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Mathematical Simulation of Lubrication Conditions in Rotary Vane Compressors

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ABSTRACT

Due to the centrifugal forces, rotary vane compressors show at high speeds vane tip friction leading to low mechanical efficiencies /1,2/.

Theoretical models which are based on the hydrodynamic lubrication theory to determine this friction loss between the vane tip and the cylinder have been earlier developed and presented /3/.

A new model has been developed and a computer program written for different types of vane tips and different shapes of the cylinder. Calculations demonstrate that it is possible to decrease the vane tip friction by approximately 10 % by optimizing the vane tip curvature with a given cylinder shape.

An observed chattering phenomenon of the vane at low speed operation can be predicted by calculation and avoided by selection of certain shapes of the vane tip.

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INTRODUCTION

In recent years, the application of double-flow rotary vane compressor in automotive air-conditioning systems has been increased because of its multiple cylinder characteristic. A certain problem is its high friction loss at higher speeds. The greatest amount of the friction is caused by the lubrication systems between the vane tip and the cylinder wall and between vane side face and rotor groove. For improving the compressor performance a minimizing of this friction is an important step.

Some work has been done in the past that included one or both of the above-mentioned losses /3 - 7/.

In this paper, the model to determine the friction loss between vane tip and cylinder wall bases on hydrodynamic lubrication theory. For the calculation of the lubrication conditions an exact knowlegde of the geometry of the clearance between both sliding parts is necessary. The description of the geometry of the clearance is complicated because the following sizes change during the rotation :

- the length of the vane outside the rotor
- the inclining angle between vane and cylinder wall
- the curvature of the cylinder wall
- the location of the minimum oil film thickness between vane tip and cylinder wall across the vane thickness.

In the following, a new model to determine friction losses is presented.

CALCULATION OF LUBRICATION CONDITIONS

For the mathematical simulation of the lubrication conditions between vane tip and cylinder wall, the following modified Reynolds' equation was used :

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)=6\eta\left(2\frac{\partial h}{\partial t}+U\frac{\partial h}{\partial x}-\frac{\rho}{2\eta}\frac{\partial U}{\partial t}\frac{\partial h^3}{\partial x}\right) \qquad (1)$$

Equation 1 describes the instationary case including variable velocities between the two surfaces, the squeeze effect and the influence of the acceleration forces /8/. This equation can be solved using by finite difference equations.

For the frictional forces between vane side face and rotor groove Coulomb's law was applied because of the low friction power as compared to the vane tip friction power loss.

S L I D I N G   G E O M E T R Y

The calculation of the lubrication pressure profile postulates a mathematical description of the geometry of the clearance. All numerical calculations concerning the lubrication gap are carried out in the complex plane by vector loops /9/. The cylinder shape contour of this double flow rotary vane compressor is given as a hypertrochoid of second order.

$$z(\varphi^*) = L e^{i\varphi^*} - R e^{-i\varphi^*} - D e^{3i\varphi^*} \tag{2}$$

with : L,R,D : design data

φ^* : angle of the hypertrochoid

$z(\varphi^*)$: vector of the cylinder wall

For the determination of the oil film height between vane tip and cylinder wall, the normal vector is an important variable.

$$n(\varphi^*) = -L \cdot e^{i\varphi^*} - R \cdot e^{-i\varphi^*} + 3D \cdot e^{3i\varphi^*} \tag{3}$$

with $n(\varphi^*)$: normal vector to $z(\varphi^*)$

A division by the amount of $n(\varphi^*)$ results in the normal unit vector.

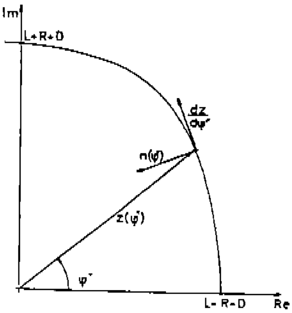


Fig. 1 : Hypertrochoid in the complex plane

Coordinates of the Vane Tip

The shape of the vane tip can be described by cartesian coordinates. The origin of this system is located in the center of the vane tip (on the vane axis).

$$y_{kup} = f(x_{kup}) \tag{4}$$

Height of the lubrication gap between the sliding surfaces

The determination of the height of the lubrication gap begins on the center of the vane tip (Index m). A start value for lubrication gap height at the center of the vane tip must be estimated.

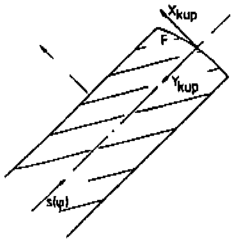


Fig. 2 : Coordinate of the vane tip

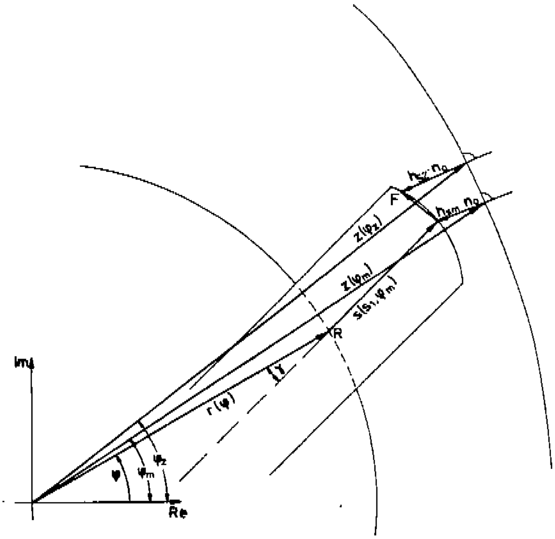


Fig. 3 : Vector loop in the complex plane for the determination of the lubrication gap height

$$r(\varphi) + s(s_1,\varphi) - h_{sm} \cdot n_o(\varphi_m) - z(\varphi_m) = 0 + i \cdot 0 \tag{5}$$

with : φ : rotor angle

r_1 : rotor radius

$r(\varphi)$: rotor radius $[r(\varphi) = r_1 \cdot e^{i\varphi}]$

s_1 : vane length outside the rotor

$s(s_1,\varphi)$: vector describing the vane length outside the rotor

h_{sm} : lubrication film height at the center of the vane tip

n_0 : normal unit vector

: inclining angle of the vane to the concerned rotor radius

The unknown values s_1 and $\Delta\varphi_m = \varphi_m - \varphi$ can be calculated by iteration.

With the calculated data for the vane length outside of the rotor (s_1) the lubrication film height for any point of the vane tip (Index z) can be calculated :

$$r(\varphi)+s(s_1,\varphi)+(ix_{kup}-y_{kup})e^{i(\varphi+\gamma)}-h_{sz}\cdot n_o(\varphi_z)-z(\varphi_z)=0+i\cdot 0 \qquad (6)$$

The values h_{sz} and $\Delta\varphi_z = \varphi_z - \varphi$ can be computed by iteration.

Transformation into a two-dimensional problem

For a prediction of the oil film pressure profile, it is practical to transform the lubrication geometry to the case of a rectilinear cylinder wall.

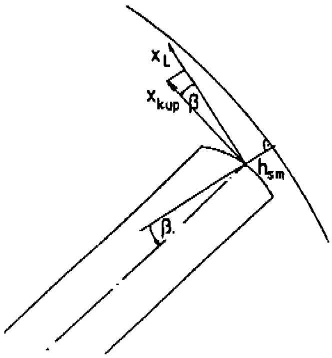


Fig. 4 : Transformation of the vane tip coordinates

As the lubrication film height depends on the arc length of the curvature, the mathematical description can be made by the following approach (Fig. 4) :

$$x_L = x_{kup} \cdot \cos\beta \qquad (7)$$

with : X_L : vane coordinate, parallel to the tangent line to $z(\varphi)$

$$\beta = \varphi + \gamma - \pi - \alpha \qquad (8)$$

$$\alpha = \frac{1}{i}\ln(n_o) \qquad (9)$$

VELOCITY OF THE VANE TIP

For the determination of the lubricant oil film pressure the velocity between the two sliding surfaces is given by :

$$U = \frac{|dz|}{dt} \qquad (10)$$

With $\varphi_m = \varphi + \Delta\varphi_m$ and $\omega = d\varphi/dt$ equation 10 results in

$$U = \frac{|dz(\varphi_m)|}{d\varphi_m} \cdot \omega \cdot \left(1 + \frac{d(\Delta\varphi_m)}{d\varphi}\right) \qquad (11)$$

The first right-hand term describes the tangent line to $z(\varphi_m)$; the second, the constant speed of rotation ($\omega = 2\pi n$ with n : rotational speed); and the third, the variation of the angular velocity of the vane tip caused by the oscillating motion of the vane in its groove. The included differential quotient can be approximated by a difference quotient.

FORCES ACTING ON THE VANE

The investigation of the power loss caused by friction of the system rotor, vane, cylinder requires the calculation of all forces acting on the vane. This includes the dynamic behaviour of the vane.

$$\Sigma\vec{F} + \vec{F}_z + \vec{F}_a + \vec{F}_c = 0 \qquad (12)$$

with :
- m : vane mass
- ΣF : sum of the reaction forces on the vane
- F_z : centrifugal force
- F_a : force due to the relative acceleration
- F_c : Coriolis force

Equation 13 to 15 were set up with regard to the specific problem of a double flow rotary vane compressor.

$$\vec{F}_z = m \cdot \omega^2 \vec{r}_{\overline{BP}} \qquad (13)$$

$$F_a = -m\,{}^{K}\vec{a}_P \tag{14}$$

$$F_C = -2m \cdot \omega \times {}^{K}\vec{v}_P \tag{15}$$

with : $\vec{r}_{BP}$: variable distance between the origin of the rotor/cylinder and the vane mass center

${}^{k}\vec{a}_P$: relative acceleration

$2\omega \times {}^{k}\vec{v}_P$: Coriolis acceleration

Both differential quotients ${}^{k}\vec{v}_P = ds_1/dt$ and ${}^{k}\vec{a}_P = d^2s_1/dt^2$ can be approximated by the following difference quotients :

$$\frac{ds_1}{dt} \approx \frac{s_{1,t} - s_{1,t-\Delta t}}{\Delta t} \tag{16}$$

$$\frac{d^2s_1}{dt^2} \approx \frac{s_{1,t} - 2 \cdot s_{1,t-\Delta t} + s_{1,t-2\Delta t}}{\Delta t^2} \tag{17}$$

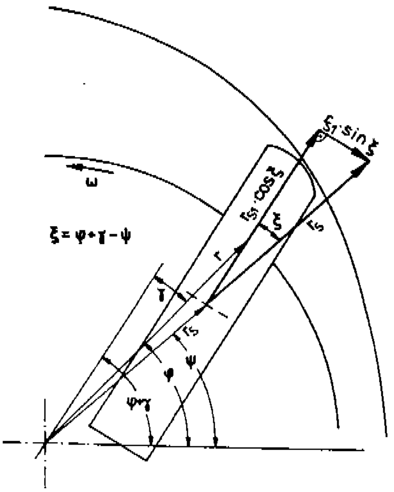


Fig. 5 : Components of the zentrifugal force

To obtain three basic equations in order to describe the vane motion the centrifugal force is divided in two parts; one in the direction of the vane axis, the other at a right angle to it (Fig. 5).

$$r_{S1} \cdot e^{i\psi} = r(\varphi) + s(\varphi) - \frac{l_F}{2} \cdot e^{i(\varphi+\gamma)} \tag{18}$$

$$r_S = r_{S1} \cdot \cos(\varphi+\gamma-\psi) - i \cdot r_{S1} \cdot \sin(\varphi+\gamma-\psi) \qquad (19)$$

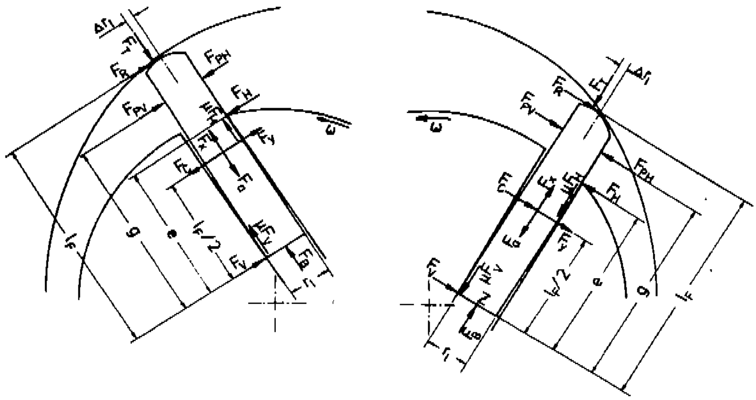
with : l_F : vane length
r_{s1} : amount of r_S

As a result of the oscillating motion of the vane the amount of r_{s1} and the difference $(\varphi-\psi)$ vary during rotation.

The direction of the vane axis is named "radial" and the other at a right angle to it "tangential" although this is not true as compared to the rotor. These specifications are correct only, if the inclining angle γ becomes zero.

The forces acting on the vane

The directions of the forces depend on the motion of the vane as well as on the angle of rotation.



- a -　　　　　　　　- b -

Fig. 6 : vane forces

with : F_T : oil film force at the vane tip
F_R : friction force (RGES in Fig. 7,8)
F_{PV}, F_{PH}: gas forces in front of and behind the vane
F_B : force at the bottom of the vane due to an oil and gas pressure
F_V, F_H : reaction forces between rotor and vane

$\mu F_V, \mu F_H$: resulting friction forces at the rotor groove (M x FV, M x FH in Fig. 7,8)

F_x, F_y : components of the centrifugal force (FUGX, FUGY in Fig. 7,8)

F_a : force due to relative acceleration (FMA in Fig. 7,8)

F_C : Coriolis force

l_F : length of the vane

r_1 : vane thickness

s_1 : length of the vane outside the rotor

e : $e = l_F - s_1 - \Delta e$

Δe : corrective factor

g : $g = l_F - s_1/2$

b_r : thickness of the vane

p_e : oil and gas pressure at the bottom of the vane

μ : friction coefficient

Fig. 6(a,b) shows the vane forces for the four quadrants. The signs of the Coriolis force and friction forces vary as shown in both drawings.

In addition, the sign of the force due to relative acceleration alternates with acceleration direction of the oscillating vane motion. According to the oil film pressure profile, the acting line of the oil film force can be located in front of or behind the vane axis.

The gas forces F_{PV} and F_{PH} can be calculated by the pressures in two adjacent compressor chambers /3,10/.

$$\begin{aligned} F_P &= F_{PV} - F_{PH} \\ &= (p_V - p_H) \cdot b_r \cdot s_1 \end{aligned} \tag{20}$$

with : p_V : chamber pressure before the vane

p_H : chamber pressure behind the vane

b_r : thickness of the vane

The next three equations are valid only for the first and third quadrant (Fig. 6b). The balance of moments is referenced to the vane axis at the bottom of the vane.

$$\Sigma F_X = 0 = F_B - \mu(F_V + F_H) - F_a + F_x - F_T \tag{21}$$

$$\Sigma F_Y = 0 = -F_V - F_C - F_y + F_H - F_P - F_R \tag{22}$$

$$\Sigma \overset{\frown}{M}_{(Z)} = 0 = \mu (F_V - F_H) \cdot \frac{r_1}{2} - (F_C + F_y) \cdot \frac{l_E}{2} + F_H \cdot e - F_P \cdot g - F_R \cdot l_F + F_T \cdot \Delta r_l \qquad (23)$$

The set of equations contains the unknown forces F_V, F_H, F_T and F_R.

First the lubrication height is estimated and so F_R and F_T are calculated directly from the oil film pressure profile. The both reaction forces F_V and F_H can be determined by the equations 22 and 23.

In the next step, the estimation of lubricant film height could be made with equation 21. The iteration is finished, if the difference of the last and actual value is neglectible.

The described equations were solved with a computer program written in FORTRAN 77 on a CDC Cyber 76 machine.

CALCULATED RESULTS

For the calculation, made for variable compressor speed and constant pressure ratio, the rotor parameter L, R, D, the dimensions of the vane and its material density were kept constant. Only the shape of the vane tip was changed. All shapes of vane tips are possible to calculate but for simplification circular, elliptic and parabolic shapes were assumed. Because of higher friction the elliptic shapes were taken out of consideration.

In the graphs, the different shapes of the vane tip are named A, B, C, ... and so on. A crank angle of 720° corresponds to the bottom dead center of the vane, an angle of 810° to the top dead center, that means the largest vane length outside the rotor groove.

Figures 7 and 8 show for the reference profile D the forces in the directions "tangential" and "radial" for a speed of 6000 min^{-1}. The described system includes the dynamic behaviour of the vane. If the speed increases, the load onto the oil film is primarily influenced by the acceleration forces F_a and F_x. In the same matter the Coriolis force influences the force equilibrium in tangential direction.

The figures 9, 10 and 11 illustrate the plot of the minimum oil film thickness, the friction forces and losses of power caused by friction of the profiles B and D. The profile B has a greater value for the average minimum oil film thickness resulting in smaller values of friction force. A substantial friction power saving seems to be possible.

Under certain conditions such as low rotational speed, a chattering phenomenon of the vane can be observed. During this phenomenon the vane accelerates in the direction of the rotor groove bottom. It appears between a crank angle between the end of the discharge valve bore and the bottom dead center of the vane. An indication for this phenomenon is the relative vane lift. (Vane lift $= |\vec{a}| / |\vec{b}|$) with

$\vec{a}$: vector from the origin to the vane tip location at the minimum oil film thickness

$\vec{b}$: vector from the origin to the cylinder wall opposite to the location of the minimum oil film thickness

In contrary to the profile B, the profile A has a chattering phenomenon at low speed operation.

Fig. 14 shows for the profiles A - G the overall frictional power losses for a speed of 6000 min^{-1} in a histogram. The diagram is referenced to shape D. Profile B shows good dynamic behaviour and liquid friction for all examined rotational speeds which results in friction power savings of about 10 %.

C O N C L U S I O N S

The lubrication systems of the vane tip and cylinder wall and also between vane side face and rotor groove was examined theoretically. Results of the developed computer program show that it is possible to save up to 10 % of the power otherwise lost by friction at the vanes. The used model includes the dynamic behaviour of the vane. An observed chattering phenomenon can be avoided by selection of certain shapes of the vane tip. It is also possible to decrease friction by a modification of the cylinder wall shape, as calculations with different hypertrochoids have shown but shall not be dealt with here.

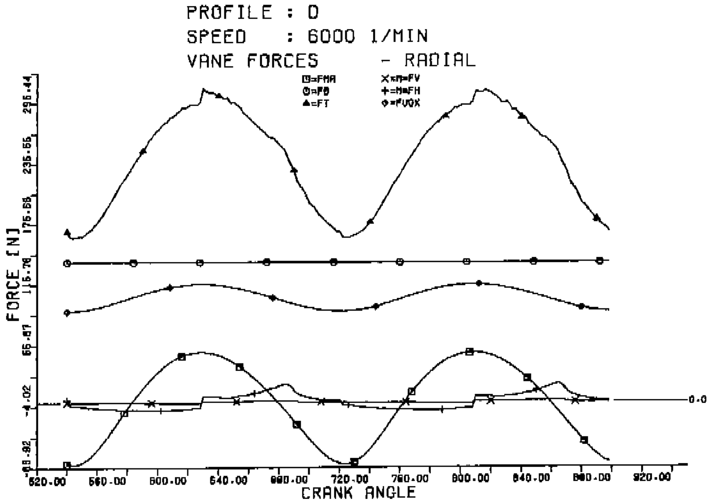


Fig. 7 : vane force "radial", n = 6000 min^{-1}

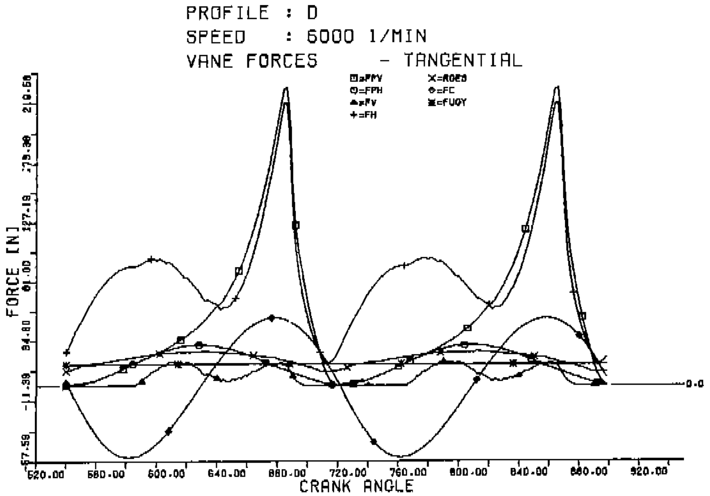
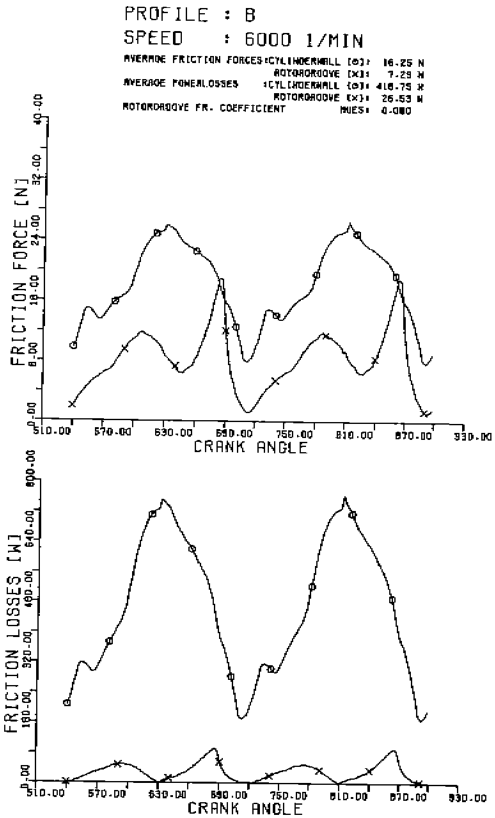
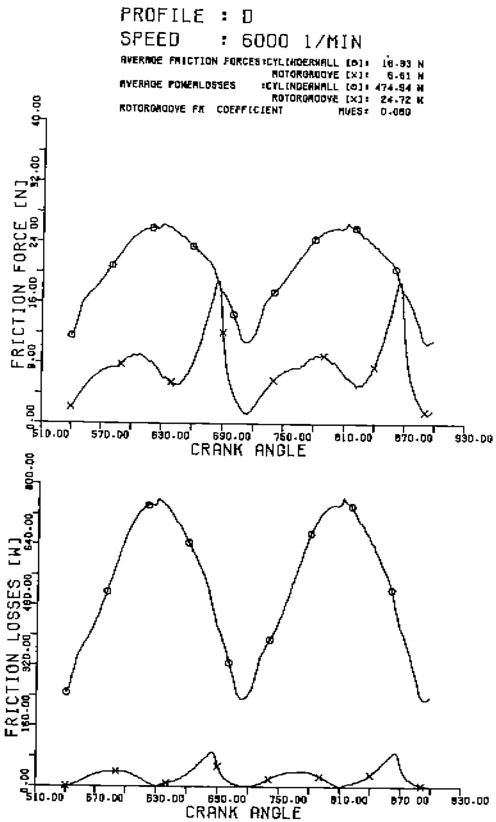


Fig. 8 : vane force "tangential", n = 6000 min^{-1}

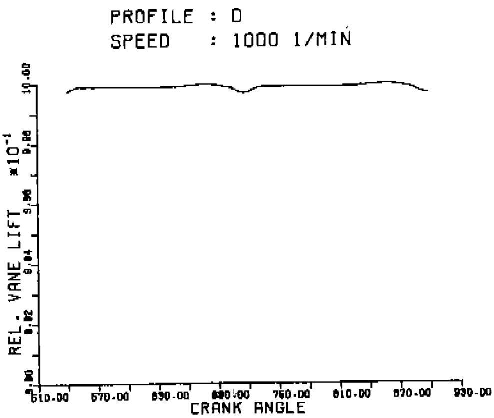




Friction forces and friction losses

Fig. 10 : Profile B

Fig. 11 : Profile D



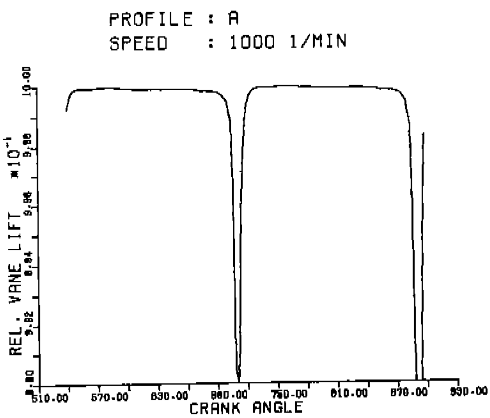
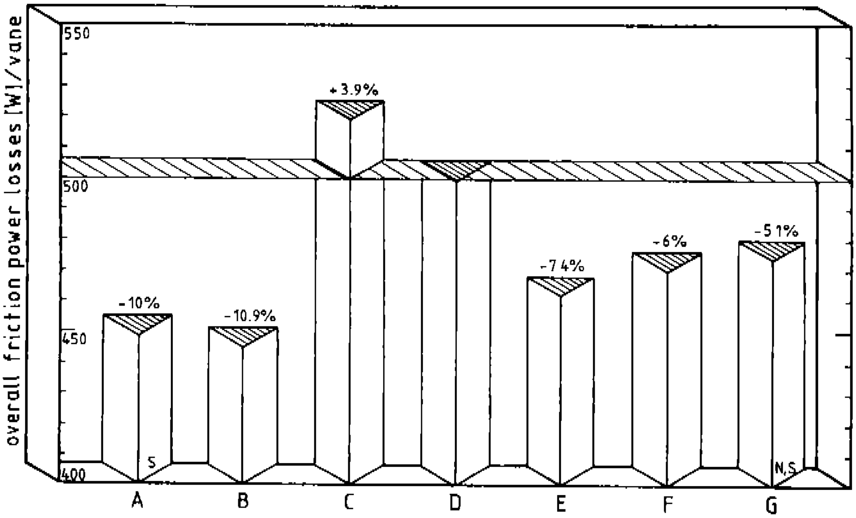


Fig. 12 + 13 : relative vane lift, profile A+D, $n = 1000\ min^{-1}$



S : scattering phenomenon by low speed operation

N,S : scattering and none fluid friction by low speed operation

Fig. 14 : overall friction power losses/vane

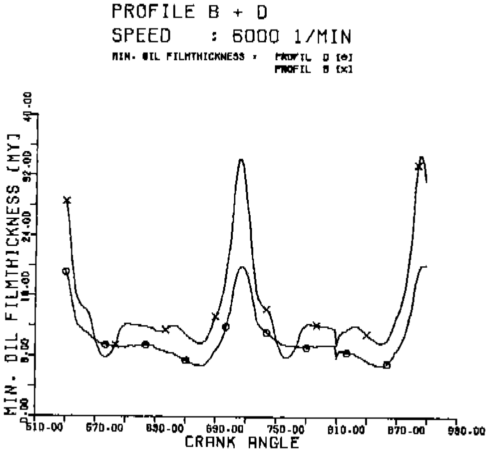


Fig. 9 : minimum oil film thickness profile B + D

R E F E R E N C E S

/1/ Kruse, Dr. H.
Experimental Investigations on Rotary Vane Compressors
Purdue Comp. Techn. Conf. 1980 (publ. 1982)

/2/ H. Kaiser, H. Kruse
An Investigation on Reciprocating and Rotary Refrigeration Compressors
Proc. Purdue Comp. Techn. Conf. 1984

/3/ H. Lindemann, H. Kaiser, M. Kuever, H. Kruse
Optimization of a Special Shaped Rotary Vane Compressor-Comparision of Theoretical and experimental results
Proc. Purdue Comp. Techn. Conf. 1982

/4/ W. Beck, R. Stern, J. Eibling
Design for Minimum Friction in Rotary-Vane Compressors
Ashrae Trans. Vol. 72, Part 1, 1966

/5/ T.C. Edwards, A.T. Mc Donald
Analysis of Mechanical Friction in Rotary Vane Machines
Proc. Purdue Comp. Techn. Conf. 1972

/6/ Platts H.H.
Hydrodynamic Lubrication of Sliding Vanes
Proc. Purdue Comp. Techn. Conf. 1976

/7/ K. Tojo, T. Kau, A. Arai
Dynamic Behaviour of Sliding Vane in Small Rotary Compressors
Proc. Purdue Comp. Techn. Conf. 1978

/8/ Constantinescu, V.N.
Gas Lubrication
The American Society of Mechanical Engineers, New York 1969

/9/ Leemhuis, R.S. & Soedel, K.
Kinematics of Wankel Compressors for Engines by Way of Vector Loops
Proc. Purdue Comp. Techn. Conf. 1976

/10/ Soedel, W. & Wolverton, S.
Anatomy of a compressor simulation program
Purdue University, West Lafayette 1974