(i)}{d\varphi} = \frac{1}{\omega(i)} A_{d.v}(i)\, p(i) \sqrt{\frac{2K}{(K-1)\, R\, T(i)}}$$

$$\times \sqrt{\left(\frac{p_{d.pl}}{p(i)}\right)^{2/K} - \left(\frac{p_{d.pl}}{p(i)}\right)^{(K+1)/K}} \qquad (14)$$

1.6.2 Valve motion equations

By neglecting the heat transfer during the Rung-kutta integration, the first low of thermodynamics can be applied.

Considering the gas flow through the valves as an adiabatic process and using continuity of gas flow, gives for the valves:

Suction valve

$$\frac{d\psi_{s.v}(i)}{d\varphi} = \frac{-K}{\omega(i)\, V(i)} \left[\psi_{s.v}(i) \frac{\omega(i)\, dV(i)}{d\varphi} - \sqrt{\frac{2RK\, T_{s.pl}}{K-1}}\, \psi_{s.v}^{1/K}(i) \sqrt{1 - \psi_{s.v}(i)^{(K-1)/K}}\right] \qquad (15)$$

Discharge valve

$$\frac{d\psi_{d.v}(i)}{d\varphi} = \frac{-K}{\omega(i)\, V(i)} \left[\psi_{d.v}(i) \frac{\omega(i)\, dV(i)}{d\varphi} + \sqrt{\frac{2KR\, T_{d.pl}}{K-1}}\, \psi_{d.v}^{\frac{K-1}{K}}(i) \sqrt{\psi_{d.v}(i)^{(K-1)/K} - 1}\right] \qquad (16)$$

1.6.3 Valve force equations

From a force balance on the valve, the inertia force equals the applied force of the gas minus the force of spring

$$\ddot{H} = \frac{1}{M_v(i)}\, \xi(i)\, \Delta p(i)\, A_v(i) - \frac{1}{M_v(i)}\, c(i)\left[H(i) + U(i)\right] \qquad (17)$$

Assuming the ratio of the gas pressure in the cylinder and the gas pressure in the plenume can be expresed as ψ, gives for the:

Suction valve

$$\frac{d^2 H_{s.v}(i)}{d\varphi^2} = \frac{(1-\psi_{s.v}(i))\, p_{s.pl}(i)\, \xi_{s.v}(i) A_{s.v}(i) - C_{s.v}(i)(H_{s.v}(i) + U_{s.v}(i))}{M_v(i)\, \omega^2(i)} \qquad (18)$$

and for the discharge valve

$$\frac{d^2 H_{d.v}(i)}{d\varphi^2} = \frac{(\psi_{d.v}(i) - 1)\, p_{d.pl}\, \xi_{d.v}(i) A_{d.v}(i) - C_{d.v}(i)\, (H_{d.v}(i) + U_{d.v}(i))}{M_v(i)\, \omega^2(i)} \qquad (19)$$

1.7 Gas leakage equation

Several references can be used for estimating the gas leakage as in [11,12,13].

The result of the calculations shows that the leakage gas is much smaller then the gas in the cylinder. In this case, it can be neglected.

2. Plenums Model

2.1 The energy conservation equation

2.1.1 Suction plenum

$$\frac{dT_{s.pl}(i)}{d\varphi} = \frac{1}{M_{s.pl}(i)\,C_v(i)}\Big[C_p(i)\Big(\frac{dM_{in}}{d\varphi}T_s - \frac{\Sigma\, dM_s(i)}{d\varphi}T_{s.pl}\Big) + \frac{dQ(i)}{d\varphi}\Big] \tag{20}$$

2.1.2 Discharge plenum

$$\frac{dT_{d.pl}(i)}{d\varphi} = \frac{1}{M_{d.pl}\;C_v(i)}\Big[C_p(i)\Big(-\frac{dM_{out}}{d\varphi}T_{d.pl} - \frac{\Sigma dM_d(i)}{d\varphi}T(i)\Big) - \frac{dQ(i)}{d\varphi}\Big] \tag{21}$$

2.2 Mass flow through plenums

2.2.1 Suction plenum

$$\frac{dM_{in}}{d\varphi} = \frac{1}{\omega(i)}\alpha_{s.pl}A_{s.pl}\sqrt{P_s - P_{s.pl}} \tag{22}$$

2.2.2 Discharge plenum

$$\frac{dM_{out}}{d\varphi} = \frac{1}{\omega(i)}\alpha_{d.pl}A_{d.pl}\sqrt{P_{d.pl} - P_d} \tag{23}$$

THE COEFFICIENT OF PERFORMANCE

Several efficiencies 14 can evaluate the compressor performance from different points of view:

1. Isentropic Efficiency

$$\eta_s = \frac{W_{isen}}{W_{actual}} \tag{24}$$

where W_{isen} is the isentropic indicated work, and W_{actual} is the actual indicated work calculated from the P - φ diagrame.

2. Coefficient of Performance

$$C.O.P = \frac{Q_E}{W_b} \tag{25}$$

where Q_E is the refrigerating capacity of the compressor at the given rating and W_b is the work into in the compressor at the same rating.

3. Excergy Efficiency

$$\eta_{er} = \frac{E_d - E_s}{W_{actual}} \tag{26}$$

where E is the excergy which was calculated by the real gas equations.

4. Volumetric Efficiency

$$\eta_v = \frac{\dot{M} v}{\dot{V}_h} \tag{27}$$

THE NUMERICAL SOLUTION

A fourth order Runge-Kutta integrating method was used for solving the above differential equations. Several subroutines were used in this program, such as the real gas properties subroutine, the suction and discharge processes subroutine, the re-expansion and compression subroutine and the gas leakage subroutine. The heat transfer equation where included in the re-expansion and compression subroutine, the process of suction gas and cylinder gas mixture and the process of heat transfer during the suction and the discharge process were included in the program.

The example compressor chosen to simulate is a two cylinder, single stage, ring valve, R12 refrigerating compressor, D=0.1 M, S=0.07 M, RPM=1440 R.P.M., the ratio between the clearance and the cylinder volume equals 0.04, the evaporating pressure p_e=1.825 Bar, (t_e=-15 °C), the condensing pressure p_k=8.453 Bar, (t_k=35 °C).

Fig.3 - Fig.7 show the p - φ diagram, T - φ diagram, M - φ diagram and H - φ diagram, which were plotted by computer.

It can be seen by comparing the Fig.3 and Fig.7 that the pressure variation will effect to the p - φ

diagram, and also effect to the efficiency of compressor.

CONCLUSIONS

1. A computer modeling for the multicylinder refrigerating reciprocating compressor has been developed. The use of real gas properties produced results closer to the real processes.

2. The general cylinder method is necessary for modeling a multicylinder compressor which has different cylinder diameters.

3. The gas parameters in the cylinder and the efficiencies are effected by the gas parameters in the suction and discharge plenums.

4. Futher investigations should be done which refer to developing the mathmatical equations of the heat transter between gas and valve passage, developing the mathmatical equations described the mechanism of refrigerant solubility in oil during the working processes, and developing the equation discribed the compressor mechanical efficiency at different rating.

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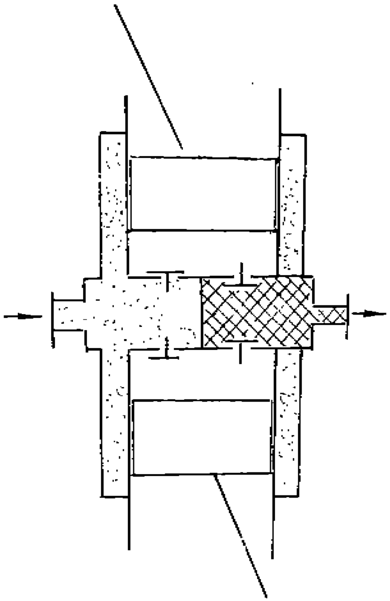

Fig.1 Two Cylinders, One Stage Compressor Scheme (The Crankangle Between Two Cylinders is 180°)

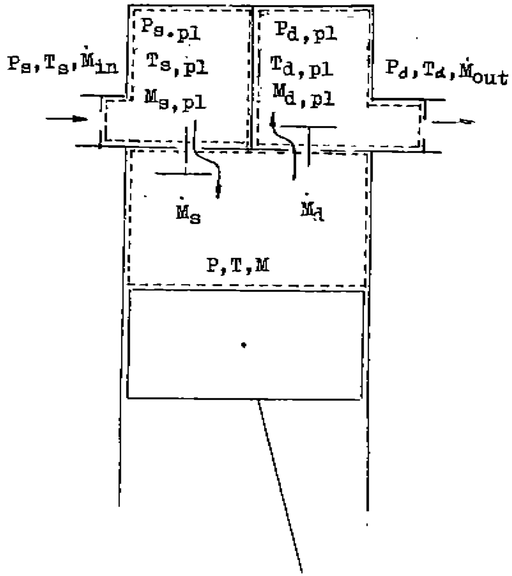

Fig.2 The Cylinder Control Volume, Suction Plenume Control Volume and Discharge Plenume Control Volume.

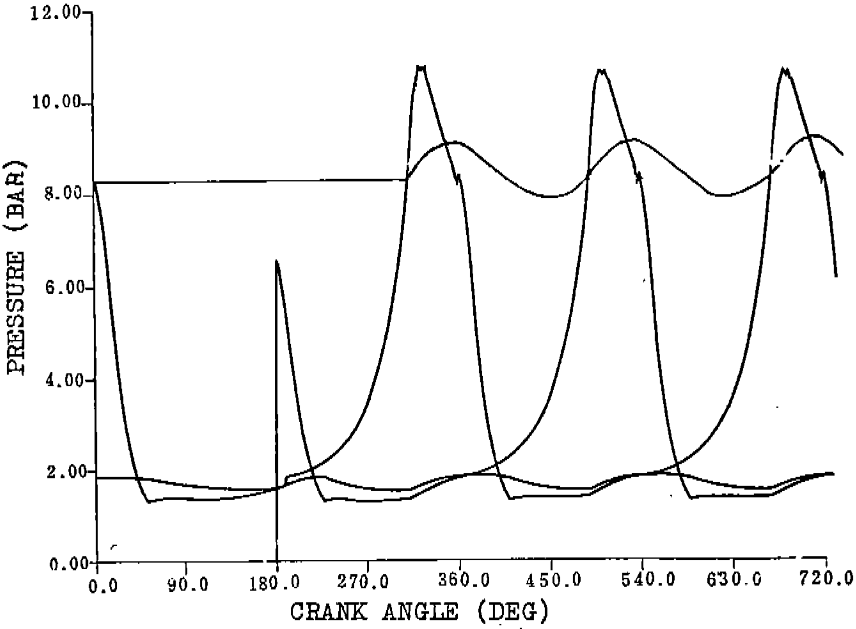

Fig.3 The P $-\varphi$ Diagram

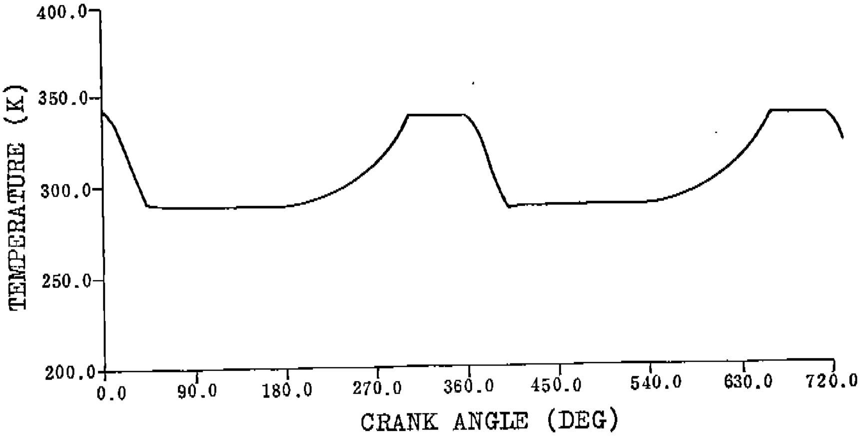

Fig.4 The T $-\varphi$ Diagram

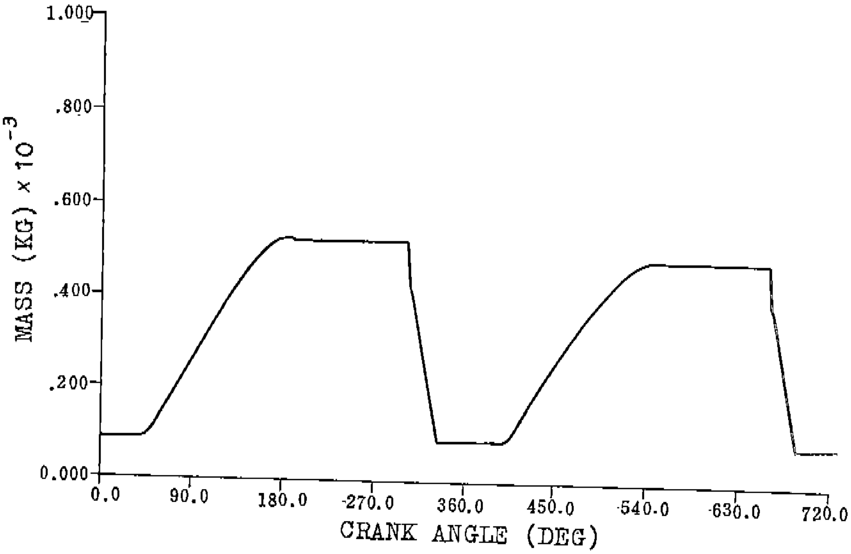

Fig.5 The M − φ Diagram

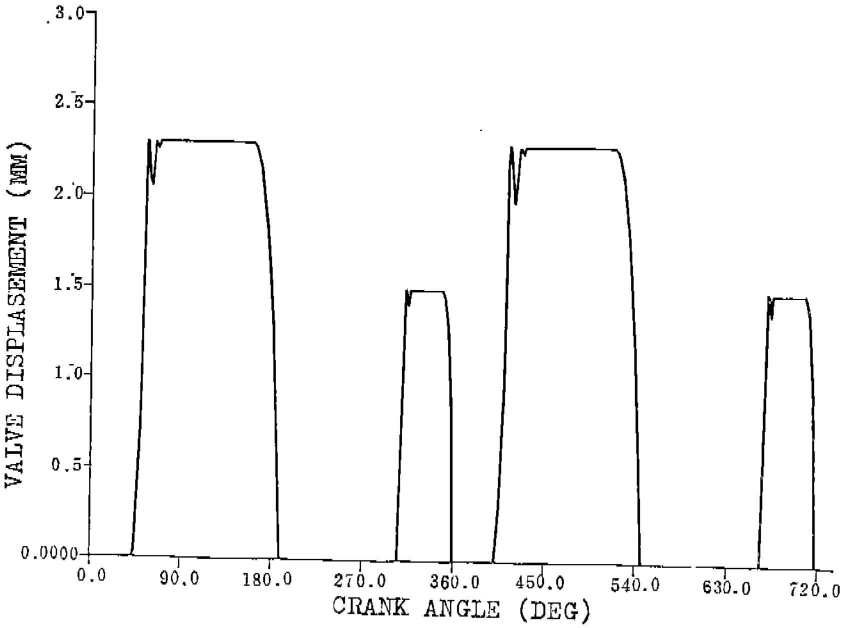

Fig.6 The Suction and Discharge Valve Displacements

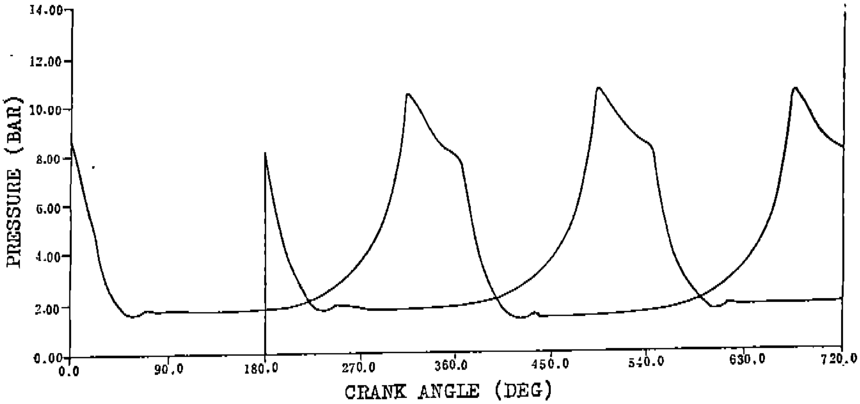

Fig.7 The P - φ Diagram (P_{pl} = const)